TMD Studies in e^+e^- Collisions

Shane Sweetman

Simulation study using PYTHIA8 & Root & FastJet

Outline

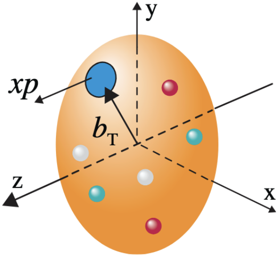
1. Theory Overview
2. Collisions and Jets
3. Mother: Daughter Particles and Leading Track
4. Anti-k_T Algorithm
5. Jet Shapes
6. Simulation Plan
7. Results

What are TMDs?

- Transverse Momentum Dependent (TMD) distributions: describe the 3D momentum structure of quarks and gluons inside hadrons.
- Extend the collinear picture by including parton transverse momentum k_T in addition to the longitudinal fraction x, i.e., $f(x, k_T)$.
- Collinear picture: partons move along the hadron's direction; only longitudinal momentum fraction x matters, transverse momentum k_T is ignored.

From Quarks to Hadrons: Key Concepts

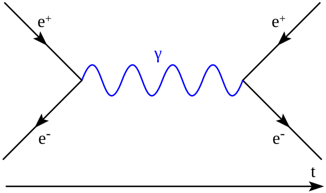
- Fragmentation functions (FF) describe how the quark's longitudinal momentum is shared among the hadrons (z).
- Transverse Momentum Distributions (TMDs) describe the sideways motion of hadrons relative to the jet direction (k_T or j_T).
- Leading hadrons carry the most momentum and thus the clearest information about the original quark.



Parton with longitudinal momentum fraction x and transverse position b_T in the proton [1].

e^+e^- Collisions: The Basics

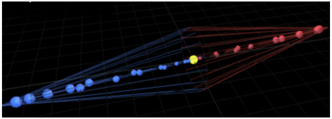
- High-energy e^+e^- collisions produce quark-antiquark pairs.
- Clean initial state: no hadrons in the beam.
- Resulting quarks hadronize into back-to-back jets.



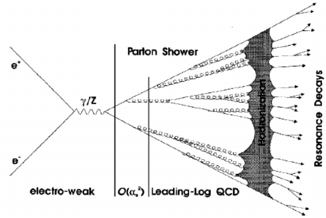
Illustrations of $e^+e^- \to q\bar{q}$.

Hadronization and Jet Formation

- Quarks and gluons produced in e^+e^- collisions cannot exist freely due to confinement.
- They undergo hadronization, forming collimated sprays of hadrons called jets.
- The jet directions reflect the original quark momenta.



Visualization of two back-to-back hadronic jets from $e^+e^- \to q\bar{q}$.



Detailed hadronization process of quarks into jets [2].

Mother/Daughter Particles & Leading Track

- Mother particle: the original particle that decays or fragments.
- Daughter particles: the resulting particles from the decay or fragmentation.
- Leading track: the particle in a jet carrying the largest fraction of momentum.
- Studying the leading track helps trace the original quark direction and momentum [3].

Anti-k_T Algorithm

- Clusters particles into jets based on transverse momentum (p_T) and distance.
- Distance measure between particles i and j:

$$d_{ij} = \min\left(\frac{1}{p_{Ti}^2}, \frac{1}{p_{Tj}^2}\right)\frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = \frac{1}{p_{Ti}^2}$$

  [4]
- Where:
  - p_{Ti} = transverse momentum of particle i
  - $\Delta R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$ (distance in rapidity-azimuth)
  - R = jet radius parameter
- Process: Combine particles with smallest d_{ij} until all clustered.
- Result: Circular, stable jets, insensitive to soft particles (low momentum).

Pseudorapidity (η)

- Pseudorapidity is a spatial coordinate describing particle angles relative to the beam axis.
- Defined as:

$$\eta = -\ln\left[\tan\left(\frac{\theta}{2}\right)\right]$$

  where θ is the polar angle.
- η is preferred over θ because differences in η are invariant under boosts along the beam direction [5].

Jet Shape Observables

Purpose: Describe how particles are distributed in a collision.

Key Observables:

- Thrust: measures how aligned the event is
- Sphericity: measures how spherical the event is in 3D
- Circularity: measures the shape in the transverse plane (2D projection)

Applications:

- Classifying events: dijet vs. multijet vs. isotropic
- Testing QCD models and tuning Monte Carlo generators

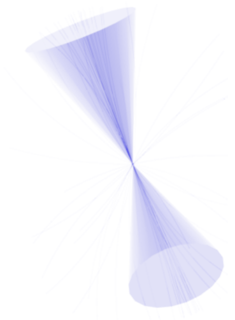
Thrust

- Thrust measures how the momentum of particles is aligned along a single axis.
- Defined as:

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

[6]

- Ranges from 0.5 (spherical distribution) to 1 (all momentum along one axis).
- Helps distinguish between narrow jets and more isotropic events.



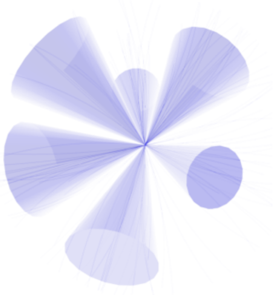
Schematic of thrust jet in an event, showing alignment along the thrust axis [7].

Sphericity

- Sphericity measures how spherical the event is in 3D.
- Calculated from the particle momenta using:

$$S = \frac{3}{2}(\lambda_2 + \lambda_3)$$

  [8] where λ_2, λ_3 are the smaller eigenvalues of the momentum tensor.
- $S = 1$: perfectly spherical event.
  $S = 0$: two-jet (linear) event.



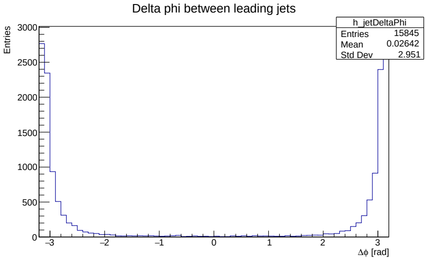
Schematic illustration of a 3D event and how sphericity quantifies its shape [7].

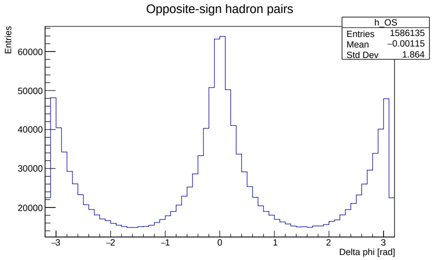
Plan for simulation

- Identify the primary quark flavor from the hard interaction and trace final-state hadrons to their origin.
- Analyze pairs of charged hadrons for azimuthal angle differences and transverse momentum imbalance.
- Reconstruct jets using the anti-k_T algorithm and compute properties: multiplicity, p_T, pseudorapidity, rapidity, and $\Delta\phi$ between leading jets.
- Evaluate global event shapes such as thrust, sphericity, and circularity.
- Study leading hadrons (pions and kaons) through p_T, rapidity, azimuthal correlations, and fragmentation variables (z, j_T) relative to jets.
- Generate flavor-tagged histograms to examine how observables depend on the quark type initiating the hadrons.

Results of simulation

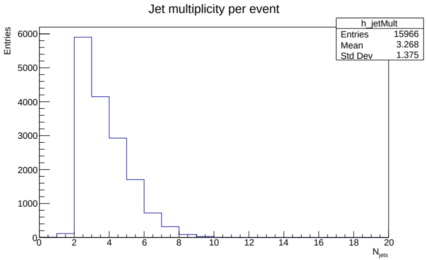
- $\Delta\phi$ is the angle between the two leading jets.
- The plot on the left (this meeting) uses the anti-k_T algorithm.
- The plot on the right (last meeting) had a spike near $\Delta\phi = 0$.
- The new result removes that spike and shows jets mostly back-to-back at $\Delta\phi \approx \pi$.





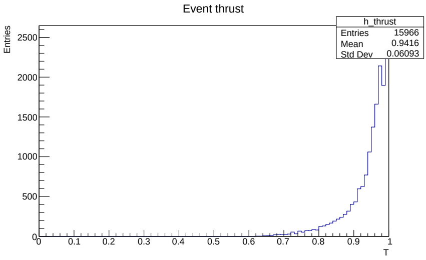
Results of simulation

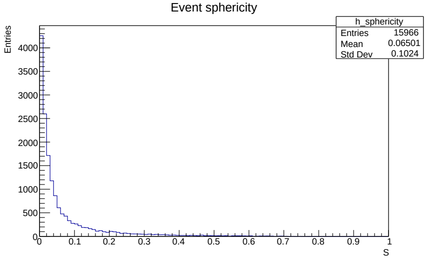
- Jet multiplicity = number of reconstructed jets in each event.
- Jets found using anti-k_T (R=0.6) with a minimum p_T cut, then counted per event.
- Result: distribution peaks at 2–3 jets, then falls roughly exponentially for larger multiplicities.
- Interpretation: most events are dijet-like; extra jets come from hard radiation or hadronization and appear less often.



Results of simulation

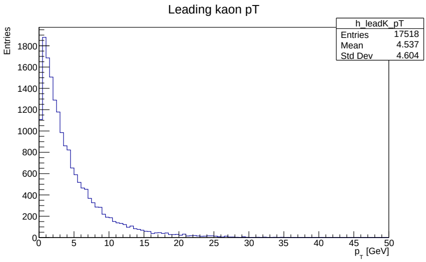
- These observables describe the overall geometry of the event in momentum space.
- Sphericity measures how isotropic an event is: values near 0 indicate jet-like events, while 1 means spherical.
- Thrust measures alignment of particle momenta: values near 1 indicate two-jet events, while smaller values are more isotropic.
- Results show: sphericity peaks near 0 and thrust peaks near 1, consistent with mostly back-to-back jet events.

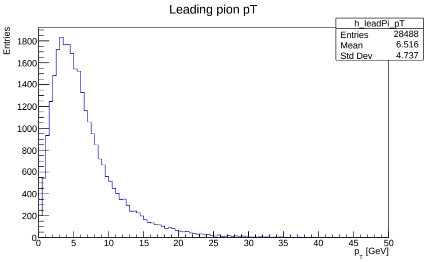




Transverse momentum of leading hadrons

- Compared p_T distributions of leading kaons and pions.
- Both peak near $p_T = 0$, as most hadrons have low transverse momentum.
- The pion distribution is noticeably broader, possibly indicating different fragmentation behavior, though this could also be due to limited event statistics.





Thank you

Questions?

References

1 D. Boer, M. Diehl, R. G. Milner, et al., *TMD factorization and evolution*, arXiv:1212.1701, p.16.

2 S. Sarkar, "Cosmic ray signatures of massive relic particles," 2000, doi:10.1142/9789812792129_0012.

3 CMS Collaboration, "CMS peers inside heavy-quark jets," *CERN Courier*, 26 March 2025, link.

4 M. Cacciari, G. P. Salam, G. Soyez, *The anti-k_T jet clustering algorithm*, Journal of High Energy Physics, JHEP04(2008)06.

References

5 Particle Data Group, "Review of Particle Physics," Prog. Theor. Exp. Phys. 2020, 083C01 (2020), section on kinematic variables, p.10.

6 S. Kluth, "Tests of Quantum Chromo Dynamics at e^+e^- Colliders," arXiv:hep-ex/0603011, p.29, link.

7 ATLAS Collaboration, "Multijet Event Isotropies," link.

8 M. Sas, J. Schoppink, "Event shapes and jets in e^+e^- and pp collisions," *Nucl. Phys. A*, 2021, doi:10.1016/j.nuclphysa.2021.122195.