```
!pip install numpy pandas emcee
import numpy as np
import pandas as pd
import emcee
import corner
import matplotlib.pyplot as plt
import time
from google.colab import files
import io
```

```
raw_data_csv = """galaxy_name,R_kpc,V_obs,err_V,V_star,V_gas
M33,0.5,30,5,25,5
M33,1.0,45,6,35,8
M33,2.0,80,8,55,15
M33,3.0,95,9,65,18
M33,5.0,110,10,70,20
M33,7.0,115,11,75,22
M33,10.0,120,12,40,25
M33,12.0,119,13,35,28
M33,15.0,118,15,30,30
M83,0.2,50,6,45,5
M83,0.5,90,8,80,10
M83,1.0,120,10,100,12
M83,2.0,135,11,110,15
M83,5.0,145,12,90,18
M83,8.0,148,13,80,20
M83,10.0,150,15,60,22
M83,15.0,149,16,50,25
M83,18.0,148,18,40,28
M83,20.0,147,20,35,30
Malin1,1.0,60,10,50,10
Malin1,5.0,100,12,70,15
Malin1,10.0,150,15,80,20
Malin1,20.0,200,18,60,25
Malin1,30.0,220,20,50,28
Malin1,50.0,240,20,40,30
Malin1,70.0,240,22,30,32
Malin1,100.0,240,25,20,30
NGC6946,0.3,40,8,35,5
NGC6946,0.8,80,10,70,10
NGC6946,1.5,120,12,110,12
```

```
NGC6946,3.0,160,15,140,15
NGC6946,5.0,190,18,160,20
NGC6946,7.0,200,20,170,22
NGC6946,10.0,205,22,140,25
NGC6946,12.0,204,23,130,26
NGC6946,15.0,203,24,110,28
NGC6946,18.0,202,25,100,30
NGC6946,20.0,201,26,90,32
NGC3198,0.5,30,5,25,5
NGC3198,1.0,60,7,50,10
NGC3198,2.0,100,9,90,12
NGC3198,4.0,130,11,120,15
NGC3198,6.0,140,13,130,18
NGC3198,8.0,145,15,135,20
NGC3198,10.0,150,16,120,22
NGC3198,12.0,149,17,110,24
NGC3198,15.0,148,18,90,26
NGC3198,18.0,147,19,80,28
NGC3198,20.0,146,20,70,30
NGC3198,22.0,145,21,60,32
NGC3198,25.0,144,22,50,35
"""

df_all = pd.read_csv(io.StringIO(raw_data_csv))
df_all["V_bary"] = np.sqrt(df_all["V_star"]**2 + df_all["V_gas"]**

galaxies = {g: d[["R_kpc","V_obs","err_V","V_bary"]].reset_index(
            for g, d in df_all.groupby("galaxy_name")}

print(f"Loaded {len(galaxies)} galaxies, {len(df_all)} data point
```

```
Loaded 5 galaxies, 51 data points total.
```

Start coding or generate with AI.

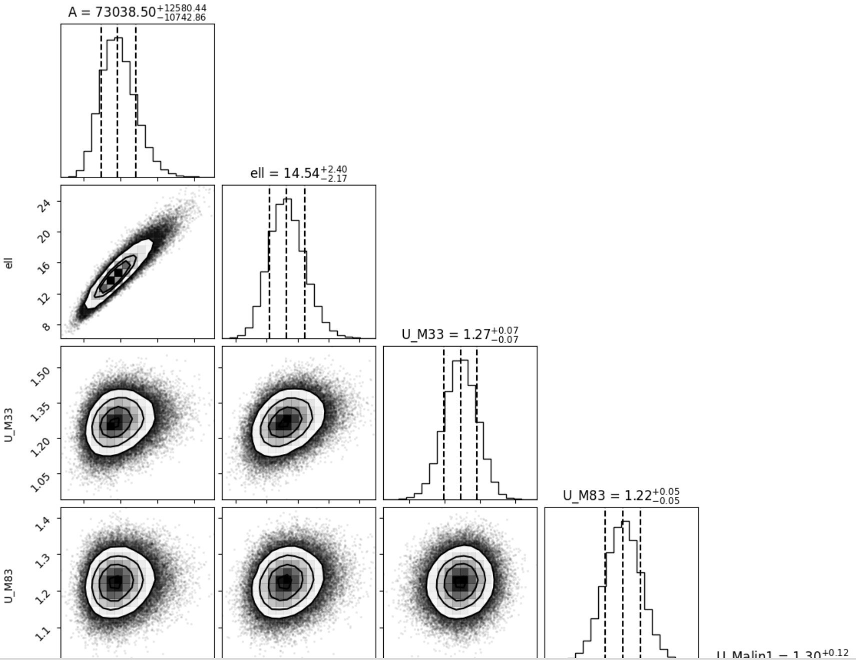
```
    def f_kernel(R, ell):
        x = R / ell
        x_safe = np.where(np.abs(x) < 1e-8, 1e-8, x)
        f = 1.0 - (1.0 / x_safe) * np.arctan(x_safe)
        small = np.abs(x) < 1e-3
        if np.any(small):
            xx = x[small]
            f[small] = (xx**2) / 3.0 - (xx**4) / 15.0
        return f

    def log_prior(theta):
        amp, ell, *Us = theta
        if not (0.0 < amp < 1e6 and 1e-4 < ell < 50.0 and all(0.1 < U
            return -np.inf
        return 0.0

    def log_like(theta):
        amp, ell, *Us = theta
        total_chi2 = 0.0
        for i, (g_name, df) in enumerate(galaxies.items()):
            R, Vobs, err, Vb = [df[c].values for c in ["R_kpc","V_obs'
            Vphi2 = amp * f_kernel(R, ell)
            Vmod = np.sqrt((Us[i]*Vb)**2 + Vphi2)
            total_chi2 += np.sum(((Vobs - Vmod)/err)**2)
        return -0.5 * total_chi2

    def log_prob(theta):
        lp = log_prior(theta)
        if not np.isfinite(lp): return -np.inf
        return lp + log_like(theta)
```

```
ndim = 7  # amp, ell, and 5 U_i
nwalkers = 64
nsteps = 20000
nburn = 5000
thin = 10

p0 = np.array([1200, 6.0, 3.5, 3.5, 4.7, 3.5, 3.0])
rng = np.random.default_rng(2025)
pos = p0 * (1 + 1e-3 * rng.standard_normal((nwalkers, ndim)))

sampler = emcee.EnsembleSampler(nwalkers, ndim, log_prob)
print("Running MCMC...")
t0 = time.time()
sampler.run_mcmc(pos, nsteps, progress=True)
print(f"Done in {time.time()-t0:.1f}s")

flat = sampler.get_chain(discard=nburn, thin=thin, flat=True)
np.save("samples_5gal.npy", flat)
print(f"Saved {len(flat)} posterior samples.")
```

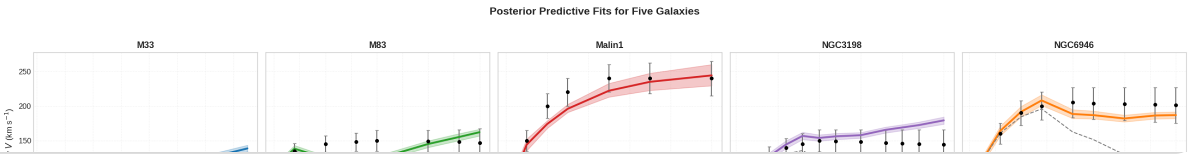
```
Running MCMC...
100%|██████████| 20000/20000 [06:50<00:00, 48.77it/s]
Done in 410.3s
Saved 96000 posterior samples.
```

```
def cred(x): return np.percentile(x, [16,50,84])
param_names = ["A","ell","U_M33","U_M83","U_Malin1","U_NGC6946","U

summary = pd.DataFrame({
    "param": param_names,
    "median": [cred(flat[:,i])[1] for i in range(ndim)],
    "16%": [cred(flat[:,i])[0] for i in range(ndim)],
    "84%": [cred(flat[:,i])[2] for i in range(ndim)]
})
print(summary)
summary.to_csv("summary_5gal.csv", index=False)
files.download("summary_5gal.csv")

fig = corner.corner(flat, labels=param_names, show_titles=True, qu
fig.savefig("corner_5gal.png", dpi=300, bbox_inches="tight")
plt.show()
files.download("corner_5gal.png")
```

```
       param        median           16%           84%
0          A  73038.503205  62295.648127  85618.942615
1        ell     14.542016     12.375306     16.937055
2      U_M33      1.268868      1.199322      1.338597
3      U_M83      1.223149      1.174103      1.271966
4   U_Malin1      1.295103      1.168869      1.416910
5  U_NGC6946      0.993962      0.946110      1.041083
6  U_NGC3198      1.142418      1.094953      1.189854
```



```
# Convert to DataFrame for easie
flat_df = pd.DataFrame(flat, col

# === Figure Setup ===
plt.style.use('seaborn-v0_8-whit
fig, axes = plt.subplots(1, 5, f

# --- Color and style palette --
colors = ['#1f77b4', '#2ca02c', 

for i, (g_name, df) in enumerate
    R = df['R_kpc'].values
    V_obs = df['V_obs'].values
    err_V = df['err_V'].values
    V_bary = df['V_bary'].values

    # --- Compute model predicti
    idx_samples = np.random.choi
    V_models = []
    for idx in idx_samples:
        amp, ell, *Us = flat_df.
        U = Us[i]
        Vphi2 = amp * f_kernel(R
        V_mod = np.sqrt((U * V_b
        V_models.append(V_mod)
    V_models = np.array(V_models
    p16, p50, p84 = np.percentil

    # --- Plot ---
    ax = axes[i]
    ax.errorbar(R, V_obs, yerr=e
                color='black', e
    ax.fill_between(R, p16, p84,
    ax.plot(R, p50, color=colors
    ax.plot(R, np.median(flat_df
            color='gray', linest

    # --- Axes formatting ---
    ax.set_xlabel('Radius $R$ (k
    ax.set_title(g_name, fontsiz
    ax.tick_params(axis='both', 
    ax.grid(alpha=0.3, linestyle
    ax.legend(fontsize=8, loc='l
    ax.legend(fontsize=8, loc='
    if i == 0:
        ax.set_ylabel('Velocity

# --- Global title and layout --
plt.suptitle('Posterior Predicti
plt.subplots_adjust(top=0.85) #
plt.tight_layout(rect=[0, 0.03,

plt.savefig("combined_predictive
plt.show()
files.download("combined_predict
```


Posterior Predictive Fits for Five Galaxies
M33
M83
Malin1
NGC3198
NGC6946
V (km s$^{-1}$)