Numerical Solution for the Fokker Planck Equation

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Introduction

The Brownian motion of particles can be treated as the rate of Brownian particles approaching a state of equilibrium which is provided by the Master equation. A simplified version of this is the Fokker Planck Equation.

Fokker Planck Equation

From the master equation, we arise with the FPE where we have some function f, λ being the restoring force, B being the modulus, T being the temperature and k being the Boltzmann constant

$$\frac{\partial f}{\partial t} = \lambda B \nabla(rf) + kBT\nabla^2 f$$

Methods

The method used was the Forward Time Centered Space (FTCS) method. Where we can write the FPE numerically as such.

Boundary and initial conditions

Considering a function where the parameters of its position x and y to be given by

$$f = f(r,t)$$
$$r = r(x,y) = \sqrt{x^2 + y^2}$$

With a grid space s and time step h. Numerically our position functions are defined as.

$$x_j = x_0 + j\Delta x$$
$$y_k = y_0 + k\Delta y$$
$$r_j^k = \sqrt{x_j^2 + y_k^2}$$

Where the change in x and y are defined by the following where we have n being the number of grid spaces in i/j. Along with a length of L = 2.

$$\Delta x = \frac{h}{n_j}$$
$$\Delta y = \frac{s}{n_k}$$
$$f(0,t) = f(L,t) = 0$$

Results

From Figure 1 and 2, our computations shows a generated solution using a exponential test function decreases the density function along the y-axis.

Conclusions

We can see that the FPE can be described for a Brownian particle approaching equilibrium over time. Further work would be to look at an animation of the function over time with a gaussian approaching to an equilibrium state.

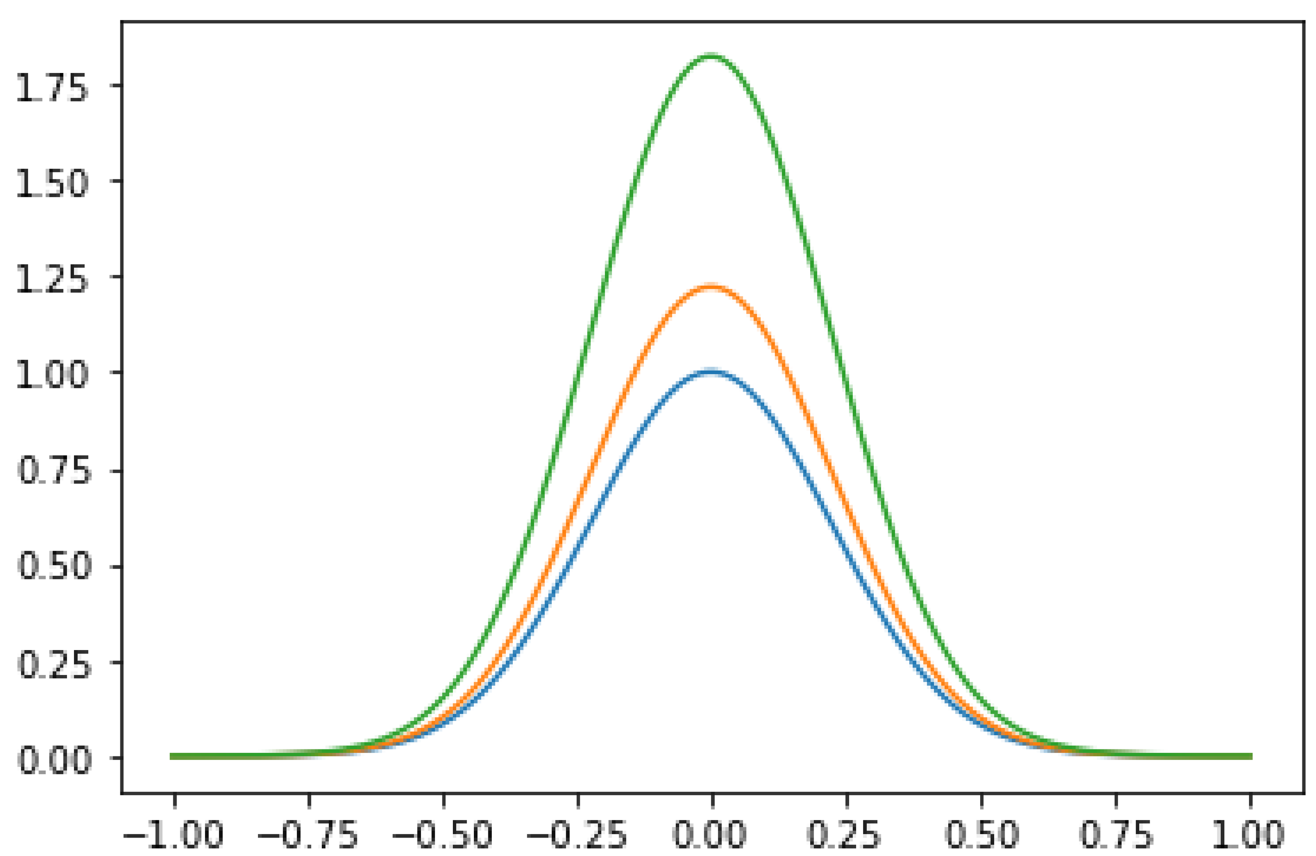

Figure 1. Numerical Sol. For FPE in 1D

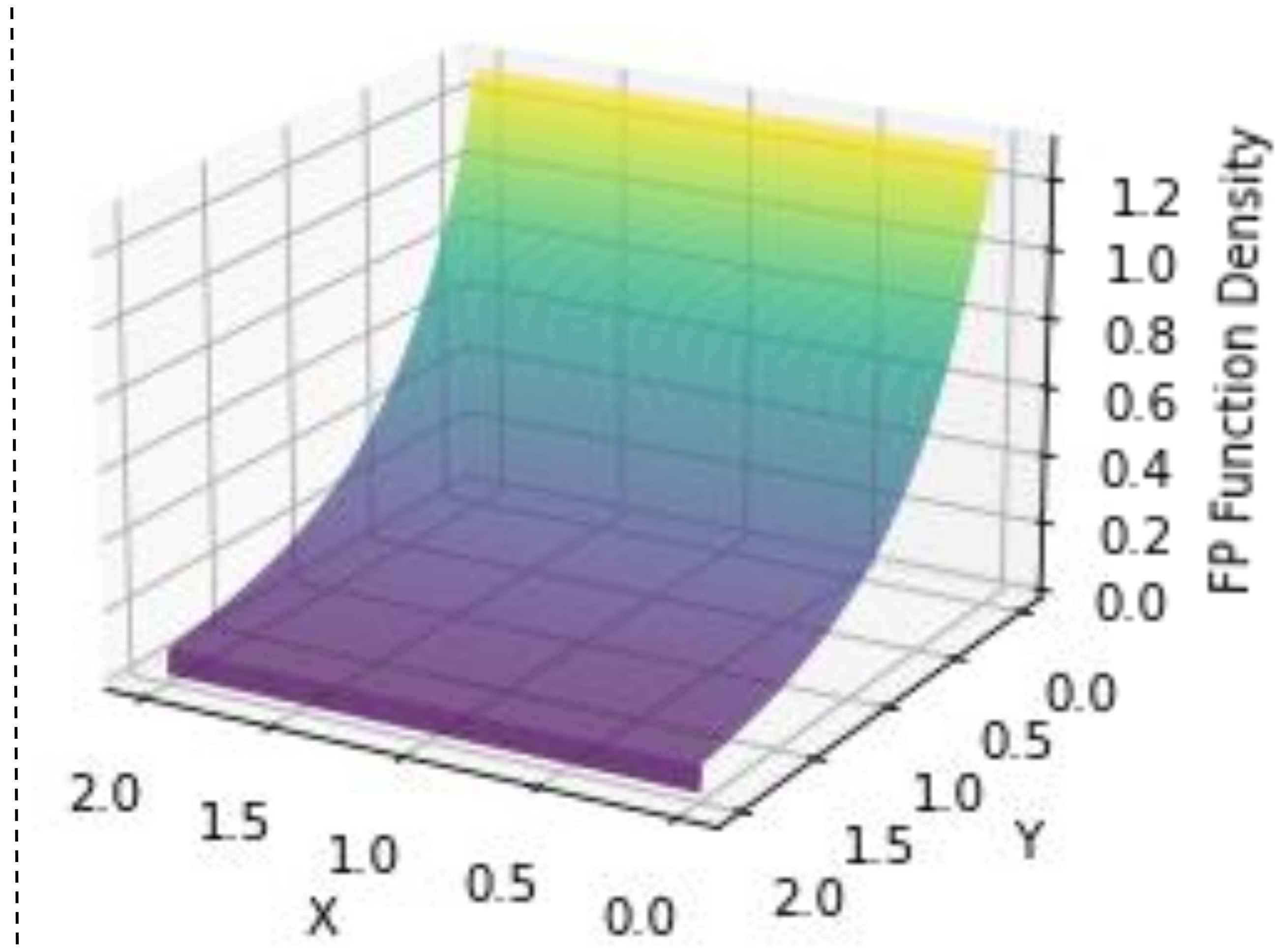

Figure 2. 3D representation of FPE

Link to Github for code:

https://github.com/ShaneGervais/Fokker-Planck-Equation-Using-FTCS-Method

References

1. Statistical Mechanics by R.K. Pathria, Butterworth Heinemann, Elsevier Science & Technology, 1996
2. Computational Physics by Mark Newmann, University of Michigan, 2012

$$\frac{f_{i+1,j}^k - f_{i-1,j}^k}{s} = \lambda B\left[\frac{r_j^k(f_{i,j+1}^k - f_{i,j-1}^k)}{2\Delta x} + \frac{f_{i,j}^k(r_{j+1}^k - r_{j-1}^k)}{2\Delta x} + \frac{r_j^k(f_{i,j}^{k+1} - f_{i,j}^{k-1})}{2\Delta y} + \frac{f_{i,j}^k(r_j^{k+1} - r_j^{k-1})}{2\Delta y}\right] + kBT\left(\frac{f_{i,j+1}^k - 2f_{i,j}^k + f_{i,j-1}^k}{\Delta x^2} + \frac{f_{i,j}^{k+1} - 2f_{i,j}^k + f_{i,j}^{k-1}}{\Delta y^2}\right)$$