GATE

CS & DA

DPP: 1

CALCULUS AND OPTIMIZATION

Q1 The domain of the function $f(x) = \sin^{-1}\left(\frac{x^2-3x+2}{x^2+2x+7}\right)$ is :

(A) $[1, \infty]$
(B) $[-1,\ 2]$
(C) $[-1, \infty)$
(D) $(-\infty\ , 2]$

Q2 What is the range of $f(x) = \cos 2x - \sin 2x$?

(A) [2, 4]
(B) $[-1,\ 1]$
(C) $\left[-\sqrt{2},\ \sqrt{2}\right]$
(D) $\left(-\sqrt{2},\ \sqrt{2}\right)$

Q3 A function f (x) is linear and has a value of 29 at x = -2 and 39 at x = 3. Find its value at x = 5.

Q4 Which of the following function is odd ?

(A) $x^2 - 2x + 3$ (B) Sin x
(C) Sin x + tan x (D) Cos x

Q5 Which of the following functions is periodic ?

(A) Sin x + cos x
(B) $e^x + log\, x$
(C) {n}
(D) [n]

Q6 Evaluate.

(i) $\lim\limits_{x\to 0} \frac{\sqrt{4+x}-2}{x}$

Q7 Evaluate :

$\lim\limits_{x\to -1} \frac{(x+2)(3x-1)}{x^2+3x-2}$

Q8 At x = 1, the function

$$f(x) = \begin{cases} x^3 - 1, 1 < x < \infty \\ x - 1, -\infty < x \leq 1 \end{cases}$$

(A) continous and differentiable
(B) continuous and non-differentiable
(C) discontinuous and differentiable
(D) discontinuous and non-differentiable

Q9 If $f(x) = x\left(\sqrt{x} - \sqrt{x+1}\right)$, then -

(A) f(x) is continuous but not differentiable at x = 0
(B) f(x) is differentiable at x = 0
(C) f(x) is not differentiable at x = 0
(D) None of these

Q10 If $\lim_{x\to\infty}\left(\sqrt{x^2-x+1} - ax\right) = b$, then the ordered pair (a,b) is:

(A) $\left(-1, \frac{1}{2}\right)$
(B) $\left(-1, -\frac{1}{2}\right)$
(C) $\left(1, -\frac{1}{2}\right)$
(D) $\left(1, \frac{1}{2}\right)$

Q11 The value of the function $f(x) = \lim\limits_{x\to 0} \frac{x^3+x^2}{2x^2-7x^2}$ is......

(A) 0 (B) $\frac{-1}{7}$
(C) $\frac{1}{7}$ (D) -1/5

Q12 $\lim\limits_{x\to 0} \frac{x-\sin x}{1-\cos x}$ is

Q13 $\underset{x\to 0}{\text{Lt}} \left(\frac{e^{2x}-1}{\sin(4x)}\right)$ is equal to

Q14 Which of the following values are correct

(A) $\frac{\sin x}{x} < 1$
(B) $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$
(C) $\lim\limits_{x\to 0} \frac{\sin x}{x} = 0$
(D) $\lim\limits_{x\to 0} \frac{\sin x}{x} = -1$

Q15 For the given function

$$f(x) = \begin{cases} \frac{x^2}{2} & ;\quad 0 \leq x < 1 \\ 2x^2 - 3x + \frac{3}{2} & ;\quad 1 \leq x \leq 2 \end{cases}$$

which of the following is (are) correct.

(A) $f(x)$ is continuous $\forall x \in [0, 2]$

(B) $f'(x)$ is continuous $\forall x \in [0, 2]$

(C) $f''(x)$ is discontinuous at $x = 1$

(D) $f''(x)$ is discontinuous $\forall x \in [0, 2]$

Q16 Let $\alpha, \beta, \in \mathbb{R}$ be such that $\lim_{x \to 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$. Then 6 $(\alpha + \beta)$ equals.

Q17 A function f(x) = 1− x^2 + x^3 is defined in the closed interval [−1, 1]. The value of x, in the open interval (−1, 1) for
which the mean value theorem is satisfied, is

(A) $-\frac{1}{2}$ (B) $-\frac{1}{3}$

(C) $\frac{1}{3}$ (D) $\frac{1}{2}$

Q18 The value of c in the lagrange's mean value theorem of the function
$f(x) = x^3 - 4x^2 + 8x + 11$ when $x \in [0, 1]$ is
:

(A) $\frac{4-\sqrt{5}}{3}$ (B) $\frac{\sqrt{7}-2}{3}$

(C) $\frac{2}{3}$ (D) $\frac{4-\sqrt{7}}{3}$

Q19 $f(x) = \frac{\sin(x)}{x}$, How many points exist such that f'(c) =0 in the interval [0,18π]

(A) 18 (B) 17

(C) 8 (D) 9

Q20 Find a point on the parabola y =$(x + 2)^2$,where the tangent is parallel to the chord joining (−2, 0) and (0, 4).

Q21 Consider the function f(x) = (x−2) log x for x ∈ [1, 2] show that the equation x log + x = 2 has at least one solution lying between 1 and 2.

Q22 If f(x) = e^x − e^{-x} and g(x) = |cos x − sin x| , then on the intveral $\left[0, \frac{\pi}{2}\right]$ Cauch's mean value theorem is -

(A) applicable

(B) not applicable as g(0) = $g\left(\frac{\pi}{2}\right)$

(C) not applicable as g' $\left(\frac{\pi}{4}\right) = 0$

(D) not applicable as g(x) contains || (i.e., mod) function

Q23 Verify Cauchy's mean value theorem for the functions $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$ in the interval [a, b], where a > 0.

Q24 If f(x) = e^x and g(x) = e^{-x} , then the value of c by cauchy mean value theorem in [a, b] is given by

(A) a + b (B) $\frac{1}{2}(a + b)$

(C) a·b (D) None of these

Q25 Cauchy's mean value theorem is applicable only

(A) for only one function

(B) for two functions

(C) for one or two functions both

(D) None of these

Q26 Use the intermediate value theorem to prove that the equation e^x = 4 − x^3 is solvable on the interval [−2, −1] .

Q27 Check whether there is a solution to the equation x^5 − 2x^3 − 2 = 0 between the interval [0,2].

Q28 The Value of c in the lagrange's mean value theorem ofr the function ***f*(*x*)** = x^3 -4x^2+8x + 11, when x ∈ [0,1] is:

(A) $\frac{4-\sqrt{5}}{3}$ (B) $\frac{\sqrt{7}-2}{3}$

(C) $\frac{2}{3}$ (D) $\frac{4-\sqrt{7}}{3}$

Q29 The expansion of f(x) = e^x cos x at x = 0.

(A) $1 + x - \frac{2x^3}{3!} + \ldots$ (B) $1 + x - \frac{x^3}{3!} + \ldots$

(C) $1 + x - \frac{x^2}{2!} + \ldots$ (D) $1 + x - \frac{2x^2}{2!} + \ldots$

Q30 The third term in the expansion of $\frac{x-1}{x+1}$ about the point x = 1 using Taylor's series is :

(A) $\frac{(x-1)^2}{2}$ (B) $\frac{(x-1)^2}{4}$

(C) $\frac{(x-1)^3}{8}$ (D) $\frac{(x-1)^3}{4}$

Q31 Find the taylor series expansion of the function cosh (x) centered at x = 0.

(A) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots.\infty$

(B) $\frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} \ldots.\infty$

(C) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots.\infty$

(D)

$1+\frac{x}{1!}+\frac{x^2}{2!}+....\infty$

Q32 Let Mclaurin series of some f(x) be given recursively, where a_n denotes the coefficient of x^n in the expansion. Also given $a_n = a_{n-1}/n$ and $a_0 = 1$, which of the following functions could be f(x)?

(A) e^x

(B) e^{2x}

(C) $c + e^x$

(D) No closed form exists

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Answer Key

Q1	(C)
Q2	(C)
Q3	43
Q4	(B, C)
Q5	(A, C, D)
Q6	$\frac{1}{4}$
Q7	1
Q8	(B)
Q9	(B)
Q10	(C)
Q11	(D)
Q12	0
Q13	0.5~0.5
Q14	(A, B)
Q15	(A, B, C)
Q16	5
Q17	(B)
Q18	(D)
Q19	(A)
Q20	(−1, 1)
Q21	Hence the proof is complete.
Q22	(C)
Q23	Thus, Cauchy's means value theorem is verified for the given functions.
Q24	(B)
Q25	(B)
Q26	Hence prooved
Q27	Yes , using IMVT we can proove.
Q28	(D)
Q29	(A)
Q30	(C)
Q31	(C)
Q32	(A)

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Hints & Solutions

Q1 Text Solution:

Since the domain of sin x= $[-1,\ 1]$

$-1 \leq \frac{x^2-3x+2}{x^2+2x+7} \leq 1$

$\Rightarrow 0 \leq \frac{2x^2-x+9}{x^2+2x+7}\ \&\ \frac{-5x-5}{x^2+2x+7} \leq 0$

$\Rightarrow x \in R\& -1 \leq x < \infty.$

Thus ,$-1 \leq x < \infty$

Q2 Text Solution:

Since, $f(x) = \cos 2x - \sin 2x$

[Since, $f(x) = a\cos x + b\sin x,$

$-\sqrt{a^2+b^2} \leq f(x) \leq \sqrt{a^2+b^2}]$

$-\sqrt{1+1} \leq \cos 2x - \sin 2x \leq \sqrt{1+1}$

$-\sqrt{2} \leq \cos 2x - \sin 2x \leq \sqrt{2}$

So, Range of f(x) is $\left[-\sqrt{2},\ \sqrt{2}\right]$.

Q3 Text Solution:

$f(x) = a\,x + b$

Given-

$x = -2$

$$\begin{aligned} -2a+b &= 29 \\ 3a+b &= 39 \\ - \quad - \quad & \quad - \end{aligned}$$

$-5a = -10$

a =2

$-2 \times 2 + b = 29$

$b = 29 + 4 = 33$

x = 5

$5 \times 2 + 33$

10 + 33 = 43

Q4 Text Solution:

(B) & (C) are odd functions

f(x) = sin x

f (-x) = sin (-x) = - sin x

f (x) = - f (-x)

Similarly

f(x) = sin x + tan x

= g (-x) = - g (x)

Q5 Text Solution:

(A), (C) & (D) are periodic functions

as sinx and cosx are periodic thus their sum is periodic.

Similarly greatest integer and fractional part are periodic.

Q6 Text Solution:

$\lim\limits_{x\to 0} \frac{\sqrt{4+x}-2}{x}$

$= \lim\limits_{x\to 0} \frac{\sqrt{4+x}-2}{x} \cdot \frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$

$= \lim\limits_{x\to 0} \frac{1}{\sqrt{4+x}+2} = \frac{1}{4}.$

Q7 Text Solution:

$\lim\limits_{x\to -1} \frac{(x+2)(3x-1)}{x^2+3x-2}$

$= \frac{\lim\limits_{x\to -1}(x+2)\ \lim\limits_{x\to -1}(3x-1)}{\lim\limits_{x\to -1}(x^2+3x-2)} = \frac{1.(-4)}{-4} = 1$

Q8 Text Solution:

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1}\left(x^3-1\right) = 0$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1}(x-1) = 0$

Also, f (1) = 0 $\Rightarrow$f is continous.

$f'(x) = \begin{cases} 3x^2, 1 < x < \infty \\ 1, -\infty < x \leqslant 1 \end{cases}$

$f'(1^+) = 3, f'(1^-) = 1$

$\Rightarrow$f is not differentiable.

Q9 Text Solution:

We have $f(x) = x\left(\sqrt{x} - \sqrt{x+1}\right)$

Let us check differentiablity of f(x) at x = 0.

$Lf'(0) = \lim\limits_{h\to 0} \frac{(0-h)[\sqrt{0-h}-\sqrt{0-h+1}]-0}{-h}$

$= \lim\limits_{h\to 0} \frac{[\sqrt{-h}-\sqrt{-h+1}]}{1}$

Rf' (0) = $= \lim\limits_{h\to 0} \frac{(0+h)[\sqrt{0+h}-\sqrt{0+h+1}]-0}{h}$

$= \lim\limits_{h\to 0} \sqrt{h} - \sqrt{h+1} = -1$

Since Lf' (0) = Rf' (0)

$\therefore$ f(x) is differentiable at x = 0

Q10 Text Solution:

$\lim_{x\to\infty}\left(\sqrt{x^2-x+1} - ax\right) = b$

$(\infty - \infty \text{ form})$

$\Rightarrow a > 0$

$\lim_{x\to\infty}\left(\frac{x^2-x+1-a^2x^2}{\sqrt{x^2-x+1}+ax}\right) = b$

$\lim_{x\to\infty} \frac{x^2(1-a^2)-x+1}{\sqrt{x^2-x+1}+ax} = b$

For existence of limit , $1-a^2 = 0$ i.e. a = 1 only

$[\because a > 0]$

$\lim_{x\to\infty} \frac{1-x}{\sqrt{x^2-x+1}+x} = b$

$\lim_{x\to\infty} \frac{\frac{1}{x}-1}{\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+1} = b$

$\Rightarrow b = \frac{-1}{2}$

So,$(a, b) = \left(1, -\frac{1}{2}\right)$

Q11 Text Solution:

$\lim_{x\to 0} \frac{x^3+x^2}{2x^2-7x^2}$

$\lim_{x\to 0} \frac{x^2(x+1)}{x^2(2-7)}$

$= \frac{1}{2-7} = \frac{1}{-5} = -\frac{1}{5}$

Q12 Text Solution:

$\underset{x\to 0}{Lt} \frac{x-\sin x}{1-\cos x}$

using L - Hospital Rule

If $x \to 0 \left\{\frac{1-\cos x}{\sin x}\right\}$

again using L- Hospital Rule

$\lim_{x\to 0} \frac{\sin x}{\cos x} = 0$

Q13 Text Solution:

$\underset{x\to 0}{\mathrm{Lt}} \left\{\frac{e^{2x}-1}{\sin(4x)}\right\}$

L-Hospital Rule

$\underset{x\to 0}{\mathrm{Lt}} \frac{e^{2x}.2}{\cos 4x\, 4}$

$\frac{1.2}{1.4} = \frac{1}{2} \to 0.5$

Q14 Text Solution:

(B) $\lim_{x\to 0} \frac{\sin x}{x}$

Using L-Hospital Rule

$\lim_{x\to 0} \frac{\cos x}{1} = 1$

(A) $\frac{\sin x}{x} < 1$

With the help of graph u can easily see that sinx<x.

Q15 Text Solution:

Continuity of ***f*** (***x***)

For x = 1, ***f*** (***x***) is a polynomial and hence is continuous.

At ***x*** = 1.

LHL $= \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{x^2}{2} = \frac{1}{2}$

RHL $= \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \left(2x^2 - 3x + \frac{3}{2}\right)$

$= 2 - 3 + \frac{3}{2} = \frac{1}{2}$

$f(1) = 2(1)^2 - 3(1) + \frac{3}{2} = \frac{1}{2}$

⇒L.H.L = R.H.L = ***f*** (1)

Therefore, ***f*** (***x***) is continuous at x = 1.

Continuity of f'(***x***)

Let ***g*** (***x***) = f'(***x***)

$\Rightarrow g(x)$

$= \begin{cases} x & ; \quad 0 \le x < 1 \\ 4x - 3 & ; \quad 1 \le x < 2 \end{cases}$

For x = 1, g (x) is linear polynomial and hence continuous.

At x = 1,

LHL = $\lim_{x\to 1^-} g(x) = \lim_{x\to 1-} x = 1$

RHL = $\lim_{x\to 1^+} g(x) = \lim_{x\to 1^+} (4x - 3) = 1$

g (1) = 4 − 3 = 1

⇒LHL = RHL = ***g*** (1)

g (x) = f' (x) is continuous at x = 1.

Continuity of $f''(x)$

Let $h\left(x\right) = f''(x)$

$= \begin{cases} 1 & ; \quad 0 \le x < 1 \\ 4 & ; \quad 1 \le x \le 2 \end{cases}$

For x $\neq$1, ***h*** (***x***) is continuous because it is a constant function.

At x = 1,

LHL $= \lim_{x\to 1^-} h(x) = 1$

RHL $= \lim_{x\to 1^+} h(x) = 4$

Thus LHL = RHL

h(***x***) is discontinuos at x = 1.

Hence ***f*** (***x***) and ***f'*** (***x***) are continuous on [0.2] but ***f*** '' (***x***) is discontinuous at ***x*** = 1.

Note : Continuity of f ' (x) is same as differentiablity of f (x).

Q16 Text Solution:

$\lim_{x\to 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$

Apply L Hospital Rule and solving we get-

GATE

Denominator needs to be zero

$\alpha = 1$

Apply L Hospital rule again to the

Apply again them

2β+2β+2β=-1

[only writing terms not containing x and sin (βx)]

β=-1/6

6(α+β)=6×5/6=5

A is correct

Q17 Text Solution:

Given $f(x) = 1 - x^2 + x^3; [-1, 1]$

By mean value theorem of f (x) in the interval [a, b]

$f'(c) = \frac{f(b)-f(a)}{b-a}$

for f(x) = $1 - x^2 + x^3$

$\Rightarrow f'(x) = 3x^2 - 2x$

⇒ By mean value theorem

$f'(c) = \frac{f(1)-f(-1)}{1-(-1)}$

$\Rightarrow 3c^2 - 2c = \frac{1-(-1)}{1-(-1)}$

$\Rightarrow 3c^2 - 2c - 1 = 0$

$\Rightarrow 3c^2 - 3c + c - 1 = 0$

$\Rightarrow 3c(c-1) + 1(c-1) = 0 \Rightarrow c = \frac{-1}{3}$ and

$c = 1$

Since $C \in (-1, 1)$, the mean value 'c' is equal to $\frac{-1}{3}$.

Q18 Text Solution:

As f(x) is polynomial so it will be continuous and differentiable in [0, 1]

$f(x) = x^3 - 4x^2 + 8x + 11$

$f(0) = 11, f(1) = 1 - 4 + 8 + 11 = 16$

$f'(x) = 3x^2 - 8x + 8$

if $c \in (0, 1)$

then $f'(c) = 3c^2 - 8c + 8$ .....(i)

Apply L.M.V.T

$f'(c) = \frac{f(1)-f(0)}{1-0} = f(1) - f(0)$

$= 16 - 11 = 5$..... (ii)

From equations (i) & (ii)

$3c^2 - 8c + 8 = 5$

$3c^2 - 8c + 3 = 0$

$\Rightarrow c = \frac{4-\sqrt{7}}{3} \leftarrow (0, 1)$ verified.

Q19 Text Solution:

We have the sine function that takes the value of zero at integral multiples of π.

But for $\frac{\sin(x)}{x}$ we have the exceptional value of $\lim_{x\to 0} \frac{\sin(x)}{x}$ reaching one.

So, leaving the first interval [0,π] , for every other interval of the form [nπ ,(n+1)π] we must have f(nπ) = f((n+1)π) by rolles theorem we have f'(c) = 0 for every interval of the from [nπ,(n+1) π] .There are 17 such intervals.

Q20 Text Solution:

Let y = f(x) = $(x+2)^2$

Here, f is a polynomial function. Hence, f is continuous in [−2, 0].

Also differentiable in (−2,0) and f'(x) = 2(x+2).

So, by Lagrange's mean value theorem, we get a, c ∈ (−2,0) such that

$f'(c) = \frac{f(0)-f(-2)}{0-(-2)}$

$or\ 2(c+2) = \frac{4-0}{2} = 2 \Rightarrow c = -1.$

and at C = −1, f(c) =1

Hence, required point = (c, f (c)) = (−1, 1)

Q21 Text Solution:

Thus can be prooved by using Rolles theorem, considering a=1,b=2.

Q22 Text Solution:

It wont be applicable as the derivative of g(x) at x=pi/4 is coming out to be 0.

Q23 Text Solution:

Here , f and g are both continuous in [a, b] .Now, $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$ and $g'(x) = -\frac{1}{2}x^{-\frac{3}{2}}$ exist for all x> 0. Hence, f and g are both differentiable on (a, b) and also g'(x)$\neq 0$ for x ∈ (a, b). Therefore, Cauchy's means value theorem is applicable for both the given functions in [a, b].

Now, $\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$

given, $\frac{\sqrt{b}-\sqrt{a}}{\frac{1}{\sqrt{b}}-\frac{1}{\sqrt{a}}} = \frac{\frac{1}{2}c^{-\frac{1}{2}}}{-\frac{1}{2}c^{-\frac{3}{2}}}$

i.e., $-\sqrt{ab} = -c\ i.e.,\ c = \sqrt{ab}$.

Here, c > a and c < b.

Thus, Cauchy's means value theorem is verified for the given functions.

Q24 Text Solution:

$$According\ to\ CMVT,$$

$$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$$

$$\frac{e^b-e^a}{e^{-b}-e^{-a}} = -\frac{e^c}{e^{-c}}$$

$$thus\ c = \frac{a+b}{2}$$

Q25 Text Solution:

Cauchy's mean value theorem is applicable only for two functions, lets say f(x) and g(x) defined on the interval [a,b].

Q26 Text Solution:

Statement 1:

If k is a value between f(a) and f(b), i.e.

either f(a) < k < f(b) or f(a) > k > f(b)

then there exists at least a number c within a to b i.e. $c \in (a, b)$ in such a way that f(c) = k

Statement 2:

The set of images of function in interval [a, b], containing [f(a), f(b)] or [f(b), f(a)], i.e.

either $f([a, b]) \supseteq [f(a), f(b)]$ or $f([a, b]) \supseteq [f(b), f(a)]$

Q27 Text Solution:

Let us find the values of the given function at the x = 0 and x = 2.

$f(x) = x^5 - 2x^3 - 2 = 0$

Substitute x = 0 in the given function

$f(0) = (0)^5 - 2(0)^3 - 2$

$f(0) = -2$

Substitute x = 2 in the given function

$f(2) = (2)^5 - 2(2)^3 - 2$

$f(2) = 36 - 16 - 2$

$f(2) = 14$

Therefore, we conclude that at x = 0, then curve is below zero; while at x = 2 it is above zero.

Since the given equation is a polynomial, its graph will be continuous.

Thus, applying the intermediate value theorem, we can say that the graph must cross at same point between (0,2).

Hence, there exists a solution to the equation $x^5 - 2x^3 - 2 = 0$ between the interval [0,2].

Q28 Text Solution:

As f(x) is polynomial so it will be continuous and differentiable in [0,1]

$f(x) = x^3 - 4x^2 + 8x + 11$

$f(0) = 11$, $f(1) = 1 - 4 + 8 + 11 = 16$

$f'(x) = 3x^2 - 8x + 8$

if $c \in (0,1)$

then $f'(c) = 3c^2 - 8c + 8$ ....(i)

Apply L.M.V.T

$$f'(c) = \frac{f(1)-f(0)}{1-0} = f(1) - f(0)$$

= 16 – 11 = 5 ....(ii)

from equation (i) & (ii)

$3c^2 - 8c + 8 = 5$

$3c^2 - 8c + 3 = 0$

$$\Rightarrow c = \frac{4-\sqrt{7}}{3} \leftarrow (0,1)\text{verified}$$

Q29 Text Solution:

$$\Rightarrow f'(x) = e^x(-\sin x) + \cos x \cdot e^x$$

$$\Rightarrow f'(x) = f(x) - e^x \cdot \sin x$$

$$\Rightarrow f''(x) = f'(x) - e^x \cdot x - e^x \sin x$$

$$\Rightarrow f''(x) = f'(x) - f(x) - e^x \sin x$$

$$\Rightarrow f'''(x) = f''(x) - f'(x) - e^x \cos x - e^x \sin x$$

$$\Rightarrow f'''(x) = f''(x) - f'(x) - f(x) - e^x \sin x$$

Now,

$$f'(0) = 1 - 0 = 1$$

$$f''(0) = f'(0) - e^0(1) - 0 = 1 - 1 = 0$$

$$f'''(0) = f''(0)f'(0) - 1 - 0 = 1 - 1 - 1 = -2$$

Taylor series expansion at x = 0 is :

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f'(0) + \frac{x^3}{3!}f'(0)$$

$$f(x) = 1 + x - \frac{2x^3}{3!} + \ldots$$

Q30 Text Solution:

Given complex function is $(x-1)/(x+1)$;

To expand about the point x = 1, let us assume $t = x - 1$;

Now the function will be

$$f(x) = \frac{x-1}{x+1} = \frac{t}{t+2} = 1 - \frac{1}{\frac{t}{2}+1} = 1 - \left(1 + \frac{t}{2}\right)^{-1}$$

Using standard Taylor's series expansion,

$$f(x) = 1 - \left[1 - \frac{t}{2} + \frac{t^2}{2^2} - \frac{t^3}{2^3}\ldots\right]$$

$$f(x) = \frac{t}{2} - \frac{t^2}{2^2} + \frac{t^3}{2^3}\ldots$$

The third term in the expansion is $\frac{t^3}{8} = \frac{(x-1)^3}{8}$

Q31 Text Solution:

We know the general expression for the expansion of the taylor series

$$\tau[f(x)] = f(a) + \frac{x\cdot f^{(1)}(a)}{1!} + \frac{x^2\cdot f^{(2)}(a)}{2!} + \ldots\infty$$

Given a = 0 we substitute in the equation to get

$$\tau[f(x)] = f(0) + f^{(1)}(0) \times \frac{x}{1!} + f^{(2)}(0) \times \frac{x^2}{2!}\ldots\infty$$

Now the n^{th} derivatives can be calculated as

$$f^{(n)}(x) = \left(\frac{e^x + e^{-x}}{2}\right)^{(n)} = \frac{e^x + (-1)^n e^x}{2}$$

Substituting x = 0 yields the final expansion

$$f^{(n)}(x) = \frac{1 + (-1)^n}{2}$$

We get

$$\tau[f(x)] = 1 + (0) \times \frac{x}{1!} + (1) \times \frac{x^2}{2!} + (0) \times \frac{x^3}{3!} + \ldots.\infty$$

Q32 Text Solution:

Observing the recurrence relation we have

$$a_n = \frac{a_{n-1}}{n} = \frac{a_{n-2}}{n(n-1)}$$

$$a_n = \frac{a_0}{n(n-1)(n-2)\ldots.3\times2\times1}$$

Thus, one could deduce that

$$a_n = \frac{1}{n!}$$

Putting this into the Mclaurin expansion we have

$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \ldots..\infty$

$$f(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots.\infty$$

Which is the well know expansion of e^x.

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