Astronomy 3303A Final Formula Sheet 2022

Before Midterm:

$p = \frac{1AU}{d}$ [radians] (Parallax for nearby stars)

$F = \frac{L}{4\pi d^2}$ [Wm^{-2}] (Flux and Luminosity)

$L_\odot = 3.86 \times 10^{26}$ [Wm^{-2}] (Luminosity of the Sun)

$L = 4\pi R^2 \sigma_{SB} T^4$ [Wm^{-2}] (Stefan-Boltzmann equation)

$\lambda_{max} T = 2.90 \times 10^6$ [nmK] (Wien's displacement law)

$m_1 - m_2 = -2.5 \log_{10}\left(\frac{F_1}{F_2}\right)$ (Difference in Magnitude in terms of flux)

$m - M = 5 \log_{10}\left(\frac{d}{10pc}\right)$ (Distance Modulus)

$M_{bol} = M_V - BC$ (Bolometric absolute magnitude, where BC is bolometric correction)

$z = \frac{\lambda_{obs}}{\lambda_e} - 1$ (Redshift)

$1 + z = \frac{\lambda_{obs}}{\lambda_e} = 1 + \frac{v_r}{c}$ (Doppler formula for speeds well below the speed of light)

$\Psi(M_V) = \Phi_{MS}(M_V)$ when $\tau_{MS}(M_V) \geq \tau_{gal}$ (Initial and current luminosity function)

$\Psi(M_V) = \Phi_{MS}(M_V) \times \frac{\tau_{gal}}{\tau_{MS}(M_V)}$ when $\tau_{MS}(M_V) \geq \tau_{gal}$

$\xi(\mathcal{M})\Delta\mathcal{M} = \xi_0 \left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{-2.35} \frac{\Delta\mathcal{M}}{\mathcal{M}_\odot}$ (Salpeter initial mass function)

$V_r = R_\odot \sin(l) \left(\frac{V}{R} - \frac{V_\odot}{R_\odot}\right)$ ($V_r > 0 \rightarrow$ Object moving away, $V_r < 0 \rightarrow$ Object moving towards us)

$\mathcal{M}(< R) = \frac{RV^2}{G}$ (Mass inside radius R)

$\vec{F}(r) = -\vec{\nabla}\Phi(r)$ (Force from potential)

$\vec{\nabla}^2\Phi = 4\pi G\rho(r)$ (Poisson's equation for potential and radial mass distribution)

$v_{esc}(r) = (2|\Phi(r)|)^2$ (Escape velocity from potential)

$v_{circ}^2 = r\vec{\nabla}\Phi(r)$ (Circular velocity from potential)

$\vec{\nabla}^2\Phi(r) = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\Phi(r)}{dr}\right)$ (r-component of laplacian in spherical coordinates)

$\mathcal{M}(r) = \int_0^r 4\pi\rho(r) r^2 dr$ (Radial mass distribution from radial density distribution)

$2\langle T\rangle + \langle V\rangle = 0$ (Virial theorem; T = Kinetic Energy, V = Potential Energy)

$v_{tot} = \sqrt{3}\sigma_r$ (Total velocity from radial velocity dispersion assuming v is isotropic)

$t_{cross} = \frac{R}{v_{tot}}$ (Crossing time for system of size R)

$t_{relax} = \frac{V^3}{8\pi G^2 m^2 n \ln\Lambda} = \frac{t_s}{2\ln\Lambda} \approx \frac{2\times10^9 yr}{\ln\Lambda} \left(\frac{V}{10kms^{-1}}\right)^3 \left(\frac{m}{\mathcal{M}}\right)^{-2} \left(\frac{n}{10^3pc^{-3}}\right)$ (Relaxation time)

$t_{cross} \approx 4\times10^{12} yr \left(\frac{V}{10kms^{-1}}\right)^3 \left(\frac{m}{\mathcal{M}_\odot}\right)^{-2} \left(\frac{n}{10^3pc^{-3}}\right)^{-1}$ (Time between strong encounters, $n = \#$ of stars per pc^3)

$r_s = \frac{2Gm}{v^2}$ (Strong encounter radius)

$\frac{1}{2}m(3\sigma_r^2) \approx -\frac{G\mathcal{M}m}{\frac{1}{2}R}$ (Virial equation rewritten to solve for mass)

After Midterm:

$\frac{I(R)}{I(0)} = e^{-R/h_R}$ (Surface brightness versus radius of a spiral galaxy)

$I(R) = I(0)e^{-(R/R_0)^{1/n}}$ (Sérsic's formula for galactic bulge of a spiral galaxy) ($R_0 = h_R$?)

$I(R,z) = I(R)e^{-|z|/h_z}$ (Dependence of surface brightness on height for a spiral galaxy)

$V_r(R,i) = V_{sys} + V(R)\sin i \cos\phi$ (Radial velocity of a spiral galaxy with inclination $i$and azimuth angle ϕ)

$L \propto (V_{max})^\alpha,\ \alpha \sim 4$ (Tully-Fisher relation for spiral galaxies)

$L \propto \sigma^4$ (Faber-Jackson relation for elliptical galaxies)

$L_* = 2\times10^{10} L_\odot$ (Typical luminosity of bright galaxy)

$I(R) = I(R_e)e^{-b[(R/R_e)^{1/n}-1]}$ (Sérsic law for elliptical galaxies with effective radius R_e)

Note Sérsic index of $n = 1$ is exponential disk, $n = 4$ is de Vaucouleurs law

$m^2 = \frac{x^2}{A^2} + \frac{y^2}{B^2} + \frac{z^2}{C^2}$ (Equation of an ellipsoid; $A \neq B \neq C$ triaxial, $C = B < A$ prolate, $A = B > C$ oblate)

$q_{obl}^2 = (b/a)^2 = (B/A)^2 \sin^2 i + \cos^2 i$ (Apparent axis ratio $q = b/a$ from intrinsic (true) axis ratio B/A)

$\frac{\mathcal{M}}{L} \propto L^{1/4}$ (Fundamental plane relation)

$z = \Delta\lambda/\lambda_{rest} = \Delta\nu/\nu_{rest}$ (Redshift in terms of wavelength or frequency)

$H_0 \approx 70$ km s^{-1} Mpc^{-1} (Hubble's constant)

$h = H_0/(100$ km s^{-1} Mpc$^{-1})$

$V_r \approx cz \approx H_0 d$ (Hubble's law, ignoring peculiar motions)

$d = h^{-1}[V_r(\text{km s}^{-1})/100]$ Mpc (Distance of a galaxy from radial velocity)

$\mathcal{M}_{BH} = f\frac{RV^2}{G}$ (Black hole mass in terms of time delay R and doppler line width V with scale factor f)