MATH 239 — Assignment 0, Question 2

a) Since there are two possible values for each digit in a binary string, for a string of length n there are 2^n possibilities.

b) There exist only two binary strings of length 2 where each digit is distinct — "10" and "01". Also, there are 2^{n-2} possible binary strings of length $n-2$ (using the same logic as shown in part (a)). Combining these two values, we see that that there are $2^{n-2} \times 2 = 2^{n-1}$ binary strings of length n where the first two digits are distinct.

c) There exist zero binary strings of length n where the first three digits are all distinct simply because it's impossible to have three unique digits in a binary string.