COMPUTER SCIENCE & IT

DIGITAL LOGIC

Lecture No. 07

BOOLEAN THEOREMS AND GATES

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NAND & NOR gate

Concept of Duality & Imp basics

Concept of Duality:

How to find out dual of a given boolean function:

→ by replacing 'OR' by 'AND' & 'AND' by OR while NOT remain as it is. And function we get after doing this is called as dual function.

eg. $f(A,B,C) = AB + BC = \Sigma(3,6,7)$

$f^D(A,B,C) = (A+B)\cdot(B+C) = B + AC$

$[f^D(A,B,C)]^D = A\cdot B + B\cdot C$

$f^D \neq f$ $\quad f \rightarrow$ is not a self dual boolean function.

$$f \underset{\text{Dual}}{\overset{\text{Dual}}{\rightleftarrows}} f^D \xrightarrow{\text{Dual}} f$$

$$\boxed{[f^D]^D = f}$$

- $f(A,B,C) = AB + BC + AC$

$f^D(A,B,C) = (A+B)\cdot(B+C)\cdot(A+C) = [B+AC][A+C]$

$= AB + BC + AC + AC = AB+BC+AC$

$f^D = f$

$f \longrightarrow$ Self dual boolean function

- $f_1(A,B) = \Sigma(1,2) = \pi(0,3)$

$= \bar{A}B + A\bar{B} = A \oplus B$

$f_1^D(A,B) = (\bar{A}+B)\cdot(A+\bar{B}) = A \odot B$

$f_1^D \neq f_1 \longrightarrow f_1$ is not a self dual boolean function.

$(0,3)\,(1,2)$

• $f_2(A,B) = \bar{A}\bar{B} + A\bar{B} = \Sigma(0,2) = \Pi(1,3)$

$= \bar{B}$

$f_2^D(A,B) = (\bar{A}+\bar{B})(A+\bar{B}) = \Pi(1,3) = \Sigma(0,2)$

1 1 0 1

$= \bar{B} + \bar{A}\cdot A = \bar{B}$

$f_2^D = f_2 \longrightarrow f_2 \rightarrow$ is a self dual boolean function.

$(0,3)\ (1,2)$

• $\overline{f_2} = \Sigma(1,3) = \Pi(0,2)$

• Note: if f is self dual boolean function then f will definitely be a self dual boolean function.

- $f(A,B,C) = AB + BC + AC = \Sigma(3,5,6,7) = \Pi(0,1,2,4)$

AB	BC	A C
$1\ 1\ 0 \rightarrow 6$	$0\ 1\ 1 \rightarrow 3$	$1\ 0\ 1 \rightarrow 5$
$1\ 1\ 1 \rightarrow 7$	$1\ 1\ 1 \rightarrow 7$	$1\ 1\ 1 \rightarrow 7$

$f(A,B,C) = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

$$f^D(A,B,C) = \underset{1\ 0\ 0}{(\bar{A}+B+C)} \cdot \underset{0\ 1\ 0}{(A+\bar{B}+C)}\ \underset{0\ 0\ 1}{(A+B+\bar{C})}\ \underset{0\ 0\ 0}{(A+B+C)}$$

$$= \Pi(0,1,2,4) = \Sigma(3,5,6,7) = f \longrightarrow f \rightarrow \text{is self dual}$$

$(0,7)\ (1,6),\ (2,5),\ (3,4)$ ✓

- $f(A,B,C,D) = \Sigma(1,2,3,8,9,10,11,15) \longrightarrow$ self dual.

- $f(A,B,C) = A\overline{B} + AC + \overline{B}C \quad = \Sigma(1,4,5,7) \longrightarrow$ self dual.

$A\overline{B}$

$1\ 0\ 0 \rightarrow 4$

$1\ 0\ 1 \rightarrow 5$

$A\ C$

$1\ 0\ 1 \rightarrow 5$

$1\ 1\ 1 \rightarrow 7$

$\overline{B}\ C$

$0\ 0\ 1 \rightarrow 1$

$1\ 0\ 1 \rightarrow 5$

- $f(A,B,C) = \overline{A}\,\overline{B} + BC = \Sigma(0,1,3,7) \longrightarrow$ Non-self dual

$\overline{A}\,\overline{B}$

$0\ 0\ 0 \rightarrow 0$

$0\ 0\ 1 \rightarrow 1$

$B\ C$

$0\ 1\ 1 \rightarrow 3$

$1\ 1\ 1 \rightarrow 7$

$$\text{AND} \underset{\text{Dual}}{\overset{\text{Dual}}{\rightleftarrows}} \text{OR}$$

$$A \cdot B \qquad (A+B)$$

$$\text{XOR} \underset{\text{Dual}}{\overset{\text{Dual}}{\rightleftarrows}} \text{XNOR}$$

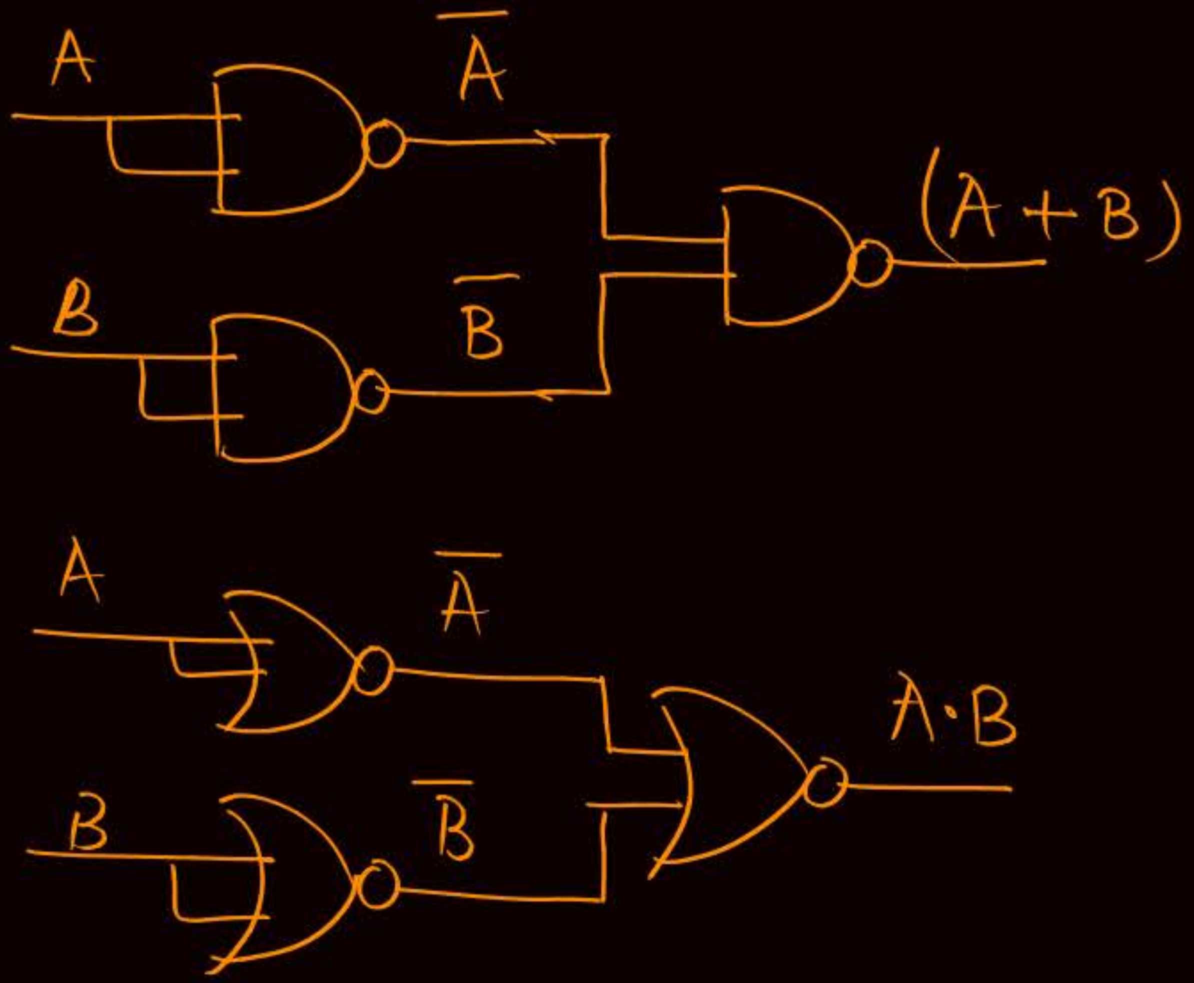
$$\bar{A}B + A\bar{B} \qquad (\bar{A}+B)(A+\bar{B})$$

$$\text{NAND} \underset{\text{Dual}}{\overset{\text{Dual}}{\rightleftarrows}} \text{NOR}$$

$$\overline{A \cdot B} \qquad \overline{A+B}$$

- $f(A,B,C) = \sum(1,2,4,7) = A \oplus B \oplus C \longrightarrow$ self dual

  $f^D = A \odot B \odot C = A \oplus B \oplus C = f \longrightarrow$ self dual.


A
$\overline{A}$
B
$\overline{B}$
$(A+B)$
A
$\overline{A}$
B
$\overline{B}$
$A \cdot B$

- n-variables $\longrightarrow$ $N = 2^n$ Comb (terms) $\quad 0 — (2^n - 1)$

$$N_B = {}^N C_0 + {}^N C_1 + {}^N C_2 + {}^N C_3 + {}^N C_4 + \cdots {}^N C_N$$

$$N_B = 2^N = 2^{2^n}$$

Self dual $\longleftarrow$ $S_D = 2^{2^{(n-1)}}$

Non self dual $\longleftarrow$ $NS_D = 2^{2^n} - S_D$

- 3-Variables $\longrightarrow$ $N = 2^3$ (Comb) Or Term $\longrightarrow$ $N_B = 2^8 = 256$

$S_D = 2^{2^2} = 16$

$NS_D = 240$

4-variables $\longrightarrow$ $N = 2^4 = 16$ (terms)

$$N_B = 2^{16} = 256 \times 256$$

$$S_D = 2^{2^3} = 2^8 = 256$$

$$NS_D = 256 \times 256 - 256 = 256 \times 255$$

- $f(A,B) = \Sigma(\text{Nil}) = \Pi(0,1,2,3)$

$$= 0 = (A+B)(A+\overline{B})(\overline{A}+B)(\overline{A}+\overline{B})$$

$$= [A + B\cdot\overline{B}]\cdot[\overline{A} + B\cdot\overline{B}]$$

$$= A\cdot\overline{A} = 0$$

- $f(A,B) = \Sigma(0,1,2,3) = \Pi(\text{Nil})$

$$= 1$$

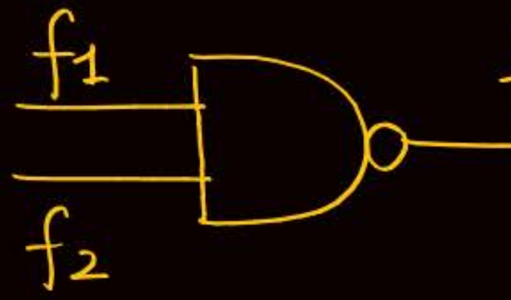
$$= \overline{A}\,\overline{B} + \overline{A}B + A\overline{B} + AB$$

$$= \overline{A}(\overline{B}+B) + A(\overline{B}+B)$$

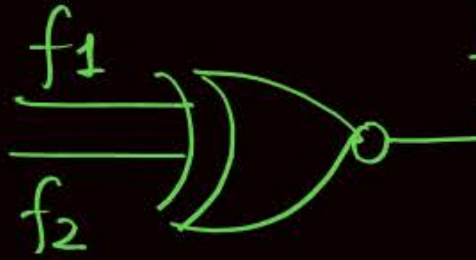
$$= \overline{A} + A = 1$$

Q. $f_1(A,B,C) = \Sigma(0,2,3,6,7) = \Pi(1,4,5)$

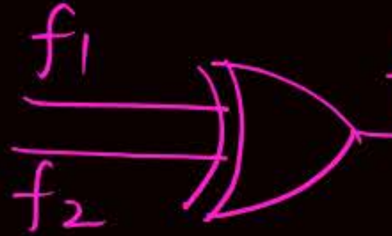
$f_2(A,B,C) = \Sigma(0,1,4,6)$

- 

$f_3(A,B,C) = \Sigma(1,2,3,4,5,7) = \Pi(0,6)$

- 

$f_4(A,B,C) = \Sigma(0,5,6) = \Pi(1,2,3,4,7)$

- 

$f_5(A,B,C) = \Sigma(1,2,3,4,7) = \Pi(0,5,6)$

- 

$f_6(A,B,C) = \Sigma(5)$

H.W.

Q.1. $f(A, B, C) = \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{C}$ $\longrightarrow$ self dual or NOT

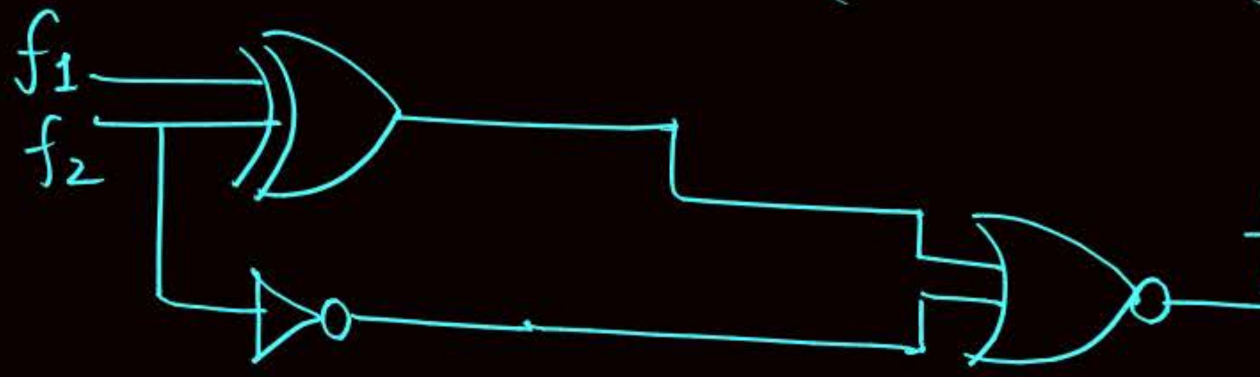
Q.2. $f(A, B, C) = \overline{\bar{A}B + \bar{A}C + BC}$ $\longrightarrow$ self dual or NOT

Q.3 $f(A, B, C) = (A + \bar{C})(B + \bar{C})(\bar{A} + C)$ $\longrightarrow$ self dual or NOT

Q.4. $f(A, B, C, D) = (A + C)(\bar{B} + \bar{D})$ $\longrightarrow$ self dual or NOT

Q.5. $f_1(A, B, C) = \prod(0, 2, 4, 6)$

$f_2(A, B, C) = \Sigma(0, 1, 2, 3, 5, 6)$



$f_3(A, B, C) = \Sigma\ ?$

2 Minute Summary

- Duality
- Imp concepts

Thank you

GW Soldiers !