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GitHub (Literature Review Repo)

Literature Review Notes: Fragmentation, Jets, and TMD Motivation

Click each paper title to open the highlighted PDF in GitHub

Scope

These notes collect short summaries of the main references I am using for my thesis on transverse-momentum-sensitive observables in back-to-back jets in e^+e^- collisions. Each paper title links to the annotated PDF stored in my GitHub repository.

Papers

Fragmentation Functions in e^+e^-, ep, and pp Collisions (PDG Review)

O. Biebel, D. de Florian, D. Milstead, W. Vogelsang (PDG), Prog. Theor. Exp. Phys. 2020, 083C01. [1]

Summary

Paper defines fragmentation functions (FFs) and parton distribution functions (PDFs). While FFs $D_i^h(x, \mu^2)$ are a probability density function for an outgoing parton i to produce a hadron h, with longitudinal momentum fraction x, where μ is the resolution/factorization scale. FFs differ from PDFs in that PDFs describe the inner structure of the initial state hadron, while FFs discuss final state hadronization, which is non-perturbative. Both appear in factorized cross sections. Paper delves into how e^+e^- is a "clean" environment for studying FFs as the hard scale is fixed by s (centre of mass energy) and how e^+e^- can be described as point like, unlike hadrons (which need PDFs). Using e^+e^- allows us to focus only on the final state by ignoring additional factors like additional soft activity or beam remnants which we would get with hadron beams. One downside is that measurements from e^+e^- cannot differentiate quark from antiquark fragmentation. It is discussed that gluon fragmentation is only constrained at higher order corrections, and since gluons typically enter indirectly (through evolution or gluon-tagged jet topologies) the gluon FF is generally less well constrained. Deep inelastic scattering (DIS), i.e. lepton–hadron scattering at high energy, in the Breit frame can provide tests of universality of fragmentation. More fragmentation measurements can theoretically be made in hadronic environments such as hadron-in-jet through the jet-based momentum fraction

$$z_h = \frac{\vec{p}_{hT} \cdot \vec{p}_{\text{jet}T}}{p_{\text{jet}T}^2} .$$

Paper concludes with some inaccuracies/uncertainties in measurements such as the gluon FF at large momentum fraction, and larger uncertainties for protons than pions.

Key points / why it matters for this thesis

- ✓ Cleanest environment for fragmentation: no hadronic initial state / no beam remnant.
- ✓ Clear statement of factorization picture and the role of the momentum fraction x.
- ✓ Good "bridge" reference for explaining why we rely on phenomenology + tuning once hadronization happens.

Jet fragmentation in e^+e^- annihilation

O. Biebel, P. Nason, B.R. Webber, arXiv:hep-ph/0109282v2 (2001). [2]

Summary

Paper outlines a definition of fragmentation functions (FFs) as non-dimensional functions that describe the final state single particle energy distribution in a hard scattering process. For e^+e^- the total fragmentation function is defined as

$$F^h(x, s) = \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{dx}(e^+e^- \to V \to hX),$$

where x is the scaled energy fraction $x = 2E_h/\sqrt{s}$ (and in practice you often see $x_p = 2|\vec{p}_h|/\sqrt{s}$ used as an approximation). The paper also directly compares FFs and PDFs by pointing out that in fragmentation, x is the fraction of a parton's momentum carried by a produced hadron, whereas in PDFs it is the fraction of a hadron's momentum carried by a parton. This is useful for the thesis because it gives a clean, measurable definition of "fragmentation" in e^+e^- that is directly connected to the basic longitudinal momentum sharing picture we use later (i.e. the same idea as introducing a momentum fraction z).

It is then discussed how parton FFs evolve with the hard scale through a DGLAP-type evolution equation (with splitting functions P_{ji}), meaning the shape of the distribution changes with energy in a predictable perturbative way once you fix an input distribution at some reference scale. A related point made is that the gluon FF only contributes at higher order (since gluons do not couple directly to the electroweak current), so gluon fragmentation is typically accessed indirectly, for example through heavy-quark three-jet topologies or via longitudinal information. The paper also explains that the angular structure of fragmentation can be written in terms of transverse, longitudinal and asymmetric pieces, and that this provides another handle on gluon fragmentation, with the specific statement that the entire $\mathcal{O}(\alpha_s)$ correction to σ_{tot} comes from the longitudinal part.

Finally, the paper discusses how the step from partons to hadrons is not perturbative and must be modelled, and it outlines two main phenomenological pictures used in practice: the string model and the cluster model. In the string model (used mainly in Monte Carlo simulations like PYTHIA) the colour field between partons is treated as a flux tube/string that grows in energy as the partons separate, and when enough energy builds up a $q\bar{q}$ pair is produced from the vacuum and the string breaks. The probability for this is described using a tunnelling form $\exp(-\pi m_{q,\perp}^2/\kappa)$, where $m_{q,\perp}^2 = m_q^2 + p_{q,\perp}^2$ is the transverse mass squared and the string tension is $\kappa \sim 1$ GeV/fm. The cluster model (mainly used by HERWIG) instead uses local colour compensation (preconfinement) where the remaining gluons at the end of the shower are split into $q\bar{q}$ pairs and grouped into colour singlet clusters. These clusters then decay into hadrons, typically two-body decays, with extra steps if a cluster is too heavy (split into two smaller clusters first) or too light (collapse into a single hadron with a small local rearrangement of energy and momentum).

Key points / why it matters for this thesis

✓ Gives a clean e^+e^- definition of the measured fragmentation observable $F^h(x, s) = \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{dx}$ and what the scaling variable $x = 2E_h/\sqrt{s}$ actually means.

✓ Makes the FF vs PDF comparison explicit, which is a nice way to motivate why fragmentation is the "final state analogue" of PDFs.

✓ Introduces DGLAP evolution for FFs and explains why gluon fragmentation is mainly a higher order / indirect effect.

✓ Gives a clear physics description of string vs cluster hadronization, which links directly to why this project compares PYTHIA (string) and HERWIG (cluster).

Parton Fragmentation Functions

A. Vossen, arXiv:1702.01329 (2017). [3]

Summary

Paper links colour confinement to fragmentation functions (FFs) by stating that FFs describe how colourless bound hadrons are formed from coloured partons. It also makes the comparison that FFs are about final-state formation, while PDFs are about initial-state structure. It highlights that the definition of the "standard" integrated (twist-2) FF $D_1^{h/q}(z)$ is chosen so that factorized cross sections take the usual hard $\otimes$ non-perturbative form across processes:

$$\text{SIDIS:} \quad \sigma(\ell p \to \ell h X) \sim \sum_q e_q^2 \, f_1^{q/p} \otimes D_1^{h/q}, \qquad pp \to hX \sim \sum_{i,j,k} f_1^{i/p_a} \otimes f_1^{j/p_b} \otimes D_1^{h/k},$$

and SIA (single-inclusive annihilation, $e^+e^- \to hX$)

$$\sim \sum_q e_q^2 \, D_1^{h/q}(z).$$

Paper outlines limitations of SIA: flavour separation becomes difficult because different FF contributions mainly differ through electroweak couplings, and gluon FFs only enter through scaling violations (so you need a good Q^2 lever arm). It also discusses why pp is harder: both protons' partonic structure enters, and cross sections require integrating over unknown initial kinematics, including regions where PDFs are not well known, which adds extra uncertainty and also makes it hard to directly access a clean z dependence.
Paper then gives a brief TMD angle: TMDs are useful for separating intrinsic transverse momentum in the nucleon from transverse momentum generated in fragmentation (especially in SIDIS). In a CSS factorization picture, the FF can be decomposed into a non-perturbative collinear part, k_T-dependent perturbative/non-perturbative pieces, and a bridging term between the non-perturbative and perturbative regimes. It lists two possible observables for intrinsic transverse momentum in fragmentation: (i) the p_T imbalance of back-to-back hadrons (sensitive to a convolution of transverse momenta), and (ii) hadron p_T measured relative to the thrust/jet axis, although identifying the quark axis with thrust/jet becomes problematic beyond LO. It also mentions that there are no measurements of the k_T dependence of unpolarized FFs in e^+e^-, but there are measurements of the Collins function $H_1^\perp(k_T)$. Since $H_1^\perp$ is a TMD, even k_T-integrated measurements still require TMD evolution.

Key points / why it matters for this thesis
✓ Lists the two key TMD-sensitive observables used here: back-to-back hadron p_T imbalance and hadron p_T w.r.t. the thrust/jet axis.
✓ Clarifies the gap in older LEP-era FF studies: they mainly constrain collinear $D_1(z)$, while TMD information needs transverse-momentum/correlation structure.
✓ Gives a short CSS-style framing of TMD fragmentation (collinear nonperturbative piece + k_T-dependent pieces + matching), motivating why the project focuses on p_T spectra and correlations, not just z.

Introduction to QCD

P. Skands, arXiv:1207.2389 (2012).[4]

Summary
Paper begins by stating that when probed at short wavelengths QCD is a theory of free 'partons' due to the consequence of asymptotic freedom. When probed at longer wavelengths such as the size of a proton, confinement dominates, and one can observe a string-like potential if one tries to separate the partonic constituents. QCD is defined as a non-Abelian gauge theory based on the special unitary group SU(3). The Lagrangian density is defined as

$$\mathcal{L} = \bar{\psi}_q^i (i\gamma^\mu)(D_\mu)_{ij} \psi_q^j - m_q \, \bar{\psi}_q^i \psi_q^i - \frac{1}{4} F_{\mu\nu}^a F^{a\,\mu\nu},$$

where ψ_q^i denotes a quark field with (fundamental) colour index i, $\psi_q = (\psi_{qR}, \psi_{qG}, \psi_{qB})^T$, μ is a Lorentz vector index, m_q allows for non-zero quark masses (from the Higgs mechanism), and $F_{\mu\nu}^a$ is the gluon field strength tensor.

D_μ is the covariant derivative in QCD,

$$(D_\mu)_{ij} = \delta_{ij}\partial_\mu - ig_s\, t^a_{ij} A^a_\mu,$$

with g_s the strong coupling (related to α_s by $g_s^2 = 4\pi\alpha_s$). Colour is not observed directly so predictions usually involve sums/averages over colour indices, and this corresponds to the number of available colour paths (red/-green/blue) that the process at hand can take. A big difference between QED and QCD is the gluon self-coupling, which is a consequence of the non-Abelian term in the field strength,

$$F^a_{\mu\nu} = \underbrace{\partial_\mu A^a_\nu - \partial_\nu A^a_\mu}_{\text{Abelian}} + \underbrace{g_s f^{abc} A^b_\mu A^c_\nu}_{\text{non-Abelian}}.$$

QCD is considered approximately scale invariant at first approximation (the "fractal" picture when zooming in on a jet), which was originally referred to as light-cone scaling or Bjorken scaling. This is only approximate because the coupling runs. The strong coupling

$$\alpha_s(Q^2) = \frac{1}{b_0 \ln(Q^2/\Lambda^2)}$$

decreases with increasing energy, which is the asymptotic freedom statement. This form also shows the rapid increase at decreasing energy where $\Lambda \sim 200\,\text{MeV}$ specifies the scale at which the perturbative coupling would nominally become infinite (the Landau pole in the one-loop expression). This region is non-perturbative and is the scale where objects such as parton densities (PDFs), fragmentation functions (FFs) and phenomenological models (such as string and cluster) are required.

A hard probe (commonly a photon in DIS) can essentially take an instantaneous snapshot of hadron structure over a much shorter timescale $1/Q \ll 1/\Lambda$. This is formalized by factorization, which allows the cross section for lepton–hadron scattering to be written as a convolution of PDFs and a perturbatively calculable partonic scattering cross section. The DGLAP equations are briefly referenced as the tool used to connect PDFs between different perturbative scales.

The anti-k_T algorithm is used for jet reconstruction, with the size parameter R typically varying between 0.4 and 0.7, generating circular-looking jets, while for jet substructure the k_T or Cambridge/Aachen algorithms are often used.

A problem arises in the need to compute integrals of the form

$$N_{\text{count}}(\Delta\Omega) = \int_{\Delta\Omega} d\Omega\, \frac{d\sigma}{d\Omega},$$

with $d\sigma/d\Omega$ a differential cross section, since in particle physics the phase space has 3 dimensions per final state particle. The dimension of phase space therefore increases rapidly and standard numerical integration methods give very slow convergence rates. For example the convergence rate of the trapezoidal rule and Simpson's rule scale like $1/n^{2/d}$ and $1/n^{4/d}$ respectively, while Monte Carlo remains $1/\sqrt{n}$ for all dimensions and quickly becomes the sensible choice for numerical integration. The only risk in the Monte Carlo method is the possibility of large deviations as a consequence of "dice rolls" (where the name comes from).

Parton showers are an iterative Markov chain governed by the evolution variable Q_E, evolving towards lower scales until the hadronization cutoff defined as $Q_{\text{had}} \sim \mathcal{O}(1\,\text{GeV})$. The so-called Sudakov factor,

$$\Delta(\Phi_F, Q_1, Q_2) = \exp\left[-\sum_r \int_{Q_2}^{Q_1} \frac{d\Phi^r_{F+1}}{d\Phi_F}\, S_r(\Phi_{F+1})\right],$$

defines the probability that there is no evolution (i.e. no emissions) between the scales Q_1 and Q_2.

The Lund string model is the basis of event generators (such as PYTHIA) for describing hadronization, where a set of coloured partons are transformed into sets of colour-singlet hadrons after the hadronization scale Q_{had}

is reached following sufficient parton showering. The Lund model operates on a 3 stage process: (1) the parton system is mapped to a continuum of strings in a colour field, (2) these strings break iteratively into a discrete set of primary hadrons, and (3) these hadrons decay into secondaries such as $\rho \to \pi\pi$, $\Lambda \to n\pi$, $\pi^0 \to \gamma\gamma$, etc. This whole process is due to the fact that as quarks move apart linear confinement implies that a potential

$$V(R) = \kappa R$$

is asymptotically reached for large distances R. Such a potential describes a string with tension (energy per unit length) κ, with the value

$$\kappa \sim (420\,\mathrm{MeV})^2 \sim 0.18\ \mathrm{GeV}^2 \sim 0.9\ \mathrm{GeV/fm},$$

which (as the paper notes) is macroscopically enormous. An appealing feature is that low energy gluons are smoothly absorbed into these strings, improving the stability of the model. The Lund model invokes the idea of quantum mechanical tunnelling, giving a Gaussian suppression of the transverse momenta and masses imparted to the produced quarks, with a characteristic Schwinger-like dependence $\propto \exp(-\pi(m_q^2 + p_{\perp q}^2)/\kappa)$. The Lund symmetric fragmentation function

$$f(z) \propto z^{-1}(1-z)^a \exp\left(-\frac{b\,m_{\perp h}^2}{z}\right)$$

is a solution to the imposed left–right symmetry requirement that the fragmentation be independent of the sequence of breakups. Rapidity is defined as

$$y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right),$$

with p_z taken along the string (or jet) direction.
Higher energies imply that parton densities can be probed at smaller x values, where the number of partons rapidly increases. Partons then become closer packed and the colour-screening distance d decreases, which ties into the underlying-event/MPI picture in hadron collisions. The paper concludes with a quick note on how Monte Carlo event generators are tuned, stating that an agreement of $\sim$ 5% should be considered the absolute sanity limit beyond which it does not make sense to tune further. It also notes that the "primordial k_T" is mainly constrained by the dilepton $p_\perp$ distribution in Drell–Yan events in hadron–hadron collisions, and gives a warning about jet $p_\perp$ sensitivity to underlying-event shifts and ISR sensitivity to PDFs.

Key points / why it matters for this thesis
✓ Gives the clean perturbative-to-nonperturbative storyline: asymptotic freedom at short distance, confinement at long distance, so jets require showers + hadronization modelling.
✓ Defines the shower mathematically through the Sudakov factor $\Delta(\Phi_F, Q_1, Q_2)$, which is the probability backbone behind PYTHIA/HERWIG event generation.
✓ Provides the explicit Lund inputs that shape hadron-level p_T and z: $V(R) = \kappa R$, tunnelling suppression $\exp(-\pi(m^2 + p_\perp^2)/\kappa)$, and $f(z) \propto z^{-1}(1-z)^a \exp(-bm_\perp^2/z)$.
✓ Gives the practical jet-definition baseline (anti-k_T, typical $R = 0.4$–0.7), which is exactly what the analysis uses when defining dijet topologies and jet axes for p_T measurements.
✓ Justifies why the project starts with MC and correlations: multi-particle final states are high-dimensional integrals, where Monte Carlo $1/\sqrt{n}$ convergence is the only scalable approach.

References

[1] O. Biebel, D. de Florian, D. Milstead, and W. Vogelsang. Fragmentation functions in e^+e^-, *ep*, and *pp* collisions. *Prog. Theor. Exp. Phys.*, 2020(8):083C01, 2020. PDG review PDF.

[2] O. Biebel, P. Nason, and B. R. Webber. Jet fragmentation in e^+e^- annihilation. 2001. arXiv:hep-ph/0109282.

[3] Anselm Vossen. Parton fragmentation functions. In David d'Enterria and Peter Z. Skands, editors, *Parton Radiation and Fragmentation from LHC to FCC-ee*, 2017. arXiv:1702.01329.

[4] Peter Z. Skands. Introduction to qcd. 2012. Lectures presented at TASI 2012 (updated July 2017). arXiv:1207.2389.