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df_all["V_bary"] = np.sqrt(df_all["V_star"]**2 + df_all["V_gas"]**2)
         galaxies = {g: d[["R_kpc","V_obs","err_V","V_bary"]].reset_index(drop=True)
                     for g, d in df_all.groupby("galaxy_name")}
         print(f"Loaded {len(galaxies)} galaxies, {len(df_all)} data points total.")
       Loaded 5 galaxies, 51 data points total.
In [ ]:
In [ ]:
         def f_kernel(R, ell):
             x = R / ell
             x_safe = np.where(np.abs(x) < 1e-8, 1e-8, x)
             f = 1.0 - (1.0 / x_safe) * np.arctan(x_safe)
             small = np.abs(x) < 1e-3
             if np.any(small):
                 xx = x[small]
                 f[small] = (xx**2) / 3.0 - (xx**4) / 15.0
             return f
         def log_prior(theta):
             amp, ell, *Us = theta
             if not (0.0 < amp < 1e6 and 1e-4 < ell < 50.0 and all <math>(0.1 < U < 5.0 for U in Us)):
                 return -np.inf
             return 0.0
         def log_like(theta):
             amp, ell, *Us = theta
             total_chi2 = 0.0
             for i, (g_name, df) in enumerate(galaxies.items()):
                 R, Vobs, err, Vb = [df[c].values for c in ["R_kpc","V_obs","err_V","V_bary"]]
                 Vphi2 = amp * f_kernel(R, ell)
                 Vmod = np.sqrt((Us[i]*Vb)**2 + Vphi2)
                 total_chi2 += np.sum(((Vobs - Vmod)/err)**2)
             return -0.5 * total_chi2
         def log_prob(theta):
             lp = log_prior(theta)
             if not np.isfinite(lp): return -np.inf
             return lp + log_like(theta)
In [ ]:
         ndim = 7 # amp, ell, and 5 U_i
         nwalkers = 64
         nsteps = 20000
         nburn = 5000
         thin = 10
         p0 = np.array([1200, 6.0, 3.5, 3.5, 4.7, 3.5, 3.0])
         rng = np.random.default_rng(2025)
         pos = p0 * (1 + 1e-3 * rng.standard_normal((nwalkers, ndim)))
         sampler = emcee.EnsembleSampler(nwalkers, ndim, log_prob)
         print("Running MCMC...")
         t0 = time.time()
         sampler.run_mcmc(pos, nsteps, progress=True)
         print(f"Done in {time.time()-t0:.1f}s")
         flat = sampler.get_chain(discard=nburn, thin=thin, flat=True)
         np.save("samples_5gal.npy", flat)
         print(f"Saved {len(flat)} posterior samples.")
       Running MCMC...
       100%
                    20000/20000 [06:50<00:00, 48.77it/s]
       Done in 410.3s
       Saved 96000 posterior samples.
In []: def cred(x): return np.percentile(x, [16,50,84])
         param_names = ["A","ell","U_M33","U_M83","U_Malin1","U_NGC6946","U_NGC3198"]
         summary = pd.DataFrame({
             "param": param_names,
             "median": [cred(flat[:,i])[1] for i in range(ndim)],
             "16%": [cred(flat[:,i])[0] for i in range(ndim)],
             "84%": [cred(flat[:,i])[2] for i in range(ndim)]
         })
         print(summary)
         summary.to_csv("summary_5gal.csv", index=False)
         files.download("summary_5gal.csv")
         fig = corner.corner(flat, labels=param_names, show_titles=True, quantiles=[0.16,0.5,0.84])
         fig.savefig("corner_5gal.png", dpi=300, bbox_inches="tight")
         files.download("corner_5gal.png")
                                            16%
              param
                         median
                     72020 502205 62205 640127 05610 042615
```

```
ש
                    A /3000.30200 C02230.040171 03010.347013
        1
                  ell
                           14.542016
                                         12.375306
                                                        16.937055
        2
                U M33
                            1.268868
                                           1.199322
                                                            1.338597
        3
                U_M83
                            1.223149
                                            1.174103
                                                            1.271966
        4
            U_Malin1
                             1.295103
                                            1.168869
                                                            1.416910
        5 U_NGC6946
                             0.993962
                                            0.946110
                                                            1.041083
        6 U_NGC3198
                                            1.094953
                                                            1.189854
                            1.142418
              A = 73038.50^{+12580.44}_{-10742.86}
                                 ell = 14.54^{+2.40}_{-2.17}
           zα
           20
           $
           Ş
                                                 U_M33 = 1.27_{-0.07}^{+0.07}
          250
          2,20
          2.05
                                                                 U_M83 = 1.22_{-0.05}^{+0.05}
           √,¤
       U_M83
           √,>
                                                                                 U_Malin1 = 1.30^{+0.12}_{-0.13}
          2.75
          2.25
          200
                                                                                                 U_NGC6946 = 0.99_{-0.05}^{+0.05}
          0.5
           √,^
           0,9
           0,8
                                                                                                                  U_NGC3198 = 1.14^{+0.05}_{-0.05}
       U_NGC3198
           √,2
                                                                                       2.25
                                                                      U M83
                                                                                                      U NGC6946
                                                                                                                      U_NGC3198
                                                     U_M33
                                                                                      U_Malin1
In [ ]:
          # Convert to DataFrame for easier column access
          flat_df = pd.DataFrame(flat, columns=param_names)
          # === Figure Setup ===
          plt.style.use('seaborn-v0_8-whitegrid')
          fig, axes = plt.subplots(1, 5, figsize=(22, 5), sharey=True)
          # --- Color and style palette ---
          colors = ['#1f77b4', '#2ca02c', '#d62728', '#9467bd', '#ff7f0e'] # Consistent palette per galaxy
          for i, (g_name, df) in enumerate(galaxies.items()):
               R = df['R_kpc'].values
               V_obs = df['V_obs'].values
               err_V = df['err_V'].values
               V_bary = df['V_bary'].values
               # --- Compute model predictions from random posterior draws ---
               idx_samples = np.random.choice(len(flat_df), 200, replace=False)
               V models = []
               for idx in idx_samples:
                   amp, ell, *Us = flat_df.iloc[idx]
                   U = Us[i]
                   Vphi2 = amp * f_kernel(R, ell)
                   V_{mod} = np.sqrt((U * V_bary)**2 + Vphi2)
                   V_models.append(V_mod)
               V_models = np.array(V_models)
               p16, p50, p84 = np.percentile(V_models, [16, 50, 84], axis=0)
               # --- Plot ---
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ax = axes[i]

ax.errorbar(R. V obs. verr=err V. fmt='o'. markersize=4.

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color='black', ecolor='gray', capsize=2, label='Observed')
    ax.fill_between(R, p16, p84, alpha=0.25, color=colors[i], label='68% band')
    ax.plot(R, p50, color=colors[i], linewidth=2.5, label='Median model')
    ax.plot(R, np.median(flat_df.iloc[:, 2 + i]) * V_bary,
            color='gray', linestyle='--', linewidth=1.3, label='Baryonic baseline')
    # --- Axes formatting ---
    ax.set_xlabel('Radius R (kpc)', fontsize=11)
    ax.set_title(g_name, fontsize=12, fontweight='semibold')
    ax.tick_params(axis='both', which='major', labelsize=10)
    ax.grid(alpha=0.3, linestyle=':')
    ax.legend(fontsize=8, loc='lower right', frameon=False)
    if i == 0:
        ax.set_ylabel('Velocity V (km s^{-1})', fontsize=11)
# --- Global title and layout ---
plt.suptitle('Posterior Predictive Fits for Five Galaxies', fontsize=14, fontweight='bold')
plt.subplots_adjust(top=0.85) # Adjust layout to prevent title overlap
plt.tight_layout(rect=[0, 0.03, 1, 0.95]) # Adjust layout to prevent title overlap
plt.savefig("combined_predictive_5gal.png", dpi=300, bbox_inches="tight")
files.download("combined_predictive_5gal.png")
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