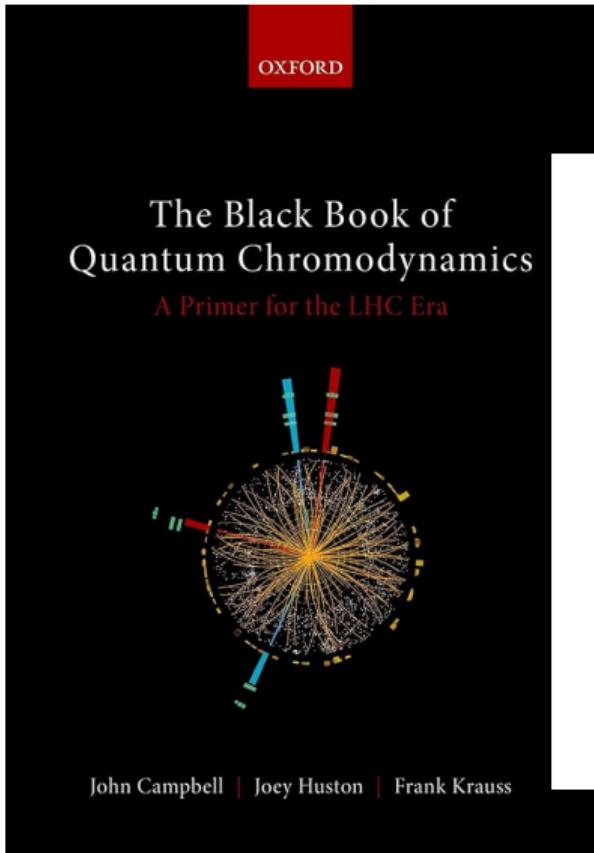




Shane Sweetman

*Project update meeting*  
Student No: 22308373

# A TMD-oriented analysis of $\pi^+\pi^-$ pairs in $e^+e^-$ collisions



**TMD Handbook**  
A modern introduction to the physics of  
Transverse Momentum Dependent distributions

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# TMD-Analysis

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## Shane Sweetman

GitHub Business Review Agent

### Literature Review Notes: Fragmentation, Jets, and TMD Motivation

Click each page side to open the highlighted PDF in GitHub

#### SCOPE

These notes collect short summaries of the main references I am using for my clean-as-must-be-motivation-sensitive observables in back-to-back jets in  $e^+e^-$  collisions. Each paper also links to the associated PDF stored in my GitHub repository.

#### PAPERS

##### Fragmentation Functions in $e^+e^-$ , $\bar{p}p$ , and $p\bar{p}$ Collisions

##### 29. Fragmentation Functions in $e^+e^-$ , $\bar{p}p$ , and $p\bar{p}$ Collisions

Received: August 2019 by B. Biedel (Ludwig-Maximilians U.), D. de Florian (CERN and ICHE, UNSAM), D. Mäntönen (Stockholm U.) and W. Vogelsang (Tübingen U.).

##### 19.1 Introduction to fragmentation

Quarks and gluons produced in hard-scattering reactions will ultimately give rise to the coherent hadronic "beam shower" that we observe in the detector. The associated hadronization process is described by fragmentation functions  $D_i^*(x, \mu^2)$  ( $i = u, d, g$ ) which are universal functions representing, in the simplest picture, a measure of the probability density that an outgoing parton produces a hadron  $A$ . Here,  $x$  is the fraction of the parton's momentum transferred to the hadron and  $\mu$  is a "resolution scale" known as factorization scale. The  $D_i^*(x, \mu^2)$  may be viewed as the hadronic analogs of the initial-state parton distributions  $f_i(x, \mu^2)$  (see Sec. 1.5 of this chapter). They are related to the fragmentation distributions since they are primarily produced via gluon-gluon fusion or indirectly made via hadron-hadron scattering at high-energy, or via the final-state parton fragmentation mechanism. (See Sec. 1.6 of this chapter.) They are also related to the fragmentation distributions since they are primarily produced via gluon-gluon fusion via a similar intermediate boson. (See Refs. [3, 2] for introductory reviews, and Refs. [3–6] for summaries of experimental and theoretical research in this field).

The closest literature to the study of fragmentation functions is provided to semiannihilation  $e^+e^- \rightarrow \bar{p}p$ . The fragmentation function for this reaction may be expressed in terms of fragmentation structure functions  $F_{1,2,A}$  that are directly related to the fragmentation functions. At center-of-mass (CM) energy  $\sqrt{s} = q^2$  we have

$$z_k = \frac{\sqrt{q^2} p_{k\perp}}{p_{tot\perp}} = \frac{1}{q} (1 + \cos^2 \theta) F_1^A(x, q^2) + \frac{3}{q} \cos \theta F_2^A(x, q^2) + \frac{3}{q} \sin \theta F_{1,2,A}(x, q^2). \quad (18.1)$$

Here,  $q$  is the four-momentum of the intermediate photon or Z-boson, with  $q^2 > 0$  and  $z = 2p/q$  is the fraction of the total energy carried by the hadron  $A$  in the center-of-mass frame of the familiar CM-Bjorken scaling. Note that  $x = 2E_k/\sqrt{s} \leq 1$  in terms of the energy  $E_k$  of the produced hadron in the CM frame of the electron-positron pair. Furthermore,  $\theta$  is the hadron's angle relative to the electron beam direction. Eq. (18.1) is the most general form for unpolarized inclusive single-particle production via vector bosons [8]. The fragmentation structure functions  $F_1$  and  $F_2$  represent the contributions from  $\gamma/\text{Z}$  photodissociation with  $\gamma$  being the incoming photon and  $\text{Z}$  the incoming vector boson of the hadron. The remaining term with the asymmetric fragmentation function  $F_{1,2}$  arises from the interference between vector and axial-vector contributions. Various fragmentation factors  $\sigma_{ik}$  are used in the literature, ranging from the total cross section  $\sigma_{ik}$  for  $e^+e^- \rightarrow$  hadrons, including all weak and QCD contributions, to  $\sigma_{ik} = 4\pi r^2 N_{ik}/(q^2 N_{tot})$  where  $N_{ik}$  is the lowest-order QED contribution and  $N_{tot}$  is the total number of hadrons in the system of  $N_{tot}$ . LEPI measurements of the three fragmentation structure functions are shown in Fig. 19.1.

Integration of Eq. (18.1) over all  $\theta$  yields the total fragmentation structure function  $F^A(q^2) = F_1^A + F_2^A$ .

$$\int_0^\pi d\theta = F^A(q^2) = \sum_i \int_0^\pi dz_i \sigma_{ik} \left( \sum_{i,j} C_{ijk} \right) F_{1,2,A}(x_k, q^2) = \sum_i \int_0^\pi dz_i \sigma_{ik} F_{1,2,A}(x_k, q^2). \quad (18.2)$$

On the right we have written the formal expansion for the structure function in terms of a sum over renormalizations of the fragmentation functions  $D_i^*$  for partons  $i = u, d, \bar{d}, \bar{u}, g$  with perturbative coefficient functions  $C_{ijk}$ . Since photons and Z-bosons do not distinguish between quarks and antiquarks,  $e^+e^-$  annihilation primarily contains the combinations  $D_u^* + D_{\bar{d}}^*$ . Gluon fragmentation contributes only at higher order in perturbation theory or by scaling violations. Corrections to the factorized expression in Eq. (18.2) are suppressed by inverse powers of  $q^2$ . They

## Recap

- Successfully graphed the pion  $p_T$  distributions for:
  - » the pion closest to the  $q\bar{q}$  pair,
  - » the highest-momentum pion.
- Did not differentiate between same-sign and opposite-sign pairs (OS/SS combined).

## Aim(s)

- Fragmentation in  $e^+e^-$  as a clean environment (final-state analogue of PDFs). [1,2,4]
- Understand the hadronisation pictures used in event generators:
  - » Lund / string model (PYTHIA). [2,4]
  - » Cluster model (HERWIG). [2,4]
- Recreate the same plots in HERWIG and compare shapes against PYTHIA. [2,4]

# Fragmentation functions: what matters

- Fragmentation functions  $D_i^h(z, Q)$  encode how an outgoing parton turns into a hadron carrying longitudinal momentum fraction  $z$  at scale  $Q$  (final-state analogue of PDFs). [1,2,3]
- Here  $Q$  is the hard (factorisation/renormalisation) scale that sets the resolution of the probe, e.g.  $Q \simeq \sqrt{s}$  in  $e^+e^-$  or  $Q \sim M_Z$  at the  $Z$  pole. [1,2,3]
- Parton distribution functions (PDFs)  $f_i(x, Q)$  describe the *initial-state* partonic content of an incoming hadron: the probability density to find a parton of flavour  $i$  carrying momentum fraction  $x$  when probed at scale  $Q$ . [1,2,3]
- In short: PDFs describe the *start* of the event (inside the incoming hadron), while FFs describe the *end* of the event (hadronisation of outgoing partons). [1,2,3]

# Why phenomenology is unavoidable: $\Lambda_{\text{QCD}}$ / Landau pole

## From perturbative QCD to hadronisation

- $\Lambda_{\text{QCD}}$  is the characteristic QCD scale ( $\sim$  a few 100 MeV) where the coupling becomes strong and confinement/non-perturbative physics sets the boundary between perturbative and hadronic descriptions. [4]
- The running coupling grows at low scales:

$$\alpha_s(Q^2) \sim \frac{1}{b_0 \ln(Q^2/\Lambda_{\text{QCD}}^2)} \quad \Rightarrow \quad Q \rightarrow \Lambda_{\text{QCD}} \text{ (Landau pole / breakdown of fixed-order pQCD)}$$

- In showers, evolution proceeds down to a cutoff  $Q_0 \sim \mathcal{O}(1 \text{ GeV})$ , where confinement dominates and hadrons must be modelled (not computed)  $\Rightarrow$  *phenomenological hadronisation models*. [4]

## What the model is doing

- After showering to a hadronisation scale, colour flow is represented as a string/flux tube with roughly linear confinement potential, so energy grows as partons separate. [2,4]
- String breaking produces  $q\bar{q}$  pairs iteratively, giving a chain of primary hadrons along the colour connection (then decays). [2,4]
- Pair creation at a string break is modelled as quantum tunnelling, giving a Schwinger-like suppression for transverse mass:

$$P \propto \exp\left(-\frac{\pi m_\perp^2}{\kappa}\right), \quad m_\perp^2 = m_q^2 + p_\perp^2$$

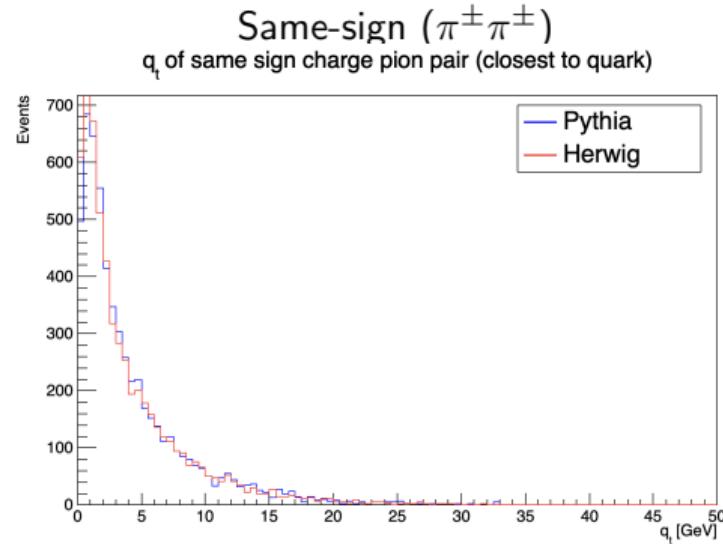
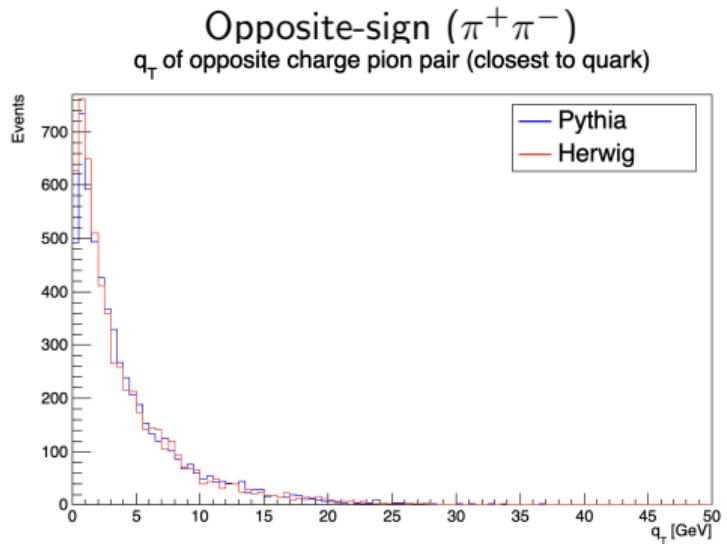
[2,4]

- Non-perturbative transverse momentum arises from the pair-creation ("string break") step, with a characteristic transverse width feeding into hadron-level  $p_T$  observables. [2,4]

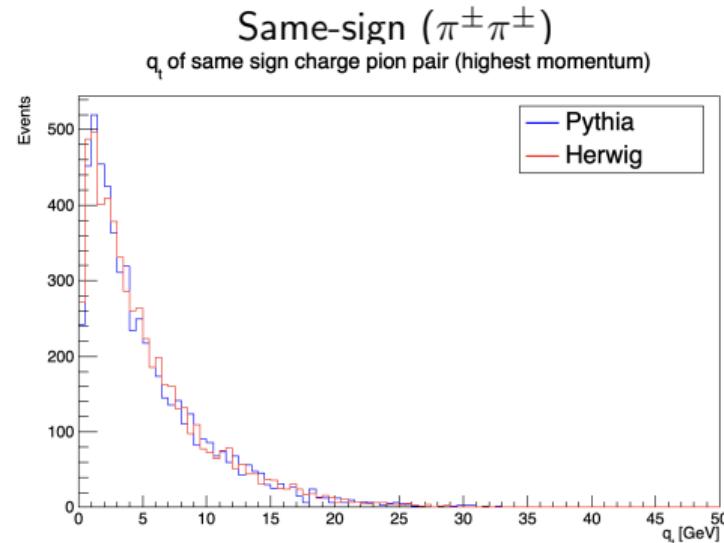
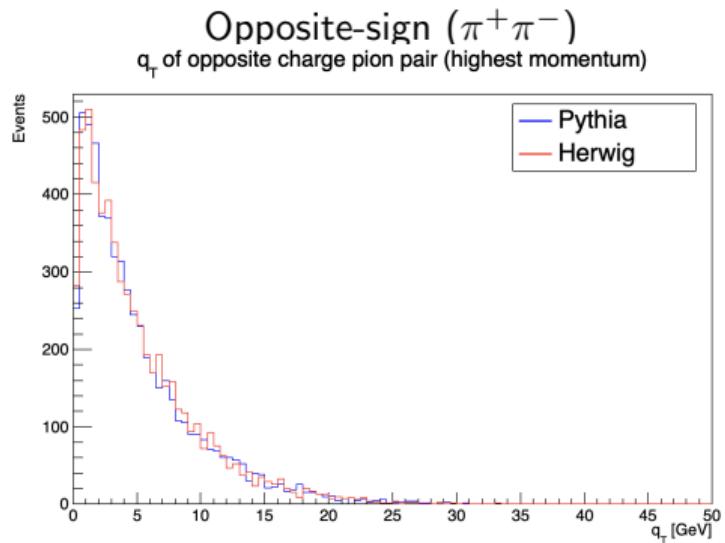
## Cluster hadronisation: what is happening

- The cluster model uses preconfinement: colour-connected partons at the shower cutoff form colour-singlet clusters with masses  $\sim$  hadronic scale. [2]
- At cutoff, remaining gluons are split  $g \rightarrow q\bar{q}$ , then colour-connected  $q$  and  $\bar{q}$  are paired into clusters. [2]
- Clusters decay into hadrons; overly massive clusters undergo fission (extra  $q\bar{q}$  creation) before decaying. [2]

# PYTHIA vs HERWIG: closest-to-quark selection

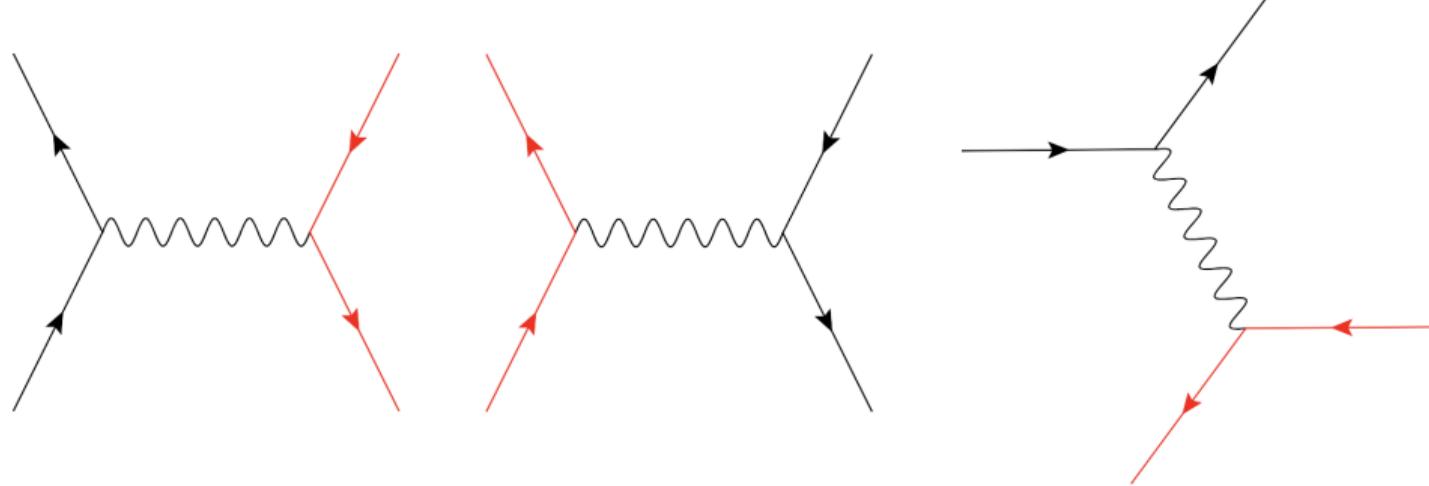


# PYTHIA vs HERWIG: highest-momentum selection



# TMDs across benchmark reactions

- Typical TMD “triad”: SIDIS, Drell–Yan, and back-to-back hadrons/jets in  $e^+e^-$ . [5]



Hadronic  $Z$  decay

$$e^-e^+ \rightarrow \gamma^*/Z^0 \rightarrow q\bar{q}$$
$$\propto N_C$$

Drell-Yan

$$q\bar{q} \rightarrow \gamma^*/Z^0 \rightarrow \ell^+\ell^-$$
$$\propto 1/N_C$$

DIS

$$\ell\bar{q} \xrightarrow{\gamma^*/Z^*} \ell\bar{q}$$
$$\propto 1$$

## When PDFs are enough (TMDH p. 26)

For inclusive Drell–Yan, integrated over the full transverse-momentum range, collinear factorization gives

$$\frac{d\sigma}{dQ^2 dY} = \sum_{i,j} \int \frac{d\xi_a}{\xi_a} \frac{d\xi_b}{\xi_b} f_{i/H_a}(\xi_a) f_{j/H_b}(\xi_b) \frac{d\hat{\sigma}_{ij}(\xi_a, \xi_b)}{dQ^2 dY} \left[ 1 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{Q^2}\right) \right]. \quad (\text{TMDH Eq. 2.2})$$

Here  $Q^2 = q^2$  is the dilepton invariant mass,  $Y$  is the dilepton rapidity, and

$$x_a = \frac{Q e^{+Y}}{\sqrt{s}}, \quad x_b = \frac{Q e^{-Y}}{\sqrt{s}}. \quad (\text{TMDH Eq. 2.3})$$

*Key point (why PDFs can be enough here):* because the observable is averaged over a large allowed range of dilepton transverse momentum, the detailed partonic transverse-momentum dependence is not numerically important; the collinear PDFs  $f_{i/H}(\xi)$  (depending only on longitudinal fractions) capture the dominant structure. [5]

## When PDFs are not enough: (TMDH p. 27)

**Small transverse momentum** ( $\Lambda_{\text{QCD}} \lesssim q_T \ll Q$ ): the cross section becomes sensitive to intrinsic transverse momentum in the incoming hadrons  $\Rightarrow$  *TMD PDFs*,

$$\frac{d\sigma}{d^4q} = \frac{1}{s} \sum_{i \in \text{flavors}} \hat{\sigma}_{i\bar{i}}^{\text{TMD}}(Q) \int d^2\mathbf{k}_T f_{i/H_a}(x_a, \mathbf{k}_T) f_{\bar{i}/H_b}(x_b, \mathbf{q}_T - \mathbf{k}_T) \left[ 1 + \mathcal{O}\left(\frac{q_T^2}{Q^2}, \frac{\Lambda_{\text{QCD}}^2}{Q^2}\right) \right]. \quad (\text{TMDH Eq. 2.6})$$

*Main point:* momentum conservation forces  $\mathbf{k}_T$  to add up to  $\mathbf{q}_T$ , so the distribution is directly sensitive to transverse structure.

**Why Fourier space is used:** one often works with the  $b_T$ -space TMD,

$$\tilde{f}_{i/H}(x, \mathbf{b}_T) = \int d^2\mathbf{k}_T e^{-i\mathbf{b}_T \cdot \mathbf{k}_T} f_{i/H}(x, \mathbf{k}_T), \quad (\text{TMDH Eq. 2.7})$$

which turns transverse-momentum convolutions into products and is convenient for resummation.

[5]

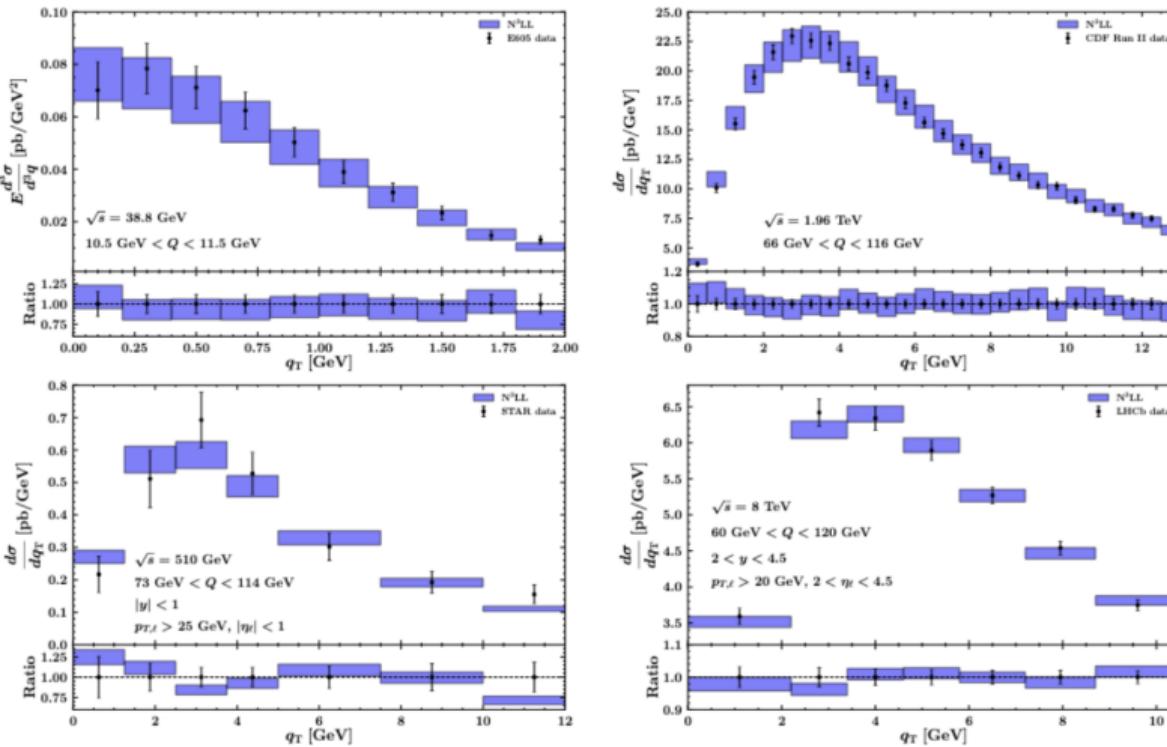
## Example paper: Bacchetta & Bertone et al. (DY) [6]

- In the notation of Bacchetta & Bertone et al. (Eq. 3.2), the fitted observable is the bin-averaged spectrum:

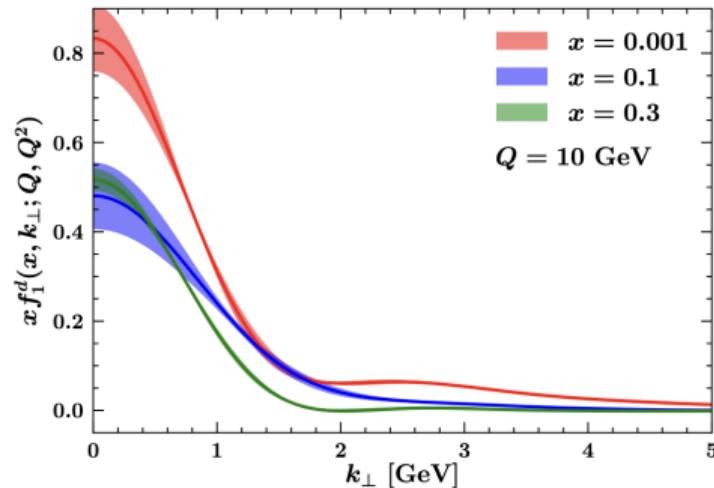
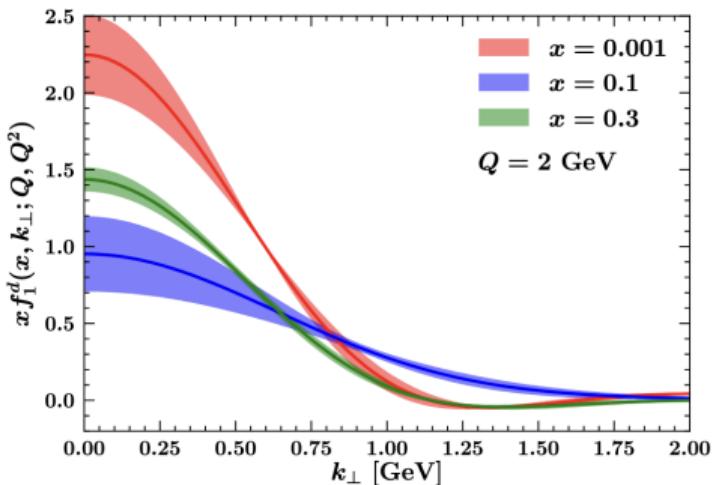
$$\left. \frac{d\sigma}{dq_T} \right|_{\text{bin}} = \frac{1}{q_{T,\max} - q_{T,\min}} \int_{y_{\min}}^{y_{\max}} dy \int_{Q_{\min}}^{Q_{\max}} dQ \int_{q_{T,\min}}^{q_{T,\max}} dq_T \frac{d\sigma}{dQ dy dq_T}.$$

- The kernel  $\frac{d\sigma}{dQ dy dq_T}$  is computed with TMD resummation (up to  $N^3 LL$ ) plus a fitted non-perturbative component, so the measured  $q_T$  shape directly constrains the unpolarised TMD PDFs. [6]

# Global fit quality across experiments ( $q_T$ spectra) [6]



# Example result: fit quality on $q_T$ spectra [6]



Plan for thesis?

## Physics / TMD background

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### Software / Herwig setup

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