

Numerical Solution for the Fokker Planck Equation

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Introduction

The Brownian motion of particles can be treated as the rate of Brownian particles approaching a state of equilibrium which is provided by the Master equation. A simplified version of this is the Fokker Planck Equation.

Fokker Planck Equation

From the master equation, we arise with the FPE where we have some function f , λ being the restoring force, B being the modulus, T being the temperature and k being the Boltzmann constant

$$\frac{\partial f}{\partial t} = \lambda B \nabla f + kBT \nabla^2 f$$

Methods

The method used was the Forward Time Centered Space (FTCS) method. Where we can write the FPE numerically as such.

Boundary and initial conditions

Considering a function where the parameters of its position x and y to be given by

$$f = f(r, t)$$

$$r = r(x, y) = \sqrt{x^2 + y^2}$$

With a grid space s and time step h . Numerically our position functions are defined as.

$$x_j = x_0 + j\Delta x$$

$$y_k = y_0 + k\Delta y$$

$$r_j^k = \sqrt{x_j^2 + y_k^2}$$

Where the change in x and y are defined by the following where we have n being the number of grid spaces in i/j . Along with a length of $L = 2$.

$$\Delta x = \frac{h}{n_j}$$

$$\Delta y = \frac{s}{n_k}$$

$$f(0, t) = f(L, t) = 0$$

Results

From Figure 1 and 2, our computations shows a generated solution using a exponential test function decreases the density function along the y -axis.

Conclusions

We can see that the FPE can be described for a Brownian particle approaching equilibrium over time. Further work would be to look at an animation of the function over time with a gaussian approaching to an equilibrium state.

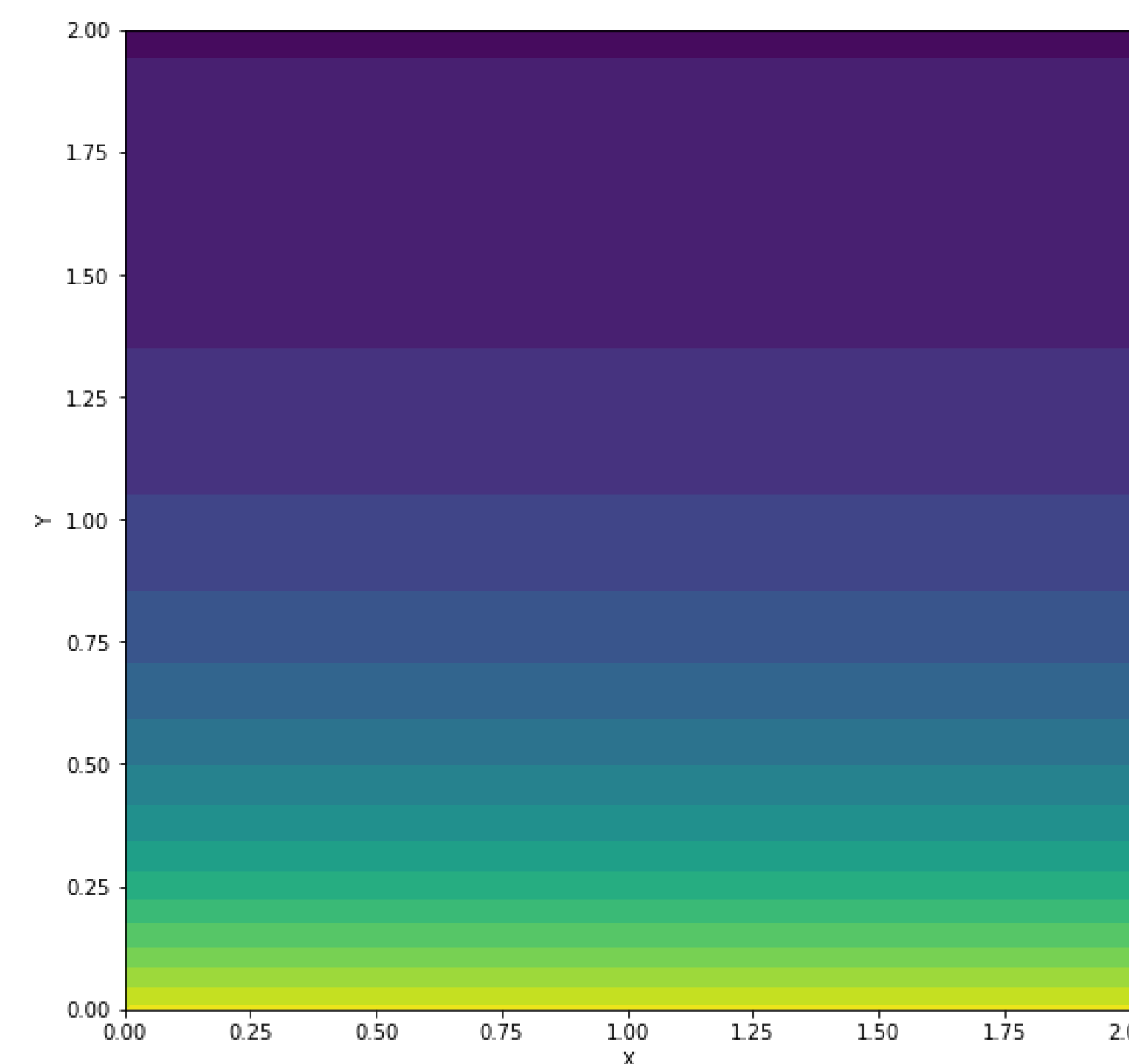


Figure 1. Numerical Sol. For FPE in 2D

Link to Github for code:

<https://github.com/ShaneGervais/Fokker-Planck-Equation-Using-FTCS-Method>

References

1. Statistical Mechanics by R.K. Pathria, Butterworth Heinemann, Elsevier Science & Technology, 1996
2. Computational Physics by Mark Newman, University of Michigan, 2012

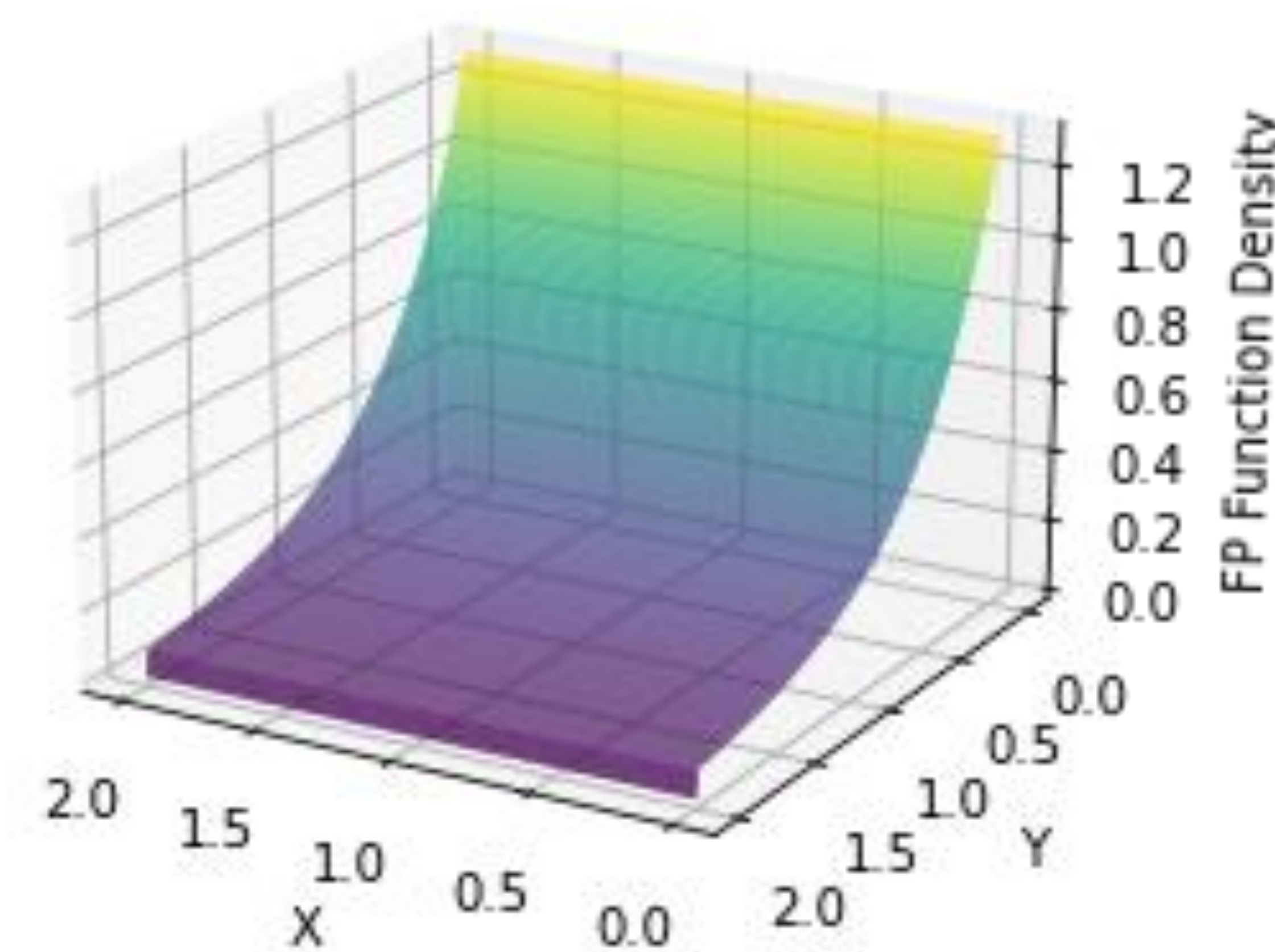


Figure 2. 3D representation of FPE

$$\frac{f_{i+1,j}^k - f_{i-1,j}^k}{s} = \lambda B \left[\frac{r_j^k (f_{i,j+1}^k - f_{i,j-1}^k)}{2\Delta x} + \frac{f_{i,j}^k (r_{j+1}^k - r_{j-1}^k)}{2\Delta x} + \frac{r_j^k (f_{i,j}^{k+1} - f_{i,j}^{k-1})}{2\Delta y} + \frac{f_{i,j}^k (r_j^{k+1} - r_j^{k-1})}{2\Delta y} \right] + kBT \left(\frac{f_{i,j+1}^k - 2f_{i,j}^k + f_{i,j-1}^k}{\Delta x^2} + \frac{f_{i,j}^{k+1} - 2f_{i,j}^k + f_{i,j}^{k-1}}{\Delta y^2} \right)$$