

MATH 239 — Assignment 1, Question 4

Bartosz Antczak (ID: 20603468)

a) Recall the handshaking lemma

$$\sum_{v \in V} \deg(v) = 2|E|$$

Also recall that the sum of an even amount of odd numbers is always even; whereas the sum of an odd amount of odd numbers is always odd. Since every vertex in G is odd, we must always have an even number of vertices in order for the sum to be even, QED.

b) By a corollary of the handshaking lemma, the average degree of the vertices of a graph is

$$\frac{\sum_{v \in V} \deg(v)}{|V|} = \frac{2|E|}{|V|}$$

Since $|E| = |V|$, we have

$$\frac{\sum_{v \in V} \deg(v)}{|V|} = 2$$

Since the average degree of the vertices is 2 where the only two possible degrees are 1 and 3, this must mean that there are an equal number of vertices of degree 1 and degree 3.