Discrete random variables...

Frequentist probability: $p = \lim_{N\to\infty} \frac{N_s}{N}$

Elementary events: $A = \{a_j | j = 1, N_A\}$

Normalization of Probability: $\sum_{j=1}^{N_A} P(a_j) = 1, \quad 0 \le P(a_j) \le 1$

Joint probability: $P(a, b)$

Marginal probability: $P_A(a) = \sum_b P(a, b)$

Conditional probability: $P(a|b) = \frac{P(a,b)}{P_B(b)}, \quad P_B(b) \neq 0$

Normalization of joint probability: $\sum_a \sum_b P(a, b) = 1, \quad 0 \le P(a, b) \le 1$

Independent variables: $P(a, b) = P_A(a)P_B(b)$

Conditional probability for independent variables: $P(a|b) = P_A(a)$

Kronecker delta: $\delta_{x,y} = \begin{cases} 1 & x = y \\ 0 & x \neq y \end{cases}$

Probability distribution: $P_F(f) = \sum_a \delta_{f,F(a)} P_A(a)$

Mean: $\langle F \rangle \equiv \sum_a F(a) P_A(a)$

Moment: $\langle F^n \rangle \equiv \sum_a F(a)^n P_A(a)$

Central moment: $\langle (F - \langle F \rangle)^n \rangle \equiv \sum_a (F(a) - \langle F \rangle)^n P_A(a)$

Variance: $\sigma^2 = \langle (F - \langle F \rangle)^2 \rangle = \sum_a (F(a) - \langle F \rangle)^2 P_A(a) = \langle F^2 \rangle - \langle F \rangle^2$

Standard deviation: $\sigma \equiv \sqrt{\langle F^2 \rangle - \langle F \rangle^2}$

Correlation function: $f_{FG} = \langle FG \rangle - \langle F \rangle \langle G \rangle$

Mean of a sum: $\langle S \rangle = \langle F \rangle N$

Variance of sum: $\sigma_S^2 = \sigma^2 N$

Standard deviation of a sum: $\sigma_S = \sigma\sqrt{N}$

Binomial coefficient: $\binom{N}{n} = \frac{N!}{n!(N-n)!}$

Binomial distribution: $P(n|N) = \binom{N}{n} p^n (1-p)^{N-n}$

Binomial theorem: $(p+q)^N = \sum_{n=0}^{N} \binom{N}{n} p^n q^{N-n}$

Binomial identities: $\binom{N}{0} = \binom{N}{N} = 1, \quad \binom{N-1}{n} + \binom{N-1}{n-1} = \binom{N}{n}, \quad \binom{N}{n+1} = \frac{N-n}{n+1}\binom{N}{n}$

Gaussian function: $g(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-x_0)^2}{2\sigma^2}\right], \quad \langle x \rangle = x_0 = x_{max}, \quad \langle (x-x_0)^2 \rangle = \sigma^2$

Gaussian approximation to binomial distribution: $P(n|N) \approx \frac{1}{\sqrt{2\pi p(1-p)N}} \exp\left[-\frac{(n-pN)^2}{2p(1-p)N}\right]$

Continuous random variables...

Probability density: $P([a,b]) = \int_a^b P(x)dx, \quad \int_\Omega P(x)dx = 1$

Marginal probability: $P_x(x) = \int_{-\infty}^{\infty} P(x,y)dy$

Conditional probability: $P(y|x) = \frac{P(x,y)}{P_x(x)}$

Baye's theorem: $P(y|x) = \frac{P(x|y)P_y(y)}{P_x(x)}$

Independence: $P(x,y) = P_x(x)P_y(y)$

Conditional probability for independent variables: $P(x|y) = P_x(x)$

Average of a function: $\langle F(x)\rangle = \int_{-\infty}^{\infty} F(x)P(x)dx$

Mean: $\langle x\rangle = \int_{-\infty}^{\infty} xP(x)dx$

Moment: $\langle x^n\rangle = \int_{-\infty}^{\infty} x^n P(x)dx$

Central moment: $\langle (x-\langle x\rangle)^n\rangle = \int_{-\infty}^{\infty} (x-\langle x\rangle)^n P(x)dx$

Variance: $\sigma^2 = \langle (x-\langle x\rangle)^2\rangle = \int_{-\infty}^{\infty} (x-\langle x\rangle)^2 P(x)dx = \langle x\rangle^2 - \langle x^2\rangle$

Standard deviation: $\sigma = \sqrt{\langle x\rangle^2 - \langle x^2\rangle}$

Gaussian integral: $G = \int_{-\infty}^{\infty} e^{-ax^2}dx = \sqrt{\frac{\pi}{a}}$

Stirling's Approximation: $\ln N! \approx N\ln N - N + 1, \quad \text{or} \quad N! \approx N^N \exp(1-N)$

Entropy...

Total entropy: $S_{total}(E,V,N) = S_q(V,N) + S_p(E,N)$

Number probability distribution: $P(N_j,N_k) = \frac{N_T!}{N_j!N_k!}\left(\frac{V_j}{V_T}\right)^{N_j}\left(\frac{V_k}{V_T}\right)^{N_k}, \quad \text{where } N_j + N_k = N_T$

$P(N_j,N_k) = \frac{\Omega_q(N_j,V_j)\Omega_q(N_k,V_k)}{\Omega_q(N_T,V_T)}, \quad \text{where } \Omega_q(N,V) = \frac{V^N}{N!}$

Configurational entropy (Boltzmann): $S_q(N,V) = k\ln\Omega_q(N,V) + kXN$

Additivity of configurational entropy: $S_q(N_j,V_j,N_k,V_k) = S_q(N_j,V_j) + S_q(N_k,V_k)$

Analytic approximation: $S_q(N,V) \approx kN\left[\ln\left(\frac{V}{N}\right) + X\right]$

Energy in subsystem of classical ideal gas: $E_\alpha = \sum_{i=1}^{N_\alpha} \frac{|\vec{p}_{\alpha,i}|^2}{2m}$

Integral over momenta of all particles: $\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}(\cdots)dp^3_{\alpha,1}\cdots dp^3_{\alpha,N_j} \equiv \int_{-\infty}^{\infty}(\cdots)dp_\alpha$

Energy probability distribution: $P(E_j,E_k) = \frac{\int_{-\infty}^{\infty}\delta\left(E_j - \sum_{i=1}^{N_j}\frac{|\vec{p}_{j,i}|^2}{2m}\right)dp_j \int_{-\infty}^{\infty}\delta\left(E_k - \sum_{\ell=1}^{N_k}\frac{|\vec{p}_{k,\ell}|^2}{2m}\right)dp_k}{\int_{-\infty}^{\infty}\delta\left(E_T - \sum_{i=1}^{N_T}\frac{|\vec{p}_i|^2}{2m}\right)dp}$

$$P(E_j, E_k) = \frac{\Omega_p(E_j, N_j)\Omega_p(E_k, N_k)}{\Omega_p(E_T, N_T)} \quad \text{where } \Omega_p(E_\alpha, N_\alpha) = \int_{-\infty}^{\infty} \delta\left(E_\alpha - \sum_{i=1}^{N_\alpha} \frac{|\vec{p}_{\alpha,i}|^2}{2m}\right) dp_\alpha$$

$$\Omega_p(E, N) = S_n m(2mE)^{3N/2-1} \quad \text{where } S_n = n\frac{\pi^{n/2}}{(n/2)!}$$

Energy-dependent entropy: $S_p(E, N) = k \ln \Omega_p(E, N)$, where $\ln \Omega_p(E, N) \approx N\left[\frac{3}{2}\ln\left(\frac{E}{N}\right) + X\right]$

Note the exact expression $\Omega_P(E, N) = \frac{3N\pi^{3N/2}}{(3N/2)!} m(2mE)^{3N/2-1}$

Entropy of a subsystem of a classical ideal gas: $S_j(E_j, V_j, N_j) = kN_j\left[\frac{3}{2}\ln\left(\frac{E_j}{N_j}\right) + \ln\left(\frac{V_j}{N_j}\right) + X\right]$

Note that traditionally $X = \frac{3}{2}\ln\left(\frac{4\pi m}{3h^2}\right) + \frac{5}{2}$

Hamiltonian for Ideal gas: $H_\alpha(q_\alpha, p_\alpha) = \sum_{i=1}^{3N_\alpha} \frac{|\vec{p}_{\alpha,i}|^2}{2m}$

Hamiltonian for interacting particles: $H_\alpha(q_\alpha, p_\alpha) = \sum_{i=1}^{3N_\alpha} \frac{|\vec{p}_{\alpha,i}|^2}{2m} + \sum_{i=1}^{N\alpha}\sum_{i'=1}^{N_\alpha} \phi(\vec{r}_{\alpha,i}, \vec{r}_{\alpha,i'})$

Entropy for interacting particles: $S_\alpha(E_\alpha, V_\alpha, N_\alpha) = k \ln \Omega_\alpha(E_\alpha, V_\alpha, N_\alpha)$

where $\Omega_\alpha(E_\alpha, V_\alpha, N_\alpha) = \frac{1}{h^{3N_\alpha} N_\alpha!}\int dq_\alpha \int dp_\alpha \delta(E_\alpha - H_\alpha)$ and H_α is the Hamiltonian for the subsystem α

Thermodynamic quantities...

Maxwell-Boltzmann distribution for momentum of a single particle: $P(\vec{p}_1) = \left(\frac{\beta}{2\pi m}\right)^{3/2} \exp\left(-\beta\frac{|\vec{p}_1|^2}{2m}\right)$

Ideal gas law: $PV = \frac{N}{\beta}$ where $\beta = \frac{1}{k_B T}$ or $PV = Nk_B T$

Celsius to Kelvin conversion: $T\,[\mathrm{K}] = T\,[^\circ\mathrm{C}] + 273.15$

Equipartition theorem: $\frac{E}{N} = \frac{3}{2}k_B T$

Equations of state: $\left(\frac{\partial S}{\partial E}\right)_{V,N} = \frac{1}{T}$, $\left(\frac{\partial S}{\partial V}\right)_{E,N} = \frac{P}{T}$, $\left(\frac{\partial S}{\partial N}\right)_{E,V} = -\frac{\mu}{T}$

Laws and postulates of thermodynamics...

0th Law: If two systems are in thermal equilibrium with a third system, they are in equilibrium with each other.

1st Law: Heat is a form of energy, and energy is conserved.

2nd Law: If a system is constrained, its entropy cannot decrease after the constraints are removed.

3rd Law: The entropy of any quantum mechanical system goes to constant as temperature goes to zero.

1st Postulate: There exist equilibrium states characterized by a small number of extensive parameters.

2nd Postulate: Without internal constraints, values of extensive parameters are those that maximize entropy.

3rd Postulate: Entropy of a composite system is additive.

4th Postulate: Entropy is a continuous and differentiable function of extensive parameters.

"Sometimes true" postulates:

5th Postulate: Entropy is an extensive function of extensive variables.

6th Postulate: The entropy is a monotonically increasing function of energy for equilibrium values of energy.

7th Postulate (The Nernst Postulate): The entropy of any system is non-negative.

Thermodynamic processes...

Energy: $U = \langle E \rangle$

First law in differential form: $dU = đQ + đW$, for fixed N

Condition for exact differential: $\dfrac{\partial f}{\partial y} = \dfrac{\partial^2 F}{\partial y \partial x} = \dfrac{\partial^2 F}{\partial x \partial y} = \dfrac{\partial g}{\partial x}$, for $dF = f(x,y)dx + g(x,y)dy$

Integrating factor: $dG = r(x,y)\, đF$

Fundamental relation: $dU = TdS - PdV + \mu dN$

Note that $đQ = TdS$, $đW = PdV$ for fixed number of particles.

Entropy in heat cycle: $dS = \dfrac{đQ_H}{T_H} + \dfrac{đQ_L}{T_L} = 0$

Work in heat cycle: $đW = \left(1 - \dfrac{T_L}{T_H}\right) đQ_H$

Efficiency of heat engine: $\eta = \dfrac{đW}{đQ_H} = 1 - \dfrac{T_L}{T_H}$

Coefficient of performance for refrigerator: $\epsilon_R = \dfrac{đQ_L}{-đW} = \dfrac{T_L}{T_H - T_L}$

Coefficient of performance for heat pump: $\epsilon_{HP} = \dfrac{-đQ_H}{-đW} = \dfrac{T_H}{T_H - T_L}$

Thermodynamic potentials...

Helmholtz free energy: $F(T,V,N) \equiv U[T] = U - TS$, $dF = -SdT - PdV + \mu dN$

Enthalpy: $H(S,P,N) \equiv U[P] = U + PV$, $dH = TdS + VdP + \mu dN$

Gibbs free energy: $G(T,P,N) \equiv U[T,P] = U - TS + PV$, $dG = -SdT + VdP + \mu dN$

Extensivity...

Condition for extensivity: $S(\lambda U, \lambda V, \lambda N) = \lambda S(U,V,N)$

Euler equation: $U = TS - PV + \mu N$

Gibbs-Duhem relation: $d\mu = -\left(\dfrac{S}{N}\right) dT + \left(\dfrac{V}{N}\right) dP$

Thermodynamic potentials for extensive systems:

$F = U - TS = -PV + \mu N$, $\quad H = U + PV = TS + \mu N$, $\quad G = U - TS + PV = \mu N$

Thermodynamic identities...

Coefficient of thermal expansion: $\alpha = \dfrac{1}{V}\left(\dfrac{\partial V}{\partial T}\right)_{P,N}$

Isothermal compressibility: $\kappa_T = -\dfrac{1}{V}\left(\dfrac{\partial V}{\partial P}\right)_{T,N}$

Specific heat per particle at constant pressure: $c_P = \dfrac{T}{N}\left(\dfrac{\partial S}{\partial T}\right)_{P,N}$

Specific heat per particle at constant volume: $c_V = \dfrac{T}{N}\left(\dfrac{\partial S}{\partial T}\right)_{V,N}$

Heat capacity: $C_P = Nc_P, \quad C_V = Nc_V$

Jacobians: $\dfrac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \dfrac{\partial u}{\partial x}\dfrac{\partial v}{\partial y} - \dfrac{\partial u}{\partial y}\dfrac{\partial v}{\partial x}$

Symmetry of Jacobians: $\dfrac{\partial(u,v)}{\partial(x,y)} = -\dfrac{\partial(v,u)}{\partial(x,y)} = \dfrac{\partial(v,u)}{\partial(y,x)} = -\dfrac{\partial(u,v)}{\partial(y,x)}$

Jacobians as partial derivatives: $\dfrac{\partial(u,y)}{\partial(x,y)} = \left(\dfrac{\partial u}{\partial x}\right)_y$

Chain rule for Jacobians: $\dfrac{\partial(u,v)}{\partial(x,y)} = \dfrac{\partial(u,v)}{\partial(r,s)}\dfrac{\partial(r,s)}{\partial(x,y)}$

Reciprocals of partial derivatives: $\left(\dfrac{\partial u}{\partial x}\right)_y = 1 \Big/ \left(\dfrac{\partial x}{\partial u}\right)_y$

Specific heat identity: $c_P = c_V + \dfrac{\alpha^2 TV}{N\kappa_T}$

Extremum principles, Stability conditions, Phase transitions...

Energy minimum princple: $dU = đW$, for constant entropy and constant number of particles
Stability with respect to volume changes: $\kappa_S \geq 0$
Stability with respect to heat transfer: $c_V \geq 0$
All stability conditions:

$$U(S,V,N) \qquad \left(\frac{\partial^2 U}{\partial S^2}\right)_{V,N} \geq 0 \qquad \left(\frac{\partial^2 U}{\partial V^2}\right)_{S,N} \geq 0$$

$$F(T,V,N) \qquad \left(\frac{\partial^2 F}{\partial T^2}\right)_{V,N} \leq 0 \qquad \left(\frac{\partial^2 F}{\partial V^2}\right)_{S,N} \geq 0$$

$$H(S,P,N) \qquad \left(\frac{\partial^2 H}{\partial S^2}\right)_{P,N} \geq 0 \qquad \left(\frac{\partial^2 H}{\partial P^2}\right)_{S,N} \leq 0$$

$$G(T,P,N) \qquad \left(\frac{\partial^2 G}{\partial T^2}\right)_{P,N} \leq 0 \qquad \left(\frac{\partial^2 G}{\partial P^2}\right)_{T,N} \leq 0$$

Helmholtz free energy of van der Waals fluid: $F_{vdW} = -Nk_BT\left[\ln\left(\dfrac{V-bN}{N}\right) + \dfrac{3}{2}\ln(k_BT) + X\right] - a\left(\dfrac{N^2}{V}\right)$

Equations of state for van der Waals fluid: $P = \dfrac{Nk_BT}{V-bN} - \dfrac{aN^2}{V^2}, \quad U = \dfrac{3}{2}Nk_BT - a\left(\dfrac{N^2}{V}\right)$

Latent heat: $\ell = \dfrac{T\Delta S}{N} = \dfrac{L}{N}$

Clausius-Clapeyron equation: $\dfrac{dP}{dT} = \dfrac{\ell N}{T\Delta V} = \dfrac{L}{T\Delta V}$

The Nernst postulate...

Specific heat at low temperatures: $\lim_{T \to 0} c_X(T) = 0$

Coefficient of thermal expansion at low temperatures: $\lim_{T \to 0} \alpha(T) = 0$

Canonical ensemble...

Partition function for canonical ensemble: $Z = \frac{1}{h^{3N} N!} \int dq \int dp \exp[-\beta H(q,p)]$

Canonical distribution: $P(p,q) = \frac{1}{h^{3N} N! Z} \exp[-\beta H(p,q)]$

Liouville theorem: $\frac{dP}{dt} = 0$

Helmholtz free energy of canonical ensemble: $F(T,V,N) = -k_B T \ln Z(T,V,N)$

Specific heat in statistical mechanics: $\frac{1}{N k_B T^2} \left(\langle E^2 \rangle - \langle E \rangle^2 \right)$

Forces independent of momentum: $Z = \frac{1}{h^{3N} N!} (2\pi m k_B T)^{3N/2} \int dq \exp\left[-\beta \sum_{j=1}^{N} \sum_{i>j}^{N} \phi(\vec{r}_i, \vec{r}_j) \right]$

Classical ideal gas (no forces): $Z = \frac{1}{h^{3N} N!} (2\pi m k_B T)^{3N/2} V^N$

Hamiltonian for SHO: $H = \frac{1}{2} K x^2 + \frac{p^2}{2m}$

Partition function for SHO: $Z = \frac{1}{\beta \hbar \omega}, \quad \omega = \sqrt{\frac{K}{m}}$

Hamiltonian for N SHOs: $H = \sum_{j=1}^{N} \left(\frac{1}{2} K_j x_j^2 + \frac{p_j^2}{2m_j} \right)$

Partition function for N SHOs: $Z = \prod_{j=1}^{N} \left(\frac{1}{\beta \hbar \omega_j} \right), \quad \omega_j = \sqrt{\frac{K_j}{m_j}}$