

COMPUTER SCIENCE & IT

DIGITAL LOGIC




Lecture No. 12.

Combinational Circuit



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A collection of various tools including pliers, wire cutters, and剥线刀, along with several multi-colored wires, arranged in a circular pattern around a central dark blue circle.

Recap of Previous Lecture

Decoder

Encoder

A collection of various tools and cables arranged in a circular pattern around a central dark blue circle. The tools include pliers, wire cutters, a screwdriver, and a pair of scissors. The cables are of various colors (red, blue, green, yellow, black) and are bundled together.

Topics to be Covered

Question Discussion

[NAT]

What is the minimum number of 2-input NOR gates required to implement a 4-variable function expressed in sum-of-minterms form as $f = \Sigma(0, 2, 5, 7, 8, 10, 13, 15)$? Assume that all the inputs and their complements are available. 3

[GATE-2019-CS: 2M]

$$BD + \overline{A}\overline{C}$$

↓₄

| | $\overline{C}\overline{D}$ | $\overline{C}D$ | CD | $C\overline{D}$ |
|----------------------------|----------------------------|-----------------|------|-----------------|
| $\overline{A}\overline{B}$ | 1 | | | 1 |
| $\overline{A}B$ | | 1 | 1 | |
| AB | | 1 | 1 | |
| $A\overline{B}$ | 1 | | | 1 |

$$BD + \overline{B}\overline{D} = (B + \overline{D})(\overline{B} + D)$$

$$= B \odot D$$

$$f = (B + \overline{D})(\overline{B} + D)$$

$$\overline{f} = \frac{1}{B + \overline{D}} + \frac{1}{\overline{B} + D}$$

$$\overline{f} = X + Y$$

$$f = \frac{1}{X + Y}$$

1 NOR \rightarrow R

[Question]

$$\underline{\underline{f(x, y, z)}}$$

A logical function $f(x, y, z)$ is given as

$$f(x, y, z) = x\bar{y} + yz$$

Another logical function

$$\underline{\underline{f_1(x, y, z) = f[\bar{f}, f, z]}}$$
 will be

a. $\bar{x} + y + z$

b. $\bar{x}y + yz$

c. $(x + \bar{y})(\bar{y} + \bar{z})$

d. $(x + \bar{y})(\bar{y} + \bar{z})(y + z)$

$$\begin{aligned} f_1(x, y, z) &= \underline{\underline{f}}[\bar{f}, f, z] = \bar{f} \cdot \bar{f} + f \cdot z \\ &= \pi(4) = (\bar{x} + y + z) = \bar{f} + f \cdot z \\ &= (\bar{f} + z) \end{aligned}$$

f_1

| | $\bar{y}\bar{z}$ | $\bar{y}z$ | $y\bar{z}$ | yz |
|-----------|------------------|------------|------------|------|
| \bar{x} | 1 | 1 | 1 | 1 |
| x | | 1 | 1 | 1 |

$$= \bar{x} + y + z$$

[Question]



A 3-input majority gate is defined by the logic function

$M(a, b, c) = ab + bc + ca$. Which one of the following gate is

$M(\overline{M(a, b, c)}, M(a, b, \bar{c}), c)$?

(a) 3-input NAND gate

(b) 3-input XOR gate

(c) 3-input NOR gate

(d) 3-input XNOR gate

$$\checkmark M[1, 1, 0] = 1$$

$$\checkmark M[0, 1, 0] = 0$$

$$M_1(a, b, c) = M[\overline{M(a, b, c)}, M(a, b, \bar{c}), c]$$

$$M_1(0, 0, 0) = M[1, 0, 0] = 0 \quad \checkmark$$

$$a \cdot b \cdot c \quad \checkmark \quad 1$$

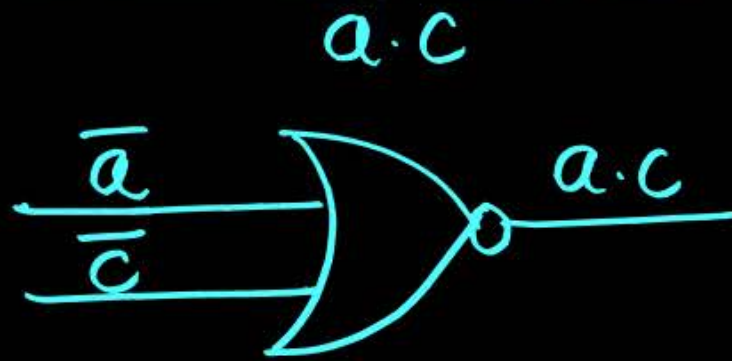
$$a \oplus b \oplus c$$

$$a + b + c \quad 1$$

[NAT]

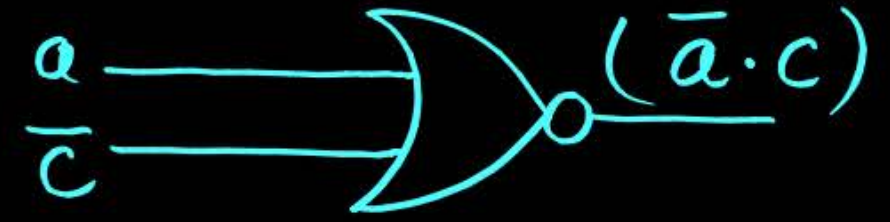


Consider the Karnaugh map given below, where X represents "don't care" and blank represents 0.



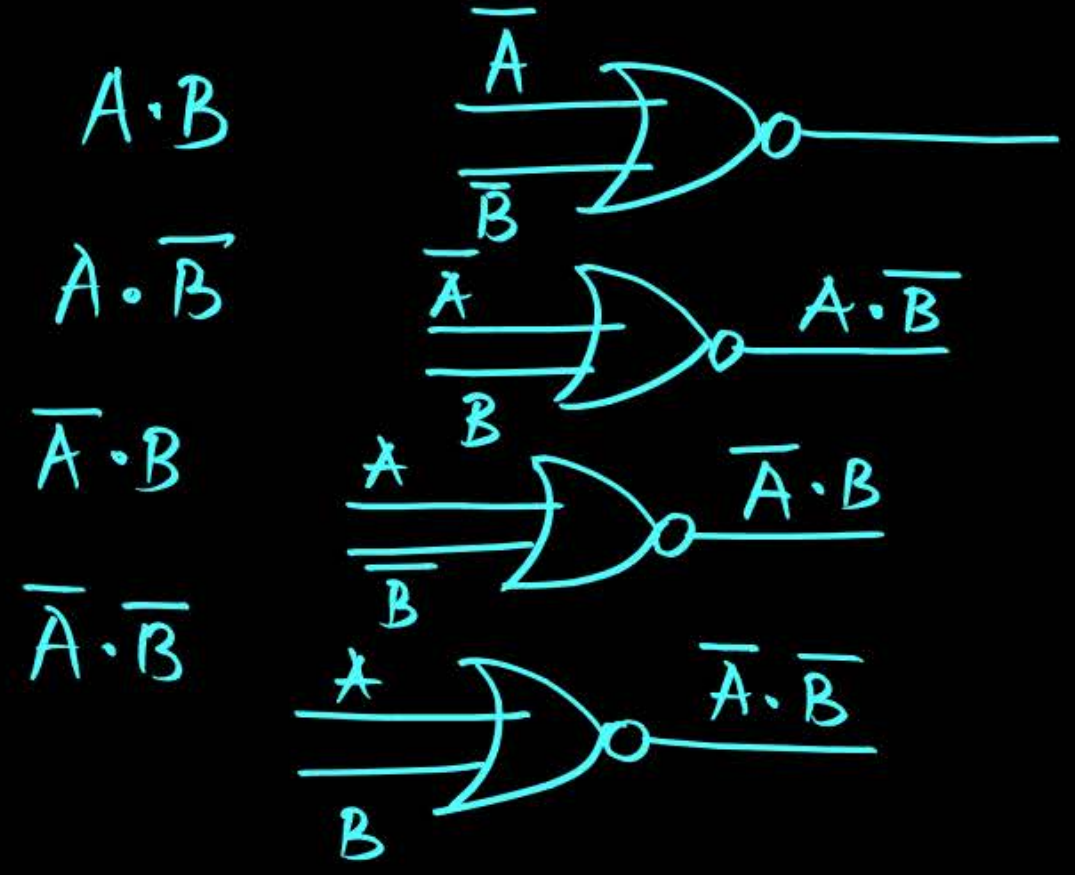
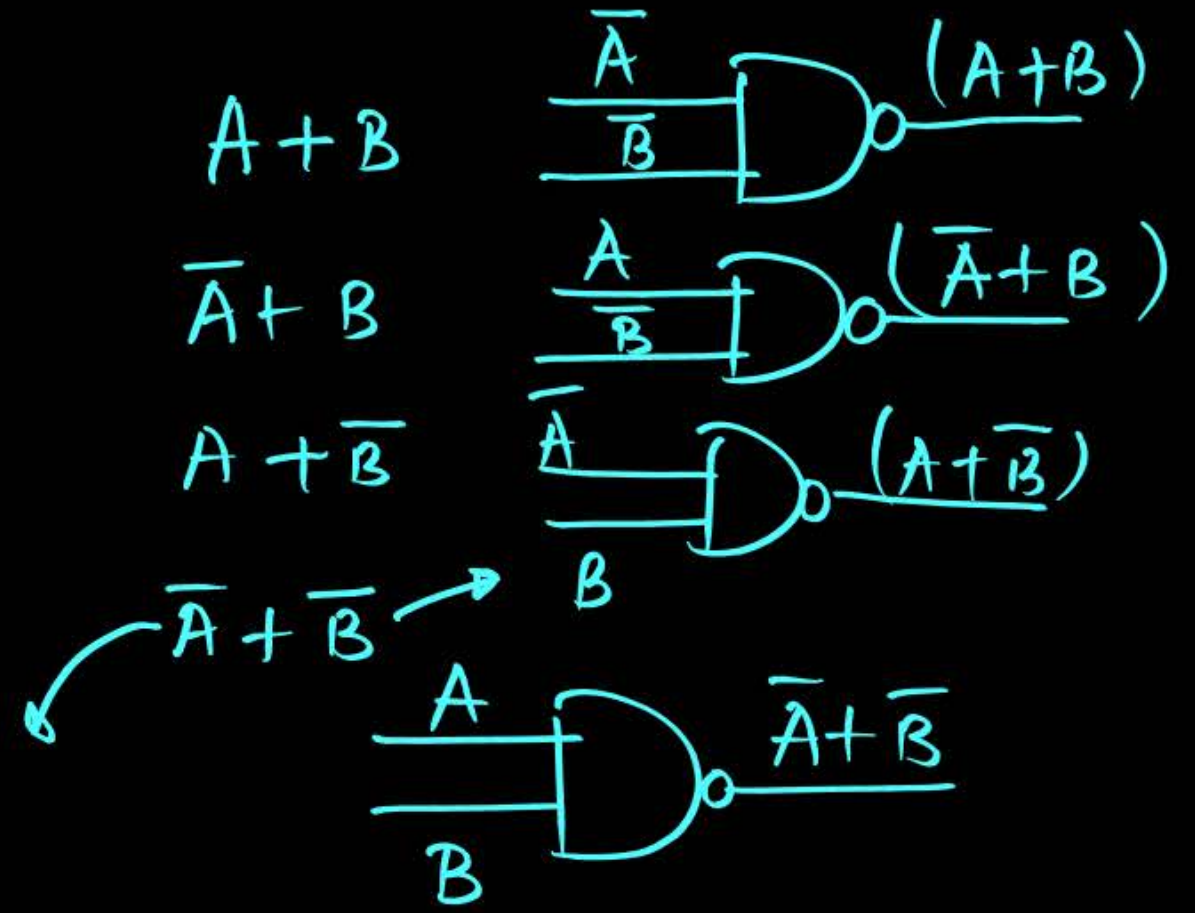
| ba \ dc | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00 | | x | x | |
| 01 | 1 | | | x |
| 11 | 1 | | | 1 |
| 10 | | x | x | |

$$f = c \cdot \bar{a}$$



Assume for all inputs(a, b, c, d) , the respective complements ($\bar{a}, \bar{b}, \bar{c}, \bar{d}$) are also available. The above logic is implemented using 2-input NOR gates only. The minimum number of gates required is 1.

[GATE-2017-CS: 1M]



[MCQ]

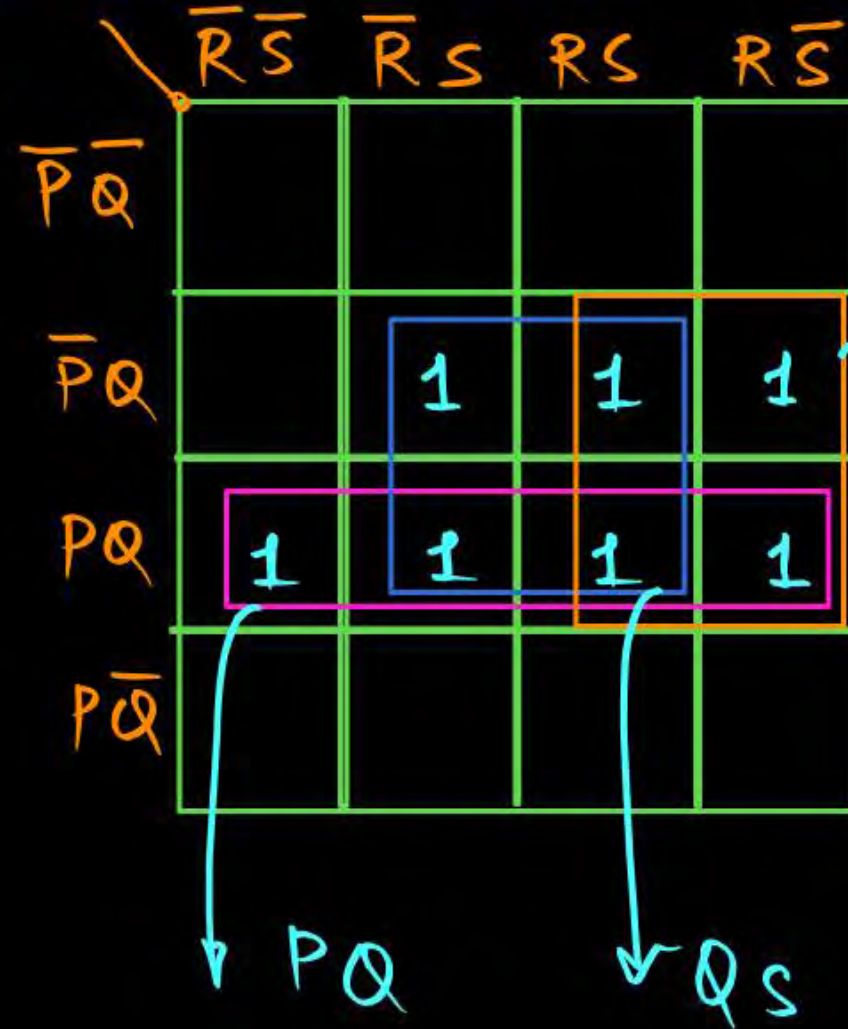
Consider the following Boolean expression for

$$F: F(P, Q, R, S) = PQ + P'QR + P'QR'S$$

The minimal sum-of-products form of F is

[GATE-2014-CS: 1M]

- ☒ A $PQ + QR + QS$
- ☐ B $P + Q + R + S$
- ☐ C $\bar{P} + \bar{Q} + \bar{R} + \bar{S}$
- ☐ D $\bar{P}R + \bar{P}RS + P$



QR

$$= PQ + QR + QS$$

[MCQ]

Consider the following minterm expression for

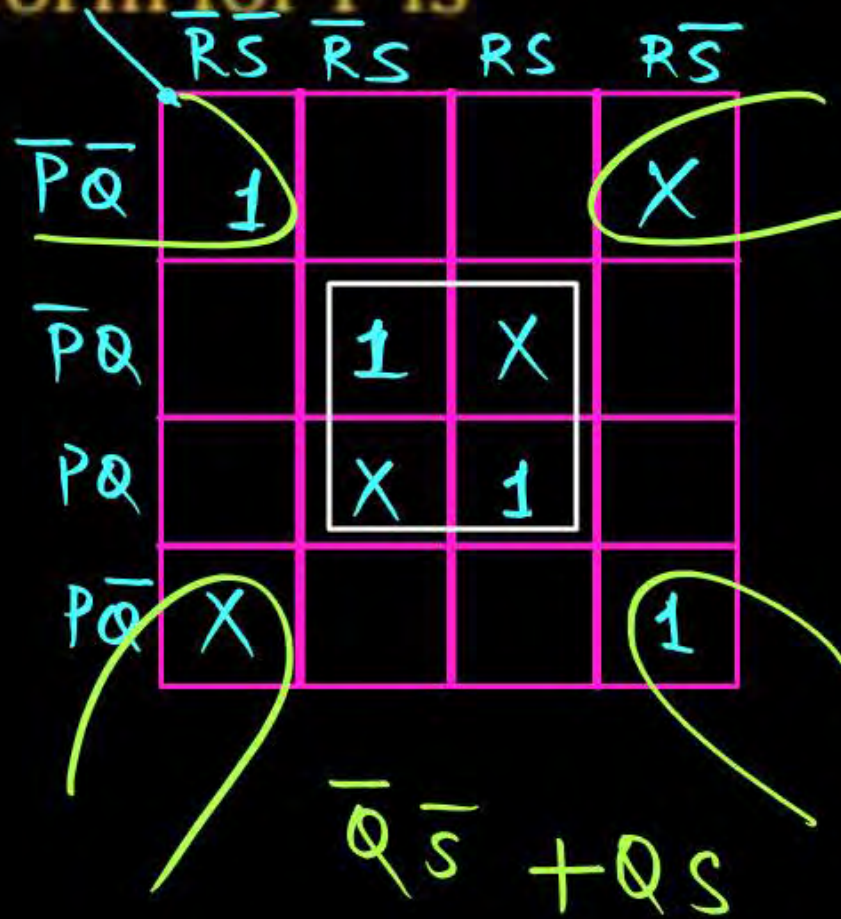
$$F: F(P, Q, R, S) = \sum 0, 2, 5, 7, 8, 10, 13, 15$$

The minterms 2, 7, 8 and 13 are 'do not care' terms.

The minimal sum-of-products form for F is

[GATE-2014-CS: 1M]

- ☐ A $Q\bar{S} + \bar{Q}S$
- ☒ B $\bar{Q}\bar{S} + QS$
- ☐ C $\bar{Q}\bar{R}\bar{S} + \bar{Q}R\bar{S} + Q\bar{R}S + QRS$
- ☐ D $\bar{P}\bar{Q}\bar{S} + \bar{P}QS + PQS + P\bar{Q}\bar{S}$



[MCQ]



What is the minimal form of the karnaugh map shown below? Assume that X denotes a don't care term

[GATE-2012-CS: M]

| cd \ ab | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00 | 1 | X | X | 1 |
| 01 | X | | | 1 |
| 11 | | | | |
| 10 | 1 | | | X |

- ☐ A $\bar{b}\bar{d}$ ✗
- ☒ B $\bar{b}\bar{d} + \bar{b}\bar{c}$
- ☐ C $\bar{b}\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d}$ ✗
- ☐ D $\bar{b}\bar{d} + \bar{b}\bar{c} + \bar{c}\bar{d}$ ✗

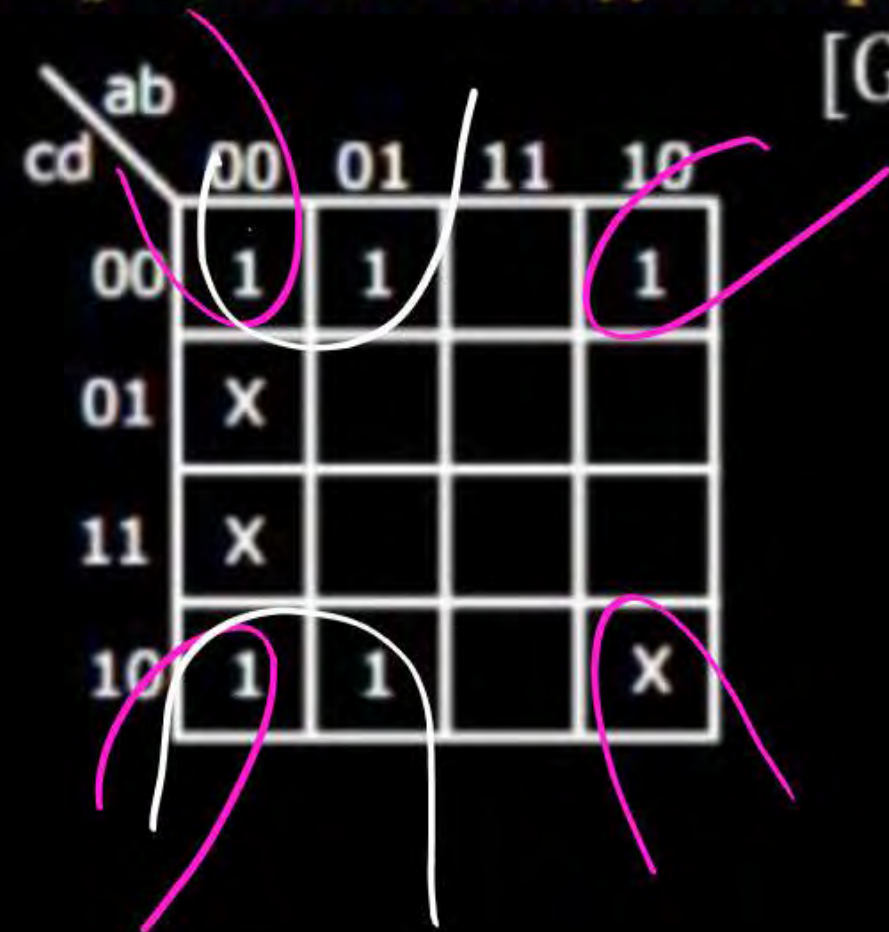
$$\bar{b}\bar{d} + \bar{c}\bar{b}$$

[MCQ]

In the Karnaugh map shown below, X denotes a don't care term. What is the minimal form of the function represented by the Karnaugh map?

[GATE-2008-CS: M]

- ☒ A $\bar{b}. \bar{d} + \bar{a}. \bar{d}$
- ☐ B $\bar{a}. \bar{b} + \bar{b}. \bar{d} + \bar{a}. b. \bar{d}$
- ☐ C $\bar{b}. \bar{d} + \bar{a}. b. \bar{d}$
- ☐ D $\bar{a}. \bar{b} + \bar{b}. \bar{d} + \bar{a}. \bar{d}$



$$\bar{a}\bar{d} + \bar{b}\bar{d}$$

[MCQ]

Consider the following Boolean function of four variables $f(A, B, C, D) = \Sigma(2, 3, 6, 7, 8, 9, 10, 11, 12, 13)$ The function is [GATE-2008-CS: 1M]

- ☐ A independent of one variable
- ☐ B Independent of two variables
- ☐ C Independent of three variables
- ☐ D Dependent on all the variables

H.W

[MCQ]

Given $f(w, x, y, z) = \sum m(0, 1, 2, 3, 7, 8, 10) + \sum d(5, 6, 11, 15)$, where d represents the don't-care condition in Karnaugh maps. Which of the following is a minimum product-of-sums (POS) form of $f(w, x, y, z)$? [GATE-2017-CS: 1M]

- ☒ A $f = (\bar{w} + \bar{z})(\bar{x} + z)$
- ☐ B $f = (\bar{w} + z)(x + z)$
- ☐ C $f = (w + z)(\bar{x} + z)$
- ☐ D $f = (w + \bar{z})(\bar{x} + z)$

| | $\bar{y}\bar{z}$ | $\bar{y}z$ | $y\bar{z}$ | yz |
|------------------|------------------|------------|------------|------|
| $\bar{w}\bar{x}$ | 1 | 1 | 1 | 1 |
| $\bar{w}x$ | | X | 1 | X |
| $w\bar{x}$ | | | X | |
| wx | 1 | | X | 1 |

$$= \bar{x}\bar{z} + \bar{w}z = (\bar{z} + \bar{x})(\bar{z} + \bar{w})$$

[MCQ]

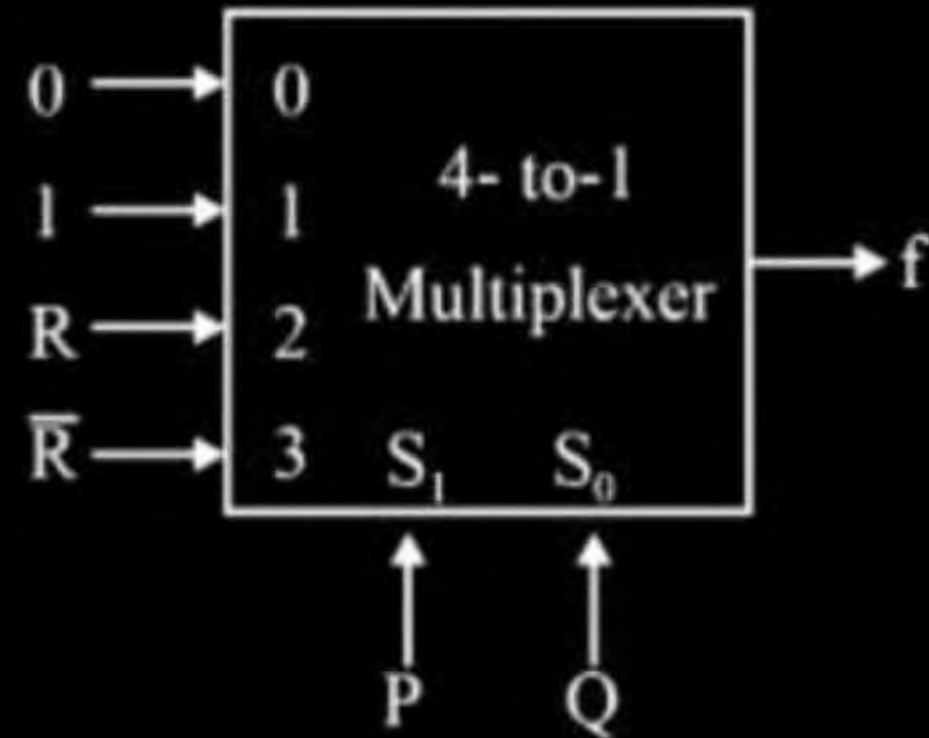


Consider the 4 – to – 1 multiplexer with two select S_1 and S_0 given below.

The minimal sum of products from the Boolean expression for the output F of the multiplexer is -

[GATE-2014-CS: 2M]

- A** $\bar{P}Q + Q\bar{R} + P\bar{Q}R$
- B** $\bar{P}Q + \bar{P}Q\bar{R} + PQR + P\bar{Q}R$
- C** $\bar{P}QR + \bar{P}Q\bar{R} + Q\bar{R} + P\bar{Q}R$
- D** $PQ\bar{R}$



H.W

[Question]

Logical circuit is implemented as

Y represents

- (A) Borrow output of full subtractor
- (B) Carry output of full adder
- (C) 3-input XOR gate
- (D) 3-input NOR gate



H.W

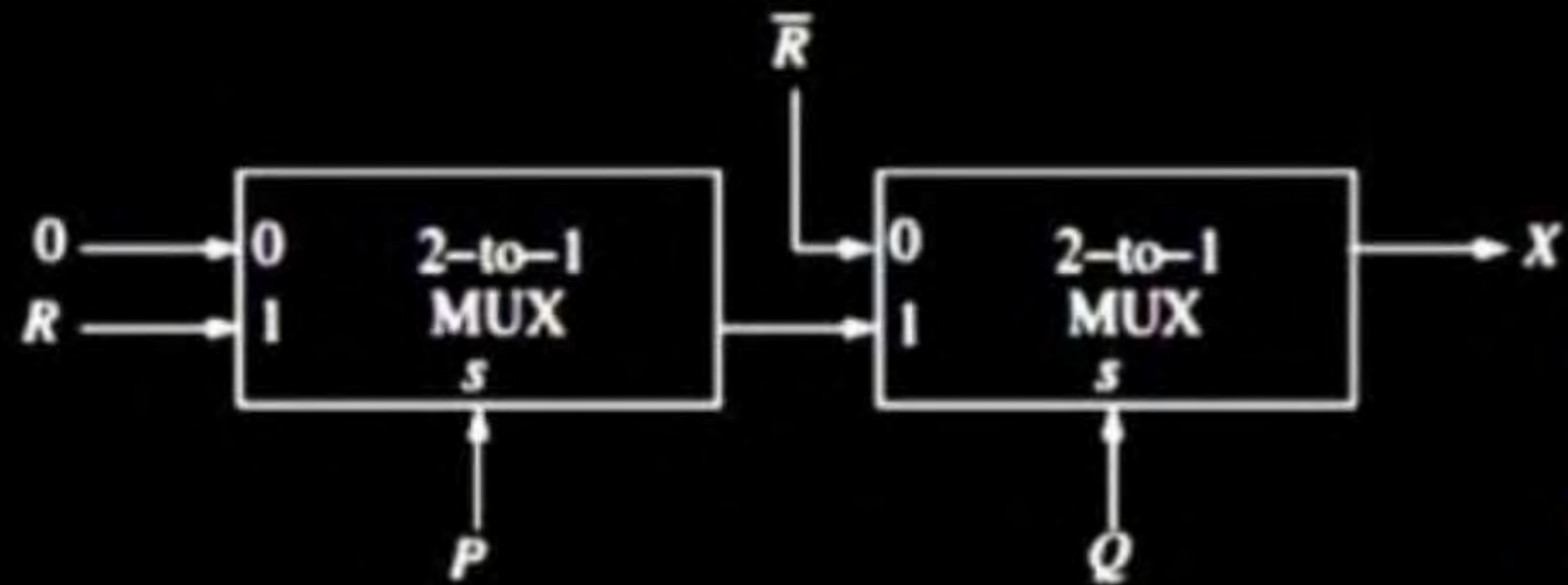
[MCQ]



Consider the two cascaded 2 to 1 multiplexers as shown in the figure.

The minimal sum of products form of the output X is : [GATE-2016-CS: 1M]

- A $\bar{P}\bar{Q} + PQR$
- B $\bar{P}Q + QR$
- C $PQ + \bar{P}\bar{Q}R$
- D $\bar{Q}\bar{R} + PQR$



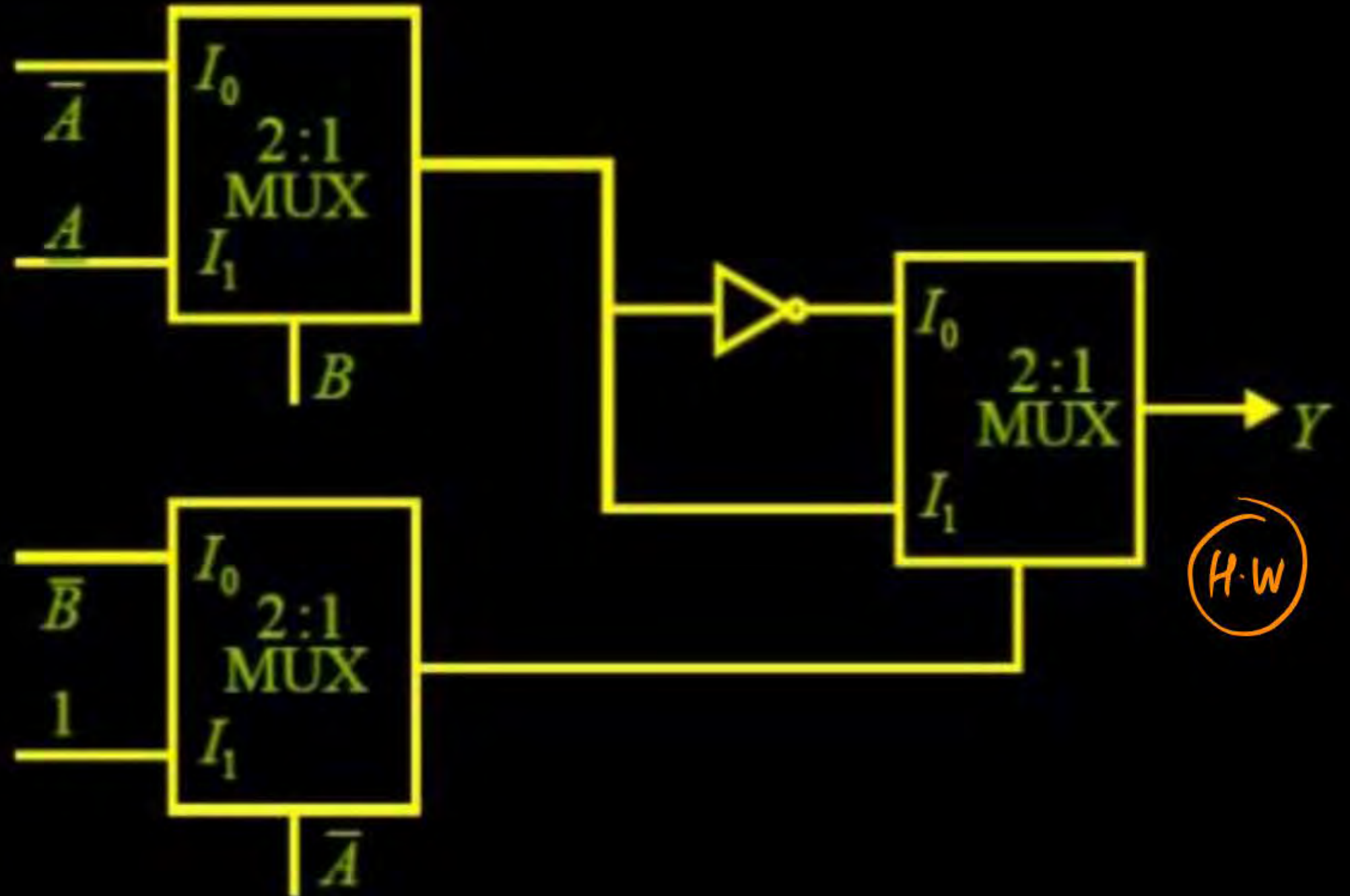
H.W

[Question]

Combination circuit is designed as shown below:-

Output Y is

- (a) 2 input NOR gate
- (b) 2 input AND gate
- (c) 2 input OR gate
- (d) 2 input NAND gate

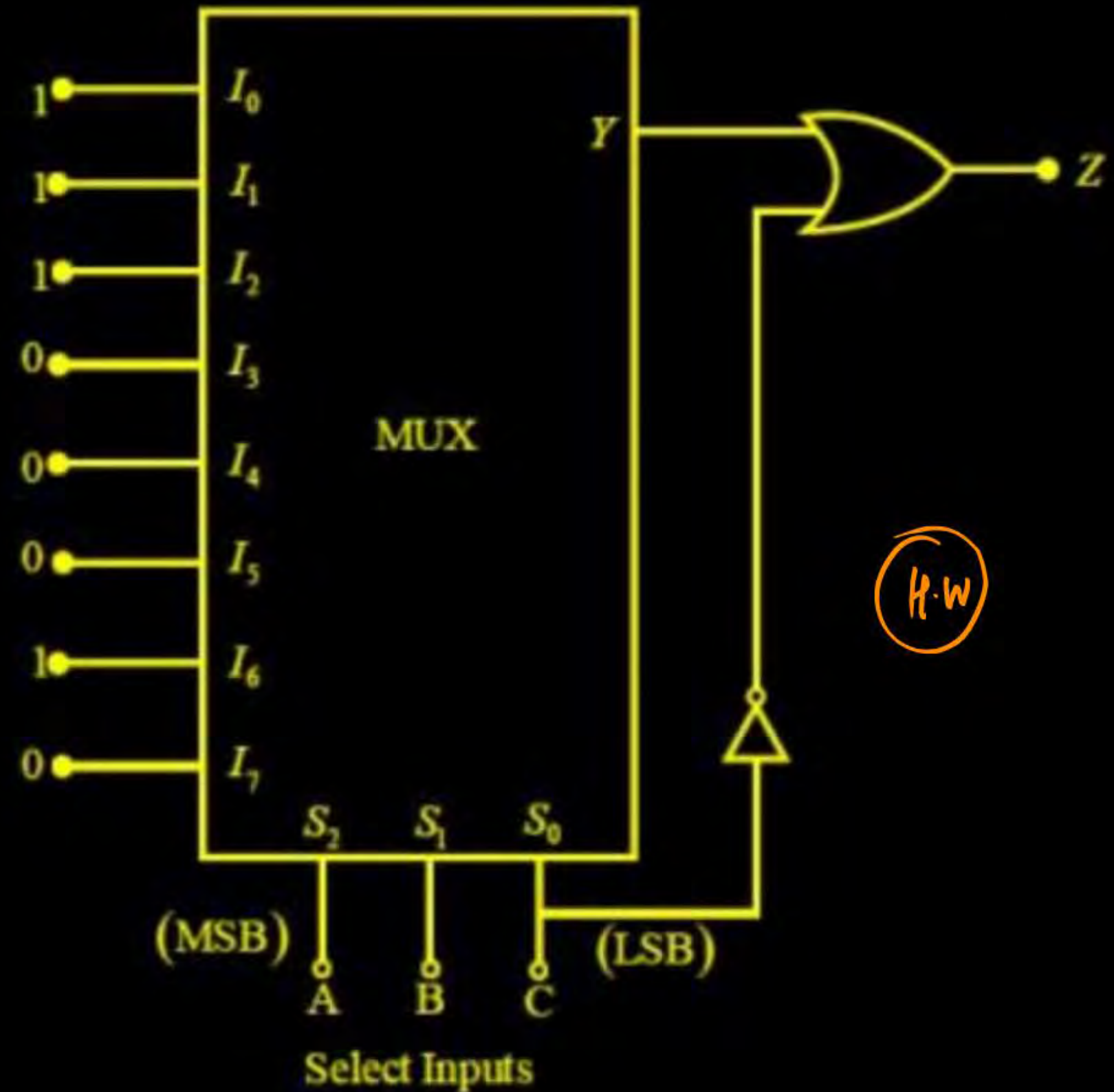


H.W

[Question]

A combinational circuit using an 8 to 1 multiplexer is shown in the following figure. The minimized expression for the output (Z) is

- (a) $C(\bar{A} + \bar{B})$
- (b) $C(A + B)$
- (c) $\bar{C} + \bar{A}\bar{B}$
- (d) $\bar{C} + AB$



H.W

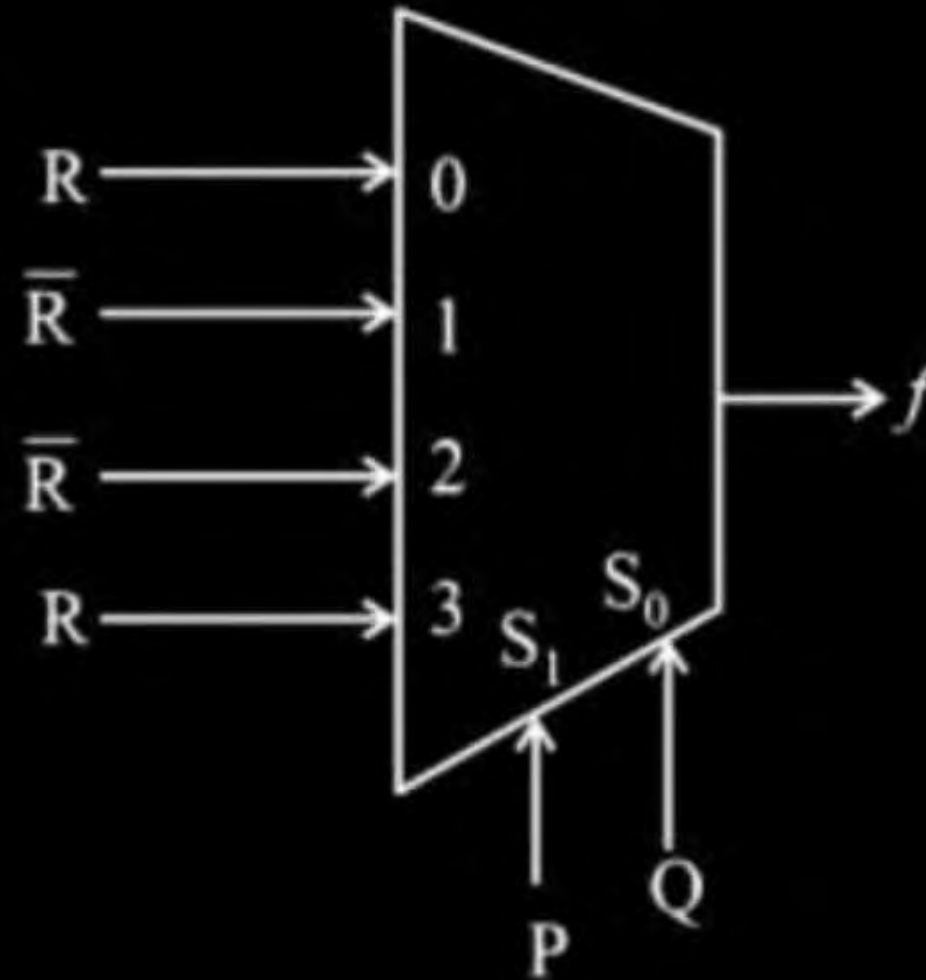
[MCQ]



The Boolean expression of the output f of the multiplexer shown below is :

[GATE-2010-CS: 1M]

- A** $\overline{P \oplus Q \oplus R}$
- B** $P \oplus Q \oplus R$
- C** $P + Q + R$
- D** $\overline{P + Q + R}$



H.W

[Question]

logical function $f(A, B, C)$ is given as

$$f(A, B, C) = \overline{AB + BC + CA}$$

Then $f_1(A, B, C) = f[AB, BC, CA]$

f_1 implemented using 2 : 1 MUX with C as select input then input I_0 & I_1 of 2: 1 MUX will be

- (a) $I_0 = 1, I_1 = \bar{A}$
- (b) $I_0 = 0, I_1 = \bar{A} + \bar{B}$
- (c) $I_0 = 1, I_1 = \bar{A} + \bar{B}$
- (d) $I_0 = 0, I_1 = \bar{A} + B$

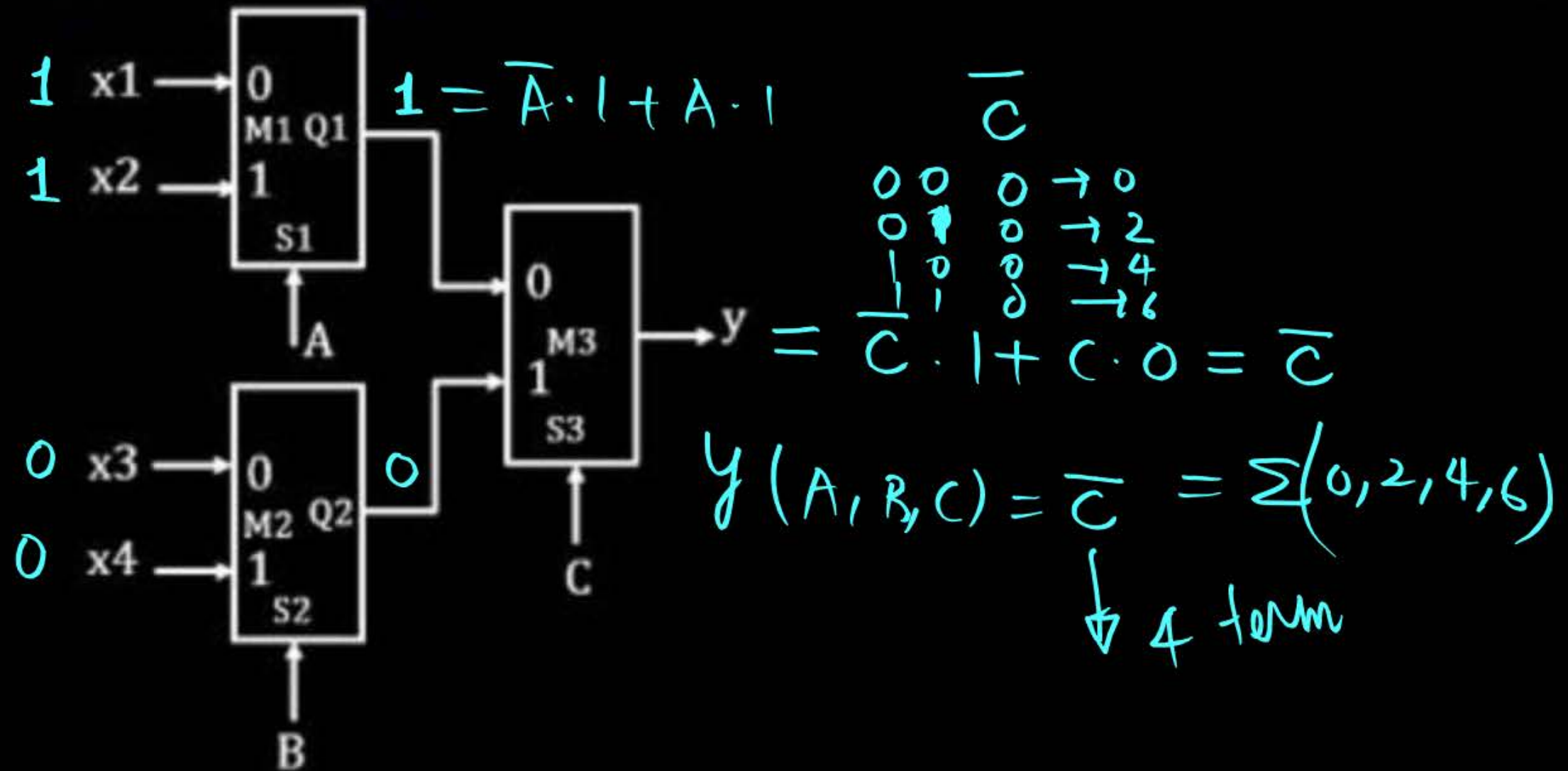
H.W

[NAT]



Consider a digital logic circuit consisting of three 2-to-1 multiplexers M1, M2, and M3 as shown below. X1 and X2 are inputs of M1. X3 and X4 are inputs of M2. A, B, and C are select lines of M1, M2, and M3, respectively.

For an instance of inputs $X1 = 1$, $X2 = 1$, $X3 = 0$, and $X4 = 0$, the number of combinations of A, B, C that give the output $Y=1$ is 4 [GATE-2024-CS: 1M]



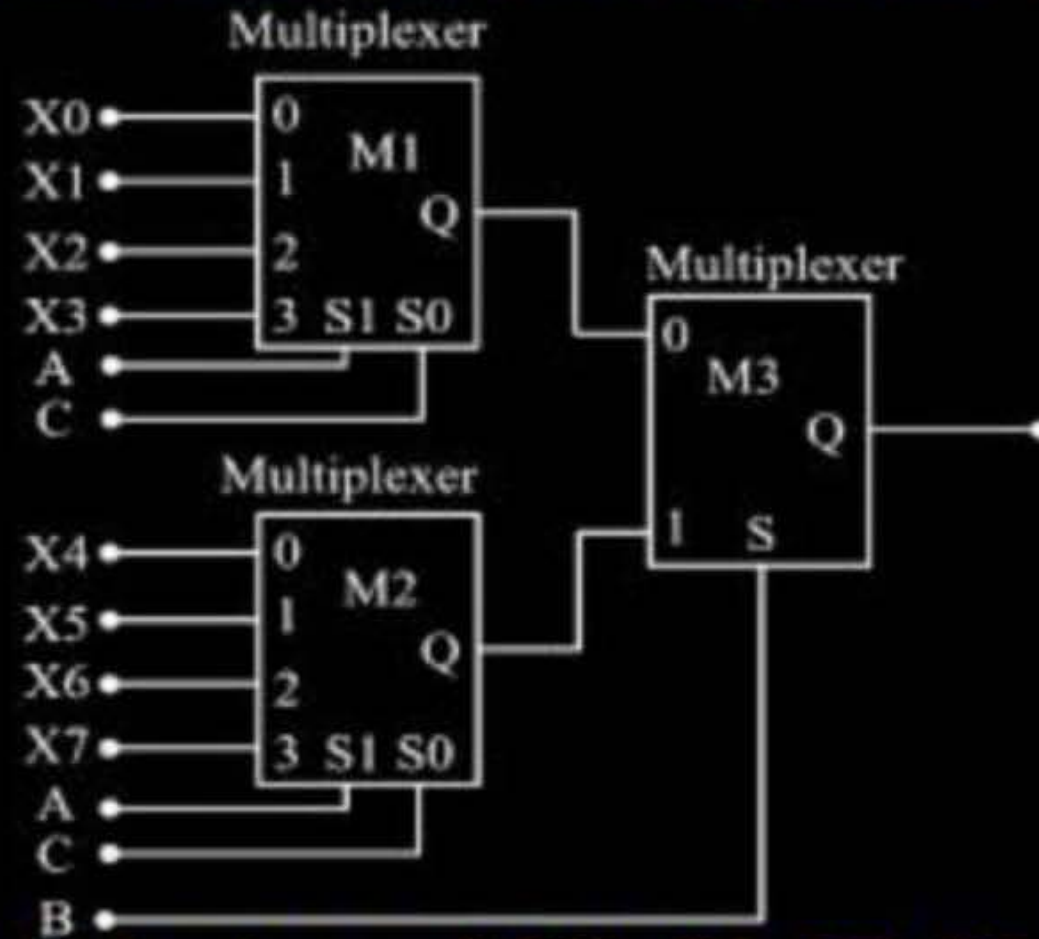
[MCQ]



A Boolean digital circuit is composed using two 4-input multiplexers (M1 and M2) and one 2-input multiplexer (M3) as shown in the figure. X0-X7 are the inputs of the multiplexers M1 and M2 and could be connected to either 0 or 1. The select lines of the multiplexers are connected to Boolean variables A, B and C as shown.

$$f(A, B, C) = \bar{A} + \bar{A}\bar{C} + A\bar{B}C$$

S_2 S_1 S_0



$$8:1 \xrightarrow{4:1} 2(4:1) + 1(2:1)$$

$$A=0, C=0, B=0$$

$$f(0,0,0) = 1, X_0 = 1$$

$$A=1, B=0, C=1 \rightarrow X_3$$

$$f(1,0,1) = 1, X_3 = 1$$

Which one of the following set of values of (X0, X1, X2, X3, X4, X5, X6, X7) will realise the Boolean function $\bar{A} + \bar{A}\bar{C} + A\bar{B}C$

[GATE-2023-CS: 2M]

- A** $(1, \underline{1}, 0, 0, 1, 1, 1, 0)$ ~~X~~
- B** $(1, \underline{1}, 0, 0, 1, 1, 0, 1)$ ~~X~~
- C** $(1, \underline{1}, 0, 1, 1, 1, 0, 0)$ ✓
- D** $(0, 0, 1, 1, 0, 1, 1, 1)$ ~~X~~

[MCQ]

In the following truth table, $V = 1$ if and only if the input is valid. What function does the truth table represent ?

What function does the truth table represent ?

[GATE-2013-CS: 1M]

- ☒ A Priority encoder
- ☐ B Decoder
- ☐ C Multiplexer
- ☐ D Demultiplexer

| Inputs | | | | Outputs | | |
|--------|-------|-------|-------|---------|-------|-----|
| D_0 | D_1 | D_2 | D_3 | X_0 | X_1 | V |
| 0 | 0 | 0 | 0 | x | x | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| X | 1 | 0 | 0 | 0 | 1 | 1 |
| X | x | 1 | 0 | 1 | 0 | 1 |
| X | x | x | 1 | 1 | 1 | 1 |

[MSQ]



Consider a Boolean expression given by $F(X, Y, Z) = \Sigma(3, 5, 6, 7)$.

Which of the following statements is/are CORRECT?

[GATE-2024-CS: 1M]

- ☐ A $F(X, Y, Z)$ is independent of input X
- ☐ B $F(X, Y, Z) = XY + YZ + XZ$
- ☐ C $F(X, Y, Z)$ is independent of input Y
- ☐ D $F(X, Y, Z) = \pi(0, 1, 2, 4)$

H.W

[MCQ]



The number of min-terms after minimizing the following Boolean expression is

____. $[D' + AB' + A'C + AC'D + A'C'D]' = \Sigma(15) = ABCD$ [GATE-2015-CS: 1M]

- ☒ A 1
- ☐ B 2
- ☐ C 3
- ☐ D 4

| | $\overline{C}\overline{D}$ | $\overline{C}D$ | CD | $C\overline{D}$ |
|----------------------------|----------------------------|-----------------|------|-----------------|
| $\overline{A}\overline{B}$ | . | . | . | . |
| $\overline{A}B$ | . | . | . | . |
| AB | . | . | 1 | . |
| $A\overline{B}$ | . | . | . | . |

[MSQ]

Let, $x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0$ where x_1, x_2, x_3, x_4 are Boolean variables, and \oplus is the XOR operator. Which one of the following must always be TRUE?

[GATE-2022-CS: 1M]

- A** $x_1 x_2 x_3 x_4 = 0$
- B** $x_1 x_3 + x_2 = 0$
- C** $\bar{x}_1 \oplus \bar{x}_3 = \bar{x}_2 \oplus \bar{x}_4$
- D** $x_1 + x_2 + x_3 + x_4 = 0$

H.W.

[MCQ]

What is the minimum number of gates required to implement the Boolean function $(AB + C)$ if we have to use only 2-input NOR gates? [GATE-2009-CS: 1M]

- ☐ A 2
- ☐ B 3
- ☐ C 4
- ☐ D 5

H.W

[Question]

A 2-bit binary multiplier can be Implemented using

- (a) 2 input AND gates only
- (b) ✓ Six 2-input AND gates and two XOR gates.
- (c) Two 2-input NORs and XNOR gate.
- (d) XOR gates and shift registers

$a_1 a_0$

$b_1 b_0$

$a_1 b_0 \quad a_0 b_0$

$a_1 b_1 \quad b_1 a_0$

c_0

$c_3 \quad s_2 \quad s_1 \quad a_0 b_0$

4 AND + 1 HA + 1 HA

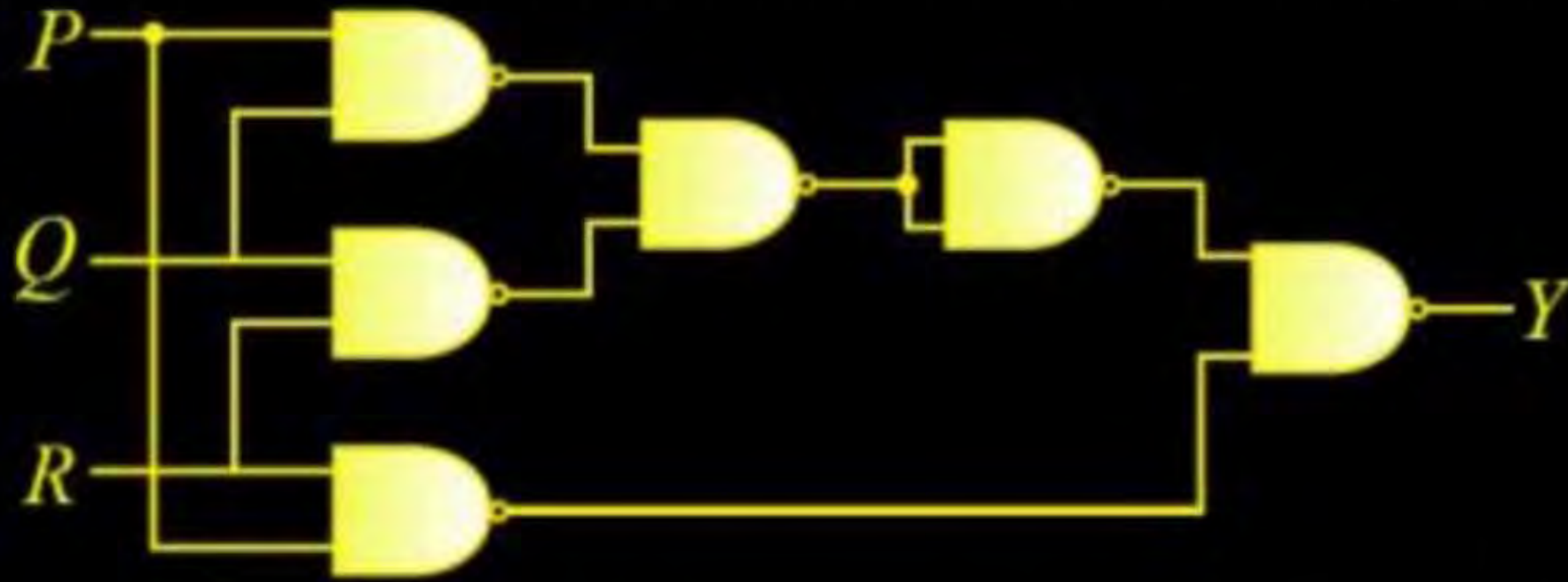
↓
1 XOR
1 AND

↓
1 XOR
1 AND

[Question]



The output Y of the circuit below is always “1” when



H.W

- (a) Two or more of the input P , Q , R are “0”.
- (b) Two or more of the inputs P , Q , R are “1”.
- (c) Any odd number of the inputs P , Q , R are “0”.
- (d) Any odd number of the inputs P , Q , R are “1”.



2 Minute Summary



Question Discussion.

Thank you

GW
Soldiers!

