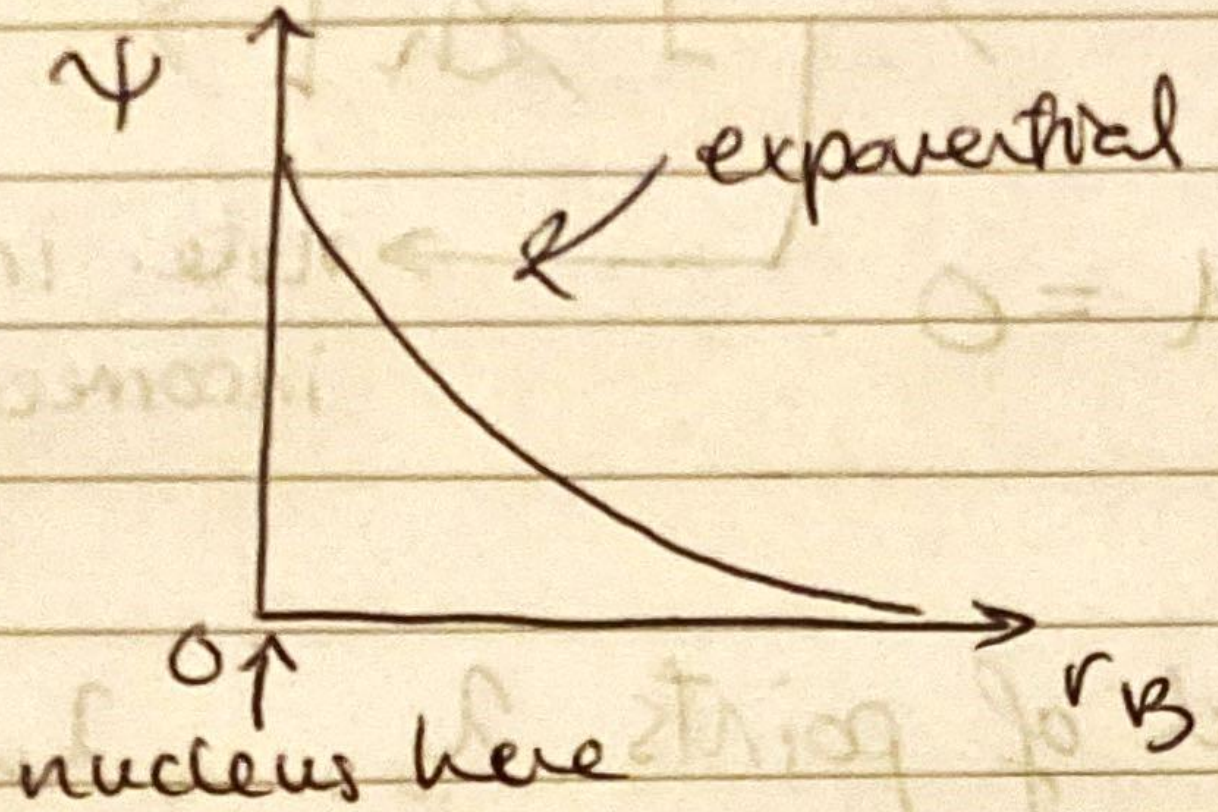
Eqn 7: $\psi = C_1 e^{-a_1 r_B}$



- sufficient for describing a bound electron about a nucleus in ground state
- symmetric spherically.

Eqn 8: similar arguments hold

General approximations in finite difference methods of function $f(x)$ at points $x_0, \dots, x_n$ on a grid, with spacing h: $(x_{i+1} = x_i + h)$

$$f'(x_i) \simeq f(x_i+h) - f(x_i-h) \cdot \frac{1}{2h}$$ [central difference]

$$f''(x_i) \simeq f(x_i+h) - 2f(x_i) + f(x_i-h) \cdot \frac{1}{h^2}$$

Therefore, for grid of points $\mu_0, \dots \mu_n$ of spacing h:

$$\frac{d}{d\mu}\left[(1-\mu^2)\frac{d}{d\mu}M\right] + \left[\beta - \frac{m_e^2}{1-\mu^2} - c^2\mu^2\right]M = 0$$

+ take $\mu = \mu_i$ here

$$= \frac{d^2M}{d\mu^2}(1-\mu^2) - 2\mu\frac{dM}{d\mu} + \left[\beta - \frac{m_e^2}{1-\mu^2} - c^2\mu^2\right]M = 0$$

$$\frac{(1-\mu^2)}{h^2}\left[M(\mu+h) - 2M(\mu) + M(\mu-h)\right] - \frac{2\mu}{2h}\left[M(\mu+h) - M(\mu-h)\right]$$

$$+ \left[\beta - \frac{m_e^2}{1-\mu^2} - c^2\mu^2\right]M = 0.$$

note: in report this is incorrectly shown as a +

Similarly for grid of points $\lambda_0, \ldots, \lambda_n$, spacing h:

$$\frac{d}{d\lambda}\left[(\lambda^2-1)\frac{d\Lambda}{d\lambda}\right] + \left[2R\lambda + c^2\lambda^2 - \frac{m_e^2}{\lambda^2-1} - \beta\right]\Lambda = 0$$

$$= (\lambda^2-1)\underbrace{\frac{d^2\Lambda}{d\lambda^2}} + 2\lambda\frac{d\Lambda}{d\lambda} + \left[2R\lambda + c^2\lambda^2 - \frac{m_e^2}{\lambda^2-1} - \beta\right]\Lambda = 0$$

$$\frac{\lambda^2-1}{h^2}\left[\Lambda(\lambda+h) - 2\Lambda(\lambda) + \Lambda(\lambda-h)\right] + \frac{2\lambda}{2h}\left[\Lambda(\lambda+h) - \Lambda(\lambda-h)\right]$$

$$+ \left[2R\lambda + c^2\lambda^2 - \frac{m_e^2}{\lambda^2-1} - \beta\right]\Lambda = 0$$

again for $\lambda = \lambda_i$ here. Due to co-ordinate system, this grid of λ, μ points would look like (very roughly)