```
!pip install numpy pandas emcee
import numpy as np
import pandas as pd
import emcee
import corner
import matplotlib.pyplot as plt
import time
from google.colab import files
import io
```

```
raw_data_csv = """galaxy_name,R_
M33,0.5,30,5,25,5
M33,1.0,45,6,35,8
M33,2.0,80,8,55,15
M33,3.0,95,9,65,18
M33,5.0,110,10,70,20
M33,7.0,115,11,75,22
M33,10.0,120,12,40,25
M33,12.0,119,13,35,28
M33,15.0,118,15,30,30
M83,0.2,50,6,45,5
M83,0.5,90,8,80,10
M83,1.0,120,10,100,12
M83,2.0,135,11,110,15
M83,5.0,145,12,90,18
M83,8.0,148,13,80,20
M83,10.0,150,15,60,22
M83,15.0,149,16,50,25
M83,18.0,148,18,40,28
M83,20.0,147,20,35,30
Malin1,1.0,60,10,50,10
Malin1,5.0,100,12,70,15
Malin1,10.0,150,15,80,20
Malin1,20.0,200,18,60,25
Malin1,30.0,220,20,50,28
Malin1,50.0,240,20,40,30
Malin1,70.0,240,22,30,32
Malin1,100.0,240,25,20,30
NGC6946,0.3,40,8,35,5
NGC6946,0.8,80,10,70,10
NGC6946,1.5,120,12,110,12
NGC6946,3.0,160,15,140,15
NGC6946,5.0,190,18,160,20
NGC6946,7.0,200,20,170,22
NGC6946,10.0,205,22,140,25
```

```
NGC6946,12.0,204,23,130,26
NGC6946,15.0,203,24,110,28
NGC6946,18.0,202,25,100,30
NGC6946,20.0,201,26,90,32
NGC3198,0.5,30,5,25,5
NGC3198,1.0,60,7,50,10
NGC3198,2.0,100,9,90,12
NGC3198,4.0,130,11,120,15
NGC3198,6.0,140,13,130,18
NGC3198,8.0,145,15,135,20
NGC3198,10.0,150,16,120,22
NGC3198,12.0,149,17,110,24
NGC3198,15.0,148,18,90,26
NGC3198,18.0,147,19,80,28
NGC3198,20.0,146,20,70,30
NGC3198,22.0,145,21,60,32
NGC3198,25.0,144,22,50,35
"""

df_all = pd.read_csv(io.StringIO
df_all["V_bary"] = np.sqrt(df_al

galaxies = {g: d[["R_kpc","V_obs
            for g, d in df_all.g

print(f"Loaded {len(galaxies)} g
```

```
Loaded 5 galaxies, 51 data points total.
```

Start coding or generate with AI.

```
    def f_kernel(R, ell):
        x = R / ell
        x_safe = np.where(np.abs(x) < 1e-8, 1e-8, x)
        f = 1.0 - (1.0 / x_safe) * np.arctan(x_safe)
        small = np.abs(x) < 1e-3
        if np.any(small):
            xx = x[small]
            f[small] = (xx**2) / 3.0 - (xx**4) / 15.0
        return f

    def log_prior(theta):
        amp, ell, *Us = theta
        if not (0.0 < amp < 1e6 and 1e-4 < ell < 50.0 and all(0.1
            return -np.inf
        return 0.0

    def log_like(theta):
        amp, ell, *Us = theta
        total_chi2 = 0.0
        for i, (g_name, df) in enumerate(galaxies.items()):
            R, Vobs, err, Vb = [df[c].values for c in ["R_kpc","V
            Vphi2 = amp * f_kernel(R, ell)
            Vmod = np.sqrt((Us[i]*Vb)**2 + Vphi2)
            total_chi2 += np.sum(((Vobs - Vmod)/err)**2)
        return -0.5 * total_chi2

    def log_prob(theta):
        lp = log_prior(theta)
        if not np.isfinite(lp): return -np.inf
        return lp + log_like(theta)
```

```
ndim = 7  # amp, ell, and 5 U_i
nwalkers = 64
nsteps = 20000
nburn = 5000
thin = 10

p0 = np.array([1200, 6.0, 3.5, 3.5, 4.7, 3.5, 3.0])
rng = np.random.default_rng(2025)
pos = p0 * (1 + 1e-3 * rng.standard_normal((nwalkers, ndim)))

sampler = emcee.EnsembleSampler(nwalkers, ndim, log_prob)
print("Running MCMC...")
t0 = time.time()
sampler.run_mcmc(pos, nsteps, progress=True)
print(f"Done in {time.time()-t0:.1f}s")

flat = sampler.get_chain(discard=nburn, thin=thin, flat=True)
np.save("samples_5gal.npy", flat)
print(f"Saved {len(flat)} posterior samples.")
```

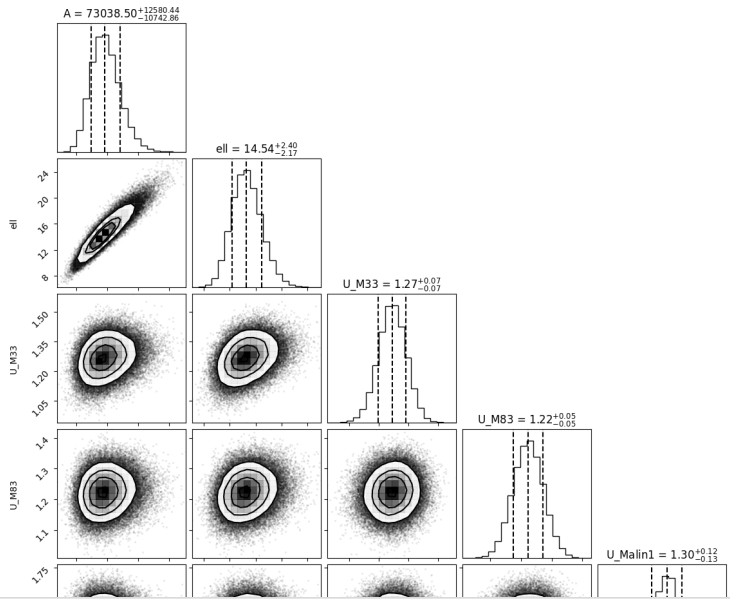
```
Running MCMC...
100%|██████████| 20000/20000 [06:50<00:00, 48.77it/s]
Done in 410.3s
Saved 96000 posterior samples.
```

```
def cred(x): return np.percentile(x, [16,50,84])
param_names = ["A","ell","U_M33","U_M83","U_Malin1","U_NGC694

summary = pd.DataFrame({
    "param": param_names,
    "median": [cred(flat[:,i])[1] for i in range(ndim)],
    "16%": [cred(flat[:,i])[0] for i in range(ndim)],
    "84%": [cred(flat[:,i])[2] for i in range(ndim)]
})
print(summary)
summary.to_csv("summary_5gal.csv", index=False)
files.download("summary_5gal.csv")

fig = corner.corner(flat, labels=param_names, show_titles=Tru
fig.savefig("corner_5gal.png", dpi=300, bbox_inches="tight")
plt.show()
files.download("corner_5gal.png")
```

	param	median	16%	84%
0	A	73038.503205	62295.648127	85618.942615
1	ell	14.542016	12.375306	16.937055
2	U_M33	1.268868	1.199322	1.338597
3	U_M83	1.223149	1.174103	1.271966
4	U_Malin1	1.295103	1.168869	1.416910
5	U_NGC6946	0.993962	0.946110	1.041083
6	U_NGC3198	1.142418	1.094953	1.189854




```
# Convert to DataFrame for easier column access
flat_df = pd.DataFrame(flat, columns=param_names)
```

```
# === Figure Setup ===
plt.style.use('seaborn-v0_8-whitegrid')
fig, axes = plt.subplots(1, 5, figsize=(22, 5), sharey=True)

# --- Color and style palette ---
colors = ['#1f77b4', '#2ca02c', '#d62728', '#9467bd', '#ff7f0

for i, (g_name, df) in enumerate(galaxies.items()):
    R = df['R_kpc'].values
    V_obs = df['V_obs'].values
    err_V = df['err_V'].values
    V_bary = df['V_bary'].values

    # --- Compute model predictions from random posterior dra
    idx_samples = np.random.choice(len(flat_df), 200, replace
    V_models = []
    for idx in idx_samples:
        amp, ell, *Us = flat_df.iloc[idx]
        U = Us[i]
        Vphi2 = amp * f_kernel(R, ell)
        V_mod = np.sqrt((U * V_bary)**2 + Vphi2)
        V_models.append(V_mod)
    V_models = np.array(V_models)
    p16, p50, p84 = np.percentile(V_models, [16, 50, 84], axi

    # --- Plot ---
    ax = axes[i]
    ax.errorbar(R, V_obs, yerr=err_V, fmt='o', markersize=4,
                color='black', ecolor='gray', capsize=2, labe
    ax.fill_between(R, p16, p84, alpha=0.25, color=colors[i],
    ax.plot(R, p50, color=colors[i], linewidth=2.5, label='Me
    ax.plot(R, np.median(flat_df.iloc[:, 2 + i]) * V_bary,
            color='gray', linestyle='--', linewidth=1.3, labe

    # --- Axes formatting ---
    ax.set_xlabel('Radius $R$ (kpc)', fontsize=11)
    ax.set_title(g_name, fontsize=12, fontweight='semibold')
    ax.tick_params(axis='both', which='major', labelsize=10)
    ax.grid(alpha=0.3, linestyle=':')
    ax.legend(fontsize=8, loc='lower right', frameon=False)
    if i == 0:
        ax.set_ylabel('Velocity $V$ (km s$^{-1}$)', fontsize=

# --- Global title and layout ---
plt.suptitle('Posterior Predictive Fits for Five Galaxies', f
plt.subplots_adjust(top=0.85) # Adjust layout to prevent titl
plt.tight_layout(rect=[0, 0.03, 1, 0.95]) # Adjust layout to
```

```
plt.savefig("combined_predictive_5gal.png", dpi=300, bbox_inc
plt.show()
files.download("combined_predictive_5gal.png")
```