Report on Facial Expression Classification

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Abstract

The abstract paragraph should be indented 1/2 inch (3 picas) on both left and right-hand margins. Use 10 point type, with a vertical spacing of 11 points. The word **Abstract** must be centered, bold, and in point size 12. Two line spaces precede the abstract. The abstract must be limited to one paragraph.

Part I

Solutions to individual problems

1 Problems from Bishop

1.1 Solution 1.1

Let $\lambda = 2$ and use (1.41) on the LHS of (1.42),

$$\begin{aligned}\prod_{i=1}^{d}\int_{-\infty}^{\infty} e^{-x_i^2}dx_i &= \prod_{i=1}^{d}\int_{-\infty}^{\infty} e^{-\frac{2}{2}x_i^2}dx_i \\ &= \prod_{i=1}^{d}(\frac{2\pi}{2})^{\frac{1}{2}} \\ &= \pi^{\frac{d}{2}}\end{aligned}$$

Compare the result with RHS of (1.42),

$$\begin{aligned}S_d &= \frac{\pi^{\frac{d}{2}}}{\int_0^\infty e^{-r^2}r^{d-1}dr} \\ &= \frac{2\pi^{\frac{d}{2}}}{\int_0^\infty e^{-r^2}r^{d-2}2rdr} \\ &= \frac{2\pi^{\frac{d}{2}}}{\int_0^\infty e^{-r^2}r^{d-2}dr^2}\end{aligned}$$

We substitute r^2 with u. Since $r > 0$, $r = u^{\frac{1}{2}}$.

$$
\begin{aligned}
S_d &= \frac{2\pi^{\frac{d}{2}}}{\int_0^{\infty} e^{-r^2} r^{d-2} dr^2} \\
&= \frac{2\pi^{\frac{d}{2}}}{\int_0^{\infty} e^{-u} u^{\frac{d}{2}-1} du} \\
&= \frac{2\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2})}
\end{aligned}
$$

Thus, we proved the correctness of (1.43).

Then, we verify S_d is the surface area of the unit sphere in d dimensions, which should be $S_2 = 2\pi$ and $S_4 = 4\pi$.

$$
\begin{aligned}
S_2 &= \frac{2\pi^{\frac{2}{2}}}{\Gamma(1)} \\
&= \frac{2\pi}{1} \\
&= 2\pi
\end{aligned}
$$

$$
\begin{aligned}
S_3 &= \frac{2\pi^{\frac{3}{2}}}{\Gamma(\frac{3}{2})} \\
&= \frac{2\pi^{\frac{3}{2}}}{\frac{\pi^{\frac{1}{2}}}{2}} \\
&= 4\pi
\end{aligned}
$$

Thus, we verified the correctness of (1.43).

1.2 Solution 1.2

Since

$$
V_d = \frac{\pi^{d/2}}{\Gamma(\frac{d}{2}+1)} a^d
$$

and

$$
\Gamma(x+1) = x\Gamma(x)
$$

so

$$
\begin{aligned}
V_d &= \frac{\pi^{d/2}}{\frac{d}{2}\Gamma(\frac{d}{2})} a^d \\
&= \frac{2\pi^{d/2}}{\Gamma(\frac{d}{2})} \frac{a^d}{d} \\
&= \frac{S_d a^d}{d}
\end{aligned}
$$

Thus, (1.45) is proved.

The volume of a d dimensional hypercube with edges a is

$$
V_d = a^d
$$

So, in d dimension, given a hypercube with edge 2a and hypersphere with radius a, we have

$$\begin{aligned}\frac{volume\ of\ a\ sphere}{volume\ of\ a\ cube} &= \frac{\frac{S_d a^d}{d}}{(2a)^d} \\ &= \frac{S_d}{d2^d} \\ &= \frac{\pi^{\frac{d}{2}}}{d2^{d-1}\Gamma(\frac{d}{2})}\end{aligned}$$

Thus, we've proved (1.46).

Using (1.46) and (1.47),

$$\begin{aligned}\lim_{d\to\infty} \frac{volume\ of\ a\ sphere}{volume\ of\ a\ cube} &= \lim_{d\to\infty} \frac{\pi^{\frac{d}{2}}}{d2^{d-1}\Gamma(\frac{d}{2})} \\ &= \lim_{d\to\infty} \frac{\pi^{\frac{d}{2}}}{d2^{d-1}\Gamma((\frac{d}{2}-1)+1)} \\ &= \lim_{d\to\infty} \frac{\pi^{\frac{d}{2}}}{d2^{d-1}(2\pi)^{\frac{1}{2}}e^{-(\frac{d}{2}-1)}(\frac{d}{2}-1)^{\frac{d+1}{2}}}\end{aligned}$$

Let $x = \frac{d}{2} - 1$,

$$\begin{aligned}\lim_{d\to\infty} \frac{volume\ of\ a\ sphere}{volume\ of\ a\ cube} &= \lim_{d\to\infty} \frac{\pi^{\frac{d}{2}}}{d2^{d-1}(2\pi)^{\frac{1}{2}}e^{-(\frac{d}{2}-1)}(\frac{d}{2}-1)^{\frac{d+1}{2}}} \\ &= \lim_{x\to\infty} \frac{\pi^{x+\frac{1}{2}}}{2^{\frac{5}{2}}(x+1)x^{\frac{3}{2}}(\frac{4x}{e})^x} \\ &= \lim_{x\to\infty} [\frac{\pi^{\frac{1}{2}}}{2^{\frac{5}{2}}(x+1)x^{\frac{3}{2}}}(\frac{e\pi}{4x})^x] \\ &= \lim_{x\to\infty} [\frac{\pi^{\frac{1}{2}}}{2^{\frac{5}{2}}(x+1)x^{\frac{3}{2}}}e^{x\ln\left(\frac{e\pi}{4x}\right)}] \\ &= (\frac{\pi}{32})^{\frac{1}{2}} \lim_{x\to\infty} \frac{(e^{\frac{e\pi}{4}})^{-(\frac{4x}{e\pi})\ln\left(\frac{4x}{e\pi}\right)}}{(x+1)x^{\frac{3}{2}}}\end{aligned}$$

For numerator, let $y = \frac{4x}{e\pi}$, then we have

$$\begin{aligned}&= \lim_{x\to\infty} (e^{\frac{e\pi}{4}})^{-(\frac{4x}{e\pi})\ln\left(\frac{4x}{e\pi}\right)} \\ &= \lim_{y\to\infty} (e^{\frac{e\pi}{4}})^{-y\ln y} \\ &= (e^{\frac{e\pi}{4}})^{\lim_{y\to\infty} -y\ln y} \\ &= 0.\end{aligned}$$

For denominator,

$$\begin{aligned}&= \lim_{d\to\infty} (x+1)x^{\frac{3}{2}} \\ &= \infty.\end{aligned}$$

Combining the limits above, it's obvious that

$$\lim_{d\to\infty} \frac{volume\ of\ a\ sphere}{volume\ of\ a\ cube} = 0.$$

Given a d dimensional hypercube with edge a, the distance from center to one corner is $\frac{1}{2}a\sqrt{d}$, and the distance from the centre to one face is $\frac{1}{2}a$. Thus the ratio is

$$\frac{distance\ from\ center\ to\ one\ corner}{distance\ from\ the\ centre\ to\ one\ face} = \sqrt{d}.$$

And it's obvious that

$$\lim_{d\to\infty} \sqrt{d} = \infty.$$

1.2.1 Solution 1.3

Using (1.45), we have

$$\begin{aligned} f &= (\frac{S_d a^d}{d} - \frac{S_d(a-\epsilon)^d}{d})/\frac{S_d a^d}{d} \\ &= 1 - (1 - \frac{\epsilon}{a})^d. \end{aligned}$$

Since $0 < \epsilon < a$, it's obvious that $0 < 1 - \frac{\epsilon}{a} < 1$. So

$$\begin{aligned} \lim_{d\to\infty} [1 - (1 - \frac{\epsilon}{a})^d] &= 1 - \lim_{d\to\infty} (1 - \frac{\epsilon}{a})^d \\ &= 1 - 0 \\ &= 1 \end{aligned}$$

Then, we evaluate the value of f as following.

Table 1: Evaluation of f

ϵ/a	**d**	**f**
0.01	2	0.01990000000000003
0.01	10	0.09561792499119559
0.01	1000	0.9999568287525893
0.02	2	0.03960000000000008
0.02	10	0.1829271931124533
0.02	1000	0.9999999983170327