

## Methodology

The Stirling engine will be based on Section 8.5 of the paper.

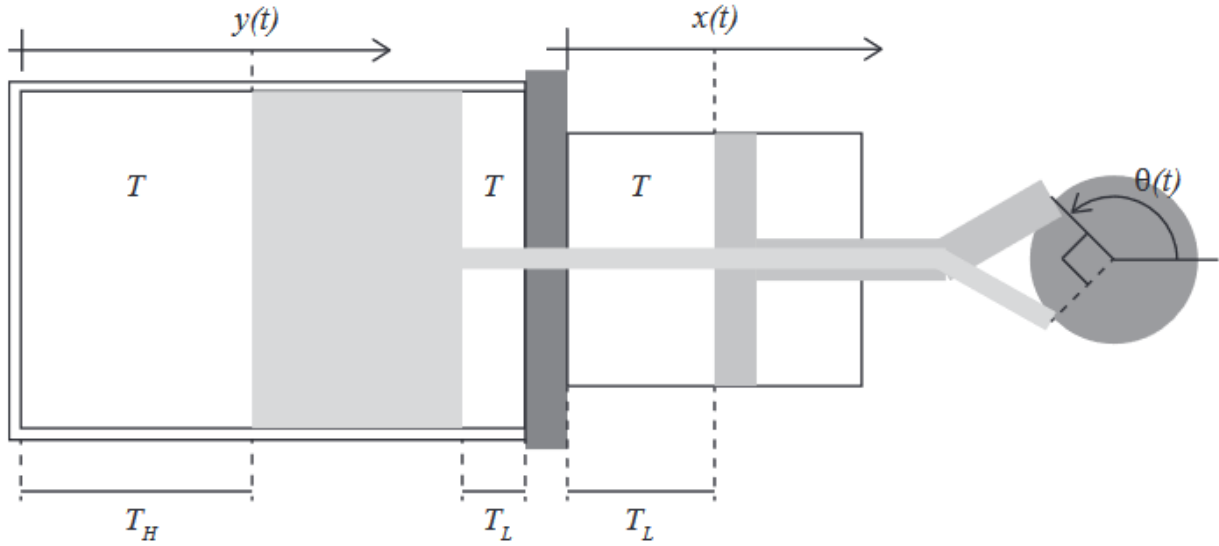


Figure 1. Realistic Stirling Engine

The following differential equation describes the piston in motion from Figure 1.

$$\ddot{\theta} = -FR\cos\theta - b\dot{\theta}$$

Where  $F$  is the force done by the piston  $F = P \cdot A_p$  ( $A_p$  surface area of the piston) and  $R$  is the radius of the piston.

Solving the ODE, we can have a solution for how the angle changes with time, therefore we can find the volume with respect to the angle of the pin during isothermal processes for when the piston is either expanding or compressing.

$$\frac{V_{exp}}{V_{comp}} = \left[ 1 + \frac{1}{2}(c_r - 1) \left( r + 1 - \cos(\theta) - (r - \sin^2(\theta))^{\frac{1}{2}} \right) \right] * V_c$$

Where  $c_r$  is the compression ratio,  $r$  is the ratio of connection rod length and crank pin radius (Stroke length/2) and  $V_c$  is the compressed volume.

In an isothermal process, which will be from A to B and C to D we know,

$$P_a V_{exp/comp} = \text{constant}$$

$$P_{exp/comp} = \text{constant} / V_{exp/comp}$$

And pressure will change as such,

$$P_a V_d = P_b V_c$$

Where  $V_d$  is the displaced volume by the piston.

Then in isochoric processes, from B to C and D to A.

$$\frac{P_b}{T_b} = \frac{P_c}{T_c}$$

Where  $T_b = T_h$  which is our maximum temperature and  $T_c = T_l$  is our lowest temperature.

We then receive the following plot.

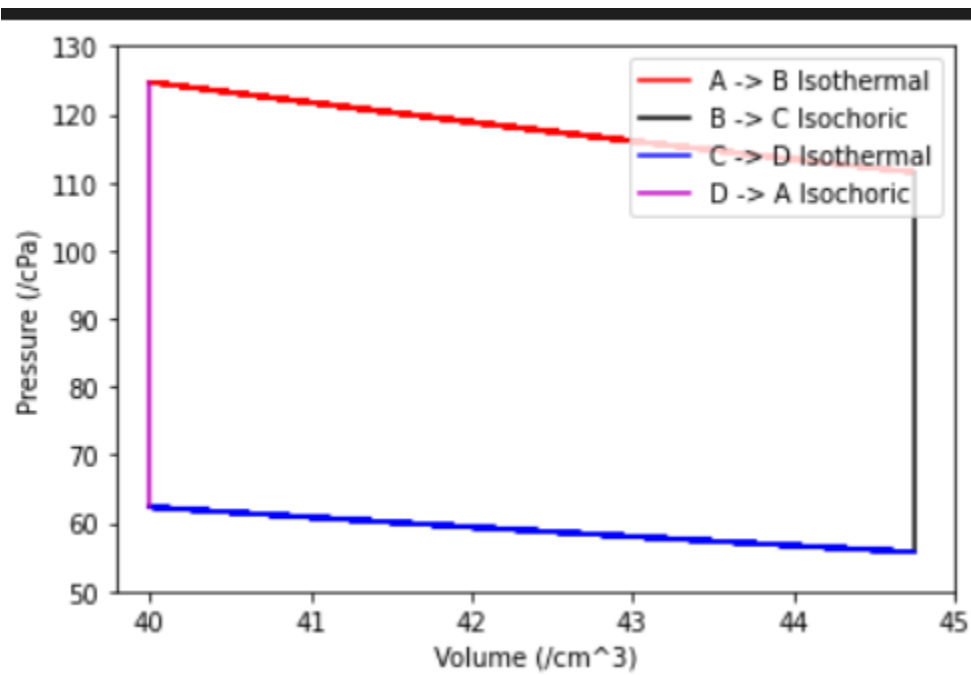


Figure 2. PV graph of the Stirling Cycle