10.9 Seasonal Models - More General Model

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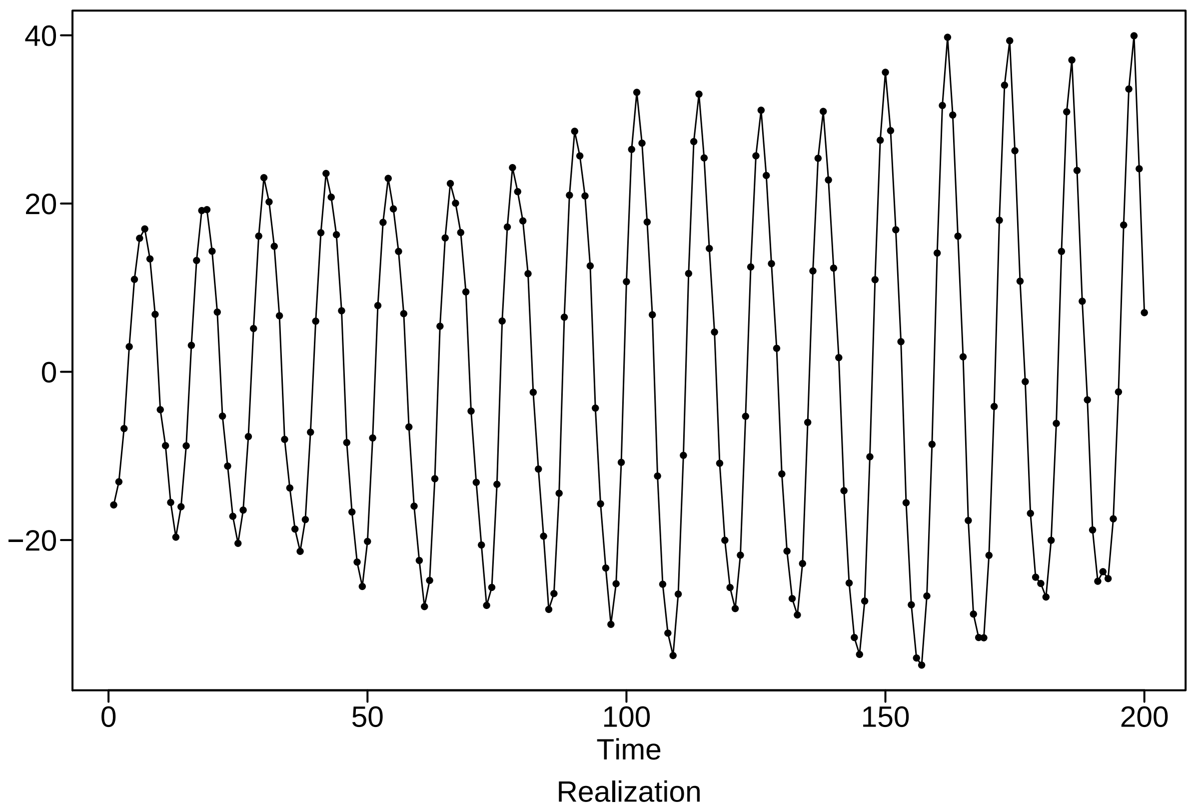
Model

$$(1 - B^{12})(1 - 1.5B + 0.8B^2)(X - 50) = a_t$$

```
x=gen.aruma.wge(n=200, s=12,phi = c(1.5,-0.8), sn=87)
```

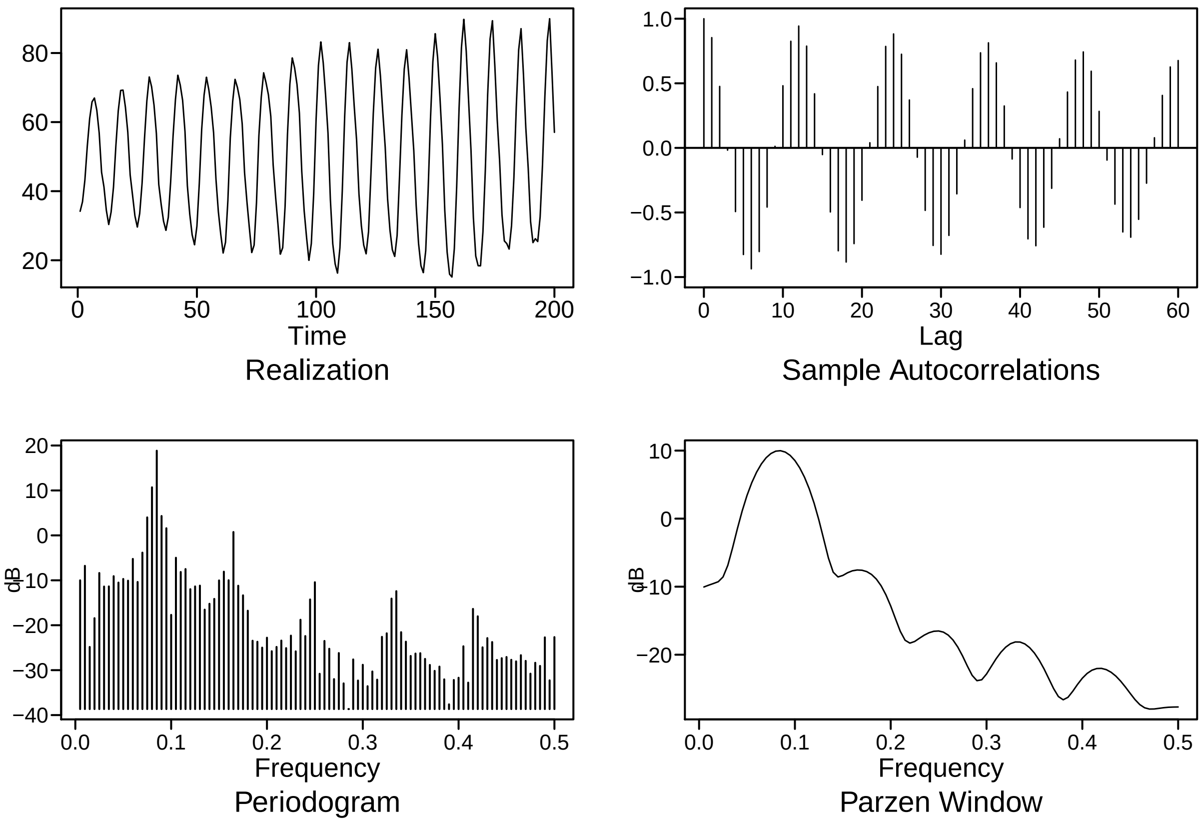



```
# add in the mean
x=x+50
```

Visualize The Data

See what we are dealing with:

```
plot=plotts.sample.wge(x,lag.max = 60)
```



Overfit Factor Table

Use a number over what you think the seasonality looks to be.

```r
d15=est.ar.wge(x,p=17, type = 'burg')
```

```
##
## Coefficients of Original polynomial:
## 1.3330 -0.5138 -0.1128 -0.1721 0.1693 -0.0499 -0.0030 -0.0388 0.0590 -0.1264 0.0688 0.8973 -1.2428 0
##
## Factor                 Roots                Abs Recip    System Freq
## 1-1.7276B+0.9985B^2    0.8651+-0.5031i      0.9992       0.0838
## 1-1.0028B+0.9955B^2    0.5036+-0.8665i      0.9977       0.1662
## 1+1.0010B+0.9936B^2   -0.5037+-0.8676i      0.9968       0.3337
## 1-0.0157B+0.9930B^2    0.0079+-1.0035i      0.9965       0.2487
## 1+1.7329B+0.9899B^2   -0.8753+-0.4941i      0.9949       0.4182
## 1+0.9836B             -1.0166               0.9836       0.5000
## 1-0.9136B              1.0945               0.9136       0.0000
## 1-1.4286B+0.7747B^2    0.9220+-0.6638i      0.8802       0.0993
## 1-0.7137B              1.4011               0.7137       0.0000
## 1+0.7516B+0.3366B^2   -1.1166+-1.3132i      0.5801       0.3621
##
##
```

Look in the factor table for patterens matching know seasonal tables.

Here is a factor table of $(1 - B^{12})$.

```r
#(1-B^12)
factor.wge(c(rep(0,11),1))
```

```
##
## Coefficients of Original polynomial:
## 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 1.0000
##
## Factor                 Roots                Abs Recip    System Freq
## 1-1.0000B+1.0000B^2    0.5000+-0.8660i      1.0000       0.1667
## 1-1.0000B              1.0000               1.0000       0.0000
## 1-1.7321B+1.0000B^2    0.8660+-0.5000i      1.0000       0.0833
## 1+1.0000B+1.0000B^2   -0.5000+-0.8660i      1.0000       0.3333
## 1-0.0000B+1.0000B^2    0.0000+-1.0000i      1.0000       0.2500
## 1+1.7321B+1.0000B^2   -0.8660+-0.5000i      1.0000       0.4167
## 1+1.0000B             -1.0000               1.0000       0.5000
##
##
```

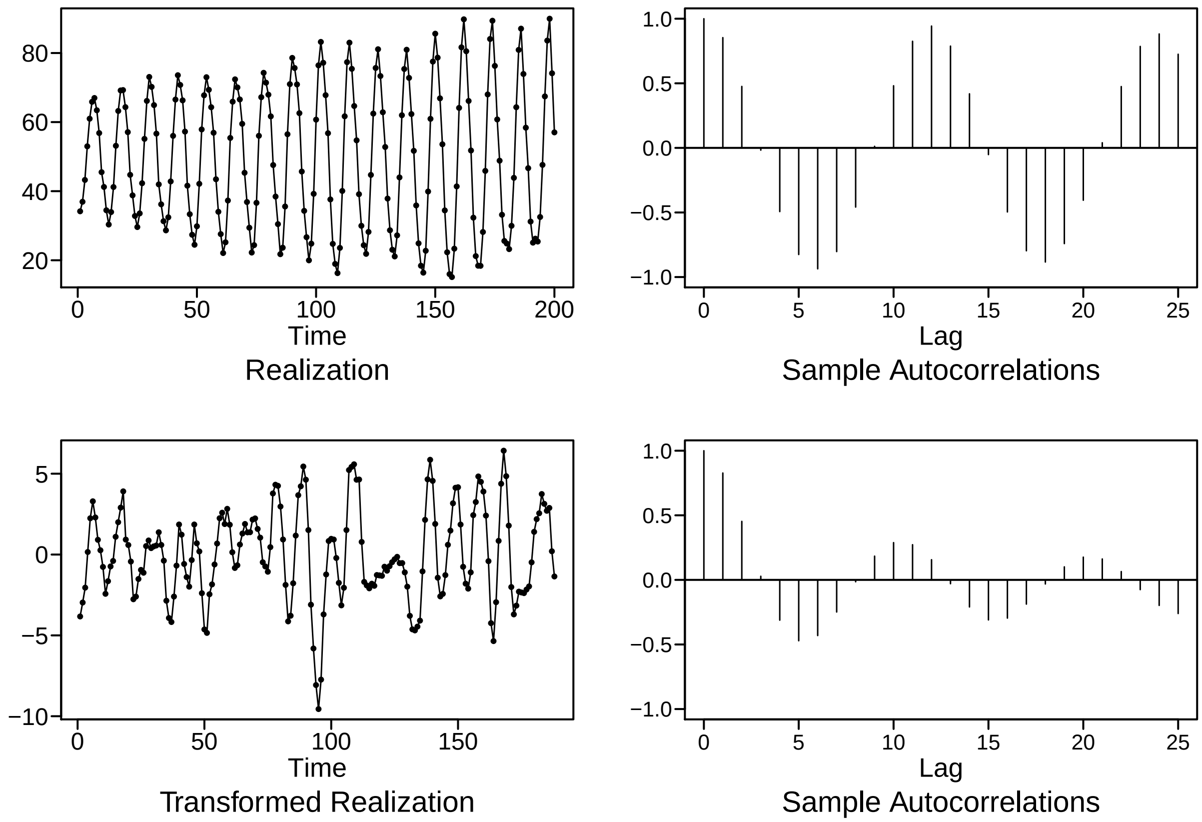
Note: The factor 1-0.9136B is not very close to the unit circle, but since all other factors are consistent with $(1 - B^{12})$ we conclude that this overfit table suggests $(1 - B^{12})$ also.

Transform

Base on the overfit table we can now transform the data to remove the seasonality.

Transform data to create $Y_t = (1 - B^{12})X_t$

```r
#rep will repeat the 11 zeros then we can add a one at the end
y=artrans.wge(x,phi.tr = (c(rep(0,11),1)))
```



The transformed data appear stationary, so we use AIC to identify a model.

We use **aic5.wge** to model the transformed data (y) When using AIC to model data that has been stationarized using the seasonal transform $(1 - B^{12})$, it is good practice to allow a range of p values to include s to uncover any seasonal stationary information that might be in the data. In R code to follow, we consider the range p=0:13 and q=0:3

```
# y is the transformed data
aic5.wge(y,p=0:13,q=0:3)
```

```
## ---------WORKING... PLEASE WAIT...
##
##
## Five Smallest Values of  aic
```

```
##        p    q        aic
## 19     4    2 0.06290065
## 9      2    0 0.06764617
## 20     4    3 0.06910374
## 13     3    0 0.07338218
## 10     2    1 0.07361012
```

AIC selects an ARMA(4,2) model. Factoring the ARMA(4,2) model we obtain:

```
est = est.arma.wge(y,p=4, q=2, factor = TRUE)
```

```
##
## Coefficients of Original polynomial:
## 1.8831 -2.1943 1.5169 -0.6321
```

```
##
## Factor                 Roots                Abs Recip    System Freq
## 1-0.4098B+0.8135B^2    0.2519+-1.0798i      0.9019       0.2135
## 1-1.4733B+0.7771B^2    0.9480+-0.6231i      0.8815       0.0925
##
##
```

```
est$phi
```

```
## [1]  1.8831035 -2.1942981  1.5169278 -0.6321408
```

```
est$theta
```

```
## [1]  0.4957013 -0.9262404
```

Note: The dominant behavior of the transformed data is a pseudo cyclic behavior of length about 10 ($f_0 = .10$). The factors associated with frequency about $f_0 = .2$ essentially cancel.

We can look for a simpler model using BIC. Use the y data.

```
aic5.wge(y,p=0:13,q=0:3,type='bic')
```

```
## ---------WORKING... PLEASE WAIT...
##
##
## Five Smallest Values of  bic
```

```
##       p    q        bic
## 9     2    0  0.1192915
## 13    3    0  0.1422426
## 10    2    1  0.1424706
## 11    2    2  0.1692218
## 17    4    0  0.1700640
```

BIC picks a simpler AR(2) model. This is consistent with:

- The pseudo cyclic data
- Damping cyclical sample autocorrelations

Decision: We choose to model the transformed data as an AR(2).

```
est2=est.ar.wge(y,p=2)
```

```
##
## Coefficients of Original polynomial:
## 1.4653 -0.7597
##
## Factor                 Roots                Abs Recip    System Freq
## 1-1.4653B+0.7597B^2    0.9644+-0.6215i      0.8716       0.0911
##
##
##
## Coefficients of Original polynomial:
## 1.4653 -0.7597
##
## Factor                 Roots                Abs Recip    System Freq
## 1-1.4653B+0.7597B^2    0.9644+-0.6215i      0.8716       0.0911
##
##
```

We can now write our model using data collected from **est.ar.wge**.

```
#Get the phi's for the model
est2$phi
```

```
## [1]  1.4652837 -0.7596714
```

```
#Get the White Noise Var
est2$avar
```

```
## [1] 1.036377
```

```
#Figure the mean with the original x data
mean(x)
```

```
## [1] 49.77867
```

Our final model:

$$(1 - B^{12})(1 - 1.47B + 0.76B^2)(x_t - 49.78) = a_y \ \sigma_a^2 = 1.04$$

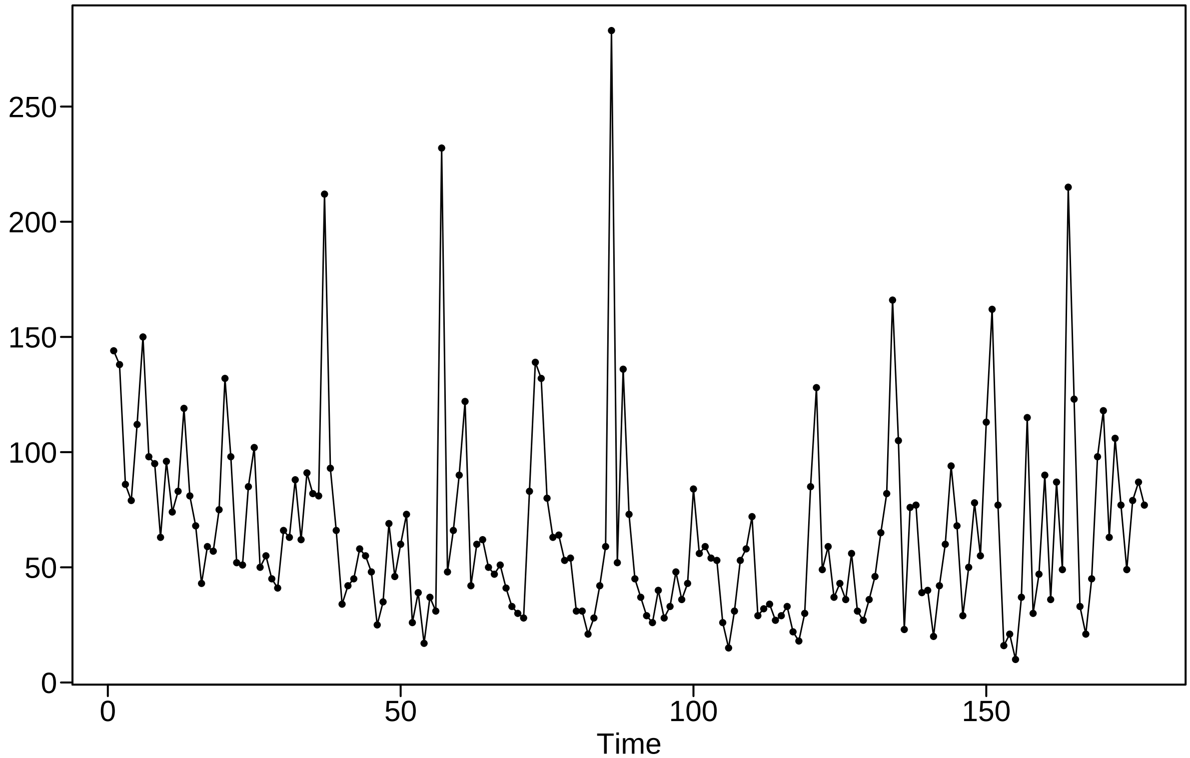
The Original Model:

$$(1 - B^{12})(1 - 1.5B + 0.8B^2)(X - 50) = a_t$$

10.9.4 Concept Check

Consider the attached Southwest Airlines flight delay data (SWADelay.csv). Use **est.ar.wge()** to overfit an **AR(15)** model with the Burg estimates. We are trying to gauge if there is sufficient evidence to suggest a $(1 - B^{12})$ factor is in the model. Look at the factor table, and answer the following questions.

```
#read in the data
SWA = read.csv("swadelay.csv",header = TRUE)
plotts.wge(SWA$arr_cancelled)
```



Realization

```r
#Get and an estimate using the arr_delay information
#Overfitting with and AR(15)
d15SW=est.ar.wge(SWA$arr_delay,p=15, type = 'burg')
```

```
##
## Coefficients of Original polynomial:
## 0.4443 0.0482 -0.0541 0.0820 0.0090 0.0304 0.0410 -0.0837 0.0588 0.0568 0.0252 0.3759 -0.0127 -0.007
##
## Factor                 Roots                Abs Recip    System Freq
## 1-0.9700B               1.0310              0.9700       0.0000
## 1-0.9946B+0.9342B^2     0.5323+-0.8872i     0.9666       0.1640
## 1-1.6636B+0.9322B^2     0.8923+-0.5259i     0.9655       0.0848
## 1+0.9229B              -1.0836              0.9229       0.5000
## 1+1.6328B+0.8443B^2    -0.9669+-0.4994i     0.9189       0.4241
## 1-0.1460B+0.8319B^2     0.0877+-1.0928i     0.9121       0.2372
## 1+0.7038B+0.7209B^2    -0.4882+-1.0719i     0.8490       0.3180
## 1+0.7885B+0.5243B^2    -0.7520+-1.1584i     0.7241       0.3416
## 1-0.7182B               1.3924              0.7182       0.0000
##
##
```

10.9.4 What is the root associated with the factor (from the factor table you generated) that would be matched with the $(1 - B)$ term from the $(1 - B^{12})$ factor table?

Express your response rounded to four decimal places. *1.0310*

10.9.5 Look at the system frequencies from the factor table you generated. What is the system frequency associated with the the factor that would be matched with the $(1 - 1.732B + B^2)$ factor from the $(1 - B^{12})$

factor table?

```
factor.wge(phi = c(rep(0,11),1))
```

```
##
## Coefficients of Original polynomial:
## 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 1.0000
##
## Factor                 Roots                Abs Recip    System Freq
## 1-1.0000B+1.0000B^2    0.5000+-0.8660i      1.0000       0.1667
## 1-1.0000B              1.0000               1.0000       0.0000
## 1-1.7321B+1.0000B^2    0.8660+-0.5000i      1.0000       0.0833
## 1+1.0000B+1.0000B^2   -0.5000+-0.8660i      1.0000       0.3333
## 1-0.0000B+1.0000B^2    0.0000+-1.0000i      1.0000       0.2500
## 1+1.7321B+1.0000B^2   -0.8660+-0.5000i      1.0000       0.4167
## 1+1.0000B             -1.0000               1.0000       0.5000
##
##
```

Express your response rounded to four decimal places. *0.0848*

10.9.6 Do you feel that there is enough evidence to suggest a $(1 - B^{12})$ factor should be present in our final model?

Yes

This is subjective, but there is only one term that doesn't match up almost exactly (the $(1 + B + B^2)$) B. No. We will take this answer, but you would almost have some domain knowledge or other evidence to not include a $(1 - B^{12})$ in this case. Note: The spectral density shows a clear peak at $1/12 = .08$, but this does not in and of itself mean that $(1 - B^{12})$ is appropriate. There are many other factors of smaller degree that will yield this frequency.

Note: The spectral density shows a clear peak at $1/12 = .08$, but this does not in and of itself mean that $(1 - B^{12})$ is appropriate. There are many other factors of smaller degree that will yield this frequency.

Forcast with the final model:

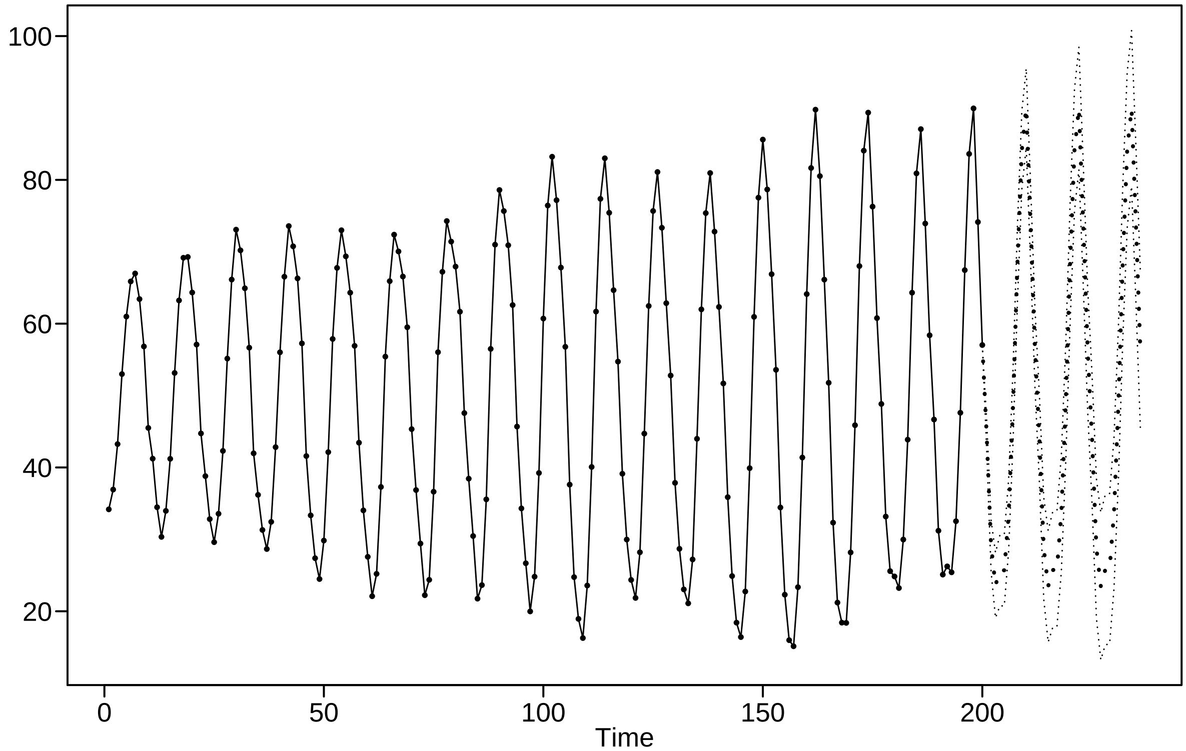
$$(1 - B^{12})(1 - 1.47B + 0.76B^2)(x_t - 49.78) = a_y \ \sigma_a^2 = 1.04$$

Generate data:

```
#This is our original data at the top of the project
x=gen.aruma.wge(n=200, s=12,phi = c(1.5,-0.8), sn=87, plot = F)
# add in the mean
x=x+50
```

Forcast:

```
fore.aruma.wge(x,s=12,phi = c(1.47,-0.76),n.ahead = 36,lastn = FALSE)
```

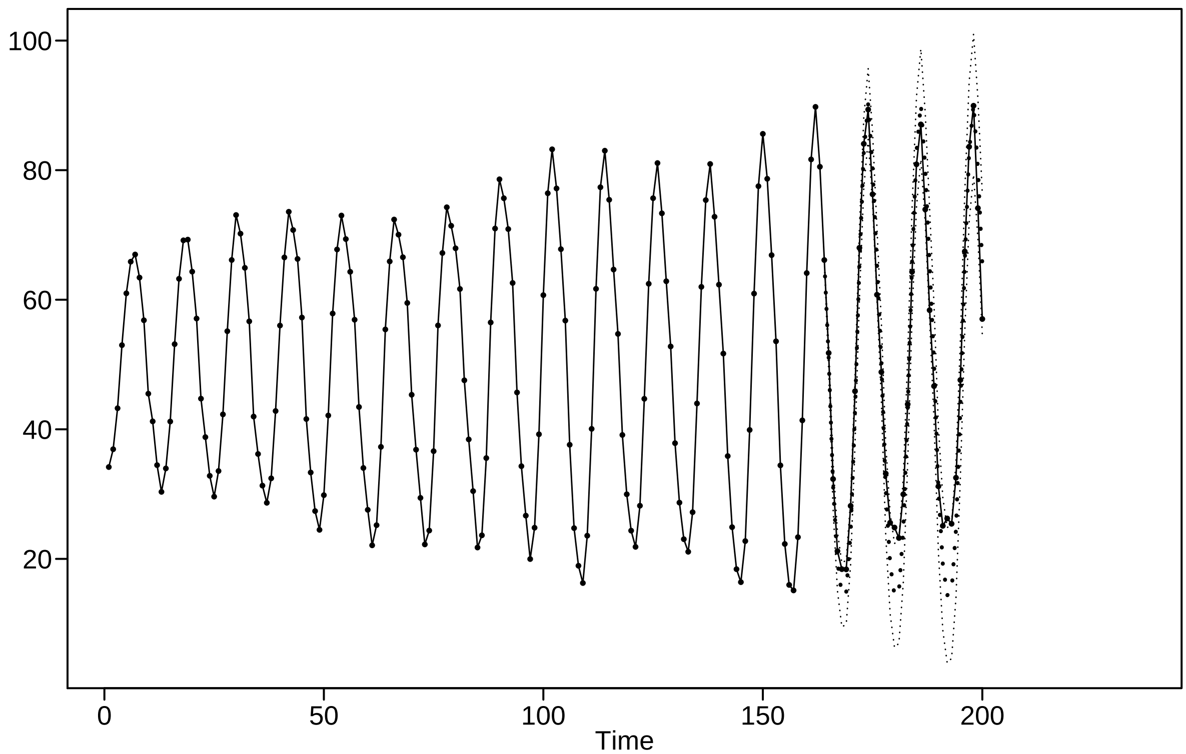



```
## $f
##  [1] 44.51711 29.07259 23.60548 25.66225 25.70131 33.38056 48.65358 68.32965
##  [9] 84.11652 90.01330 73.85623 56.56400 44.04619 28.73346 23.46486 25.71329
## [17] 25.88320 33.60915 48.85137 68.44668 84.13823 89.95626 73.75590 56.45985
## [25] 43.96935 28.69966 23.47357 25.75178 25.93316 33.65334 48.87836 68.45277
## [33] 84.12667 89.93464 73.73290 56.44247
##
## $ll
##  [1] 42.51188 25.50750 19.06663 20.74594 20.74324 28.39816 43.52612 63.01281
##  [9] 78.67735 84.53921 68.38203 51.07396 37.95225 21.67015 15.69402 17.67706
## [17] 17.82827 25.51937 40.62009 60.04894 75.64186 81.43634 65.23583 47.92171
## [25] 34.98904 18.99868 13.23355 15.31231 15.48203 23.17027 38.27729 57.71671
## [33] 73.31266 79.10314 62.90110 45.59484
##
## $ul
##  [1]  46.52235  32.63768  28.14432  30.57857  30.65939  38.36297  53.78105
##  [8]  73.64650  89.55569  95.48738  79.33043  62.05403  50.14012  35.79677
## [15]  31.23571  33.74952  33.93813  41.69894  57.08265  76.84442  92.63460
## [22]  98.47618  82.27596  64.99800  52.94965  38.40064  33.71359  36.19124
## [29]  36.38429  44.13642  59.47944  79.18883  94.94067 100.76614  84.56469
## [36]  67.29011
##
## $resid
##    [1]  0.000000000  0.000000000  0.000000000  0.000000000  0.000000000
##    [6]  0.000000000  0.000000000  0.000000000  0.000000000  0.000000000
##   [11]  0.000000000  0.000000000  0.000000000  0.000000000 -0.595229145
```

```
##  [16]  0.911784987  0.456564996  0.114340182 -0.843491301  0.043296259
##  [21]  0.669629379 -0.464404620 -1.100671556  1.339597434 -0.158378369
##  [26] -0.568541786  1.122241277  0.084642636  0.796184805  1.162178965
##  [31] -2.615622998  2.200796105 -0.597117905 -1.679998978  1.132321304
##  [36]  0.211005728 -0.703732248 -0.879112722  1.454723765 -0.751873520
##  [41] -0.485884402  0.579532259  0.120942857  0.938922035 -1.006850760
##  [46] -0.205589896 -1.847263129 -0.016093264 -0.572709205  0.550554119
##  [51] -0.035960089  0.890851002 -2.023985608 -0.976845991  0.395501927
##  [56] -0.379730136  1.519501736  0.849516957 -2.294358085  0.579929563
##  [61] -2.143821625 -0.971456055  0.145644078  1.146006319 -1.918085254
##  [66]  0.234253750  0.179066095  0.779177639 -0.197422137 -0.210137868
##  [71]  2.019882017 -0.877733636 -0.420944688  0.363981737  0.663174431
##  [76]  0.963050988 -0.114177480  0.451195165 -0.421309868  0.802835920
##  [81]  1.172372846  0.107673656 -0.059111574  0.416692871 -0.811425388
##  [86]  0.760834943 -0.344933883  1.461922806  2.296955277 -0.887464488
##  [91]  0.772844332  0.003971943 -0.205153352 -0.989127644 -0.665786838
##  [96]  0.863917956  0.644805907  0.908761850  0.596508701 -0.285640933
## [101]  2.031987759 -0.168137192 -1.152258027 -1.810628428 -0.099313438
## [106] -1.888742174 -2.106852868  0.177801264  0.405477929 -1.667660307
## [111] -0.168954684 -1.192328585  0.135145503 -0.841749547 -0.730730350
## [116] -0.731030806  1.234015597  2.145525917  1.439742427 -1.113922504
## [121]  1.586929454  0.542577071  2.080093060 -2.525440961  0.688339141
## [126]  1.171108370 -0.568364784 -0.185994762 -0.866730518  0.202440113
## [131] -0.896081194 -0.383249967  0.200194420 -0.890199304  0.173001402
## [136] -0.172491505 -0.142845265 -0.074380625 -0.544286476  0.146121980
## [141] -0.731984289 -0.767739890 -1.706488834 -0.565776833 -0.772150557
## [146] -1.076047281 -1.101256531  1.579337273  0.570295191  0.705407107
## [151]  0.655674769 -0.525920349 -0.349430707 -0.755395075  0.960453213
## [156]  0.279781522  0.338578988  0.624093378 -0.373393399  1.451024612
## [161]  0.596923139  0.499047313 -1.122916982 -0.323498787  0.723610352
## [166] -0.037184593  0.628756256  2.452846435 -1.169190743  1.905225176
## [171] -0.132863392  0.954924575  0.099683438 -0.989699370 -1.811654301
## [176]  0.574822467  1.695689231  1.111144991  0.894778148  0.628729326
## [181] -1.267664710 -0.452512670 -0.959019449  0.614320200  0.753217095
## [186] -0.463355958 -1.372517199 -0.683937789 -0.440442706 -0.602035367
## [191]  0.770883099  0.613989787 -0.240956104  0.401685341  1.653065637
## [196] -0.424383053  0.941612085  1.285321502 -1.976209020  0.536327859
##
## $wnv
## [1] 1.046692
##
## $se
##  [1] 1.023080 1.818925 2.315739 2.508323 2.529629 2.542044 2.616054 2.712676
##  [9] 2.775085 2.792899 2.792959 2.801039 3.109151 3.603729 3.964716 4.100116
## [17] 4.109660 4.127442 4.199632 4.284563 4.334883 4.346898 4.346974 4.356196
## [25] 4.581788 4.949480 5.224499 5.326257 5.332210 5.348508 5.408710 5.477582
## [33] 5.517349 5.526275 5.526427 5.534505
##
## $psi
##  [1]  1.47000000  1.40090000  0.94212300  0.32023681 -0.24526537 -0.60392007
##  [7] -0.70136082 -0.57202115 -0.30783687 -0.01778413  0.20781336  1.31900157
## [13]  1.78099416  1.61562022  1.02140616  0.27359569 -0.37408302 -0.75783476
## [19] -0.82971400 -0.64372517 -0.31569335  0.02516190  0.27691494  1.38794192
## [25]  1.82981927  1.63499846  1.01278510  0.24619526 -0.40780964 -0.78658857
```

```
## [31] -0.84634987 -0.64632700 -0.30687478  0.04010258  0.29217564  1.39902022
##
## $ptot
## [1] 14
##
## $phitot
##  [1]  1.47 -0.76  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  1.00
## [13] -1.47  0.76
```

```
forcast = fore.aruma.wge(x,s=12,phi = c(1.47,-0.76),n.ahead = 36,lastn = TRUE)
```



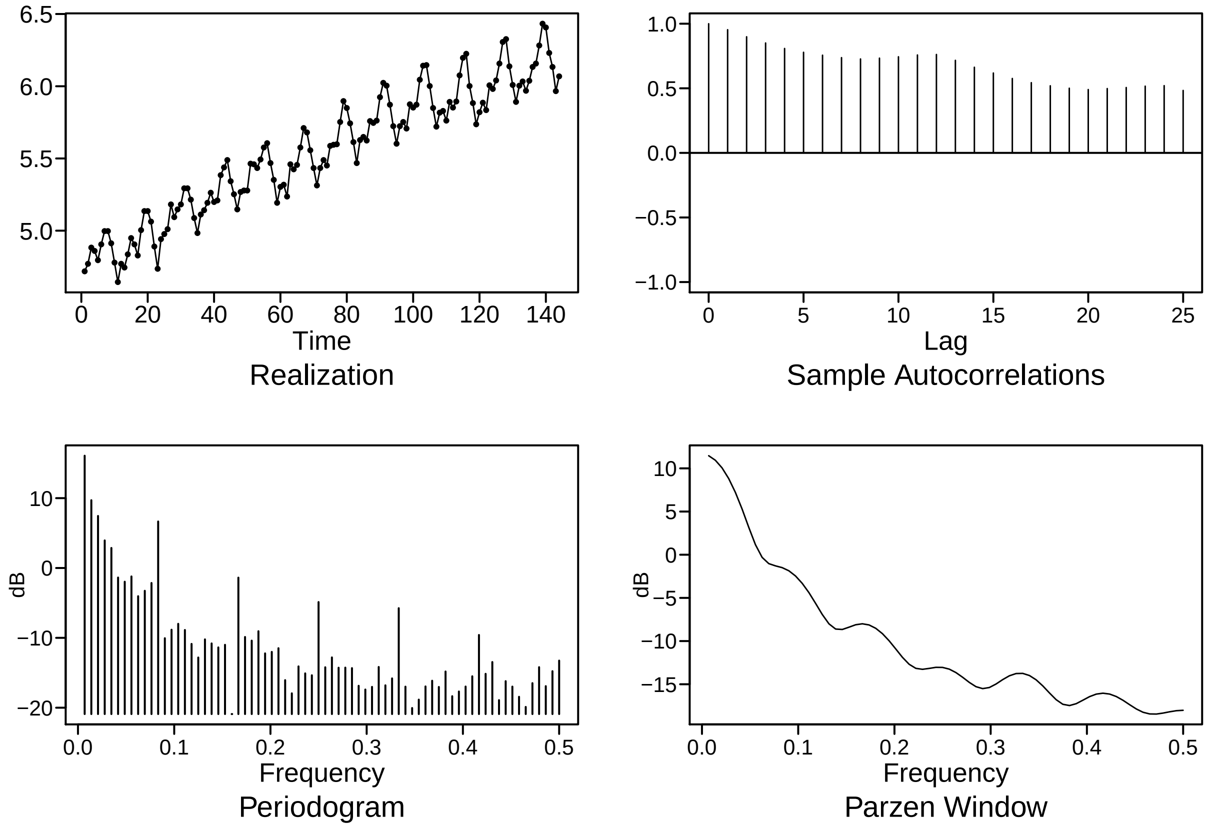

```
ase = mean((x[(200-36+1):200] - forcast$f)^2)
```

Summary:

- The forecasts (open circles) are very close to the true values.
- We were able to fit a model very close to the true model.
- Forecasts (ahead and last 36) are quite good.

10.10 Seasonal Models: Example-Airline Data

```
data("airlog")
lair=ts(airlog)
plotts.sample.wge(airlog)
```




```
## $autplt
##  [1] 1.0000000 0.9537034 0.8989159 0.8508025 0.8084252 0.7788994 0.7564422
##  [8] 0.7376017 0.7271313 0.7336487 0.7442552 0.7580266 0.7619429 0.7165045
## [15] 0.6630428 0.6183629 0.5762087 0.5438013 0.5194561 0.5007029 0.4904028
## [22] 0.4981819 0.5061666 0.5167434 0.5204897 0.4835237
##
## $freq
##  [1] 0.006944444 0.013888889 0.020833333 0.027777778 0.034722222 0.041666667
##  [7] 0.048611111 0.055555556 0.062500000 0.069444444 0.076388889 0.083333333
## [13] 0.090277778 0.097222222 0.104166667 0.111111111 0.118055556 0.125000000
## [19] 0.131944444 0.138888889 0.145833333 0.152777778 0.159722222 0.166666667
## [25] 0.173611111 0.180555556 0.187500000 0.194444444 0.201388889 0.208333333
## [31] 0.215277778 0.222222222 0.229166667 0.236111111 0.243055556 0.250000000
## [37] 0.256944444 0.263888889 0.270833333 0.277777778 0.284722222 0.291666667
## [43] 0.298611111 0.305555556 0.312500000 0.319444444 0.326388889 0.333333333
## [49] 0.340277778 0.347222222 0.354166667 0.361111111 0.368055556 0.375000000
## [55] 0.381944444 0.388888889 0.395833333 0.402777778 0.409722222 0.416666667
## [61] 0.423611111 0.430555556 0.437500000 0.444444444 0.451388889 0.458333333
## [67] 0.465277778 0.472222222 0.479166667 0.486111111 0.493055556 0.500000000
##
## $db
##  [1]  16.074652   9.707151   7.450389   3.951430   2.870990  -1.362910
##  [7]  -1.963401  -1.207527  -4.037011  -3.261281  -2.144731   6.659726
## [13] -10.049663  -8.832691  -7.989512  -8.870613 -10.863837 -12.810353
## [19] -10.215250 -10.798307 -11.364355 -10.996384 -20.907381  -1.379644
## [25]  -9.873929 -10.397306  -9.049324 -12.219998 -12.007972 -11.480775
```

```
## [31] -16.052737 -17.938246 -14.082488 -15.083055 -15.354353  -4.875903
## [37] -14.208957 -12.796969 -14.274424 -14.257255 -14.330363 -16.861169
## [43] -17.397169 -17.013422 -14.168538 -16.792001 -15.802334  -5.751597
## [49] -16.984530 -20.043553 -18.839272 -16.953054 -16.136041 -17.027935
## [55] -14.816991 -18.351344 -17.674782 -16.968754 -15.500680  -9.585170
## [61] -15.148437 -13.462485 -18.912460 -16.196325 -16.978206 -18.433174
## [67] -19.887023 -16.486009 -14.193681 -16.910723 -14.772011 -13.254041
##
## $dbz
##  [1]  11.4654366  10.9402294  10.0590071   8.8164678   7.2162262   5.2946980
##  [7]   3.1768189   1.1618879  -0.3081259  -1.0306219  -1.2919915  -1.4941980
## [13]  -1.8606232  -2.4646056  -3.3131502  -4.3858382  -5.6296105  -6.9173670
## [19]  -8.0045606  -8.6064604  -8.6554265  -8.3893942  -8.1090657  -7.9990125
## [25]  -8.1318582  -8.5188051  -9.1418634  -9.9633188 -10.9162381 -11.8823737
## [31] -12.6853883 -13.1569739 -13.2679842 -13.1632662 -13.0423784 -13.0491791
## [37] -13.2490611 -13.6450446 -14.1898757 -14.7847644 -15.2784729 -15.5045413
## [43] -15.3786493 -14.9725277 -14.4620652 -14.0193387 -13.7595675 -13.7426596
## [49] -13.9886449 -14.4865070 -15.1914119 -16.0104400 -16.7868134 -17.3199722
## [55] -17.4658590 -17.2504902 -16.8437217 -16.4316935 -16.1411403 -16.0357959
## [61] -16.1342649 -16.4226496 -16.8585671 -17.3711175 -17.8654359 -18.2428288
## [67] -18.4379904 -18.4500786 -18.3378652 -18.1841683 -18.0614831 -18.0155070
```

```r
est.lair=est.ar.wge(lair,p=15,type='burg')
```

```
##
## Coefficients of Original polynomial:
## 0.6996 0.2599 0.0079 -0.0646 0.1381 -0.0953 0.0235 -0.0969 0.1770 -0.1191 0.1030 0.7754 -0.4590 -0.4
##
## Factor                 Roots                Abs Recip    System Freq
## 1-1.7253B+0.9953B^2    0.8667+-0.5035i      0.9976       0.0838
## 1-0.9861B+0.9950B^2    0.4955+-0.8715i      0.9975       0.1677
## 1+0.9869B+0.9850B^2   -0.5010+-0.8742i      0.9925       0.3328
## 1-0.0375B+0.9788B^2    0.0192+-1.0106i      0.9893       0.2470
## 1-1.9697B+0.9704B^2    1.0149+-0.0218i      0.9851       0.0034
## 1+1.6891B+0.9504B^2   -0.8886+-0.5124i      0.9749       0.4168
## 1+0.8540B             -1.1709               0.8540       0.5000
## 1+0.6000B             -1.6667               0.6000       0.5000
## 1-0.1110B              9.0072               0.1110       0.0000
##
##
```

Matching up the factor tables it could have a (1-B^12).

Note:

- Factor tables for other high orders are similar
- The factor $1 - 1.9697B + 0.9704B^2$ is associated with a frequency of $f_0 = 0.0034$ or a period of 1/.0034 = 294 (longer than the data record). This suggests aperiodic data or a very long period.
- Also this factor is very close to $1 - 2B + B2 = (1 - B)2$ which is associated with frequency $f_0 = 0$
- We encountered a similar situation with an ARIMA model with $d = 2$.
- $(1 - B)^2$ provides the last "piece" needed for a factor of $(1 - B^{12})$ plus an extra $(1 - B)$ factor
- That is, the factor table suggests the presence of nonstationary factors $(1 - b)(1 - B^{12})$
- Although $(1 + .85B)$ is not as close to the unit circle "as we would expect for s=12 data, the"total picture" suggests a factor of $(1 - B^{12})$