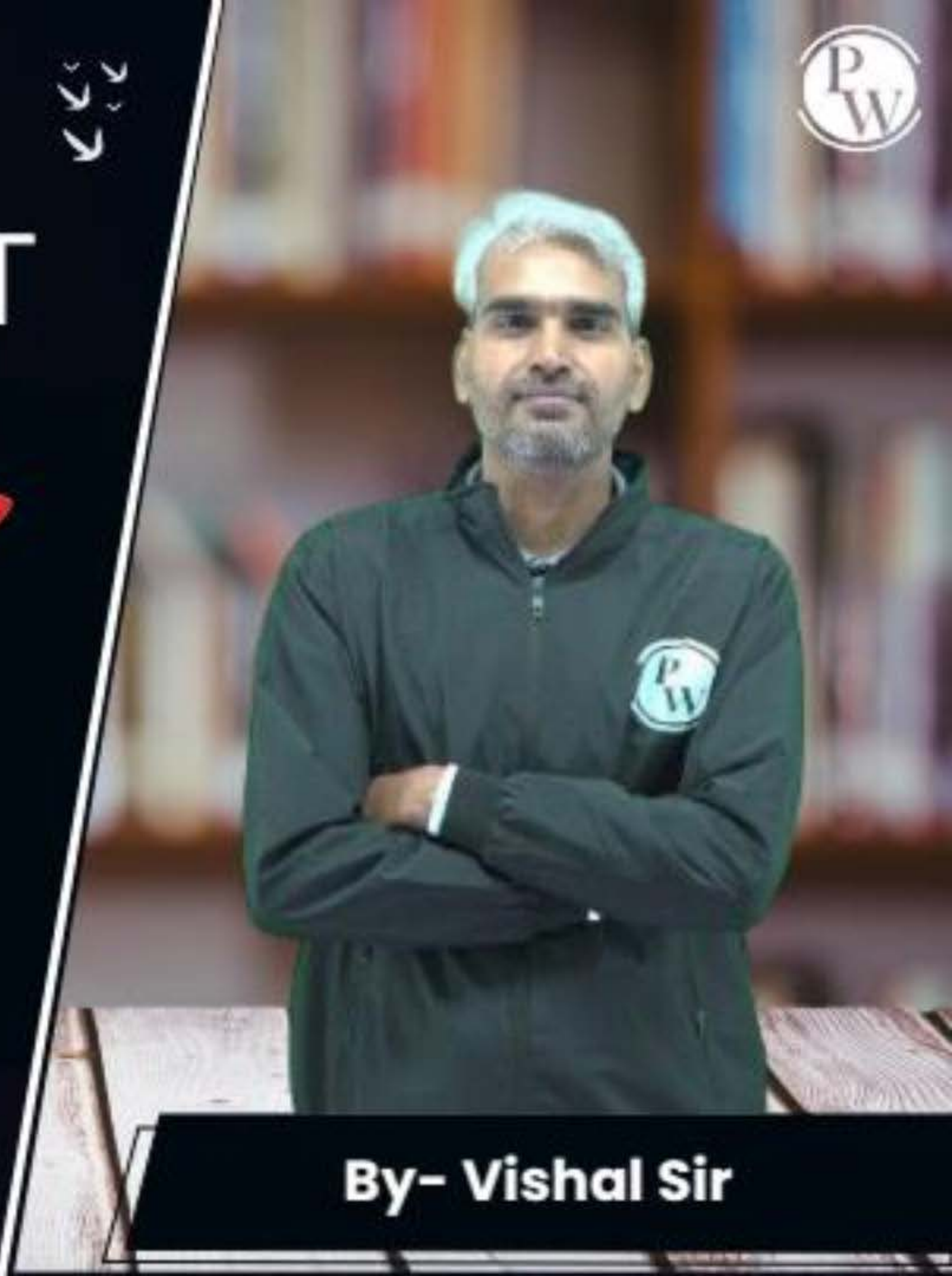
Computer Science & IT

Discrete Mathematics

Graph Theory

Lecture No. 06



By- Vishal Sir

Recap of Previous Lecture

- **Topic** Graph isomorphism
- **Topic** Self-complementary graph
- **Topic** Planar graphs

Slide

Topics to be Covered

- **Topic** Simple connected planar graph
- **Topic** Euler's equation for connected planar graph
- **Topic** Polyhedral graph
- **Topic** Euler's equation for disconnected planar graph
- **Topic** Vertex coloring

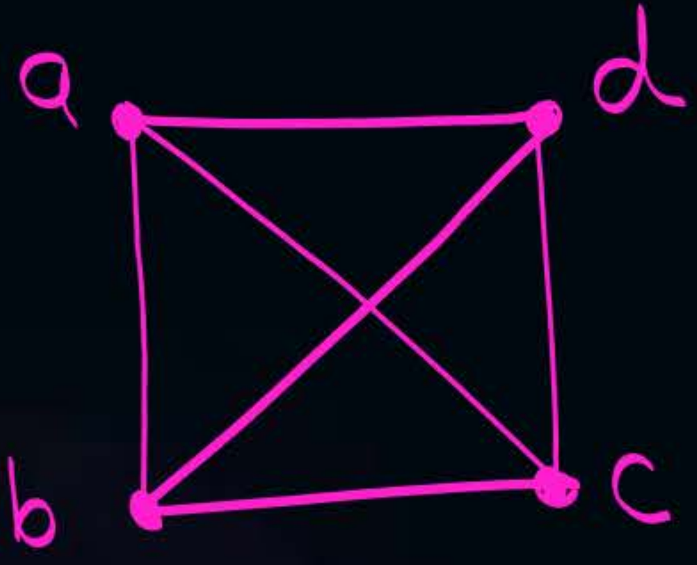
Slide

Topic : Planar graph

A graph G= (V, E) is said to be planar if it can be drawn in the plane so that no two edges of G intersect each other at a non-vertex point.

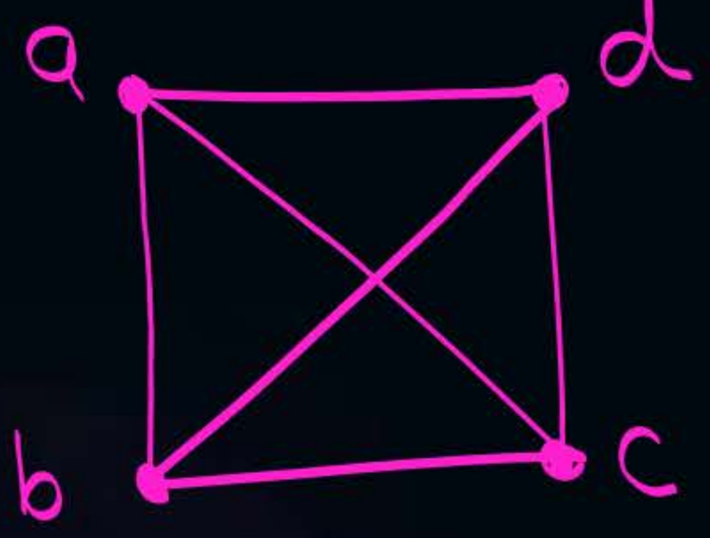
Such a drawing of a planar graph is called a planar embedding of the graph.

Topic : Planar graph



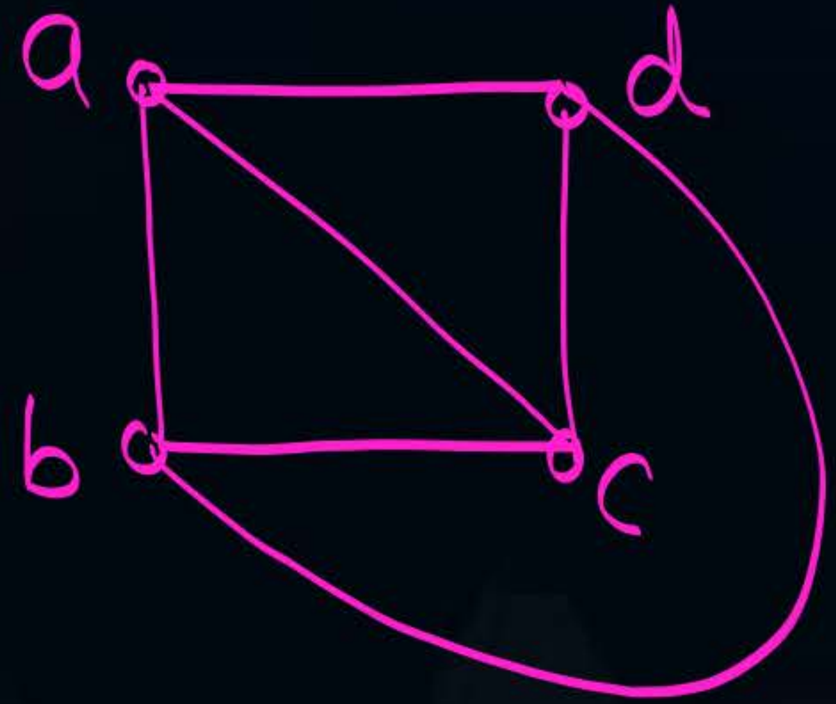
K_4

Topic : Planar graph



K_4

$\Longrightarrow$

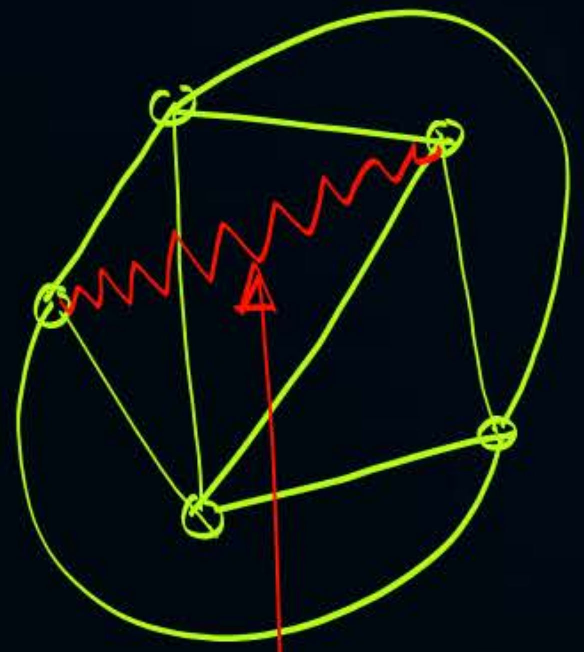


We can draw K_4 on a plane s.t. no two edges cross each other at a non-vertex point.

$\therefore K_4$ is planar

Topic : Planar graph

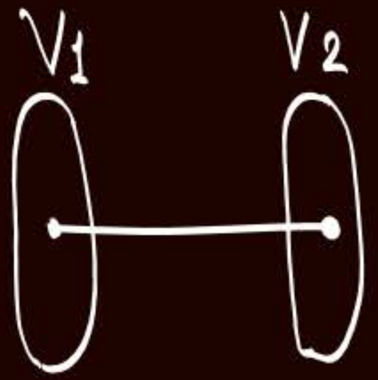
K_1 K_2 K_3 K_4



K_5 is non-planar

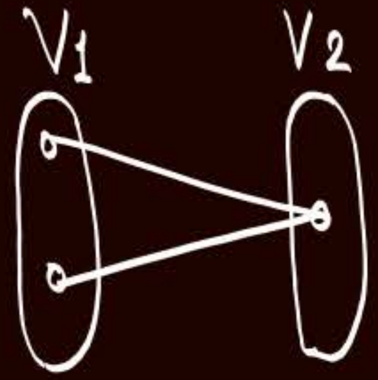
Complete graph K_n is planar if and only if $n \le 4$

- K_5 is a non-planar graph with 5 vertices & 10 edges



$K_{1,1}$

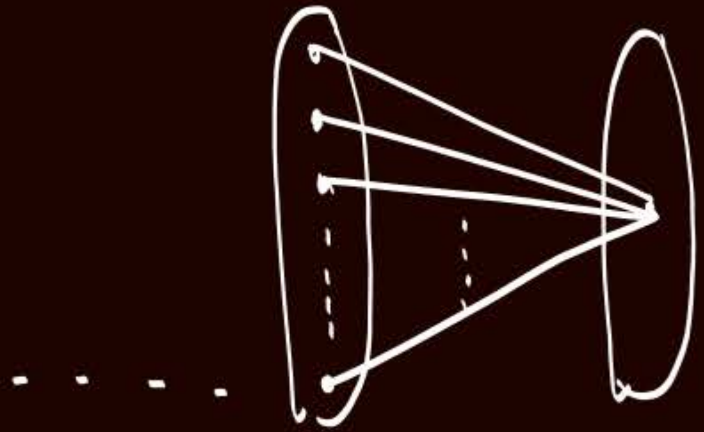
is planar



$K_{2,1}$

is planar

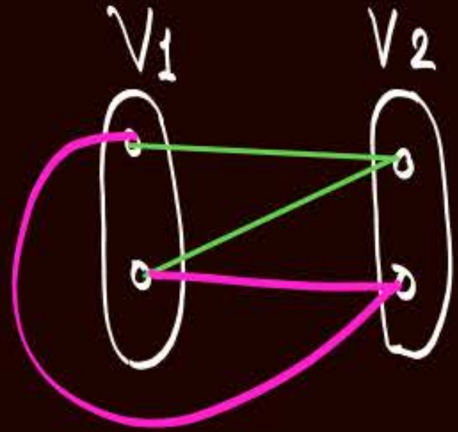
$\therefore K_{1,2}$ is also planar



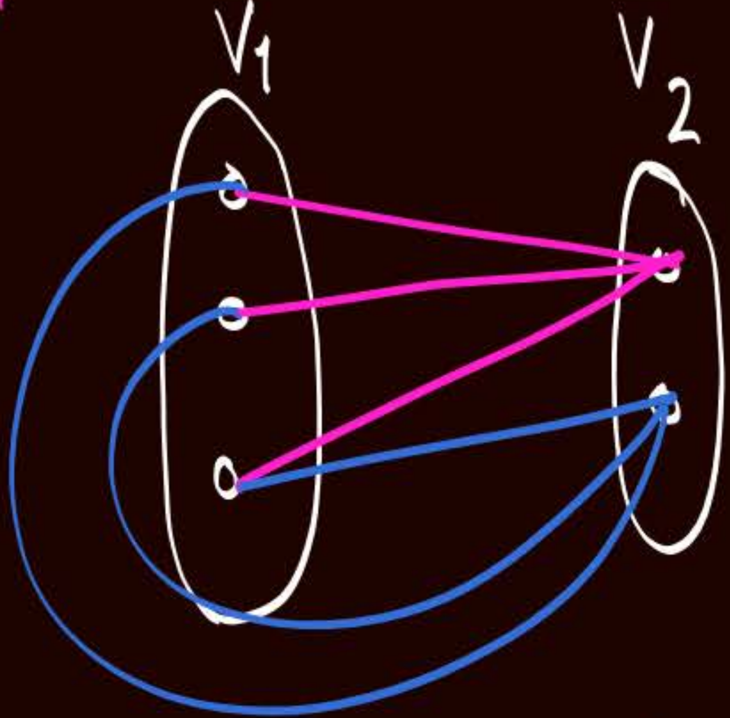
$K_{n,1}$

is planar

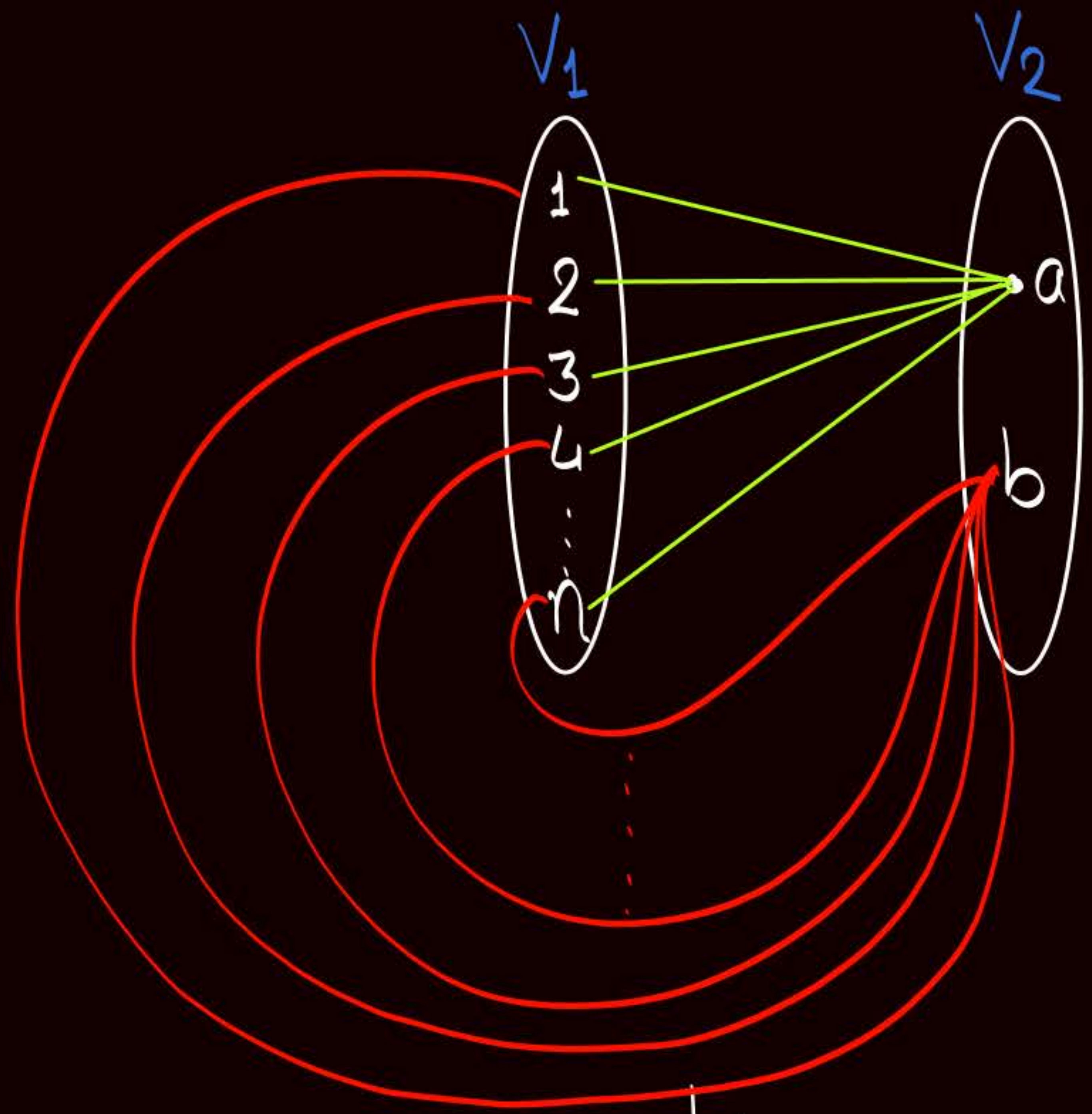
$\therefore K_{1,n}$ is also planar



$K_{2,2}$ is planar



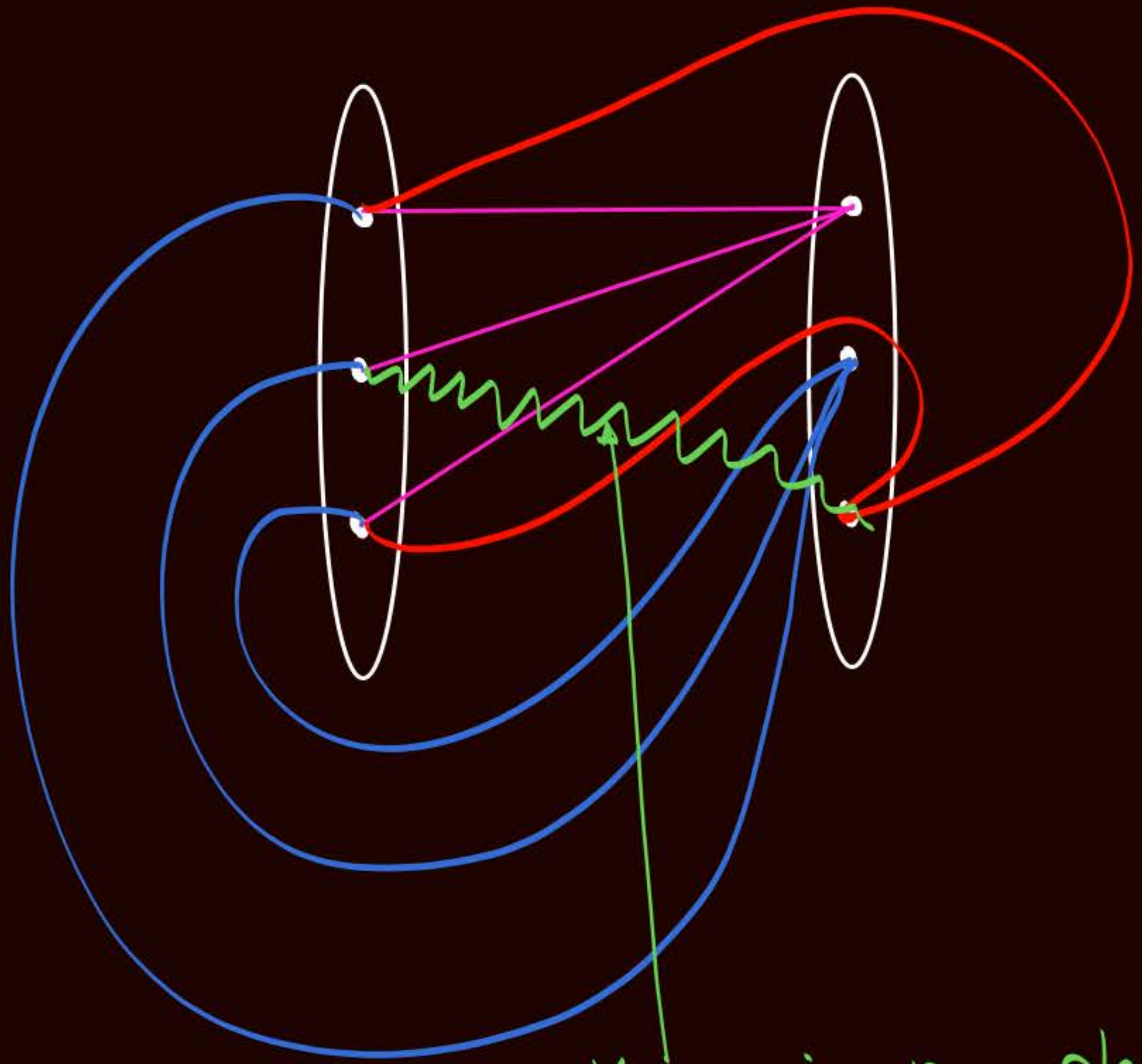
$K_{3,2}$ is planar



$K_{n,2}$ is planar

$\therefore K_{2,n}$ is also planar

→ Check whether $K_{3,3}$ is a planar graph or not?



$K_{3,3}$ is non-planar

$K_{3,3}$ is a non-planar graph with '6' vertices but only '9' edges

Note: Complete bi-partite graph $K_{m,n}$ is planar if and only if $m \leq 2$ or $n \leq 2$

Note :-

- ① K_5 is the non-planar graph with minimum number of vertices.

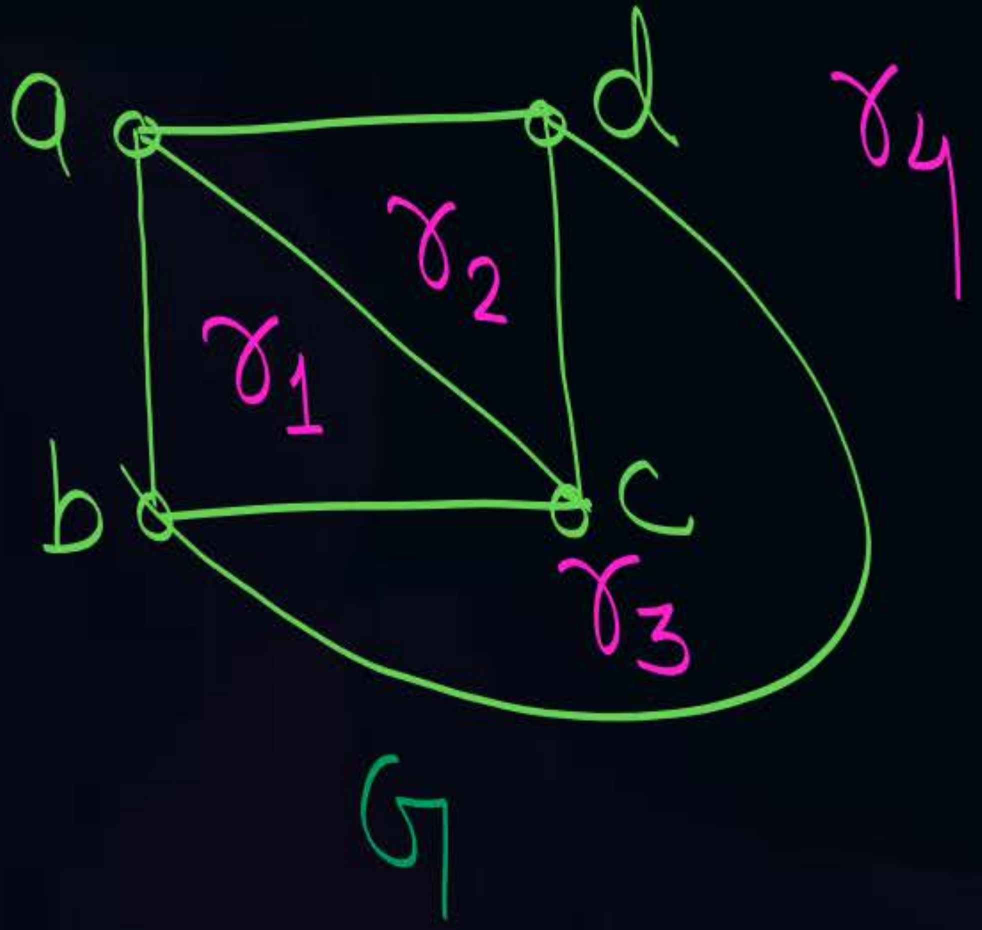
  it has only 5 vertices but 10 edges

- ② $K_{3,3}$ is a non-planar graph with minimum number of edges.

  it has only 9 edges, but 6 vertices

Topic : Region / Face

- Every planar graph divides a plane into connected areas called as regions of the plane or faces of the planar graph.



* r_1, r_2 and r_3 are bouded regions (interior regions) of planar graph G

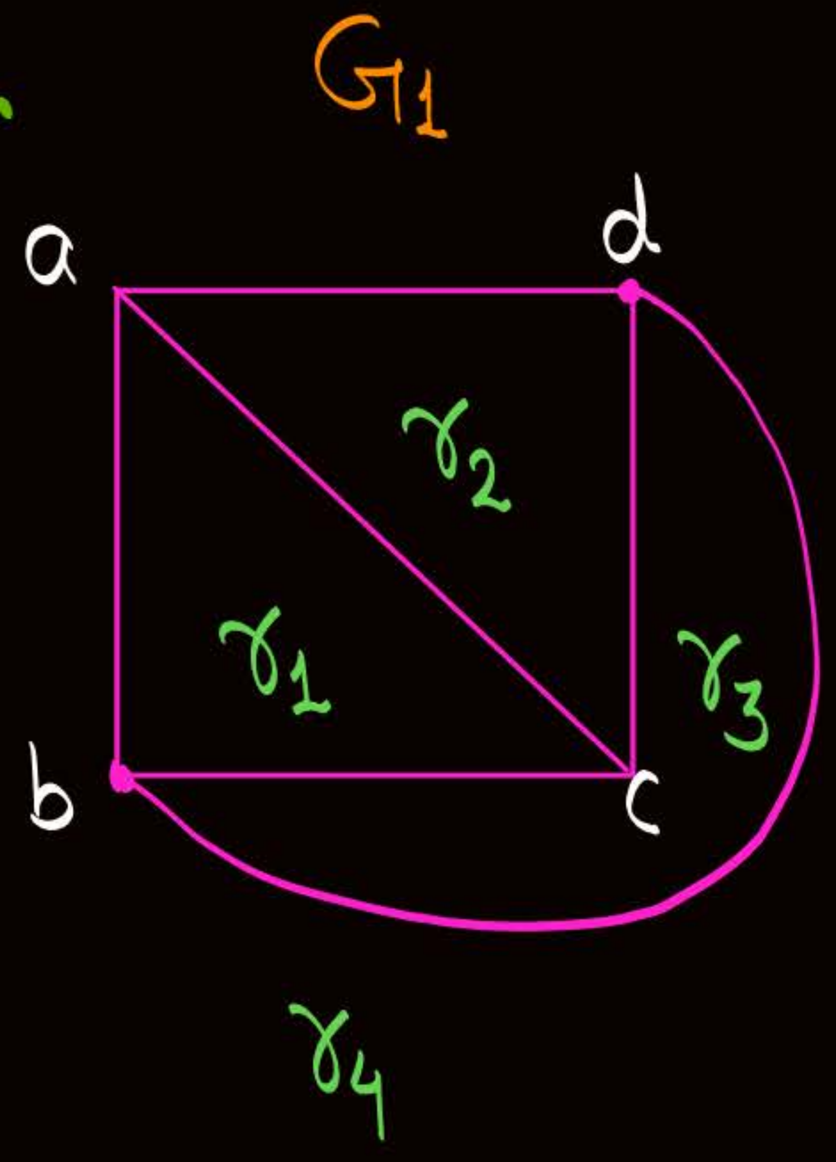
Whereas r_4 is an Unbounded (Exterior) region of Planar graph G.

Topic : Region / Face

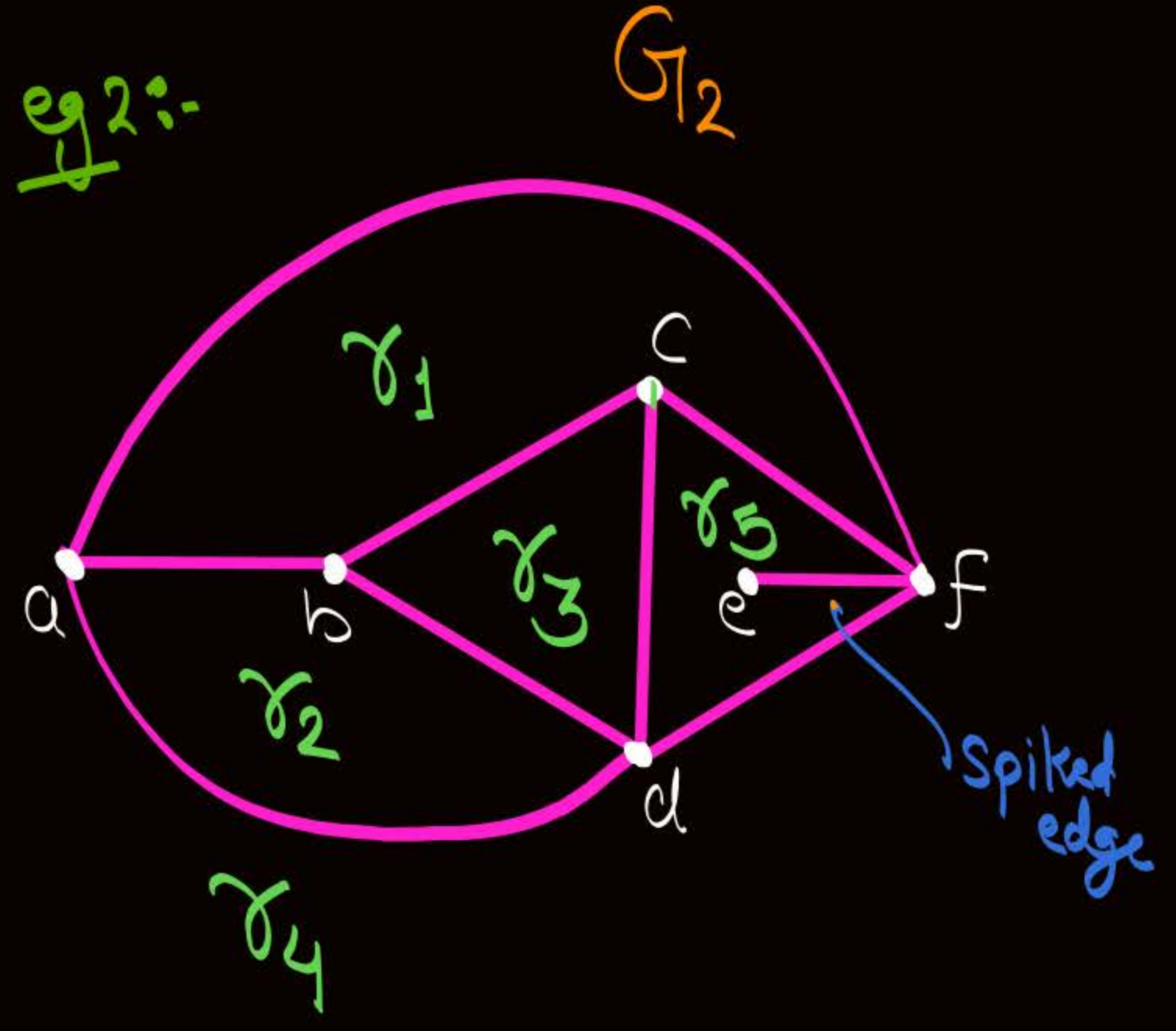
- In any planar graph there will be exactly '1' unbounded (Exterior) region, all other regions of the planar graph will be bounded (interior) regions.

+ $$\boxed{\text{Number of bounded regions (faces) in a planar graph} = \text{Total no. of regions in that planar graph} - 1}$$

eg 1:



eg 2:-



Topic : Degree of a region/face

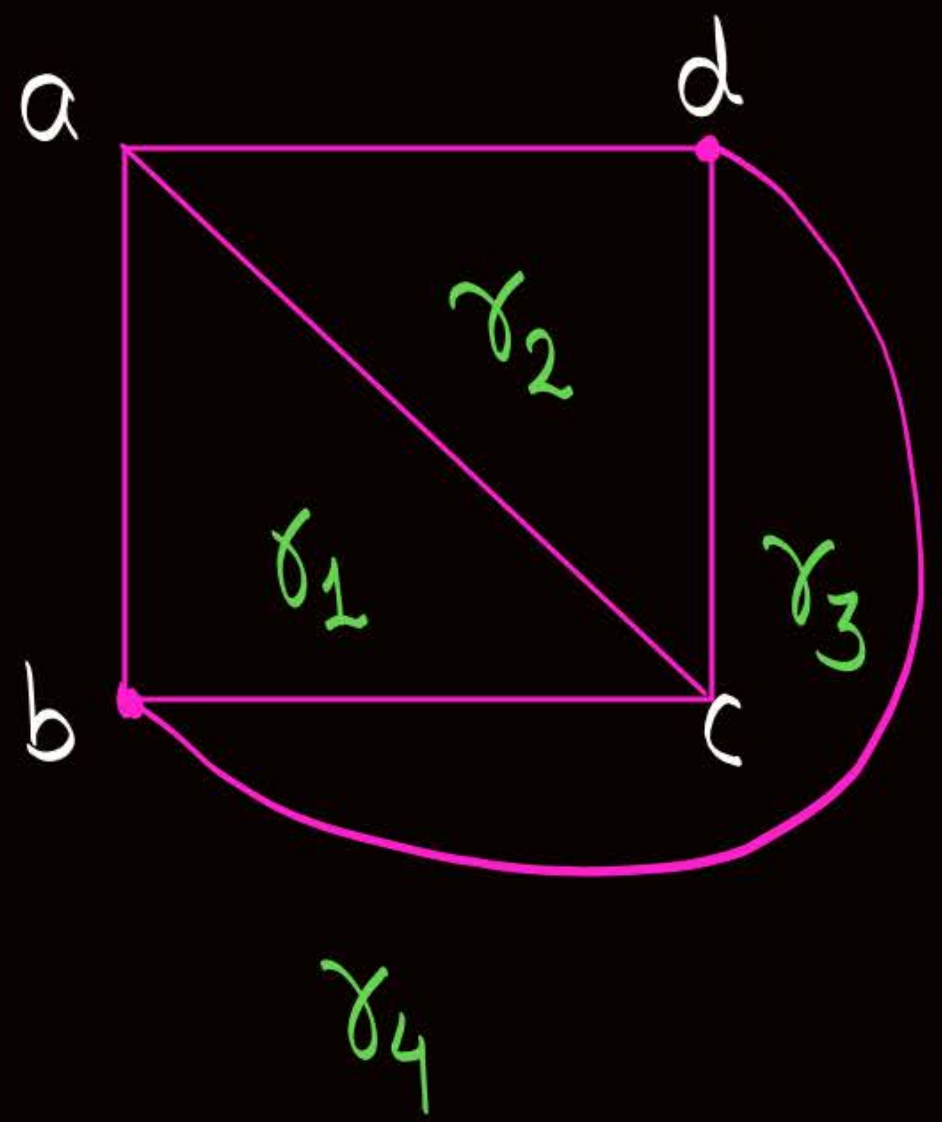
* Degree of a region "r" can be denoted by $\deg(r)$, and it is defined as.

$\deg(r) =$ No. of edges Enclosing the region 'r'

(or)

No. of Edges Exposed to region 'r'

eg:-



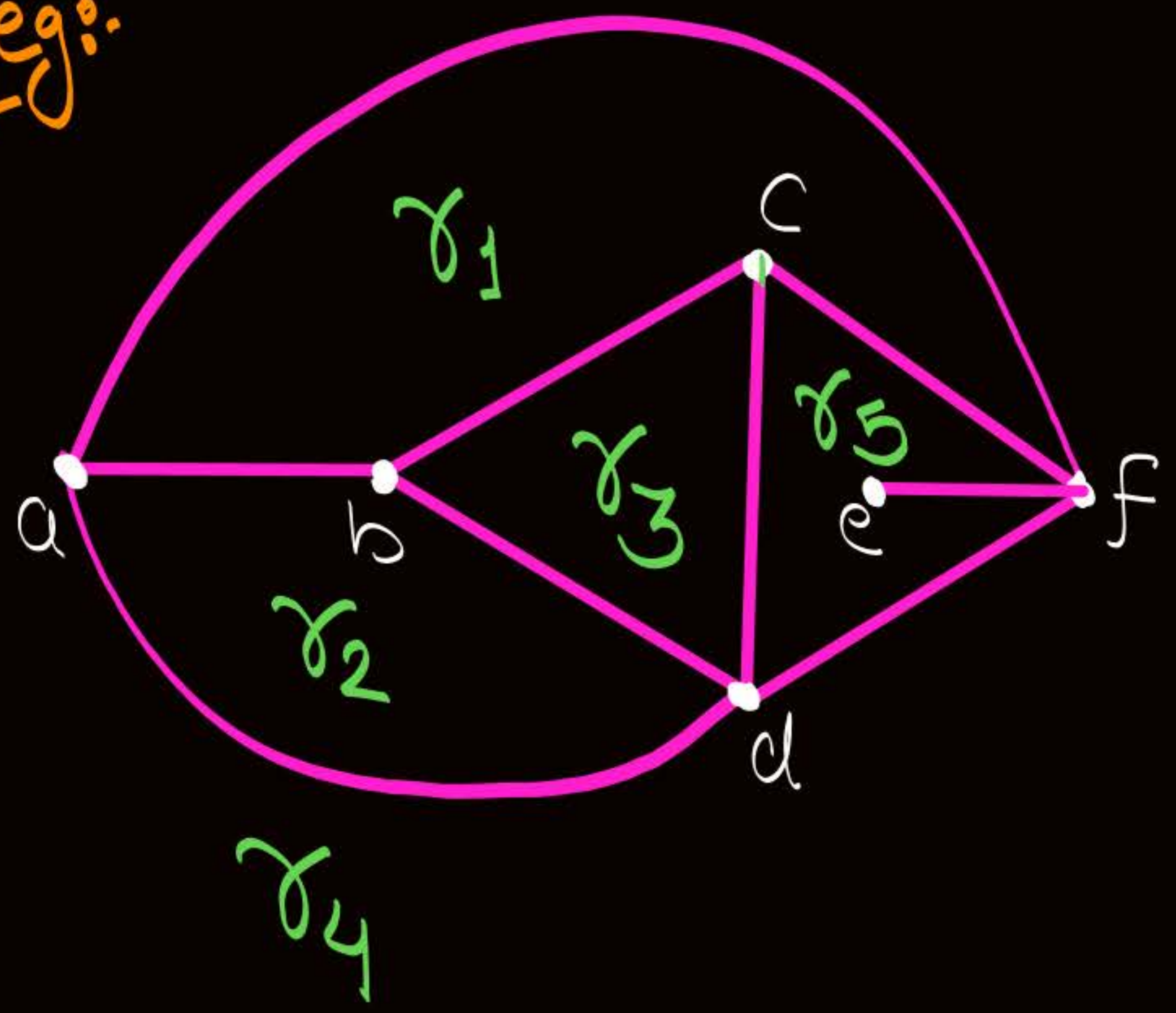
$\deg(r_1) = 3$

$\deg(r_2) = 3$

$\deg(r_3) = 3$

$\deg(r_4) = 3$

eg:



Note: Spiked edge will be Counted as two edges to obtain the degree of Corresponding region

$\deg(r_1) = 4$

$\deg(r_2) = 3$

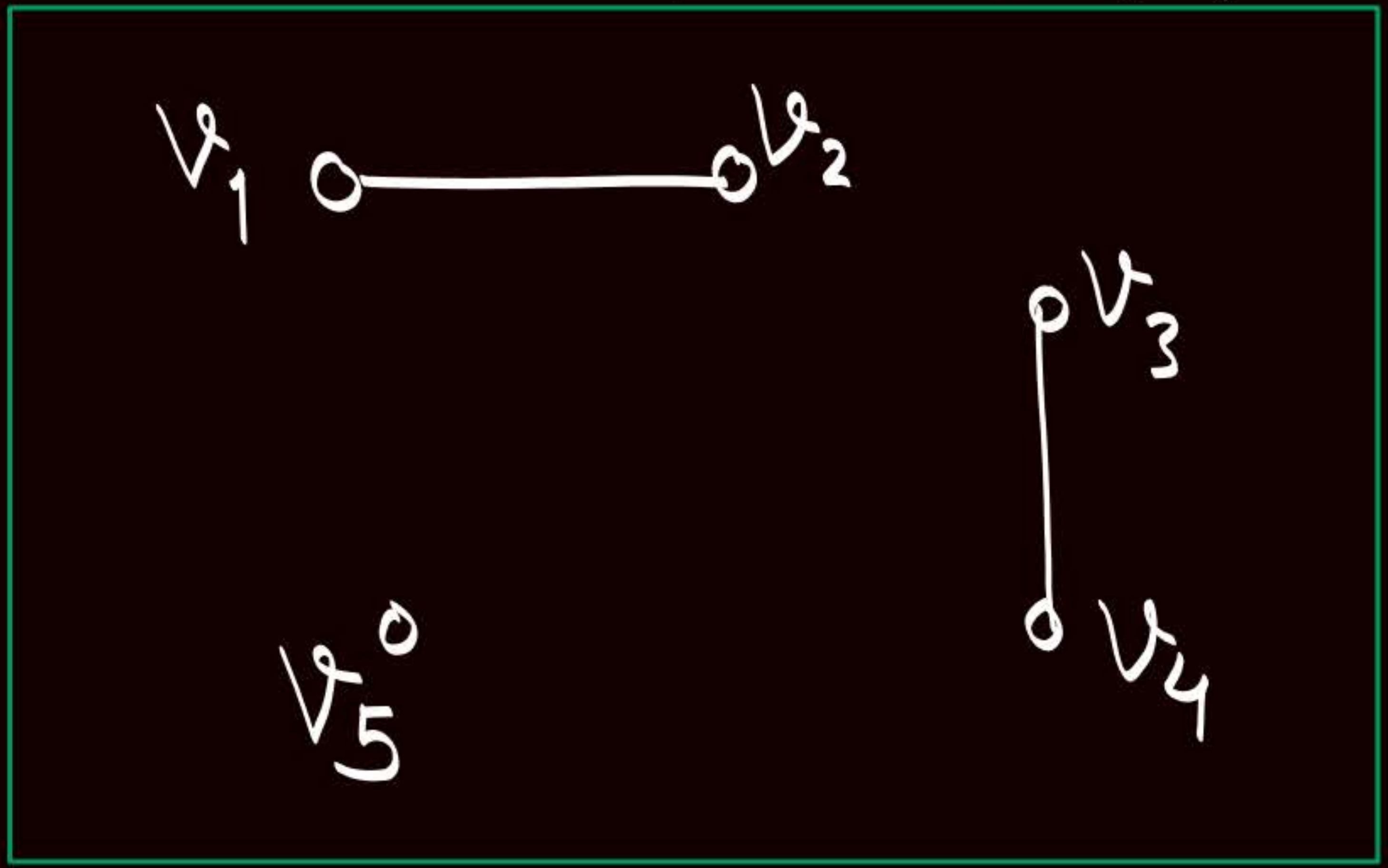
$\deg(r_3) = 3$

$\deg(r_4) = 3$

$\deg(r_5) = 3 + 2 = 5$

w.r.t. Spiked Edge

Consider the following graph G



it is a planar graph with only '1' region, let's call it region 'r', then

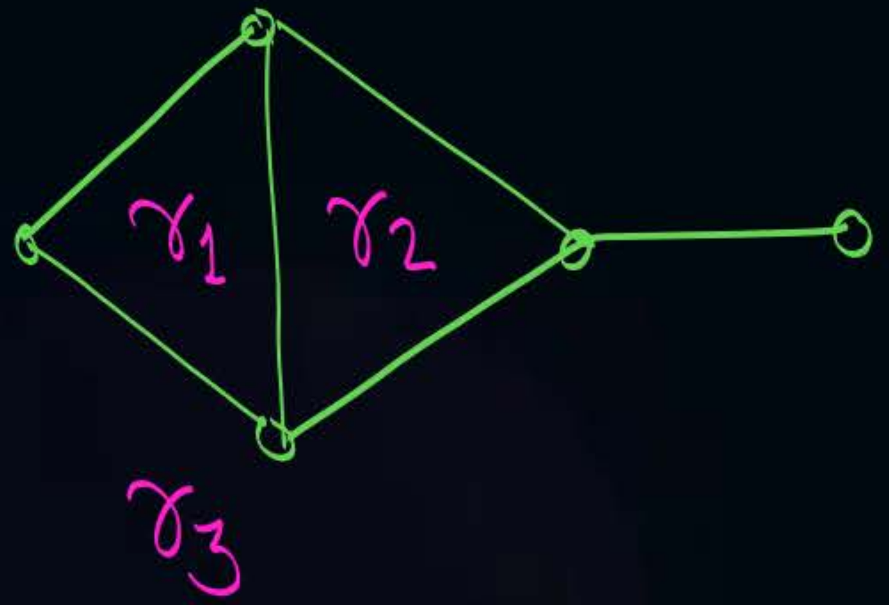
$$\deg(r) = 2 + 2 = 4$$

No two edges cross each other at a non-vertex point,

$\therefore$ graph G is a planar graph

Topic : Sum of degree theorem

W.r.t. degree of regions



$\deg(r_1) + \deg(r_2) + \deg(r_3)$

$3 + 3 + 6 = 12$

$= 2 \times 6 = 2 \times |E|$

* Let $|R|$ denotes the number of regions in the planar graph $G = (V, E)$, then

$$\sum_{i=1}^{|R|} \deg(r_i) = 2|E|$$

Some authors denotes the no. of regions by $|R|$, and some author denote the no. of faces by 'f' and $|R| = f$

Topic : Sum of degree theorem

- Corollary 1 :- No. of regions of odd degree are always even

- Corollary 2 :- In a planar graph G, if degree of each region is Exactly 'K' then

$$\boxed{K \cdot |R| = 2|E|}$$

Corollary 3: In a planar graph G if degree of each region is at least 'K' {i.e. $\geq K$} then

$$\boxed{K|R| \leq 2|E|}$$

Topic : Sum of degree theorem

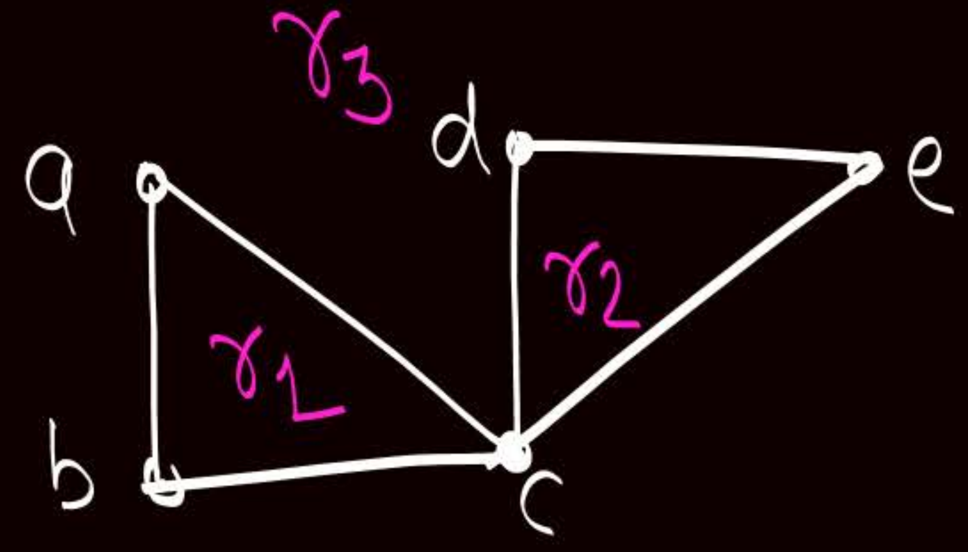
Corollary 4: In a planar graph G if degree of each region is at most 'K' {i.e $\leq K$} then

$$\boxed{K|R| \geq 2|E|}$$

Today's Topic

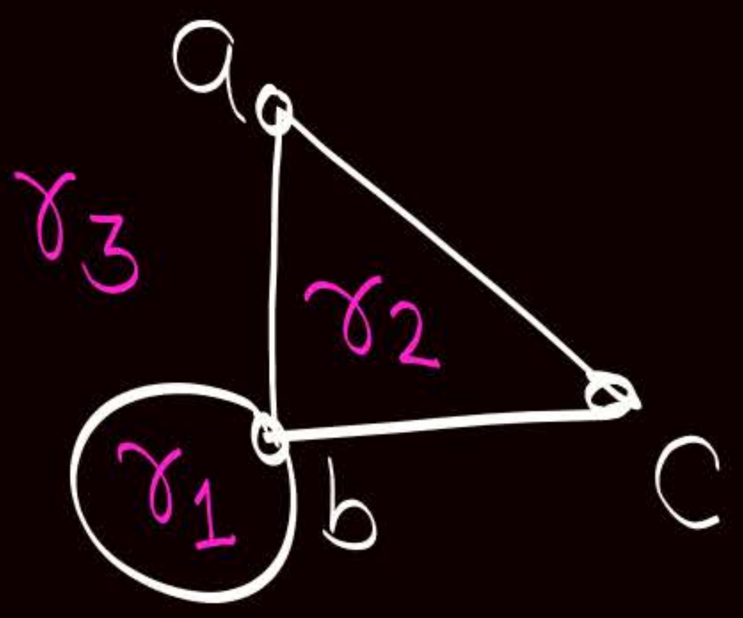
Note:. Every planar graph need not be simple,

eg.



Simple planar graph

(No loop + No parallel edges)



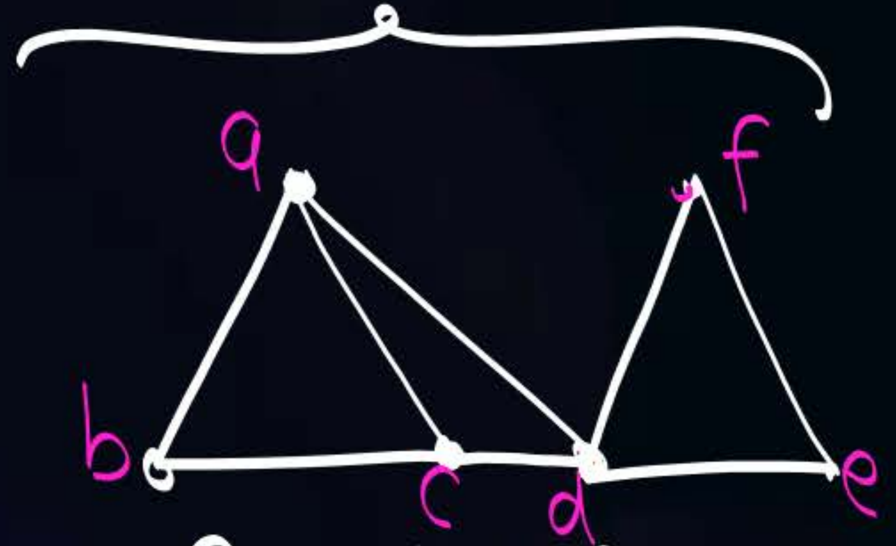
It is also a planar graph, but it is not simple graph

Topic : Simple planar graph

* A planar graph with no loop and no parallel edges is called a simple planar graph {It may or may not be Connected.}

G_1

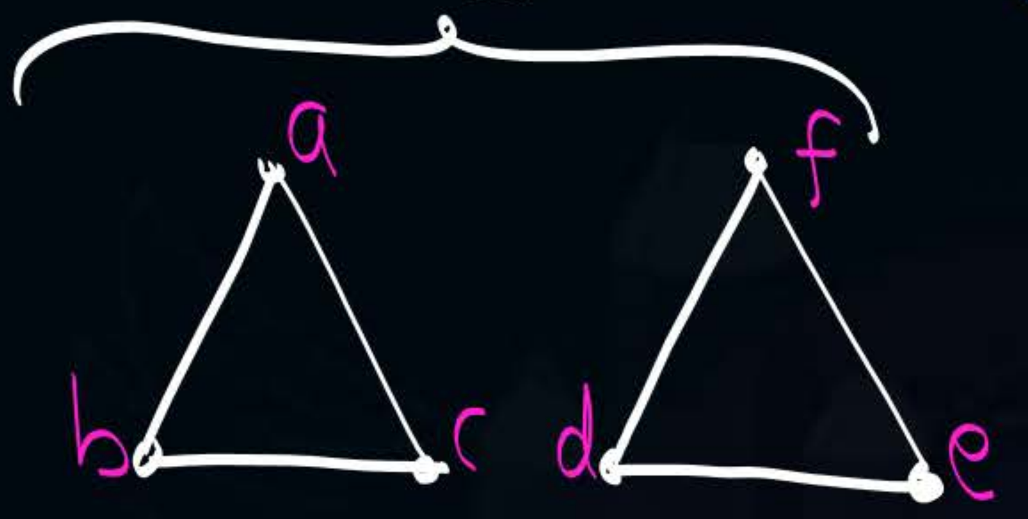
eg:



Connected Simple Planar graph

G_2

eg:



It is a Simple planar graph, but not a Connected graph

Topic : Simple planar graph

Note:- In a planar graph with two or more regions.
{i.e, in a planar graph with at least two regions}

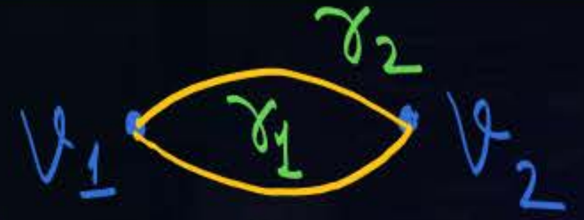
(i) $\deg(r) = 1$, if and only if there is a loop

eg:

$\deg(r_1) = 1$, & $\deg(r_2) = 1$

& (ii) $\deg(r) = 2$, if and only if region is w.r.t. Parallel edges

eg:



$\deg(r_1) = 2$

& $\deg(r_2) = 2$

Topic : Simple planar graph

- In a simple planar graph with two or more regions degree of each region will be at least '3' { it may or may not be Connected }

∴ In a planar graph if degree of each region is at least '3', then

$$\boxed{3|R| \leq 2|E|}$$

will hold true

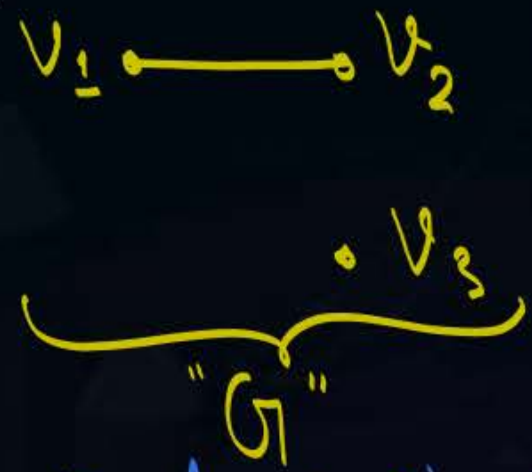
Topic : Simple planar graph

- In a simple planar graph with two or more regions degree of each region will be at least '3'

eg 1:-

It is a simple planar graph with only '1' region and deg(region) = 0

eg 2:-



It is also a simple planar graph with only '1' region and deg(region) = 2

Topic : Euler's equation (For Connected Planar graph)

- For any connected planar graph G, {may or may not be a simple graph} following Equation will hold true

$$\boxed{|V| + |R| = |E| + 2}$$

It is called Euler's Equation for Connected planar graph

Where $|V|$ = No. of vertices
$|E|$ = No. of edges
$|R|$ = No. of regions

$|V|+|R|=|E|+2$ will not hold true, if Planar graph is not a Connected graph

Some author represent the Euler's Equation using the following Format,

$$\boxed{n+f-e=2}$$

Where,
n = No. of vertices
e = No. of edges
f = No. of faces = No. of regions

Note 1:- In a simple planar graph {may or may not be Connected}

$$3|R| \le 2|E| \text{ ——— eq}^n \text{ (1)}$$

will always hold true

Note 2: In a Connected planar graph {may or may not be simple}

$$|V| + |R| = |E| + 2 \text{ ——— eq}^n \text{ (2)}$$

will always hold true

Topic : Simple connected planar graph

Simple Connected planar graph $\Rightarrow$ Simple planar graph + Connected Planar graph

$\downarrow$ ($\therefore$ $3|R| \le 2|E|$ will hold true) and ($\therefore$ $|V|+|R| = |E|+2$ will also hold true)

$|R| \le \frac{2|E|}{3}$ $\qquad$ $|E| \ge \frac{3|R|}{2}$

1. In a simple planar graph only eq^n 1 is guaranteed to hold true, but eq^n 2 may or may not hold true

2. In a Connected Planar graph only eq^n 2 is guaranteed to hold true, but eq^n 1 may or may not hold true

3. If graph is a simple Connected Planar graph, then both eq^n 1 & eq^n 2 will hold true

Topic : Simple connected planar graph

Simple Connected planar graph $\Rightarrow$ Simple planar graph + Connected Planar graph

Simple planar graph $\downarrow$ $\therefore 3|R| \leq 2|E|$ — eq^n (1)

Connected Planar graph $\downarrow$ $\therefore |V| + |R| = |E| + 2$ eq^n (2)

From Equation (1) $3|R| \leq 2|E|$ $\therefore |R| \leq \frac{2|E|}{3}$

Put this value of $|R|$ in Eq^n (2)

$$|V| + \frac{2|E|}{3} \geq |E| + 2$$

$$|V| \geq \frac{|E|}{3} + 2 \Rightarrow \boxed{|E| \leq 3|V| - 6} \text{ — eq}^n \text{ (3)}$$

$$\boxed{e \leq 3n - 6}$$

From Eq^n (1) $3|R| \leq 2|E|$ $\therefore |E| \geq \frac{3|R|}{2}$

Put this value of $|E|$ in Eq^n (2)

$$\therefore |V| + |R| \geq \frac{3|R|}{2} + 2$$

$$\boxed{|V| \geq \frac{|R|}{2} + 2} \text{ — eq}^n \text{ (4)} \quad \text{or} \quad \boxed{|R| \leq 2|V| - 4}$$

Topic : Note

In a simple connected planar graph all four equations will always hold true

$$3|R| \leq 2|E| \quad \text{——— } (1)$$

$$|V| + |R| = |E| + 2 \quad \text{——— } (2)$$

$$|E| \leq 3|V| - 6 \quad \text{——— } (3)$$

$$|R| \leq 2|V| - 4 \quad \text{——— } (4)$$

Topic : Polyhedral graph

* A polyhedral graph is a **simple connected planar graph** in which degree of each 'vertex' is at least '3'.

$\therefore 3|V| \leq 2|E|$ will also hold true in a polyhedral graph

$$\boxed{3|V| \leq 2|E|} \text{ — eq}^n \text{ (5)}$$

- In a polyhedral graph, following equations will hold true

$$\begin{cases} 3|R| \le 2|E| & \text{— (1)} \\ |V|+|R| = |E|+2 & \text{— (2)} \\ |E| \le 3|V|-6 & \text{— (3)} \\ |R| \le 2|V|-4 & \text{— (4)} \end{cases}$$

$$3|V| \le 2|E| \quad \text{— (5)}$$

From eq^n (5) $3|V| \le 2|E|$ $\therefore$ $|V| \le \frac{2|E|}{3}$

Put this value of $|V|$ in Euler's eq^n

i.e. $\frac{2|E|}{3} + |R| \ge |E|+2$

$$|R| \ge \frac{|E|}{3}+2$$

$$\boxed{|E| \le 3|R|-6} \quad \text{— eq}^n \text{ (6)}$$

From eq^n (5) $3|V| \le 2|E|$ $\therefore$ $|E| \ge \frac{3|V|}{2}$

Put this value of $|E|$ in Euler's equation

i.e. $|V| + |R| \ge \frac{3|V|}{2} + 2$

$$|R| \ge \frac{|V|}{2}+2$$

$$\boxed{|V| \le 2|R|-4} \quad \text{— eq}^n \text{ (7)}$$

Note:- In a polyhedral graph all seven equations from eq^n 1 to eq^n 7 will hold true

→ Euler's equation for a disconnected planar graph with 'K' Connected Components

$G \Rightarrow$ $|V|$ = No. of vertices

$|R|$ = No. of regions

$|E|$ = No. of Edges

K = No. of Connected Components

Euler's equation for disconnected Planar graph

{may or may not be simple}

→ Let 'G' is a planar graph with 'K' Connected Components.

* Let $|V|$ = Total no. of vertices in G.
  $|R|$ = Total no. of regions in G = (No. of all interior regions + 1)
  $|E|$ = Total no. of edges in G
  K : No. of Connected Components in G.

graph is planar ∴ Each Connected Component will also be planar

Graph 'G' with 'K' Connected Components

1st Connected Component + Planar	2nd Connected Component + Planar	3rd Connected Component + planar	- - - - - - -	Kth Connected Component + Planar
let $n = \lvert V_1 \rvert$, $e = \lvert E_1 \rvert$, $f = \lvert R_1 \rvert$	let $n = \lvert V_2 \rvert$, $e = \lvert E_2 \rvert$, $f = \lvert R_2 \rvert$			
∴ $\lvert V_1 \rvert + \lvert R_1 \rvert = \lvert E_1 \rvert + 2$	∴ $\lvert V_2 \rvert + \lvert R_2 \rvert = \lvert E_2 \rvert + 2$			

$$|V_1| + |R_1| = |E_1| + 2$$
$$|V_2| + |R_2| = |E_2| + 2$$
$$|V_3| + |R_3| = |E_3| + 2$$
$$\vdots$$
$$|V_k| + |R_k| = |E_k| + 2$$

$$\sum \quad \underbrace{(|V_1| + |V_2| + \cdots |V_k|)}_{|V|} + \underbrace{(|R_1| + |R_2| + \cdots |R_k|)}_{|R| + (k-1)} = \underbrace{(|E_1| + |E_2| + \cdots |E_k|)}_{|E|} + 2k$$

$$|V| + |R| + k - 1 = |E| + 2k$$

$$\boxed{|V| + |R| = |E| + k + 1}$$

$\boxed{\text{Euler's equation for disconnected Planar graph}}$ {may or may not be simple}

Let 'G' is a $\underbrace{\text{planar graph}}_{\text{\{May not be simple\}}}$ with 'K' Connected Components.

then following Equation will hold true

$$\boxed{|V| + |R| = |E| + K + 1}$$

it is called Euler's Equation for disconnected planar graph

When, $|V|$ = No. of Vertices in G

$|E|$ = No. of edges in G

$|R|$ = No. of regions in G = (Total no. of interior regions + '1' Exterior region)

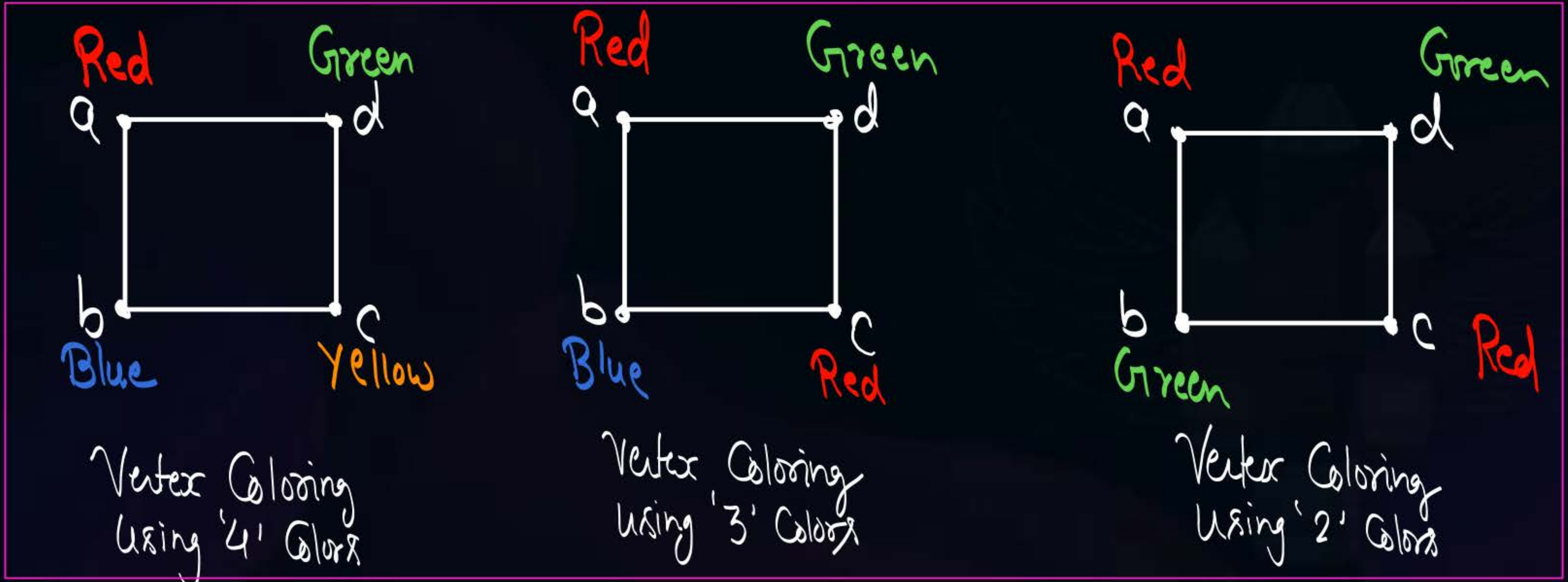
K = No. of Connected Components in G

Coloring

Topic : Vertex coloring

- An assignment of colors to the vertices of graph G, such that no two adjacent vertices of the graph have the same color is called vertex coloring of graph G.

All of them are valid Vertex Coloring



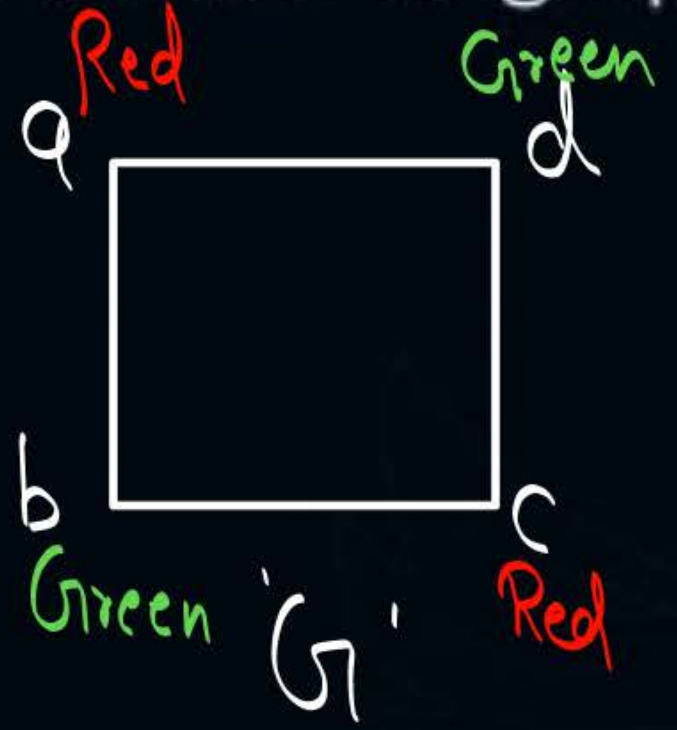
* If we have a graph with 'n' vertices, then we can always vertex color that graph using 'n' colors { i.e. one color per vertex }

"But we are interested in the minimum number of colors required for the vertex coloring of the given graph"

Topic : Chromatic number

Minimum number of colors needed for the vertex coloring of graph G is called chromatic number of graph G,

It is denoted by $\chi(G)$.



We can not vertex color the graph using less than two colors

$\therefore$ $\boxed{\chi(G) = 2}$

Topic : K-colorable vs K-chromatic

K-Colorable	K-Chromatic
* If there exist a vertex coloring of graph 'G' that uses 'K' colors, then graph G is called 'K' colorable	If chromatic number of graph 'G' is 'K', then graph 'G' is said to be K-chromatic
If graph G is K-colorable, then $\chi(G) \leq K$	If graph 'G' is K-chromatic, then $\chi(G) = K$

Note:-

① If graph G is K-Colorable, then graph G is $(K+1)$-Colorable as well,

Provided $(K+1) \leq$ No. of vertices in graph G.

② If graph G is K-Colorable, then graph G may or may not be $(K-1)$-Colorable

③ If graph G is K-chromatic, then it is neither $(K+1)$-chromatic nor $(K-1)$-chromatic

Note:-

★ (4) If graph G is K-Chromatic then graph G is K-Colorable as well, but

if graph is K-Colorable then it need not be K-Chromatic

Topic : Four-Color Theorem

Every planar graph G is four colorable.

For any planar graph G, $\chi(G) \leq 4$

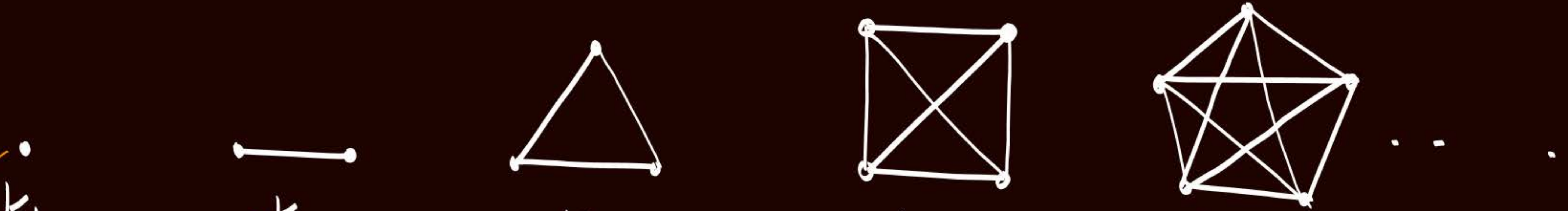
Note: →

1. Chromatic number of graph G can be '1' if and only if graph 'G' is a NULL graph

   i.e. $\boxed{(\chi(G)=1) \iff \text{'G' is NULL graph}}$

2. $\boxed{\text{If graph } G \text{ is not a NULL graph then } \chi(G) \geq 2}$

Q:- Find the Chromatic number of Complete graph 'K_n'



K_1 K_2 K_3 K_4 K_5

$\chi(K_1)=1$ $\chi(K_2)=2$ $\chi(K_3)=3$ $\chi(K_4)=4$ $\chi(K_5)=5$

$\boxed{\text{For a Complete graph } K_n,\ \chi(K_n)=n}$

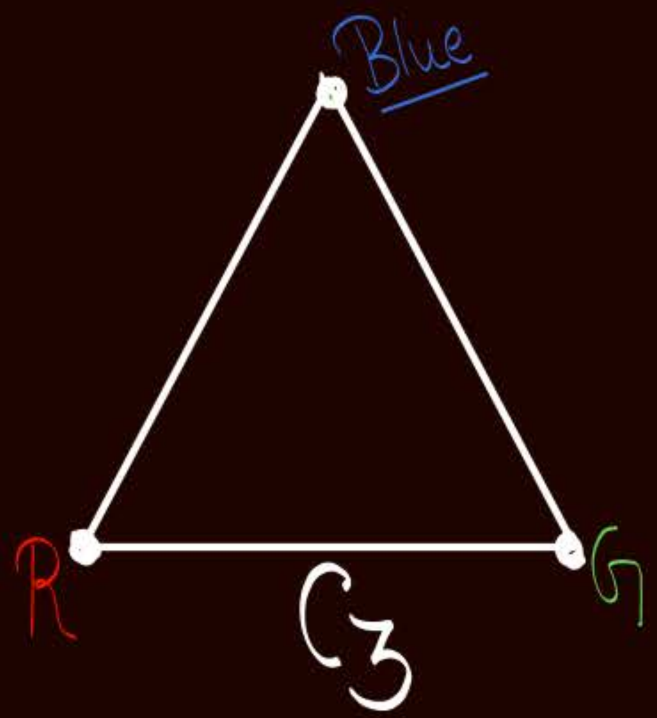
Max degree $\Delta(K_1)=0$ $\Delta(K_2)=1$ $\Delta(K_3)=2$ $\Delta(K_4)=3$ $\Delta(K_5)=4$

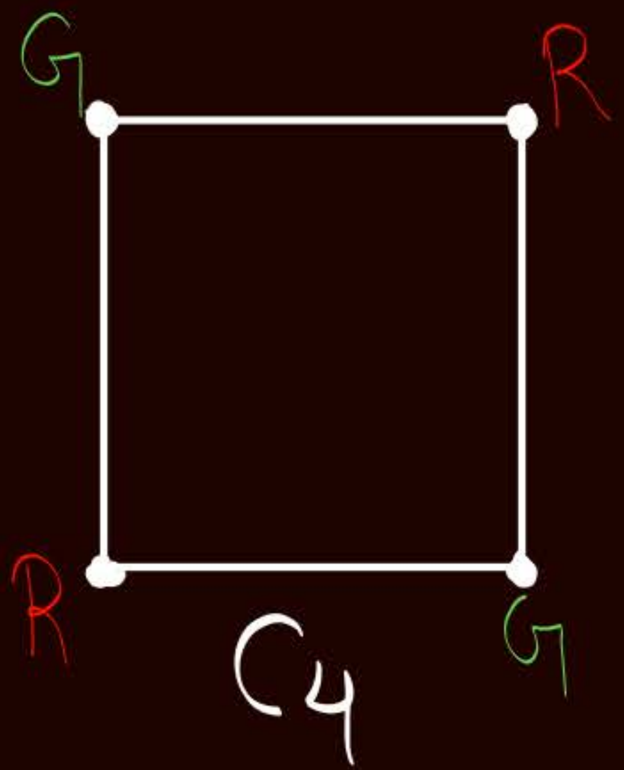
$\chi(K_1)=1+\Delta(K_1)$ $\chi(K_2)=1+\Delta(K_2)$ $\chi(K_3)=1+\Delta(K_3)$ $\chi(K_4)=1+\Delta(K_4)$ $\chi(K_5)=1+\Delta(K_5)$

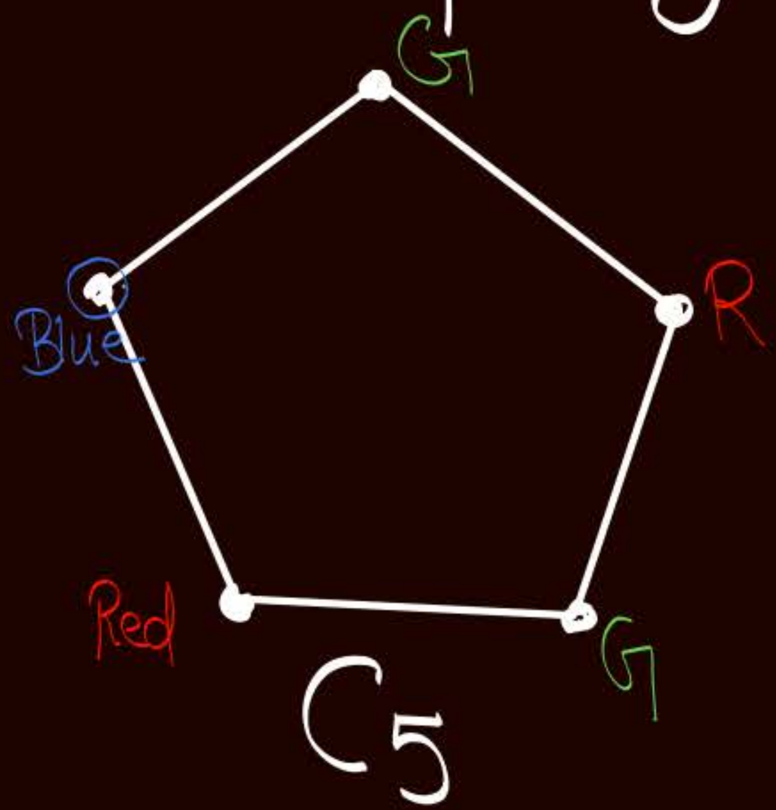
For any graph G,

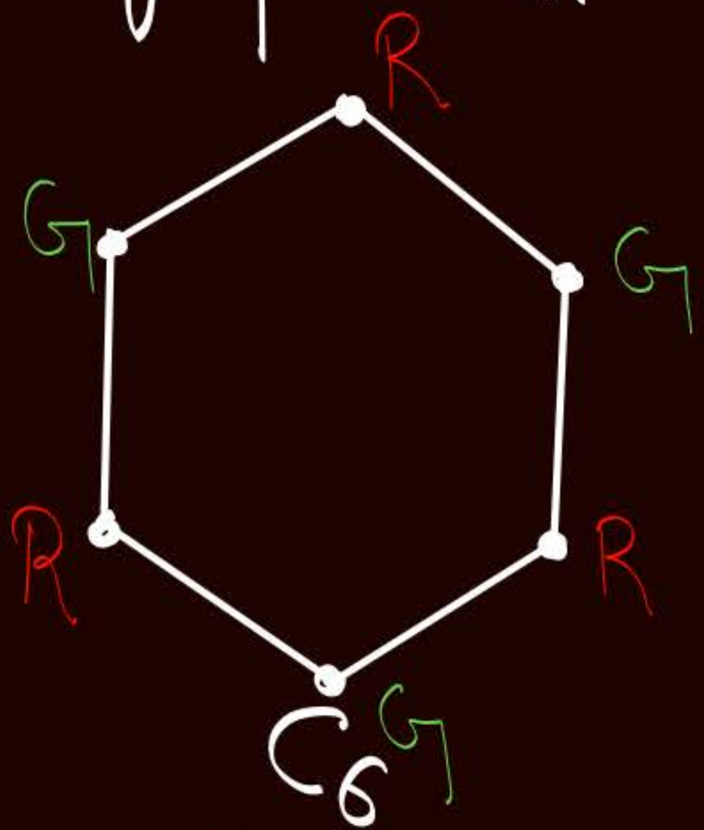
$$\chi(G) \leq 1 + \Delta(G)$$

Q:- Find the Chromatic number of Cycle graph 'C_n'





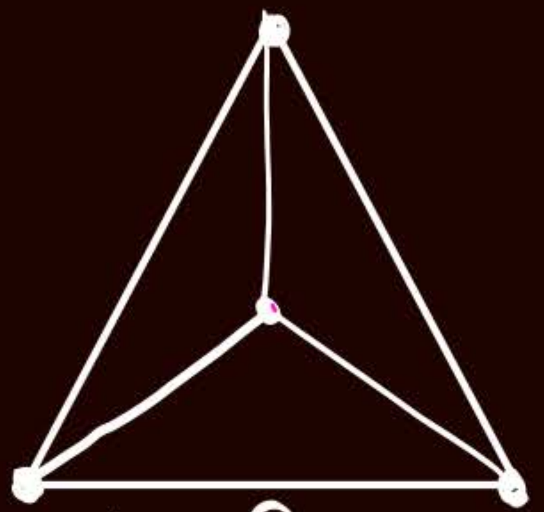




$\chi(C_3) = 3$ $\chi(C_4) = 2$ $\chi(C_5) = 3$ $\chi(C_6) = 2$

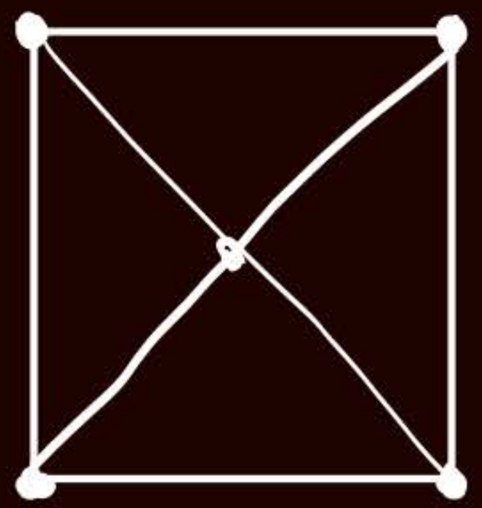
$$\boxed{\chi(C_n) = \begin{cases} 2, & \text{if } n = \text{Even} \\ 3, & \text{if } n = \text{odd} \end{cases}}$$

Q:- Find the Chromatic number of Wheel graph W_n



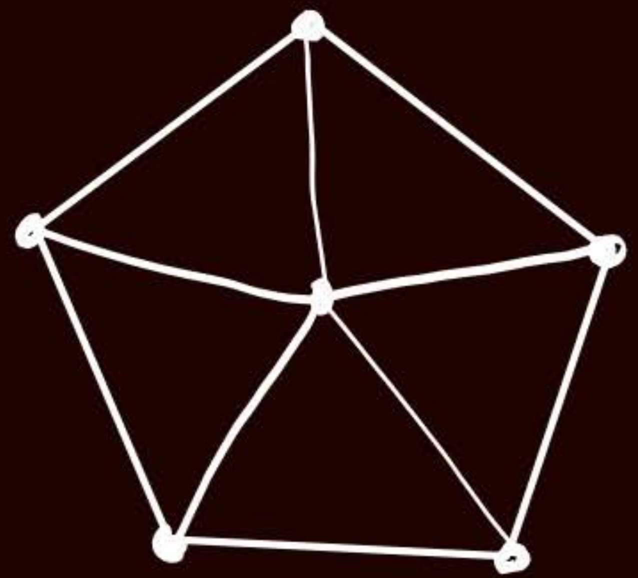
$W_4 = C_3 + 1\text{ hub}$

$\chi(W_4) = 3 + 1 = 4$



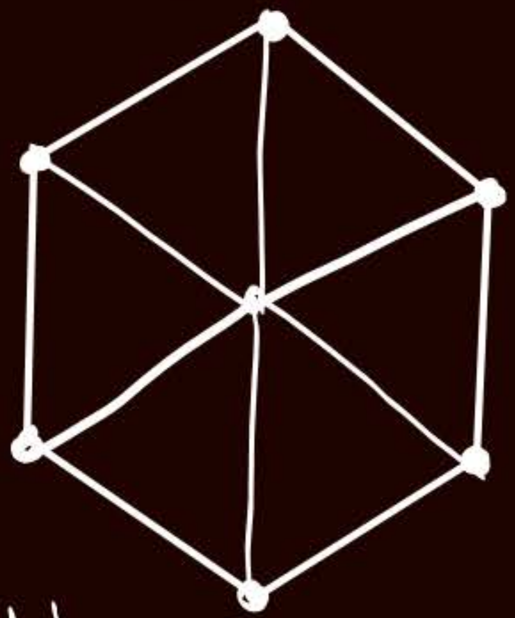
$W_5 = C_4 + 1\text{ hub}$

$\chi(W_5) = 2 + 1 = 3$



$W_6 = C_5 + 1\text{ hub}$

$\chi(W_6) = 3 + 1 = 4$



$W_7 = C_6 + 1\text{ hub}$

$\chi(W_7) = 2 + 1 = 3$

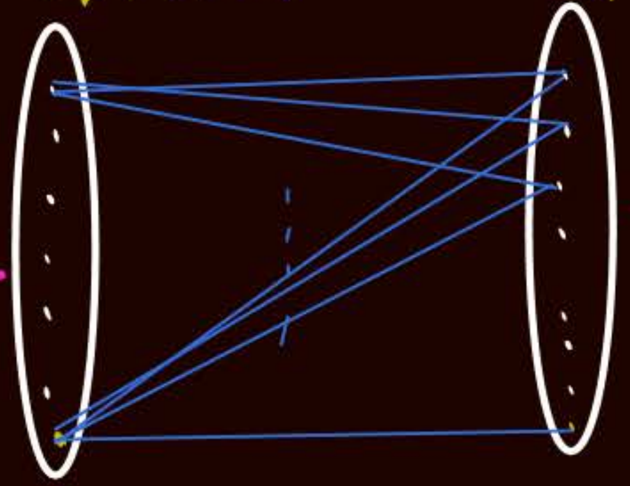
Hub vertex will always require a different Color.

$$\chi(W_n) = \begin{cases} 4, & \text{if } n = \text{even} \\ 3, & \text{if } n = \text{odd} \end{cases}$$

Q:- Find the Chromatic number of Complete bipartite graph $K_{m,n}$

$K_{m,n}$

V_1, $|V_1| = m$ $\qquad$ V_2, $|V_2| = n$



all vertices of subset 'V_1' Can be Colored Using the Same Color

all vertices of subset 'V_2' Can be Colored Using the Same Color

i.e. One Color per Subset of Partition

$$\therefore \boxed{\chi(K_{m,n}) = 2}$$

Note :-

① Complete bipartite graph can never be a NULL graph because no subset of partition can be Empty. and Each vertex of 1^{st} non-Empty subset will connect with each vertex of 2^{nd} non-Empty subset.
Hence at least one Edge will be present in the graph

But, a bipartite graph may be a NULL graph

eg:

it is a bipartite graph, and no Edge ∴ NULL graph

① For a complete bipartite graph Chromatic no is always $= 2$

② But for a bipartite graph Chromatic no. is 1 or 2

When NULL graph → 1

Otherwise → 2

Note :- ① Chromatic no. of a bipartite graph is 1 or 2

∴ $\chi(\text{Bipartite graph}) \leq 2$

② Bipartite graph are also known as bi-colorable graph {i.e. 2-colorable}, but not 2-chromatic

③ If Chromatic number of a graph 'G' is '2' then graph 'G' is a bipartite graph.

i.e. if $\chi(G) = 2$, then G is bipartite

Note:

1. In a graph G, if there exists at least one cycle of odd length, then $\chi(G) \geq 3$

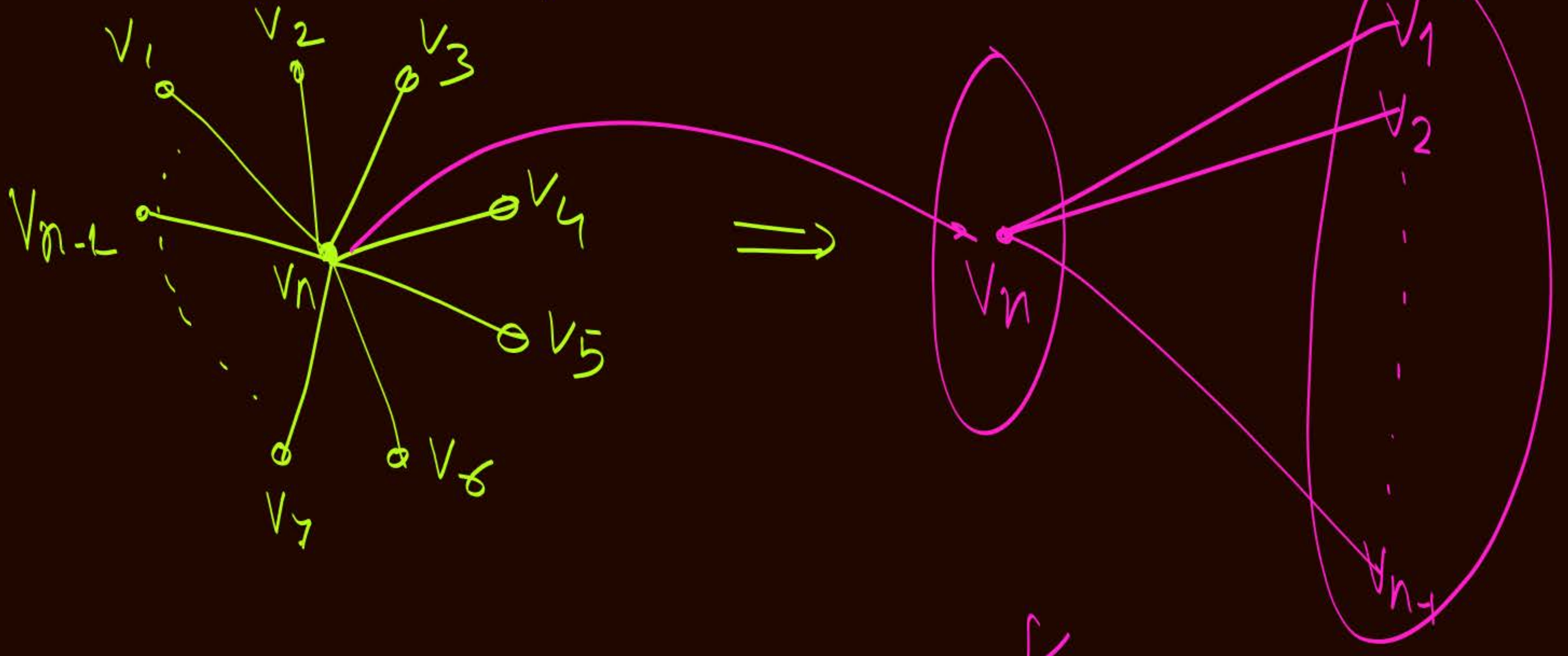
2. In a non-NULL graph G, if Every cycle is of Even length then $\chi(G) = 2$

3. In a graph G, if all cycles are of even length then graph G is a bipartite graph {Provided $|V| \geq 2$}

4. If $\chi(G) = 2$, then 'G' is a bipartite graph And hence Every cycle in graph G is of Even length

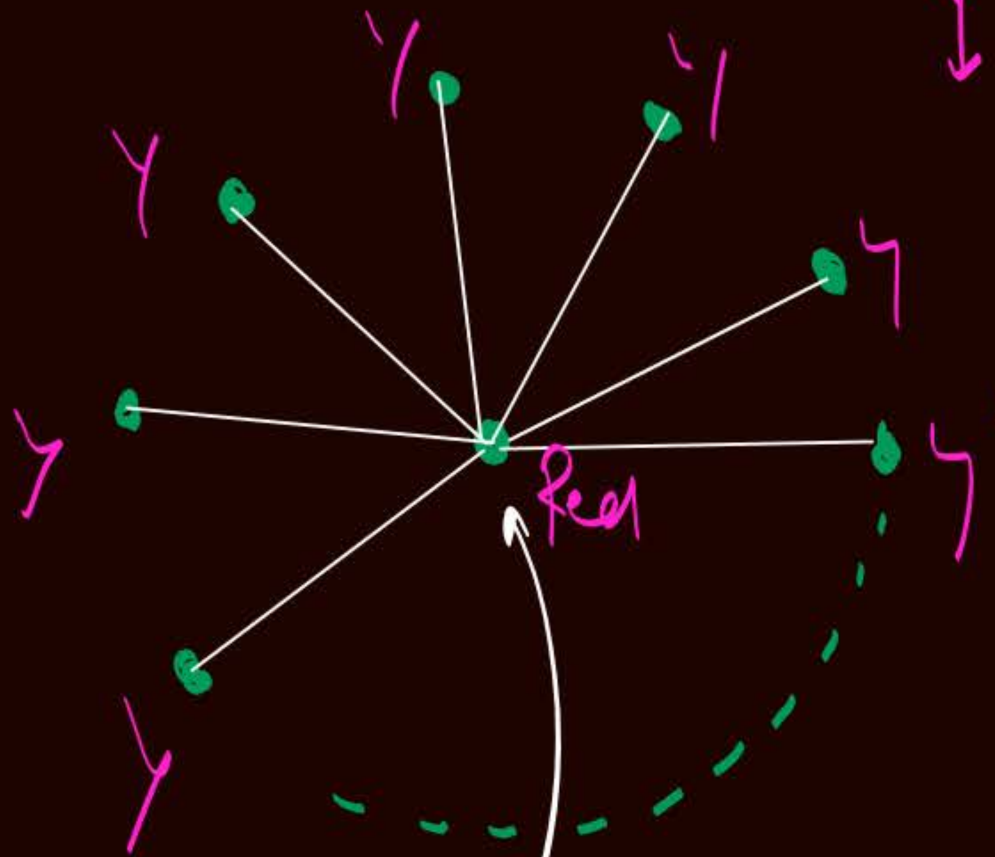
5. For a non-NULL tree (ie, for any tree with two or more vertices) the Chromatic number = 2

Star graph with 'n' vertices



$K_{1,n-1}$ or $K_{n-1,1}$

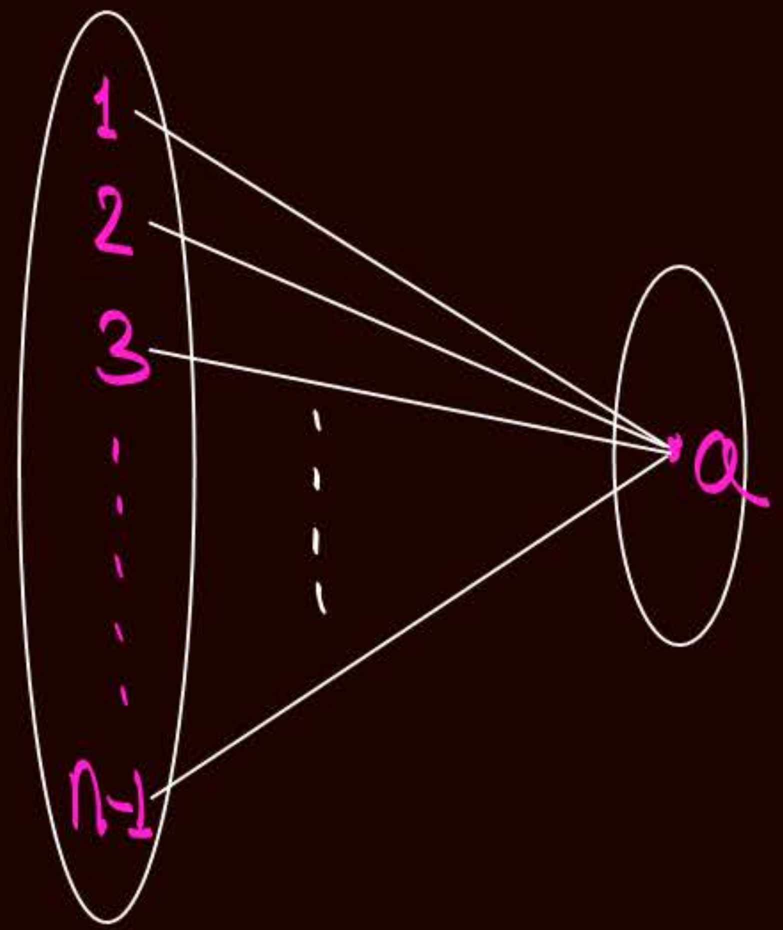
Q:- Find the Chromatic number of a star graph with 'n' vertices



1 hub vertex
& (n-1) vertices at Peripheri

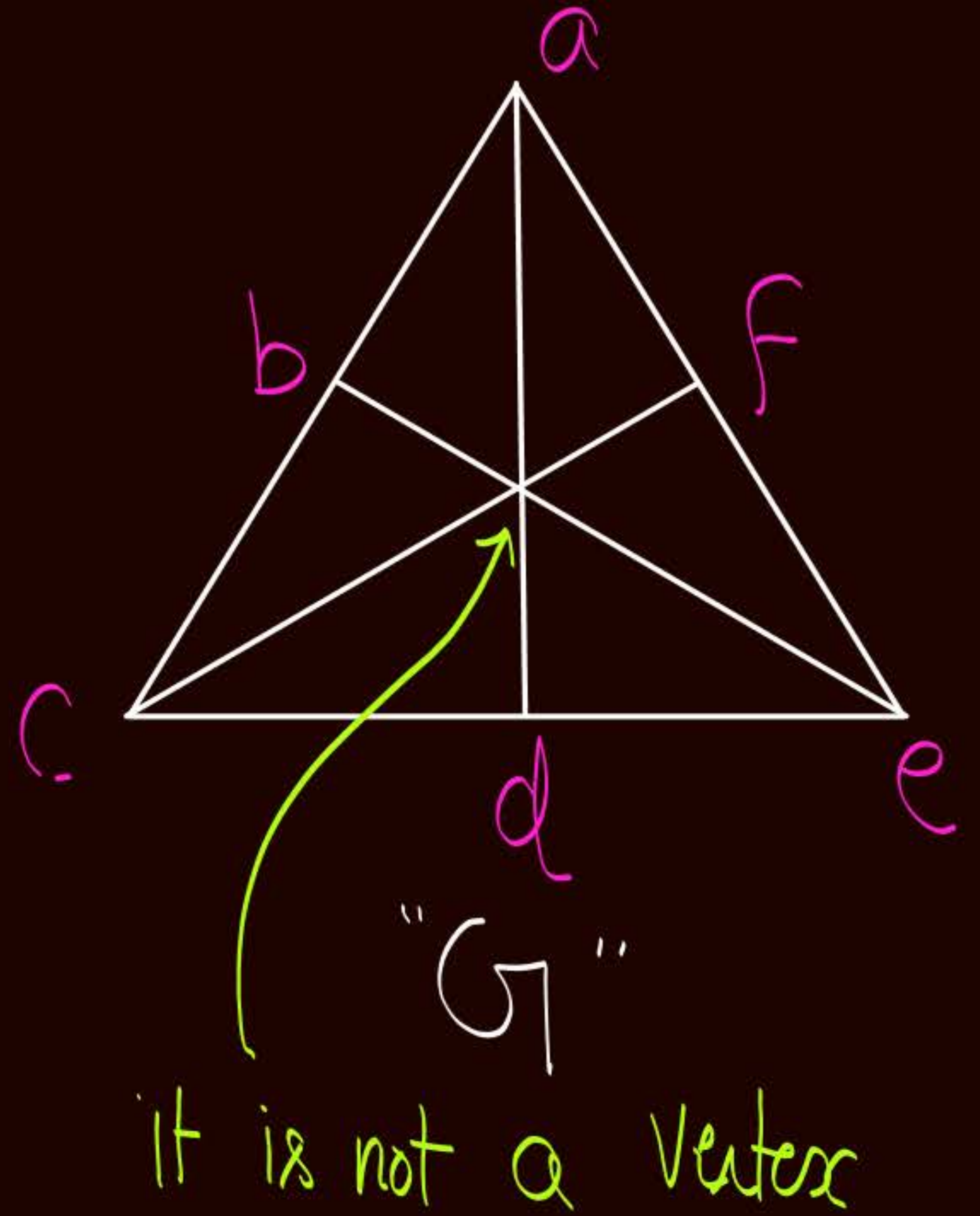
A star graph with n-vertices Can be visualized as

$K_{1,n-1}$ or $K_{n-1,1}$

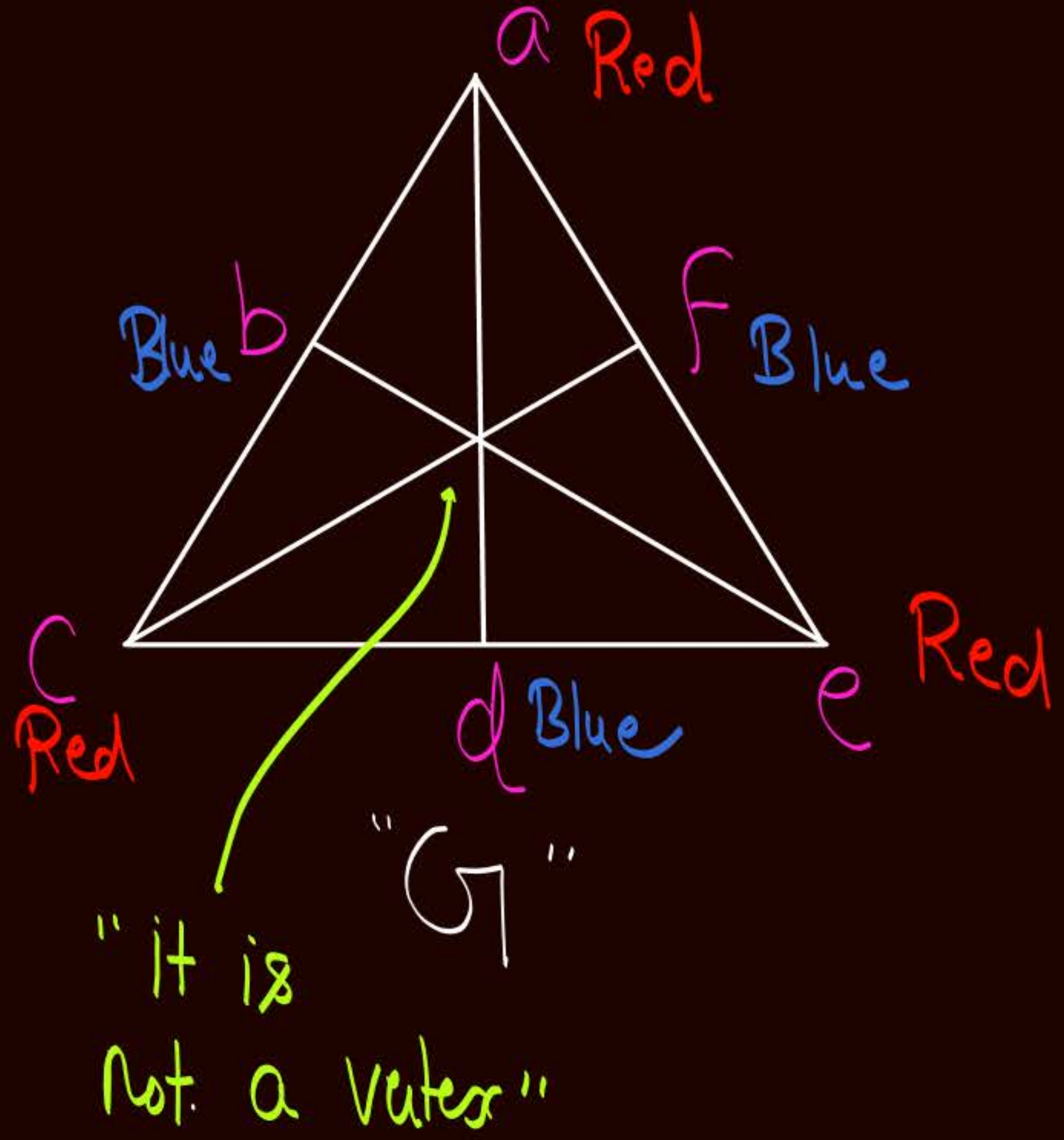


$\therefore \chi\left(\begin{matrix}\text{Star graph} \\ \text{with n-vertices}\end{matrix}\right) = 2$

eg:- Find the Chromatic number of following graph "G"



eg:- Find the Chromatic number of following graph "G"



- We can vertex color the graph using two colors.

$\therefore \chi(G) \leq 2$ —— eq^n ①

- Graph G is not a NULL graph

$\therefore \chi(G) \geq 2$ —— eq^n ②

By eq^n ① & eq^n ②

$$\boxed{\chi(G) = 2}$$

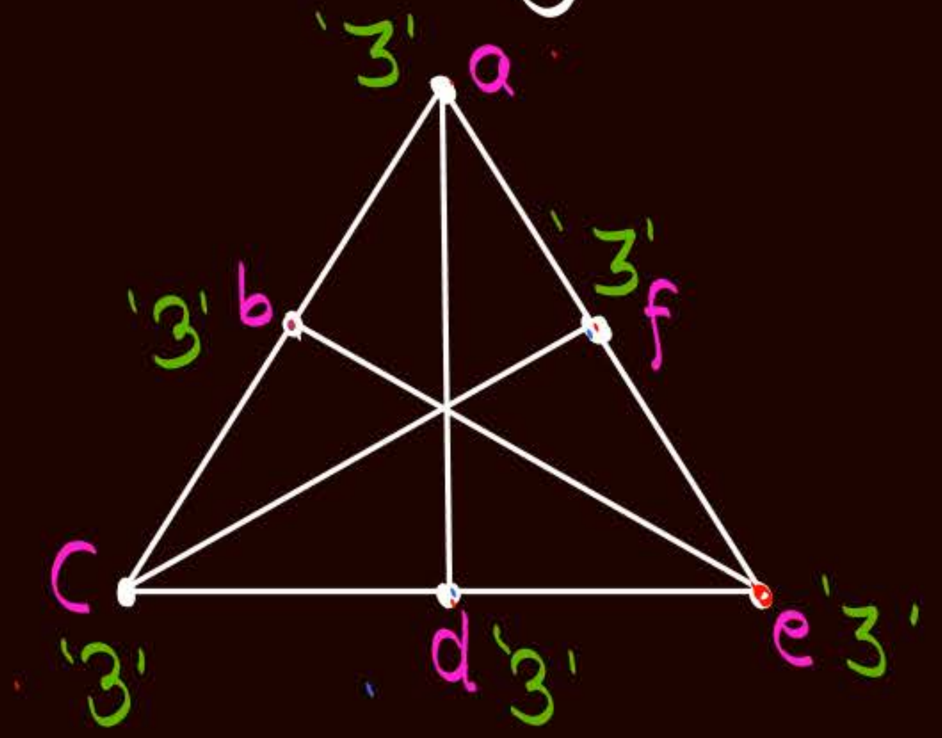
Topic : Welsh Powell's Algorithm

- It is used to $\boxed{\text{check the upper bound on chromatic number}}$ of a graph G.

1. Arrange the vertices of the graph G in non increasing order of their degrees.
2. If two or more vertices are having the same degree then use alphanumeric order.
3. Assign the colors to the vertices in non increasing order of their degrees such that no two adjacent vertices have the same color.
4. If Welsh Powell's algorithm uses "m" colors for a graph G, then $\chi(G) \le m$. {i.e. graph G is m-Colorable}

   'm' is the upper bound on Chromatic No.

eg:- Find the Chromatic number of following graph "G" using "Welsh Powell's algo"



"G"

All vertices are of same degree
∴ Use alphanumeric order

Vertices in non-increasing order of their degrees	a	b	c	d	e	f
Color Used	Red	— Green	Red	— Green	Red	— Green

↳ Welsh Powell's algorithm requires '2' Colors

$$\therefore \chi(G) \leq 2 \text{ ——— } (1)$$

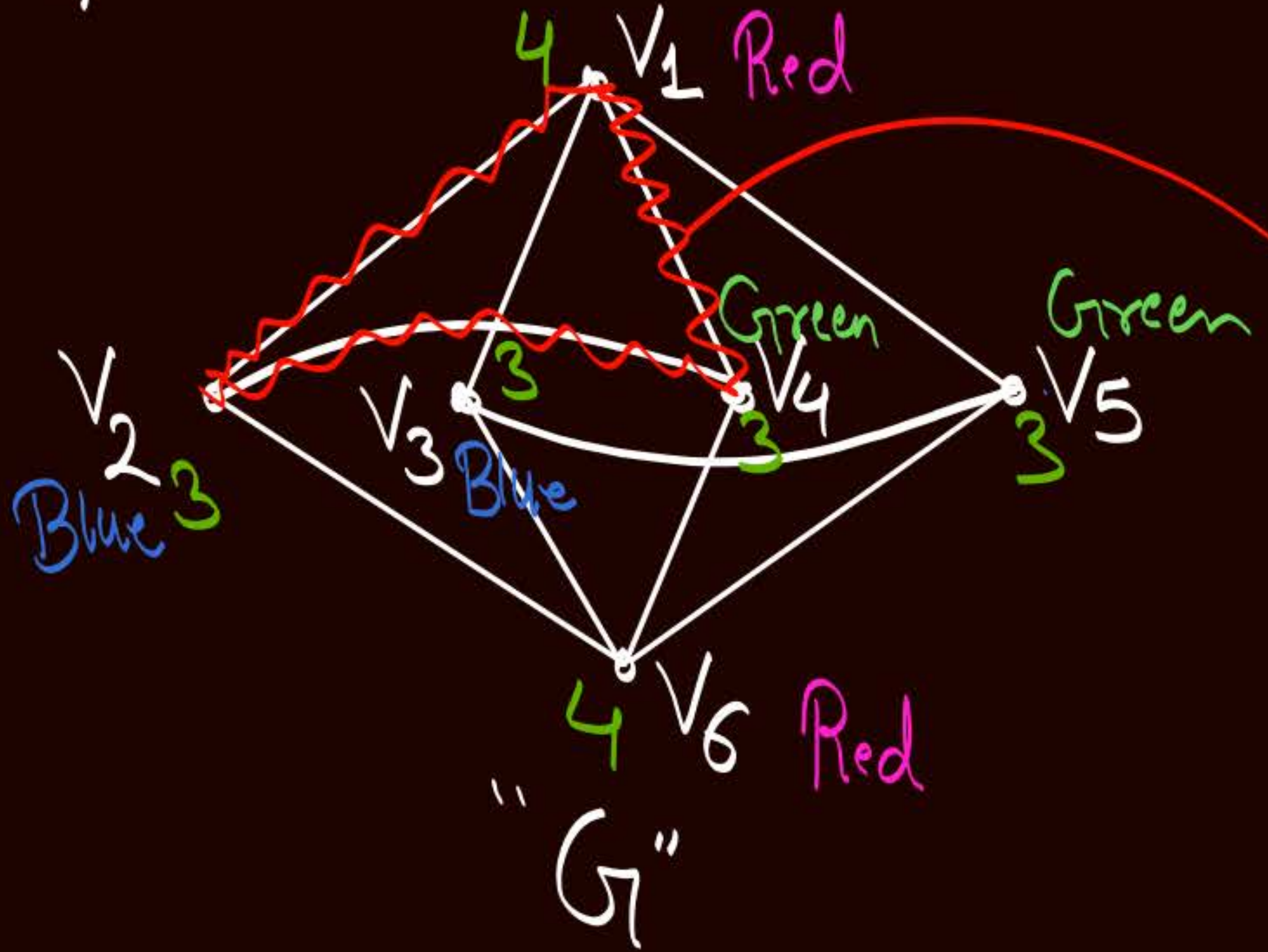
Graph G is not a NULL graph

$$\therefore \chi(G) \geq 2 \text{ ——— } (2)$$

By eq^n (1) & eq^n (2)

$$\chi(G) = 2$$

Q:- Find the Chromatic number of the following graph 'G'



Graph G is 3-Colorable

$\therefore \chi(G) \leq 3$ —— ①

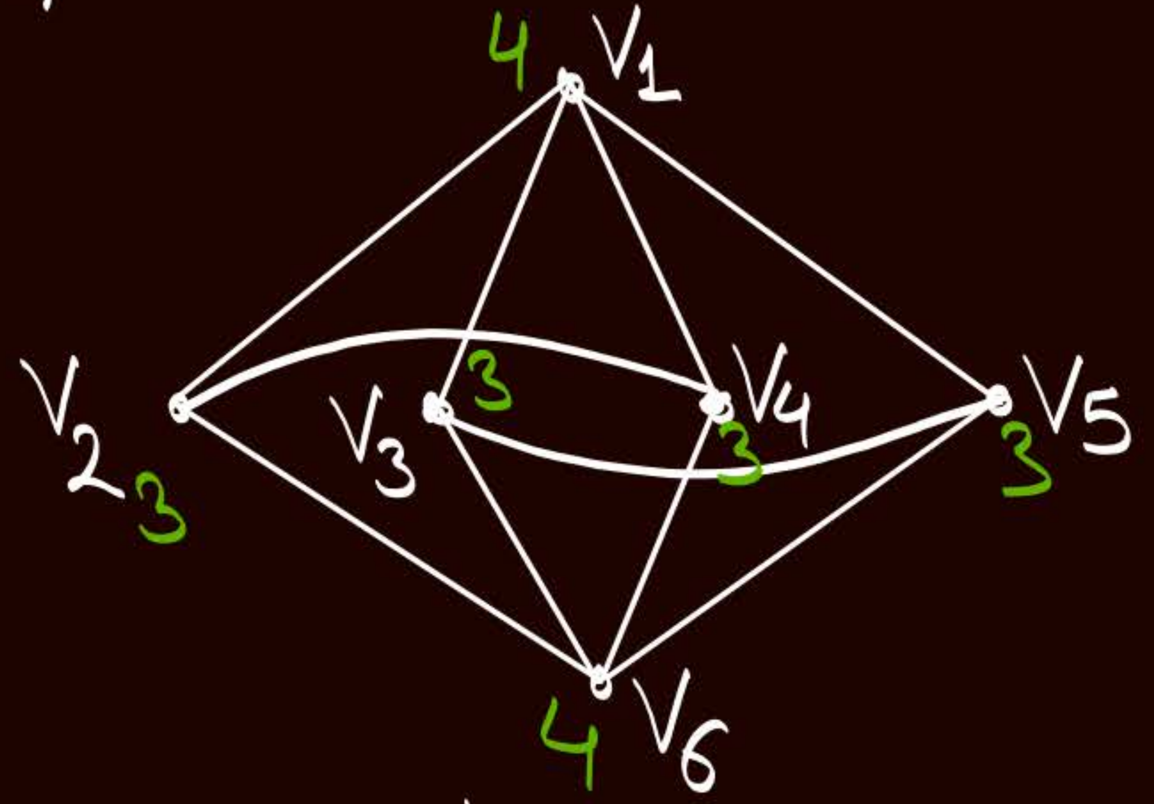
There exist an odd length cycle

$\therefore \chi(G) \geq 3$ —— ②

By eqⁿ ① & eqⁿ ②

$\chi(G) = 3$

Q:- Find the Chromatic number of the following graph 'G'



"G"

	deg = 4		deg = 3			
Vertices in non-increasing order of deg.	V_1	V_6	V_2	V_3	V_4	V_5
Colors	Red	Red	– G	– G	– – B	– – B

'3' color, $\therefore \chi(G) \leq 3$ — eqⁿ ①

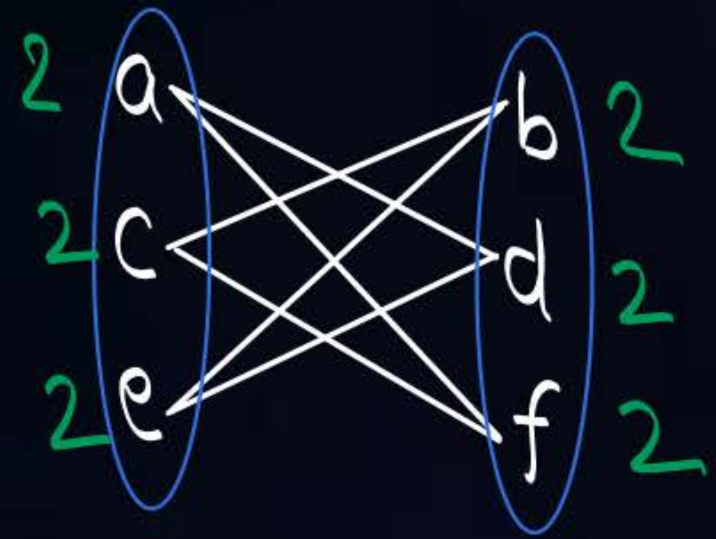
odd length cycle $\therefore \chi(G) \geq 3$ — eqⁿ ②

$\therefore \chi(G) = 3$

Topic : Welsh Powell's Algorithm

→ Welsh Powell's algo need not produce optimal result in all cases

eg: Find the chromatic number of following graph G



G

Same degree ∴ Use alphanumeric order

Vertices in non-increasing order of deg	a	b	c	d	e	f
Color used	Red	Red	– Green	– Green	– – Blue	– – Blue

Given graph is a bipartite graph ∴ $\chi(G) = 2$

But Welsh Powell's algorithm uses '3' colors.

Edge Coloring:- An assignment of Colors to the Edges of graph G, such that no two adjacent edges are Colored using the same color is called "Edge Coloring" of graph G.

* Minimum number of Colors required for Edge Coloring of graph G is Called "Index number" of graph G.

Q. Let, $G = (V, E)$ be a graph. Define $\xi(G) = \sum_d i_d \times d$ where i_d is the number of vertices of degree d in G. If S and T are two different trees with $\xi(S) = \xi(T)$, then

A. $|S| = 2|T|$

B. $|S| = |T| - 1$

C. $|S| = |T|$

D. $|S| = |T| + 1$

Q. In binary tree with n nodes, every node has an odd number of descendants. Every node is considered to be its own descendant. What is the number of nodes in the tree that have exactly one child?

(A) 0

(B) 1

(C) $(n-1)/2$

(D) $n-1$

Q.

The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequences cannot be the degree sequence of any graph?

I. 7, 6, 5, 4, 4, 3, 2, 1

II. 6, 6, 6, 6, 3, 3, 2, 2

III 7, 6, 6, 4, 4, 3, 2, 2

IV 8, 7, 7, 6, 4, 2, 1, 1

(A) I and II

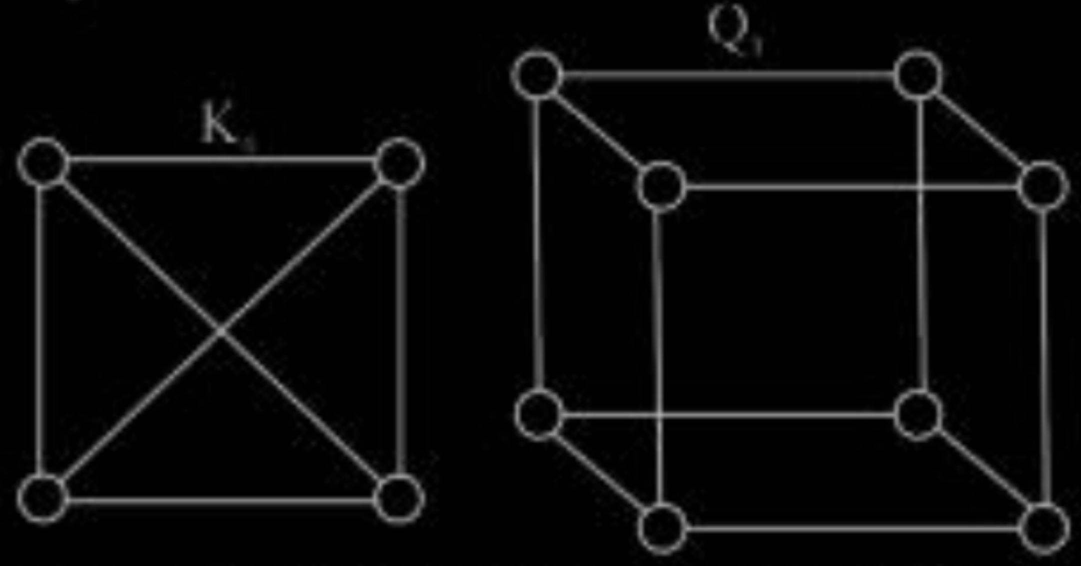
(B) III and IV

(C) IV only

(D) II and IV

Q.

K_4 and Q_3 are graphs with the following structures:



Which one of the following statements is TRUE in relation to these graphs?

(A) K_4 is planar while Q_3 is not

(B) Both K_4 and Q_3, are planar

(C) Q_3 is planar while K_4 is not

(D) Neither K_4 nor Q_3, is planar

Q.

Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is a connected graph, then the number of bounded faces in any embedding of G on the plane is equal to

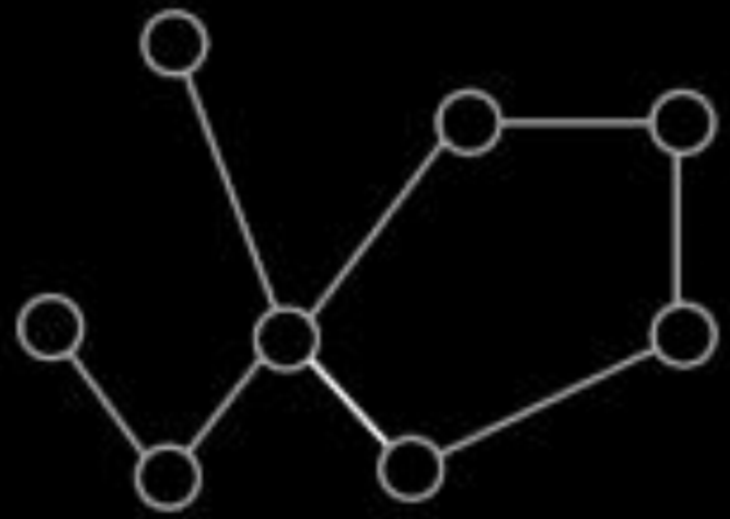
(A) 3

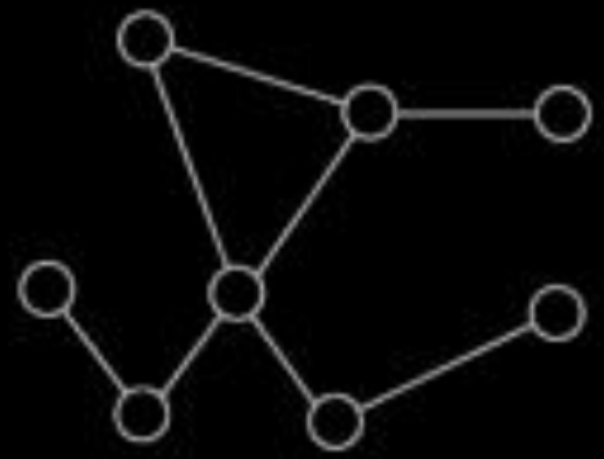
(B) 4

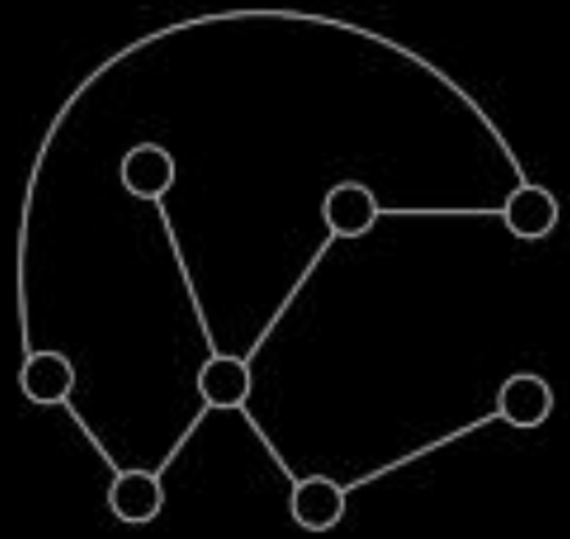
(C) 5

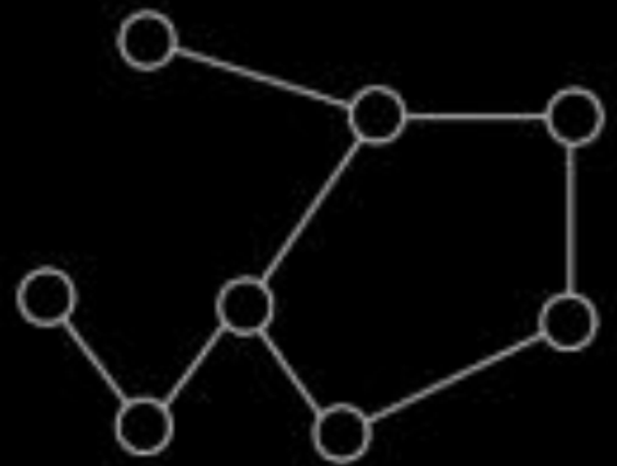
(D) 6

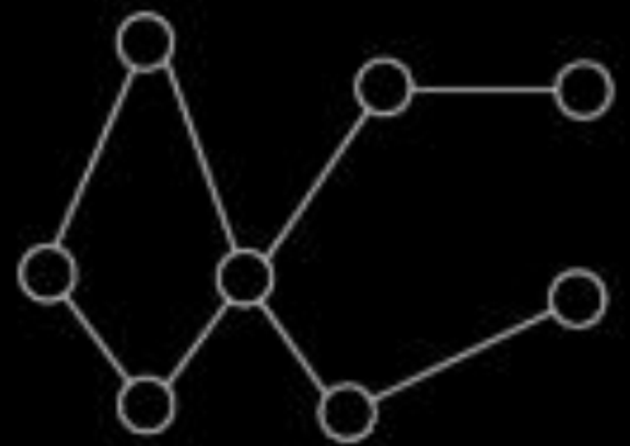
Q. Which of the following graphs is isomorphic to



A


B


C


D


Q. Consider an undirected random graph of eight vertices. The probability that there is an edge between a pair of vertices is $\frac{1}{2}$ What is the expected number of unordered cycles of length three?

(A) $\frac{1}{8}$

(B) 1

(C) 7

(D) 8

Q. Which of the following statements is/are TRUE for undirected graph?

P: Number of odd degree vertices is even.

Q: Sum of degrees of all vertices is even.

(A) P only

(B) Q only

(C) Both P and Q

(D) Neither P nor Q

Q.

Consider an undirected graph G where self-loops are not allowed. The vertex set of G is $\{(i, j): 1 \leq i \leq 12, 1 \leq j \leq 12)$. There is an edge between (a, b) and (c, d) if $|a - c| \leq 1$ and $|b - d| \leq 1$. The number of edges in this graph is_____

Q. An ordered n-tuple $(d_1, d_2, \ldots, d_n)$ with $d_1 \geq d_2 \geq \ldots \geq d_n$, is called graphic if there exists a simple undirected graph with n vertices having degrees $d_1, d_2, \ldots, d_n$ respectively. Which of the following 6-tuples is NOT graphic?

A. (1, 1, 1, 1, 1, 1)

B. (2, 2, 2, 2, 2, 2)

C. (3, 3, 3, 1, 0, 0)

D. (3, 2, 1, 1, 1, 0)

Q.

The maximum number of edges in a bipartite graph on 12 vertices is______.

Q.

A cycle on n vertices is isomorphic to its complement.
The value of n is______.

Q. If G is a forest with n-vertices and k connected components, how many edges does G have?

(A) $\lfloor n/k \rfloor$

(B) $\lceil n/k \rceil$

(C) $n - k$

(D) $n - k + 1$

Q. Let G be a connected planar graph with 10 vertices. If the number of edges on each face is three, then the number of edges in G is _____.

Q. A graph is self-complementary if it is isomorphic to its complement. For all self-complementary graphs on n vertices, n is?

(A) A multiple of 4

(B) Even

(C) Odd

(D) Congruent to 0 mod 4, or 1 mod 4

Q. Consider a binary tree T that has 200 leaf nodes. Then, the number of nodes in T that have exactly two children are_______

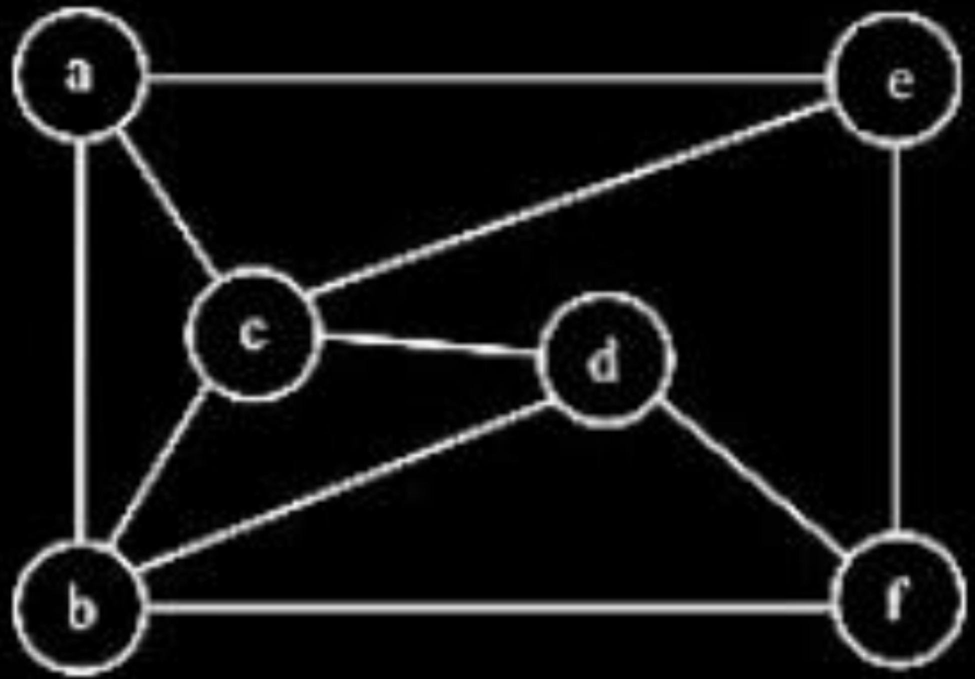
Q. The minimum number of colours that is sufficient to vertex-colour any planar graph is______.

Q. Let T be a tree with 10 vertices. The sum of the degrees of all the vertices in T is____

Q.

G is an undirected graph with n vertices and 25 edges such that each vertex of G has degree at least 3. Then the maximum possible value of n is_____.

Q. The chromatic number of the following graph is_____.



Q.

Graph G is obtained by adding vertex s to $K_{3,4}$ and making s adjacent to every vertex of $K_{3,4}$ The minimum number of colours required to edge-colour G is ________.

Q.

In an undirected connected planar graph G, there are eight vertices and five faces. The number of edges in G is _____

Q.

Consider a simple undirected graph of 10 vertices. If the graph is disconnected, then the maximum number of edges it can have is______.

2 mins Summary

- **Topic**: Simple connected planar graph
- **Topic**: Euler's equation for connected planar graph
- **Topic**: Polyhedral graph
- **Topic**: Euler's equation for disconnected planar graph
- **Topic**: Vertex coloring

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THANK - YOU

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