

Supplementary Material for AMA3020 Project: Loose Chippings

Supplementary material to aid in the understanding and explanation of the project "Assessment of the 20 mph Speed Limit on Newly Surfaced Roads".

Derivations

Solving for t using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (1)$$

Thus,

$$t = \frac{-(-u_y) \pm \sqrt{(-u_y)^2 - 4(\frac{1}{2}g)(y)}}{2(\frac{1}{2}g)}$$

$$t = \frac{u_y \pm \sqrt{u_y^2 - 2gy}}{g} \quad (2)$$

For there to be real solutions, the discriminant must be greater than 0. To prove this we derived the following:

$$u_y^2 - 2gy \geq 0$$

$$u_y \geq \sqrt{2gy} \quad (3)$$

Calculations

For real solutions, Equation 3 must hold, after substituting $g = 9.81$, $u_y = 8.94 \sin \theta$ and $y = 0.75$, that is:

$$8.94 \sin \theta \geq \sqrt{2(9.81)(0.75)}$$

$$\sin \theta \geq \frac{1}{8.94} \sqrt{2(9.81)(0.75)}$$

$$\sin \theta \geq 0.429$$

$$\theta \geq \arcsin 0.429 = 25.41^\circ \quad (4)$$

Impact of Road Gradients on Stone Trajectory

When a vehicle travels on an inclined road, the trajectory of an ejected stone is influenced by the slope angle α , which modifies both velocity components and the effect of gravity. The initial velocity components along the incline are given by:

$$u_x = u \cos(\theta - \alpha) + u \cos \alpha \quad (5)$$

$$u_y = u \sin(\theta - \alpha) - g \sin \alpha \quad (6)$$

where u is the vehicle's speed (assumed to be the stone's initial speed), θ is the stone's ejection angle relative to the horizontal, and g is gravitational acceleration (9.81 m/s^2).

The total flight time of the stone is derived from Equation 2. The horizontal displacement along the incline is:

$$x = u_x t \quad (7)$$

For $\alpha > 0$ (uphill travel), the stone is projected downhill, and gravity increases its range, raising the risk of impact. Conversely, for $\alpha < 0$ (downhill travel), the stone is projected uphill, and gravity reduces its range, making impacts less likely. This suggests that on steep uphill roads, a lower speed limit may be necessary, whereas on downhill roads, the 20 mph limit may be conservative. [1]

Bibliography

- [1] Vedantu, Projectile Motion on Inclined Plane <https://www.vedantu.com/jee-main/physics-projectile-motion-on-inclined-plane> (Accessed on 09/02/2025)