GATE

CS & DA

DPP: 1

Linear Algebra

Q1 Consider the following two statements with respect to the matrices $A_{m\times n}$, $B_{n\times m}$, $C_{n\times n}$ and $D_{n\times n}$.

Statement 1: tr(AB) = tr(BA)

Statement 2: tr(CD) = tr(DC)

Where tr() represents the trace of a matrix. Which one of the following holds?

(A) Statement 1 is correct and Statement 2 is wrong.

(B) Statement 1 is wrong and Statement 2 is correct.

(C) Both Statement 1 and Statement 2 are correct.

(D) Both Statement 1 and Statement 2 are wrong.

Q2

Calculate the determinant of the following matrix-

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 7 & 3 \\ 4 & 10 & 14 & 6 \\ 3 & 4 & 2 & 7 \end{vmatrix}$$

(A) 4 (B) 5

(C) 0 (D) 7

Q3

The determinant of the matrix $A = \begin{bmatrix} x & 4 & y+z \\ y & 4 & z+x \\ z & 4 & x+y \end{bmatrix}$ is equal to.

(A) 4x (B) x+y+z

(C) xyz (D) 0

Q4 Find the area of triangle in determinant form whose vertices are A(O, O), B(O, –5), and C(8,O).

(A) 20 (B) 22

(C) 23 (D) 24

Q5

Let $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then |2A| is equal to.

(A) $4\cos 2\theta$ (B) 1

(C) 2 (D) 4

Q6

If A, B, C are non-singular n × n matrices, then $(ABC)^{-1}$ = ________.

(A) $A^{-1}C^{-1}B^{-1}$

(B) $C^{-1}B^{-1}A^{-1}$

(C) $C^{-1}A^{-1}B^{-1}$

(D) $B^{-1}C^{-1}A^{-1}$

Q7 Let A, B, C, D be n × n matrices, each with non zero determinant and ABCD = I then B =

(A) $A^{-1}D^{-1}C^{-1}$ (B) CDA

(C) ABC (D) Does not exist

Q8 The value of the determinant of the matrix $A = \begin{bmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{bmatrix}$ is equal to.

(A) (x – y) (y – z) (z – x)

(B) (x – y) (y – z) (z – x) (x + y+ z)

(C) (x + y+ z)

(D) (x – y) (y – z) (z – x) (xy + yz+ zx)

Q9 If A is 3 × 3 matrix and |A| = 4, then $|A^{-1}|$ is equal to-.

(A) $\frac{1}{4}$ (B) $\frac{1}{16}$

(C) 4 (D) 2

Q10 If |A| = 0 where A is defined as the matrix $\begin{bmatrix} 4 & -4 & 0 \\ a & b+4 & c \\ a & b & c+4 \end{bmatrix}$, then a + b + c is equal to.

(A) 41 (B) 116

(C) 628 (D) – 4

Q11 If I_3 is the identity matrix of orders, the value of $(I_3)^{-1}$ is :

(A) 0

(B) $3I_3$

(C) I_3

(D) Does not exist.

Q12 If A is any square matrix, then

(A) A + A^T is skew symmetric

(B) A – A^T is symmetric

(C) A A^T is symmetric

(D) A A^T is skew symmetric

Q13 Each diagonal element of a skew symmetric matrix is -

(A) Zero

(B) Positive and equal

(C) Negative and equal

(D) Any real number.

Q14 If A is a singular matrix, then adj A is

(A) Singular (B) Non-singular

(C) Symmetric (D) Non defined

Q15 If A + B = $\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$ and A − 2B = $\begin{bmatrix}-1 & 0\\1 & -1\end{bmatrix}$, then B is equal to.

(A) $\frac{1}{3}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$ (B) $\frac{1}{3}\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$

(C) $\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}$ (D) $\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$

Q16 If x + $\begin{bmatrix}2 & 1\\6 & 1\end{bmatrix} = \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$, then 'X' is equal to

(A) $\begin{bmatrix}1 & 1\\0 & 6\end{bmatrix}$ (B) $\begin{bmatrix}0 & -1\\0 & -6\end{bmatrix}$

(C) $\begin{bmatrix}-1 & 0\\-6 & 0\end{bmatrix}$ (D) $\begin{bmatrix}1 & 0\\6 & 0\end{bmatrix}$

Q17 If $\begin{bmatrix}x & 1\\-1 & -y\end{bmatrix} + \begin{bmatrix}y & 1\\3 & x\end{bmatrix} = \begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$, then

(A) x = −1, y = 0

(B) x = 1, y = 0

(C) x = 0, y = 1

(D) x = 1, y = 1

Q18 Let $A = \begin{bmatrix}2 & 3 & 5\\1 & 0 & 2\\3 & 4 & 5\end{bmatrix}$ and A + B − 4I = 0, then B is equal to.

(A) $\begin{bmatrix}2 & -3 & -5\\-1 & 4 & -2\\-3 & -4 & -1\end{bmatrix}$

(B) $\begin{bmatrix}2 & 3 & 5\\1 & -4 & 2\\3 & 4 & 1\end{bmatrix}$

(C) Both of them

(D) None of them

Q19 $\begin{bmatrix}7 & 1 & 2\\9 & 2 & 1\end{bmatrix}\begin{bmatrix}3\\4\\5\end{bmatrix} + 2\begin{bmatrix}4\\5\end{bmatrix}$ is equal to.

(A) $\begin{bmatrix}45\\44\end{bmatrix}$ (B) $\begin{bmatrix}43\\45\end{bmatrix}$

(C) $\begin{bmatrix}44\\43\end{bmatrix}$ (D) $\begin{bmatrix}43\\50\end{bmatrix}$

Q20 If f(x) = x^2 + 4x − 5 and A = $\begin{bmatrix}1 & 2\\4 & -3\end{bmatrix}$, then f(A) is equal to.

(A) $\begin{bmatrix}0 & -4\\8 & 8\end{bmatrix}$ (B) $\begin{bmatrix}2 & 1\\2 & 0\end{bmatrix}$

(C) $\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}$ (D) $\begin{bmatrix}8 & 4\\8 & 0\end{bmatrix}$

Q21 If A is a symmetric matrix and B is a skew-symmetrix matrix such that $A + B = \begin{bmatrix}2 & 3\\5 & -1\end{bmatrix}$, then AB is equal to.

(A) $\begin{bmatrix}-4 & 2\\1 & 4\end{bmatrix}$

(B) $\begin{bmatrix}4 & -2\\1 & -4\end{bmatrix}$

(C) $\begin{bmatrix}4 & -2\\-1 & -4\end{bmatrix}$

(D) $\begin{bmatrix}-4 & -2\\-1 & 4\end{bmatrix}$

Q22 If A is involutory matrix and I is unit matrix of same order, then (I − A) (I + A) is.

(A) Zero matrix (B) A

(C) I (D) 2A

Q23 If $A = \begin{bmatrix}3 & -6\\a & -2\end{bmatrix}$ is an idempotent matrix, then which of the following is/are TRUE.

(A) a = 4 (B) a =1

(C) |A| = 0 (D) |A| = 2

Q24 If $A = \begin{pmatrix}2 & 4\\-1 & k\end{pmatrix}$ is a nilpotent matrix of index 2, then k equals to.

(A) 2 (B) − 3

(C) 4 (D) − 2

Q25 A square matrix A is said to be orthogonal if A 'A = AA '= I_n, A' is transpose of A

If A and B are orthogonal matrices, of the same order, then which one of the following is an orthogonal matrix

(A) AB (B) A+B

(C) A+iB (D) (A+B)

Q26 Check the nature of the following matrices.

$$A = \begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}.$$

Q27 Check the Nature of the following matrices.

$$A = \begin{bmatrix}\cos\theta & 0 & \sin\theta\\0 & 1 & 0\\-\sin\theta & 0 & \cos\theta\end{bmatrix}.$$

Q28 Check the Nature of the following matrices.

$$A = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\i & -i\end{bmatrix}$$

Q29 Check the Nature of the following matrices.

$$A = \begin{bmatrix}i & 0 & 0\\0 & i & 0\\0 & 0 & i\end{bmatrix}.$$

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Answer Key

Q1	(C)
Q2	(C)
Q3	(D)
Q4	(A)
Q5	(D)
Q6	(B)
Q7	(A)
Q8	(B)
Q9	(A)
Q10	(D)
Q11	(C)
Q12	(C)
Q13	(A)
Q14	(A)
Q15	(B)
Q16	(C)
Q17	(B)
Q18	(A)
Q19	(D)
Q20	(D)
Q21	(C)
Q22	(A)
Q23	(C)
Q24	(D)
Q25	(A)
Q26	The matrix is an Orthogonal matrix as AA^T is coming out to be an identity matrix.
Q27	Orthogonal Matrix
Q28	Unitary Matrix
Q29	Unitary matrix,A unitary matrix is **a complex square matrix whose columns (and rows) are orthonormal.**

Hints & Solutions

Q1 Text Solution:

Given,

order of 'A" is m × n; order of 'B' is 'n × m'

Order of 'C' is n × n; order of 'D' is 'n × n'

For any two matrices A and B, if both AB and BA exist, then tr(AB) = tr(BA)

∴ Both statements 1 and statements 2 are correct

Q2 Text Solution:

As you can see that the third row is a multiple of second row so carrying out the elementary row operation.

$R_3 \rightarrow R_3 - 2R_2$

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 7 & 3 \\ 4 & 10 & 14 & 6 \\ 3 & 4 & 2 & 7 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 7 & 3 \\ 0 & 0 & 0 & 0 \\ 3 & 4 & 2 & 7 \end{vmatrix}$$

Now as all the elements of 3rd row of the determinant is 0, thus the value of determinant is 0.

Thus 'C' is the correct option.

Q3 Text Solution:

$$\begin{bmatrix} x & 4 & y+x \\ y & 4 & z+x \\ z & 4 & x+y \end{bmatrix}$$

using elementary operation -

$C_1 \rightarrow C_1 + C_3$

$$\begin{bmatrix} x+y+z & 4 & y+x \\ x+y+z & 4 & z+x \\ x+y+z & 4 & x+y \end{bmatrix}$$

Now calculating the determinant -

$C_1 \rightarrow C_1 \times \frac{1}{x+y+z}$

$$x+y+z \begin{vmatrix} 1 & 4 & y+z \\ 1 & 4 & z+x \\ 1 & 4 & x+y \end{vmatrix}$$

$C_2 \rightarrow C_2 \times \frac{1}{4}$

$$(x+y+z).\ 4 \begin{vmatrix} 1 & 1 & y+z \\ 1 & 1 & z+x \\ 1 & 1 & x+y \end{vmatrix}$$

As two colums are equal , thus the determinant will be 0.

D is correct options.

Q4 Text Solution:

The area of triangle is calculated by using the formula .

$$\frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

Here, $(x_1, y_1) = (0, 0)$

$(x_2, y_2) = (0, -5)$

$(x_3, y_3) = (8, 0)$

$$\frac{1}{2}\begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & -5 \\ 1 & 8 & 0 \end{vmatrix}$$

Now expanding the determinant using first element of first row we get.

$$\frac{1}{2}\left\{+1\begin{vmatrix} 0 & -5 \\ 8 & 0 \end{vmatrix}\right\} = +\frac{5 \times 8}{2} = 20$$

thus 20 is the correct option.

Q5 Text Solution:

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$|A| = Determinant\ of \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$= \cos^2\theta + \sin^2\theta = 1$

Now using the formula.

$|2A| = 2^2\,|A|$

$= 4 \cdot |A|$

$= 4 \times 1 = 4.$

$|KA| = K^n\,|A|$ where n is the order of determinant.

Q6 Text Solution:

A, B, C are non- singular matrices, thus the inverse of A, B, C. exists. Now, we have to find $(A\,B\,C\,)^{-1}$.

using the reversal law -

$(AB)^{-1} = B^{-1}\,A^{-1}$

Treating BC = M (As a single matrix).

$(ABC)^{-1} = (AM)^{-1} = M^{-1}A^{-1}$

$(BC)^{-1}\,A^{-1} = C^{-1}\,B^{-1}\,A^{-1}.$

Thus B is the correct answers.

Q7 Text Solution:

A, B, C, D are n ×n matrices with non - zero determinant & ABCD = I, As they have non-zero determinant thus the inverse of every matrix exists.

ABCD = I

Post multiply with D^{-1}.

(ABCD) D^{-1} = I. D^{-1}

(ABC)D D^{-1} = D^{-1}

ABC . I = D^{-1} as, D. D^{-1} =I

ABC = D^{-1}

Post mulitiply with C^{-1}

(ABC).

$C^{-1} = D^{-1}\,C^{-1} = AB\left(C\,C^{-1}\right) = D^{-1}$

C^{-1}

$AB\ .\ I = D^{-1}\,C^{-1}$

$AB = D^{-1}\,C^{-1}$

Pre multiply with A^{-1}

$(A^{-1}\,A)B = A^{-1}\,D^{-1}\,C^{-1}$

$I\ .\ B = A^{-1}\,D^{-1}\,C^{-1}$

Thus B = $A^{-1}\,D^{-1}\,C^{-1}$

Thus A is the correct option.

Q8 Text Solution:

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$$A = \begin{bmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{bmatrix}$$

$$|A| = Determinant\ of \begin{bmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$R_2 \to R_2 - R_1$

$$\begin{vmatrix} 1 & x & x^3 \\ 0 & y-x & y^3-x^3 \\ 0 & z-y & z^3-y^3 \end{vmatrix}$$

$$\begin{vmatrix} 1 & x & x^3 \\ 0 & y-x & (y-x)\left(x^2+y^2+xy\right) \\ 0 & z-y & (z-y)\left(y^2+z^2+zy\right) \end{vmatrix}$$

Taking (y − x) common form R_2 & (z − y) commom form R_3

$(y-x)$
$(z$
$-y)$

$$\begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & \left(x^2+y^2+xy\right) \\ 0 & 1 & \left(y^2+z^2+zy\right) \end{vmatrix}$$

expanding through 1st element of 1st column we get -

$$(y-x)\ (z-y)$$

$$\begin{vmatrix} 1 & x^2+y^2+xy \\ 1 & y^2+z^2+zy \end{vmatrix}$$

$(y-x)\ (z-y)$

$\left(y^2+z^2+zy-x^2-y^2-zy\right)$

$(y-x)\ (z-y)\ \left(z^2 - x^2 + y\ (z-x)\right).$

$(y-x)\ (z-y)$

$(\ (z-x)\ (z+x)\ +\ y\ (z-x))$

$(y-x)\ (z-y)\ (z-x)\ (x+y+z)$

$(x-y)\ (y-z)\ (z-x)\ (x+y+z)$

Thus B is the correct options.

Q9 Text Solution:

A = 3 × 3 Matrix.

$|A| = 4,$ thus the determinant of $|A^{-1}| = |A|^{-1} = (4)^{-1} = \frac{1}{4}.$

Thus (a) is the correct option.

Q10 Text Solution:

$$\begin{bmatrix} 4 & -4 & 0 \\ a & b+4 & c \\ a & b & c+4 \end{bmatrix}$$

$|A| = 0.$

Determinant of $\begin{bmatrix} 4 & -4 & 0 \\ a & b+4 & c \\ a & b & c+4 \end{bmatrix} = 0$

$C_2 \to C_2 + C_1$

$$\begin{bmatrix} 4 & 0 & 0 \\ a & b+4+a & c \\ a & b+a & c+4 \end{bmatrix} = 0$$

expanding through first elements of 1 Row :-

$$4\begin{vmatrix} b+4+a & c \\ b+a & c+4 \end{vmatrix} = 0$$

$\cancel{bc} + 4b + 4c + 16 + \cancel{ac} + 4a - \cancel{bc} - \cancel{ac}$
$= 0$

4 (a + b + c) + 16 = 0

a + b + c = -4

(d) is correct options.

Q11 Text Solution:

I_3 is the identity matrix.

Thus as we know that the inverse of every identity matrix is the identity matrix, thus the inverse of I_3 is I_3 its So,c is the correct option.

Q12 Text Solution:

A is square matrix , and A is said to be symmetric if transpose of A is A.

Now; $(AA^T)^T = (A^T)^T . A^T$ as $(AB)^T = B^T A^T$

and $(A^T)^T . A^T$, thus $(A^T)^T\ A^T = A . A^T$

thus option c is correct.

Q13 Text Solution:

For a skew symmetric matrix -

$(A^T) = -A$

Thus $a_{ii} = -a_{ii}$ as the diagonal elements are same after taking transpose.

$2a_{ii} = 0$

$a_{ii} = 0$

Thus, option (a) is correct.

Q14 Text Solution:

A is a singular matrix.

thus as we know that the adjoint follows the same property thus the determinant of adjoint of matrix is also singular thus (A) is correct.

Q15 Text Solution:

$$A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \quad .... (1)$$

$$A - 2B = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} \quad ....(2)$$

Substracting eq (1) & (2) we get -

$$+3B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$B = \frac{1}{3}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Q16 Text Solution:

$$x + \begin{bmatrix} 2 & 1 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 6 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ -6 & 0 \end{bmatrix}$$

thus option (c) is correct.

Q17 Text Solution:

$$\begin{bmatrix} x & 1 \\ -1 & -y \end{bmatrix} + \begin{bmatrix} y & 1 \\ 3 & x \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

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$$\begin{bmatrix} x+y & 2 \\ 2 & -y+x \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

Comparing elements :

x + y = 1

−y + x = 1

Adding both the equations.

2x = 2

x = 1

y = 0

thus x = 1, y = 0 thus option B is correct .

Q18 Text Solution:

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix}$$

Now, $A + B - 4I = 0$

$B = 4I - A$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$4I = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$4I - A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & -2 \\ -3 & -4 & -1 \end{bmatrix}$$

Q19 Text Solution:

$$\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix}_{2\times3} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}_{3\times1} + 2\begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 21+4+10 \\ 27+5+5 \end{bmatrix} + 2\begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 35 \\ 40 \end{bmatrix} + 2\begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 35 \\ 40 \end{bmatrix} + \begin{bmatrix} 8 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 43 \\ 50 \end{bmatrix}$$

Thus options (D) is correct.

Q20 Text Solution:

$f(x) = x^2 + 4x - 5$

$f(A) = A^2 + 4A - 5I$

$$A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix}$$

$$A^2 + 4A = \begin{bmatrix} 13 & 4 \\ 8 & 5 \end{bmatrix}$$

$$A^2 + 4A - 5 = \begin{bmatrix} 13 & 4 \\ 8 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$$

Option (D) is correct.

Q21 Text Solution:

The correct option is C *that is* $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

Given $A = A^T$ and $B = -B^T$

$$\therefore A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} \quad ...(i)$$

$$(A+B)^T = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}^T = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$$

$$A^T + B^T = A - B = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix} \quad ...(ii)$$

Solving (i) and (ii) we get.

$$A = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}.$$

Q22 Text Solution:

The correct option is A zero matrix.

$(I - A)(I + A) = I - A^2 = O$,

{Since A is involuntary, therefore $A^2 = I$}.

Q23 Text Solution:

The correct option is C that is |A| = 0

Given $A = \begin{bmatrix} 3 & -6 \\ a & -2 \end{bmatrix}$ is an idempotent matrix.

We know that for an idempotent matrix, $A^2 = A$.

$$A^2 = \begin{bmatrix} 3 & -6 \\ a & -2 \end{bmatrix} \cdot \begin{bmatrix} 3 & -6 \\ a & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-6a & -6 \\ a & 4-6a \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ a & -2 \end{bmatrix}$$

Equating the terms, we got a = 1.

Also, $|A| = \begin{vmatrix} 3 & -6 \\ 1 & -2 \end{vmatrix} = 0.$

Q24 Text Solution:

The correct option is D and is -2.

Nilpotency of matrix is 2, so square of given matrix will be Null matrix :

$$\begin{pmatrix} 2 & 4 \\ -1 & k \end{pmatrix} \times \begin{pmatrix} 2 & 4 \\ -1 & k \end{pmatrix} \text{ Null matrix}$$

$$= \begin{pmatrix} 0 & 8+4k \\ -2-k & -4+k^2 \end{pmatrix} =$$

By comparing we can say that $k = -2$.

Q25 Text Solution:

The correct option is A that is AB

(A+B)' (A+B) =$(A' + B')$(A+B)

= A 'A + $A'B$+ B' A + B'B = $2I_n$ + A 'B + B'A

(AB)' (AB) = (B'A ')(AB)

= B' (A 'A)B = B'I_n B = B'B = I_n

Thus only AB is an orthogonal .

Q26 Text Solution:

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$$A^T = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Q27 Text Solution:

$$A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$A^T = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$AA^T$$

$$= \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q28 Text Solution:

$$A = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}$$

$$A^\theta = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix}$$

$$AA^\theta = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Q29 Text Solution:

$$AA^\theta = I$$

$$A = \begin{bmatrix} i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & i \end{bmatrix}$$

$$A^\theta = \begin{bmatrix} -i & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & -i \end{bmatrix}$$

$$AA^\theta = \begin{bmatrix} i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & i \end{bmatrix} \begin{bmatrix} -i & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & -i \end{bmatrix} = I$$