Parton Fragmentation Functions

Anselm Vossen[1]

[1]Indiana University, USA

Abstract: This contribution gives an overview of the field of fragmentation functions. The emphasis is on recent experimental results on light quarks and gluons fragmenting into light hadrons. Some possibilities for the study of fragmentation functions at a future FCC-ee are discussed.

Introduction

Perturbative QCD can be used to describe many high energy scattering processes. In most cases factorization theorems enable a separation of the respective cross-sections into parts dominated by short distances, which are perturbatively calculable and long distance parts which have to be measured experimentally [1].

If hadrons in the final state are identified, the non-perturbative functions describing the formation of these colorless bound final states are called fragmentation functions (FFs). For a more in depth overview of this field and a detailed list of available datasets and experimental results, see our recent review [2]. The study of fragmentation functions is complementary to the study of parton distribution functions (PDFs), which describe the initial state long distance behaviour of the collisions, i.e. the structure of the hadronic systems before the scattering. At variance to PDFs, FFs cannot be computed on the lattice, a challenge which is rooted in the difficulty to integrate over the spectators in the final state. Since FFs describe the formation of colorless bound states from colored partons they are conceptually a consequence of color confinement, one of the most intriguing problems in QCD.

In the following we will concentrate on leading twist (twist 2) FFs which have a probabilistic interpretation. For the exact field theoretic definition see again [2]. Here we will just give a working definition. The integrated fragmentation function $D_1^{h/q}(z)$ gives the probability that a quark q fragments into hadron h with h carrying the fraction z of the parent parton momentum. The subscript indicates here the leading twist and the superscript might be omitted if the meaning is otherwise clear. The definition is chosen such that the cross-section for semi-inclusive hadron production can be written using QCD factorization theorems as $\sigma(lp \to lhX) = \sum_q e_q^2 f_1^{q/p} \otimes D_1^{h/q}$ for Semi-Inclusive Deep-Inelastic Scattering (SIDIS), and $\sigma(pp \to hX) = \sum_{i,j,k} f_1^{i/p_a}(x_a) f_1^{j/p_b}(x_b) \otimes D_1^{h/k}$ for pp and $\sigma(e^+e^- \to hX) = \sum e_q^2 D_1^{h/q}(z)$ for single-inclusive annihilation.

H \ q	U	L	T
U	$D_1^{h/q}$		$H_1^{\perp\, h/q}$
L		$G_1^{h/q}$	$H_{1L}^{\perp\, h/q}$
T	$D_{1T}^{\perp\, h/q}$	$G_{1T}^{h/q}$	$H_1^{h/q}$ $H_{1T}^{\perp\, h/q}$

Table 1: Interpretation of FFs for quarks, see text for more details. The columns indicate the quark polarization — unpolarized (U), longitudinally polarized (L), transversely polarized (T). The rows indicate the hadron polarization.

Extending the integrated fragmentation function D_1 to different quark and hadron polarizations and allowing the FFs to depend on the transverse momentum $\vec{k}_T$ of the hadron with respect to the parent quark, we can classify fragmentation functions according to the so-called Amsterdam notation for FFs [3][4][5][6] as given in tbl. 1. Here we only introduce functions describing quark fragmentation into single hadrons and the integrated gluon FF. Quite a bit of theoretical and experimental work has been devoted recently to polarized gluon fragmentation functions and di-hadron fragmentation functions, both polarized and unpolarized. We will not discuss these much further in this write-up and instead refer to the literature. Note also that some of the FFs vanish if we integrate over $\vec{k}_T$. These are denoted with the superscripts $\perp$. Even though fragmentation functions, enter virtually all semi-inclusive cross-sections of hard-scattering processes, the precise knowledge of spin and $\vec{k}_T$ dependent FFs in the intermediate z region plays an especially important role in hadronic physics, where the nuclear structure encoded in the PDFs has to be disentangled from fragmentation contributions. For some chiral-odd parton distribution functions, like the transversity $h_1(x)$, the only precise experimental data is sensitive to the combination with a chiral-odd FFs, like the transverse polarization dependent Collins FF $H_1^{\perp}$. In the following we will however concentrate on the quark polarization integrated FF D_1.

Experimental Access to FFs

As alluded to in the introduction, FFs enter many hard scattering processes. Current extractions use Single Inclusive Annihilation (SIA), SIDIS and pp data. The different configurations are complementary to each other, each having different advantages and disadvantages. The cleanest access is provided by SIA, where the factorized cross-section can be written as

$$\frac{1}{\sigma_{\text{tot}}}\frac{d\sigma^{e^+e^- \to hX}}{dz} = \frac{1}{\sum_q e_q^2}\left(2F_1^h(z,Q^2) + F_L^h(z,Q^2)\right) \tag{1}$$

and the structure function F_1^h can be written in terms of FFs at NLO as

$$\sum_q e_q^2\left(D_1^{h/q}(z,Q^2) + \frac{\alpha_S(Q^2)}{2\pi}\left(C_1^q \otimes D_1^{h/q} + C_1^g \otimes D_1^{h/g}\right)(z,Q^2)\right). \tag{2}$$

Here the superscript q signifies quark related functions, g gluonic functions. The C are Wilson coefficients. Looking at the cross-sections, the advantages of SIA are the clean and direct access to the FFs. There is no contribution by the nucleon structure. Theory calculations are well under control. In this venue new NNLO calculations have been shown [7].

However, there are also obvious shortcomings. The separation of flavors is nontrivial, since the contribution of each FF differs only by the coupling constants. And the access to the gluon FFs comes only from scaling violations which require a large lever am in Q^2.

Some of these shortcomings can be mitigated somewhat. More flavor sensitivity can be reached by including data on the Z^0 pole as well as below, using polarized beams [8] and detecting two back-to-back identified hadrons in the final state. Other techniques, such as using three jet events to access the gluon FF at leading order, or using displaced vertices to identify charm and bottom production are ambiguous beyond leading order calculations. However, in current fits, the heavy quark tagged data is commonly used.

SIA Data

Experimental data on SIA exists over a wide range of energies, from $\sqrt{s} = 3$ GeV up to $\sqrt{s} = 209$ GeV [9][10]. For our knowledge of FFs the datasets taken by the B-factories on or near the $\Upsilon(4S)$ resonance from Belle [11][12] and BaBar experiments [13] as well as the data taken by LEP and SLC on the Z^0 resonance [14][15][16][17] are the most important. Many older datasets taken below the $\Upsilon(4S)$ resonance or in between the resonance and the Z^0 pole lack statistics and sometimes are not documented very well. Above the Z pole, the e^+e^- annihilation cross-section drops quite quickly, which will also mean that the Z^0 pole data collected by FCC-ee will play a dominant role compared to other planned center-of-mass energies. LEP collected over 200 pb^{-1} on the Z^0 pole whereas Belle at KEKB collected about 1 ab^{-1} on the $\Upsilon(4S)$ resonance (BaBar at PEP-II collected about half that). This corresponds to orders of magnitude more data than collected by other experiments, other than LEP, and allowed for the first time to measure the SIA cross-section for identified pion, kaons and protons at large $z > 0.5$. From both, the LEP experiments and the B-factories, we have precision data on π, K and p production. Belle recently [12] showed the first measurement of the cross-section for back-to-back production of identified hadrons.

Fits using SIA Data

The SIA data have been used to extract FFs in a number of fits. Here we note the HKNS fit [18] which was pioneering in the estimation of uncertainties of fragmentation functions. The SIA data used in this fit does not allow a flavor separation of the FFs beyond a distinction between favored and unfavored FFs and the gluon FF obtained from the evolution equation is only weakly constrained (the last publication does not use the large Belle dataset yet). During this workshop, a new fit by the NNPDF group showed a first extraction of FFs using a neural network approach but also restricted to SIA data [19]. It is encouraging that this is now available since it presumably gives uncertainties free of bias by the parametrization, e.g. in areas where there is no data.

Access to Flavor information and Gluon FFs

Data beyond SIA is needed to achieve flavor separation and access to the gluon FF. For flavor separation, in particular SIDIS data on identified hadron production is important. The main challenge in the use of the data is that many SIDIS experiments are at quite low $\sqrt{s}$. This is in particular true for the new JLab data which would offer unprecedented statistics. Recent efforts in theory are aimed at getting threshold, higher order and target mass effects under control such as the JAM collaboration which presented at this workshop [20].

In pp collisions the gluon FF can be accessed at leading order. However, calculations are much more challenging since the cross-section includes the partonic structure of both colliding protons. Since the initial partonic kinematics in the process $pp \to hX$ are not known, integration over all x and z is necessary. This includes kinematic areas in which the PDFs might not be known so well and does not allow access to the z dependence of the FFs. A new development to address this problem has been the formalism to extract fragmentation from hadrons-in-jets [21]. These calculations, currently available at NLO, allow the extraction of the z dependence of the FFs.

The first fit to include SIDIS data along with the latest e^+e^- data from Belle and BaBar as well as the latest pp data from the LHC (but not yet hadron-in-jet measurements) is the DSS fit [23]. This fit uses a parametrized approach and also extracts uncertainties of FFs. As is HKNS, the DSS fit is done at NLO accuracy. Agreement with all datasets is good in general. However, to solve the disagreement of the previous, pre-LHC, version of the fit [22], a p_T cut of 5 GeV on the pp data was introduced. This removes the region in which the fit does not converge to a consistent description of PHENIX and ALICE data due to the ALICE data preferring a smaller contribution by the gluon fragmentation function. The disagreement of QCD calculations using a representative set of FFs with newly published LHC data was a surprise at the time [24] and is a reminder that even though a lot of progress has been made in the extraction FFs, they are still fits and the potential for surprises exists with new data. With the use of both, SIDIS and pp data, the DSS fit extracts flavor separated FFs and a precise extraction of the gluon FF.

Challenges at low and high z

Similar to PDFs, the extraction of FFs at low and high z is challenging for theory. At high z, roughly above 0.8, theoretical uncertainties rise due to threshold effects, whereas at low z target mass corrections have to be applied and time like splitting functions for the FFs diverge. Target mass corrections are an issue in particular for data sets at lower $\sqrt{s}$ and as mentioned above, there are efforts to address this [20]. The divergence of the splitting functions in principle makes a resummation to all orders necessary. In contrast to PDFs, the divergence happens already at higher z. Therefore fits usually employ cuts of z greater than about 0.1. This regime is of course of interest in particular at high $\sqrt{s}$, like at the LHC, because the majority of the particle production happens there. However, approximation schemes to the resummation have been known for quite some time , in particular the so-called Modified Leading Log Approximation (MLLA) and these were also discussed during this workshop [25]. For spin dependent FFs, this region is usually not that interesting, since the multiple splittings tend to average out any spin dependence. Modern fits, like DSS, do not always do a good job in describing the low z region. However, as we have learned in this workshop [7] this is not necessarily caused by the insufficiency of the fixed order calculation including data below the usual cutoff in calculations at NNLO or even NLO leads to a good description even at low z where one would describe the data with the MLLA.

Transverse Momentum Dependence and Evolution

Recently, significant experimental and theoretical efforts have been focused on the intrinsic transverse momentum dependence of the FFs. Here, in addition to the z dependence, the dependence on $\vec{k}_T$ is considered, where $\vec{k}_T$ is the transverse momentum of the detected hadron with respect to the parent quark direction. One important motivation for the precision mapping of the transverse momentum dependence is the necessity to disentangle the intrinsic transverse momentum of partons in the nucleon from transverse momentum generated in the fragmentation in SIDIS experiments. But also beyond nuclear physics applications, it is interesting to explore the $\vec{k}_T$ dependence of FFs because the tools used to describe the $\vec{k}_T$ spectrum have certain universal aspects that apply to PDFs and FFs. Following one possible factorization scheme, the Collins-Soper-Sterman (CSS) formalism [1], one can decompose the FFs in a nonperturbative collinear part, $\vec{k}$ dependent perturbative and non-perturbative parts and a term that bridges the non-perturbative and the perturbative part. Since the non-perturbative parts are universal and spin independent, exploring Transverse Momentum Dependent (TMD) evolution effects in FFs would have an impact beyond the studies of FFs. Currently, data on the transverse momentum spectrum of Z^0 and Drell-Yan production has arguably the largest impact on TMD evolution studies due to the large lever arm in $\sqrt{s}$ [26]. Having e^+e data covering a large range in Q^2 would add complimentary information from a process that is theoretically well under control. There are in principle two observables, which give access to the intrinsic transverse momentum in fragmentation. One is the p_T imbalance of back-to-back hadrons, which is sensitive to the convolution of the transverse momenta. The other is measuring the transverse momentum relative to the thrust or jet axis. The later method has the caveat that identifying the quark axis with the thrust or jet axis is problematic beyond LO. However, given the efforts to measure TMDs in jets, e.g. in pp, it will be important to do both measurements and compare. At this moment, there are no measurements of the $\vec{k}_T$ dependence of unpolarized FFs in e^+e^- available. There are publications of the $\vec{k}_T$ dependence of the Collins FF $H_1^\perp$ [27][28] (also see the next section). Since $H_1^\perp$ is a TMD, even the $\vec{k}_T$ integrated measurements need TMD evolution. Other results that are sensitive the transverse momentum dependence come from COMPASS and HERMES which measured the p_T spectra in SIDIS [29][30].

Spin Dependent Fragmentation

We already mentioned spin dependent fragmentation functions several times in the previous sections. Here we will summarize some recent results that are sensitive to transverse quark spin dependent FFs and FFs where the produced hadron is polarized. The main motivation to study transverse spin dependent FFs is that they can give access to the chiral-odd transversity PDF h_1 [35] which cannot be accessed in inclusive measurements. Instead, h_1 can be measured by using a transverse spin dependent FF as a quark polarimeter. In unpolarized e^+e^- annihilation one can exploit the fact that in $e^+e^- \to q\bar{q}$ production, the spins of the quarks are correlated and therefore spin dependent FFs can be measured in correlation measurements of back-to-back hadron pairs. The Collins FF $H_1^\perp$, which describes a correlation between the transverse polarization of the fragmenting quark and the transverse momentum of the produced unpolarized hadron, has recently been measured by Belle [31][32], Babar [27] and BES-III [28]. If two hadrons are detected in the final state, the correlation of the relative transverse momentum between the two hadrons and the quark polarization is described by the di-hadron fragmentation function $H_1^\sphericalangle$. Due to the additional degree of freedom provided by the other hadron, this effect survives an integration over the intrinsic transverse momentum in the fragmentation, so $H_1^\sphericalangle$ can be treated in a collinear framework. It has been extracted recently [36] from Belle measurements [33] and has been used for the first measurement sensitive to transversity in pp [37]. Recently, Belle also showed the first measurement sensitive to $G_1^\perp$, which describes the azimuthal correlation of the relative momentum of an unpolarized hadron pair with the parent quark helicity [39]. While the previous examples are FFs, which are sensitive to the parent quark spin and where the produced hadron is spinless, one can also have FFs which describe correlations of the hadron polarization with the spin and/or momentum of the parent quark. Considering this, quite a large number of FFs can be constructed [38] but most remain unmeasured today. A notable exception is the polarizing FF $D_{1T}^{\perp\Lambda/q}$., which describes he transverse polarization of Λ hyperons in the fragmentation of unpolarized quarks. This effect has been measured by Belle [39].

QCD Vacuum effects on Fragmentation

The FFs discussed previously describe correlations between microscopic quark properties and "macroscopic" properties in the final state. However, there is also a suggestion that fluctuations in the QCD vacuum can leave an imprint on the final state. In particular, coupling to sphalerons or instantons that mediate between QCD vacuum states with different winding numbers could lead to a measurable effects. Quite some time ago jet handedness correlations were suggested as a sensitive observable [40]. However, since these average over many events, they are not unambiguously connected to QCD vacuum fluctuations. More recently, event-by-event fluctuations have been proposed [41]. A precision measurement of this observable would need sufficient statistics of high multiplicity e^+e annihilation which could be made available for the first time by the FCC-ee.

Opportunities with future datasets from Belle II and FCC-ee

Before the turn-on of a FCC-ee, several other facilities will have collected large datasets that can be used for the precision study of FFs. Probably most relevant will be Belle II at SuperKEKB [42]. The successor of Belle will collect about 50 ab 1 at the $\Upsilon(4S)$ resonance over a decade. This is a similar amount as FCC-ee aims for at the Z^0 resonance and would therefore be a good opportunity to study evolution effects in FFs and extract gluon FFs from scaling violations, even though they are not very strong in the relevant Q^2 region. The planned EIC will allow the study of FFs in SIDIS at much higher $\sqrt{s}$ than was achieved at previous experiments. For example, the plan for the proposed realization at BNL, the eRHIC [43] , is to have a staged approach with $\sqrt{s}$ starting at 25 GeV and reaching 140 GeV after some time. At these energies, higher order effects are heavily suppressed and the collider avoids nuclear effects that are present in fixed targets. In addition, fragmentation in jets can be studied using data from the current pp facilities, RHIC and LHC. This will allow precision measurements of the gluon fragmentation functions. The advantage of an e^+e machine is the degree of theoretical control. Particularly the existence of factorization proofs and availability of calculations at NNLO. Therefore, even though the most precise data on gluon FFs or transverse momentum in jet fragmentation might not come from the FCC-ee, the extraction of gluon FFs from FCC-ee and Belle II via evolution equations and the study of fragmentation in jets at FCC-ee would be a crucial input to our understanding of these FFs. Combined with the Belle II data or possibly with even lower energy data from BES III, the study of evolution, which is currently a topic of very active theoretic study in the nuclear physics community, would be very interesting at FCC-ee. The FCC-ee could be instrumental in studying FFs of heavy mesons that are non-trivial to reconstruct in pp environments, for example heavy Λ baryons. Together with Belle II data, the flavor structure of heavy mesons that can also be produced at Belle II could be studied as well. Since FCC-ee is accessing the same phase space as the LEP experiments but with much more precision, similar topics can be addressed in kinematic regions that were not accessible by LEP due to lack of statistics. Probably the most prominent example are heavy flavor FFs which we know mostly from LEP data. FCC-ee would help to measure those at mid to high z and possibly access the p_T dependence. In general, FCC-ee would allow unpolarized and polarized FFs to be studied at very high precision. It would add data at high Q^2 and high z. Due to the high $\sqrt{s}$, lower values of z can also be accessed that remain out of reach at lower energies or come with stronger mass effects. Given the statistics, it is also expected that flavor separation using back-to-back hadrons or polarized beams, would be vastly improved compared to LEP. The exact impact of FCC-ee data on FF extraction can obviously only be evaluated doing a more quantitative study, such as using pseudo-data in a global fit. The FCC-ee is also is also expected to produce a significant amount of Higgs bosons (several 10k), which would give clean access to gluon or b-FFs. However, the collected statistics is still small in comparison with other ways to access this FF, but these measurements would certainly be complementary. Due to the large statistics and higher multiplicity, event-to-event fluctuations due to QCD vacuum effects in e^+e could be studied for the first time at an FCC-ee.

I want to thank Emanuele Nocera, Ingazio Scimeni and Marco Stratmann for helpful discussions.

References

[1] J. Collins, "Foundations of perturbative QCD," (Cambridge monographs on particle physics, nuclear physics and cosmology. 32)

[2] A. Metz and A. Vossen, Prog. Part. Nucl. Phys. **91**, 136 (2016) [arXiv:1607.02521 [hep-ex]].

[3] P. J. Mulders and R. D. Tangerman, Nucl. Phys. B **461**, 197 (1996) Erratum: [Nucl. Phys. B **484**, 538 (1997)] 6 Dec 2016

[4] P. J. Mulders and J. Rodrigues, Phys. Rev. D **63**, 094021 (2001)

[5] A. Bacchetta, M. Diehl, K. Goeke, A. Metz, P. J. Mulders and M. Schlegel, JHEP **0702**, 093 (2007)

[6] S. Meissner, A. Metz and K. Goeke, Phys. Rev. D **76**, 034002 (2007)

[7] D. Anderle *et al.* , Fragmentation Functions at NNLO* and NNLL, *these proceedings*, p. 37.

[8] M. Kalelkar *et al.* [SLD Collaboration], Nucl. Phys. Proc. Suppl. **96**, 31 (2001)

[9] M. T. Ronan *et al.*, Phys. Rev. Lett. **44**, 367 (1980).

[10] G. D. Lafferty, P. I. Reeves and M. R. Whalley, J. Phys. G **21**, A1 (1995).

[11] M. Leitgab *et al.* [Belle Collaboration], Phys. Rev. Lett. **111**, 062002 (2013)

[12] R. Seidl *et al.* [Belle Collaboration], Phys. Rev. D **92**, no. 9, 092007 (2015)

[13] J. P. Lees *et al.* [BaBar Collaboration], Phys. Rev. D **88**, 032011 (2013)

[14] P. Abreu *et al.* [DELPHI Collaboration], Eur. Phys. J. C **5**, 585 (1998).

[15] D. Buskulic *et al.* [ALEPH Collaboration], Z. Phys. C **66**, 355 (1995).

[16] R. Akers *et al.* [OPAL Collaboration], Z. Phys. C **63**, 181 (1994).

[17] K. Abe *et al.* [SLD Collaboration], Phys. Rev. D **59**, 052001 (1999)

[18] M. Hirai, S. Kumano, T.-H. Nagai and K. Sudoh, Phys. Rev. D **75**, 094009 (2007)

[19] V. Bertone *et al.* , Neural Network Fragmentation Functions, *these proceedings*, p. 19.

[20] N. Sato *et al.*, First Monte Carlo analysis of FFs from single inclusive e^+e annihilation, *these proceedings*, p. 26.

[21] T. Kaufmann, A. Mukherjee and W. Vogelsang, Phys. Rev. D **92**, no. 5, 054015 (2015)

[22] D. de Florian, R. Sassot and M. Stratmann, Phys. Rev. D **75**, 114010 (2007)

[23] D. de Florian, R. Sassot, M. Epele, R. J. Hernndez-Pinto and M. Stratmann, Phys. Rev. D **91**, no. 1, 014035 (2015)

[24] D. d'Enterria, K. J. Eskola, I. Helenius and H. Paukkunen, Nucl. Phys. B **883** (2014) 615 doi:10.1016/j.nuclphysb.2014.04.006

[25] D. d'Enterria and R. Perez-Ramos, JHEP **1408**, 068 (2014) doi:10.1007/JHEP08(2014)068; arXiv:1408.2865 [hep-ph]; arXiv:1505.02624 [hep-ph]; and *these proceedings*, p. 44.

[26] U. D'Alesio, M. G. Echevarria, S. Melis and I. Scimemi, JHEP **1411**, 098 (2014)

[27] J. P. Lees *et al.* [BaBar Collaboration], Phys. Rev. D **90**, no. 5, 052003 (2014)

[28] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **116**, no. 4, 042001 (2016)

[29] C. Adolph *et al.* [COMPASS Collaboration], Eur. Phys. J. C **73**, no. 8, 2531 (2013) Erratum: [Eur. Phys. J. C **75**, no. 2, 94 (2015)]

[30] A. Airapetian *et al.* [HERMES Collaboration], Phys. Rev. D **87**, 074029 (2013)

[31] R. Seidl *et al.* [Belle Collaboration], Phys. Rev. D **78**, 032011 (2008) Erratum: [Phys. Rev. D **86**, 039905 (2012)]

[32] K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. **96**, 232002 (2006)

[33] A. Vossen *et al.* [Belle Collaboration], Phys. Rev. Lett. **107**, 072004 (2011)

[34] A. Abdesselam *et al.* [Belle Collaboration], arXiv:1611.06648 [hep-ex].

[35] J. C. Collins, Nucl. Phys. B **396**, 161 (1993)

[36] A. Courtoy, A. Bacchetta, M. Radici and A. Bianconi, Phys. Rev. D **85**, 114023 (2012)

[37] L. Adamczyk *et al.* [STAR Collaboration], Phys. Rev. Lett. **115**, 242501 (2015)

[38] D. Pitonyak, M. Schlegel and A. Metz, Phys. Rev. D **89**, no. 5, 054032 (2014)

[39] A. Abdesselam *et al.* [Belle Collaboration], arXiv:1611.06648 [hep-ex].

[40] A. Efremov and D. Kharzeev, Phys. Lett. B **366**, 311 (1996)

[41] Z. B. Kang and D. E. Kharzeev, Phys. Rev. Lett. **106**, 042001 (2011)

[42] T. Abe *et al.* [Belle-II Collaboration], arXiv:1011.0352 [physics.ins-det].

[43] E. C. Aschenauer *et al.*, arXiv:1409.1633 [physics.acc-ph].