Problem A: It's Raining Space Dust

Team 1011
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Motivation

- Cosmic dust originates from comets, interstellar debris, and asteroid collisions.
- Studying it provides clues about:
  - chemical abundance in the early solar system
  - interplanetary material flux
  - Earth's geological deposition record
- **Challenge**: Model different sized particles at different velocities as they fall through earth's atmosphere.

Problem Statement

Goal: Determine whether observed cosmic dust distribution in a geological layer can be used to infer the incoming dust distribution at the top of the atmosphere.

We model:

- Atmospheric entry dynamics
- Heating and mass loss via thermal ablation and sputtering
- Fragmentation events
- Lateral diffusion along Earth's surface after landing

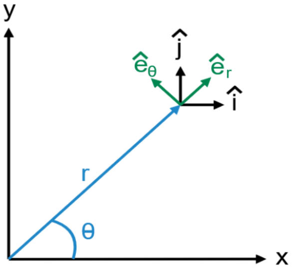
Simplifying Assumptions

- Particle modeled as a sphere of density ρ.
- Particle composed of FeO, SiO_2, MgO, Al_2O_3, CaO, $Na_2O^{(1)}$
- Earth is modeled as a point mass centered at the origin
- Coefficient of drag is constant during the decent$^{(2)}$

Particle Composition	
Oxides	**Mass Percentage**
FeO	**36.336 %**
SiO_2	**34.034 %**
MgO	**24.224 %**
Al_2O_3	**2.503 %**
CaO	**1.902 %**
Na_2O	**1.001 %**

(1) Vondrak, T., Plane, J. M. C., Broadley, S., & Janches, D. (2008). *A chemical model of meteoric ablation. Atmos. Chem. Phys.*, 8, 7015–7031. https://doi.org/10.5194/acp-8-7015-2008
(2) DeLuca, M., & Sternovsky, Z. (2019, May 6). *High-speed drag measurements of aluminum particles in free molecular flow. Journal of Geophysical Research: Space Physics*, 124(5), 3743–3751. https://doi.org/10.1029/2019JA026583

Polar Coordinate Background/Definitions



$\hat{e}_r$: the radial unit vector, which points directly outward from the origin in the direction of increasing radius.

$\hat{e}_\theta$: the tangential unit vector, which points normal to the radial unit vector in the direction of increasing angle.

$$\begin{cases} \hat{e}_r = cos(\theta)\hat{i} + sin(\theta)\hat{j} \\ \hat{e}_\theta = -sin(\theta)\hat{i} + cos(\theta)\hat{j} \\ \frac{d\hat{e}_r}{d\theta} = \hat{e}_\theta, \qquad \frac{d\hat{e}_\theta}{d\theta} = -\hat{e}_r \end{cases} \tag{1}$$

Position, Velocity, and Acceleration

Position

$$\vec{r} = r(t)\,\hat{e}_r = r(t)\cos\theta\,\hat{i} + r(t)\sin\theta\,\hat{j}$$

Velocity

$$\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta,$$

$$v_r = \dot{r}, \quad v_\theta = r\dot{\theta} = r\omega.$$

Acceleration

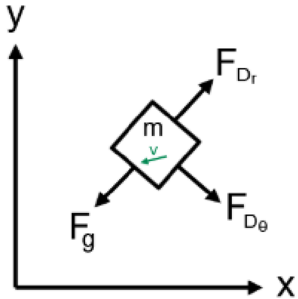
$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\,\hat{e}_\theta,$$

$$a_r = \ddot{r} - r\dot{\theta}^2, \quad a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta}.$$

Note: $\hat{e}_r$ and $\hat{e}_\theta$ are the local polar unit vectors.

Forces

$$
\begin{cases}
\vec{F}_g = -G\dfrac{Mm}{r^2}\hat{e}_r, \\
\vec{F}_D = -\dfrac{1}{2}\rho C_d A|v|\vec{v}, \\
\vec{F}_{D_r} = -\dfrac{1}{2}\rho C_d A|v|\dfrac{dr}{dt}, \\
\vec{F}_{D_\theta} = -\dfrac{1}{2}\rho C_d A|v| r\dfrac{d\theta}{dt}
\end{cases}
\tag{2}
$$



Where:

G: Universal gravitational constant
M: Mass of Earth
m: Mass of particle
C_D: Coefficient of drag
A: Cross-sectional area of particle
ρ: Density of atmosphere

Newton's Second Law

$$\text{Radial:} \begin{cases} \sum F_r = ma_r \\ -G\frac{Mm}{r^2} - \frac{1}{2}\rho C_D A\sqrt{(\frac{dr}{dt})^2 + (r\frac{d\theta}{dt})^2}\frac{dr}{dt} = m(\frac{d^2r}{dt^2} - r(\frac{d\theta}{dt})^2) \\ -\frac{1}{2}\rho C_D A\sqrt{(\frac{dr}{dt})^2 + (r\frac{d\theta}{dt})^2}\, r\frac{d\theta}{dt} = m(\frac{d^2r}{dt^2} - r(\frac{d\theta}{dt})^2) \end{cases} \tag{3}$$

$$\text{Tangential:} \begin{cases} \sum F_\theta = ma_\theta \\ -\frac{1}{2}\rho C_D A\sqrt{(\frac{dr}{dt})^2 + (r\frac{d\theta}{dt})^2}\, r\frac{d\theta}{dt} = m(2\frac{dr}{dt}\frac{d\theta}{dt} + r\frac{d^2\theta}{dt^2}) \end{cases} \tag{4}$$

Equations of Motion

We solve a coupled system:

$$\begin{cases} \frac{dr}{dt} = v_r \\ \frac{dv}{dt} = r\omega^2 - \frac{GM}{r^2} - \frac{1}{2m}\rho_{\text{air}} C_d A v_{\text{rel}} v_r \\ \frac{d\omega}{dt} = -\frac{2v_r\omega}{r} - \frac{1}{2m}\rho_{\text{air}} C_d A \omega v_{\text{rel}} \\ \frac{dm}{dt} = \underbrace{-\dot{m}_{\text{sput}}}_{\text{sputtering}} + \underbrace{-\dot{m}_{\text{thermal}}}_{\text{thermal ablation}} \end{cases} \tag{5}$$

where:

$\boldsymbol{v}_{\text{rel}} = \sqrt{v^2 + (r\omega)^2}$

$\boldsymbol{M}$: mass of Earth

$\boldsymbol{A} = \pi r_{\text{obj}}^2$

$\boldsymbol{C_d}$: coefficient of drag

Mass Loss - Compositional Ablation

Mass Percentage per Particle Temperature Stage						
Oxide (Melting Point in K)	Na_2O (1548)	FeO (1650)	SiO_2 (1984)	Al_2O_3 (2303)	CaO (2845)	MgO (3073)
Stage 1 298 - 1547 K	1.001	36.336	34.034	2.503	1.902	24.224
Stage 2 1548 - 1649 K	0	36.703	34.378	2.528	1.921	24.469
Stage 3 1650 - 1983 K	0	0	54.313	3.994	3.035	38.658
Stage 4 1984 - 2303 K	0	0	0	8.743	6.644	84.614
Stage 5 2303 - 2844 K	0	0	0	0	7.280	92.720
Stage 6 2845 - 3072 K	0	0	0	0	0	100

Kim, S., Chen, J., Cheng, T., Gindulyte, A., He, J., He, S., Li, Q., Shoemaker, B. A., Thiessen, P. A., Yu, B., Zaslavsky, L., Zhang, J., & Bolton, E. E. (2025). *PubChem 2025 update*. *Nucleic Acids Res.*, 53(D1), D1516–D1525. https://doi.org/10.1093/nar/gkae1059

Model 1: Assumptions

- Temperature varies smoothly with altitude.
- Fragmentation occurs when mass loss reaches fixed thresholds.
- Surface diffusion modeled via angular diffusion kernel.
- We assume the atmosphere is composed primarily of N, N_2, O, Ar, He, H(1)

(1) Vondrak, T., Plane, J. M. C., Broadley, S., and Janches, D. (2008). A chemical model of meteoric ablation. *Atmos. Chem. Phys.*, 8, 7015–7031. `https://doi.org/10.5194/acp-8-7015-2008`

Model 1: Mass Loss

Sputtering Mass Loss:(1)

$$\dot{m}_{\text{sput}} = 2\pi r_{\text{obj}}^2 v_{\text{rel}} M_t \sum_i n_i Y_i(E)$$

Thermal Ablation (Fragments):(2)

$$\dot{m}_{\text{thermal}} = \frac{C_h \rho_{\text{air}} A v_{\text{rel}}^3}{2Q_{\text{abl}}}$$

$\boldsymbol{M_t}$: average atomic mass
$\boldsymbol{n_i}$: number density of species i
$\boldsymbol{Y_i}$: sputtering yield of species i

$\boldsymbol{C_h}$: heat transfer coefficient
$\boldsymbol{Q_{abl}}$: latent heat of ablation
$\boldsymbol{\rho_{air}}$: density of air components.

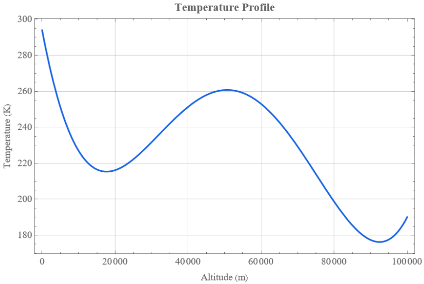
- Both depend strongly on velocity.
- Different species contribute differently to sputtering.

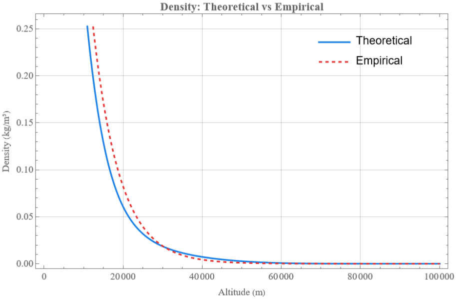
(1) Vondrak, T., Plane, J. M. C., Broadley, S., and Janches, D. (2008). A chemical model of meteoric ablation. *Atmos. Chem. Phys.*, 8, 7015–7031. `https://doi.org/10.5194/acp-8-7015-2008`

(2) Baldwin, B., and Y. Sheaffer (1971). Ablation and breakup of large meteoroids during atmospheric entry. *J. Geophys. Res.*, 76(19), 4653–4668. `https://doi.org/10.1029/JA076i019p04653`

Model 1: Atmospheric Pressure Calculation and Model

- Modeled atmospheric pressure through scale height calculations.
- Used empirical data to calculate temperature at given altitude.





Sources:
R. Stull (2019). *Standard Atmosphere – Pressure and Density.* UBC ATSC 113 – Weather for Sailing, Flying & Snow Sports.
`https://www.eoas.ubc.ca/courses/atsc113/flying/met_concepts/02-met_concepts/02a-std_atmos-P/index.html`

Space Math. (n.d.). *Exponential Functions and the Atmosphere.* NASA Goddard Space Flight Center.
`https://spacemath.gsfc.nasa.gov/weekly/7Page15.pdf`

Model 1: Fragmentation

- When cumulative mass loss exceeds threshold Δm_f, a fragment is shed.
- Each fragment inherits:

$$(r, v, \omega), \quad m \mapsto \Delta m_f$$

- Each fragment is subjugated to the same governing equations as the main body.

Model 1: Fragment Diffusion

After impact, fragments spread along the surface via:

$$\begin{cases} D(T) = D_o e^{\frac{E_A}{RT}} \\ K(\phi, T) = \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-D(T)\frac{n^2\pi^2 T}{R^2}\right)\cos(n\phi) \end{cases} \tag{6}$$

where

$\boldsymbol{E_A}$:activation energy for diffusion
$\boldsymbol{T}$: temperature of particle
$\boldsymbol{\phi}$: angle of landing
$\boldsymbol{R}$: Earth's radius

- This yields a smooth angular distribution.
- Allows backward inference of incoming flight angle distribution.

Vondrak, T., Plane, J. M. C., Broadley, S., and Janches, D. 2008.A chemical model of meteoric ablation. Atmos. Chem. Phys. 8, 7015–7031. https://doi.org/10.5194/acp-8-7015-2008

Model 1: Simulation Workflow

1. Initialize particle (entry height, mass, tangential velocity, angular velocity).
2. Integrate descent ODE system.
3. Detect fragmentation and create new fragments.
4. Compute landing angles for all fragments.
5. Apply angular diffusion to obtain surface distribution.

This model was developed using python libraries.

Model 2: Atmospheric Density Model

- The atmospheric density function was derived from empirical data of air density at different altitudes[1], and was fit to an exponential curve using Mathematica.

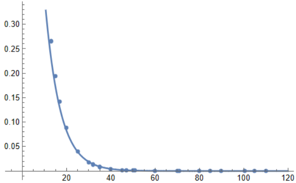
```
In[•]:= FindFit[logdensitydata, lna + b h, {lna, b}, h]
Out[•]= {lna → 0.44847, b → -0.147354}

In[•]:= a = Exp[0.44847029702262264`]
        b = -0.14735354093060496`
Out[•]= 1.56591
Out[•]= -0.147354

In[•]:= density[x_] := a Exp[b (x)]

In[•]:= modelplot = Plot[density[x], {x, -5, 120}];
        Show[modelplot, dataplot]
```

Out[•]=



(1) UBC ATSC 113. (2019, January). Standard atmosphere — Pressure and density [Web page]. University of British Columbia. https://www.eoas.ubc.ca/courses/atsc113/flying/met$_c oncepts/02 - met_c oncepts/02a - std_a tmos - P/index.html$

Model 2: Mass Loss

Power of Drag:

$$\begin{aligned} P_D = \vec{F}_D \cdot \vec{v} = F_{Dr} * v_r + F_{D\theta} * r\omega \\ P_D = -\frac{1}{2}\rho C_D A|v|v_r^2 - \frac{1}{2}\rho C_D A|v|r^2\omega^2 \\ P_D = -\frac{1}{2}\rho C_D A|v|(v_r^2 + r^2\omega^2) \end{aligned} \tag{7}$$

Temperature Rate of Change:

$$\begin{aligned} Q = mc\Delta T \\ \frac{dT}{dt} = \frac{dQ}{dt}\frac{1}{mc} = \frac{\alpha P_D}{mc} \\ T = \int \frac{\alpha P_D}{mc} dt + T_0 \end{aligned} \tag{8}$$

Model 2: Mass Loss (contd.)

Mass Loss:

- Once the particle reaches the melting temperature of one of the oxides, the temperature stops increasing and the oxide will melt away.

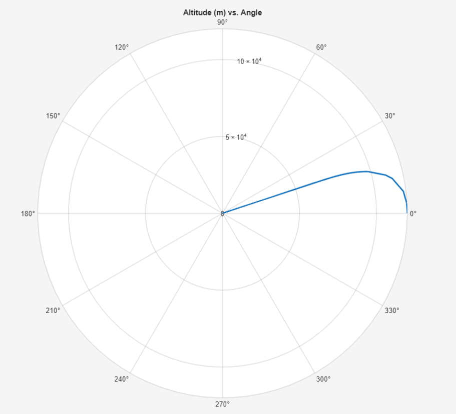
$$\begin{gathered} \alpha P_D / \Delta H_{fus_{oxide}} = \dot{m}_{oxide} \\ \int \alpha P_D / \Delta H_{fus_{oxide}} dt = \Delta m_{oxide} \end{gathered} \tag{9}$$

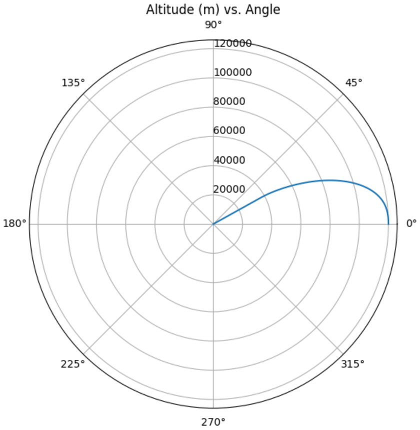
- Once the mass of that oxide reaches zero, the particle will begin to increase in temperature again until the next oxide melting temperature is reached.

Example Particle

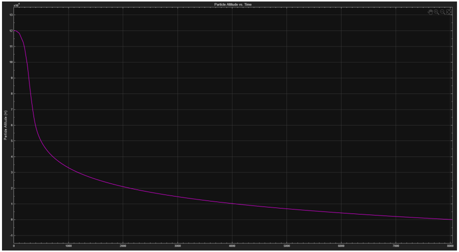
- Entry height: 120 Km above Earth's Surface
- Initial radial velocity: 0 Km/s
- Initial angular velocity: 0.0012 rad/s
- Initial mass: 0.0138 μm
- Density: 3310.16 kg/m^3

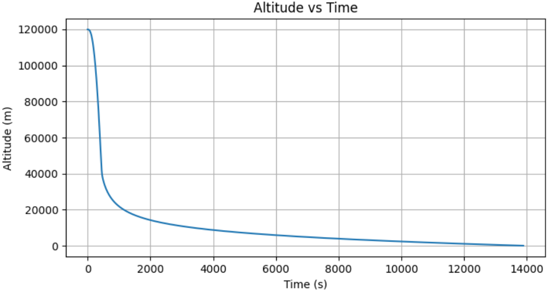
Polar Plot of Path



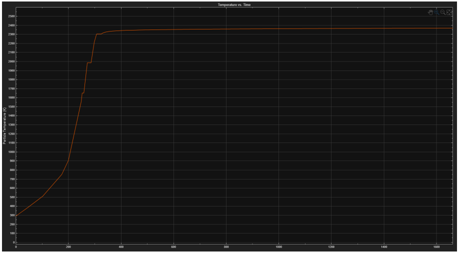


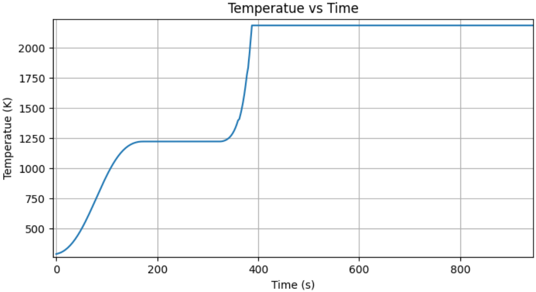
Particle Altitude



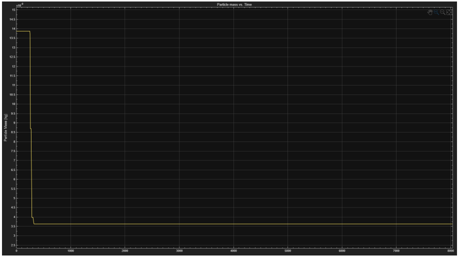


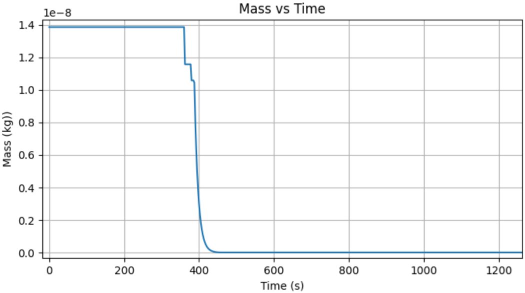
Particle Temperature



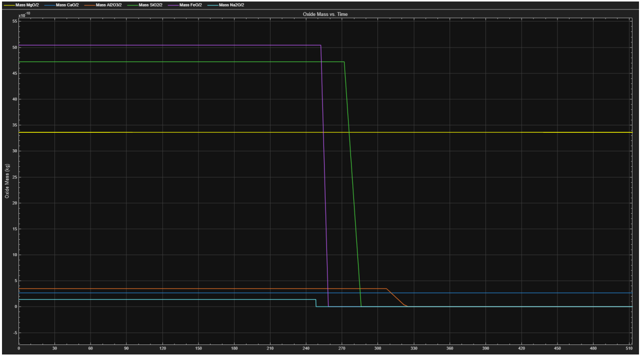


Particle Mass



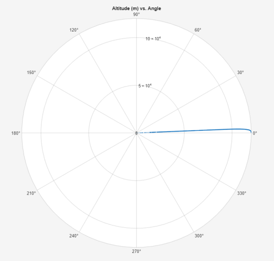


Oxide Mass

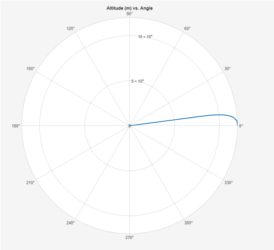


Mass MgO/2
Mass CaO/2
Mass Al2O3/2
Mass SiO2/2
Mass FeO/2
Mass Na2O/2
Oxide Mass vs. Time
x10^-10
Oxide Mass (kg)
55
50
45
40
35
30
25
20
15
10
5
0
-5
0
30
60
90
120
150
180
210
240
270
300
330
360
390
420
450
480
510

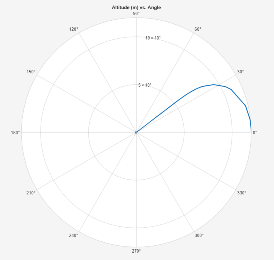
Changing mass - Trajectories



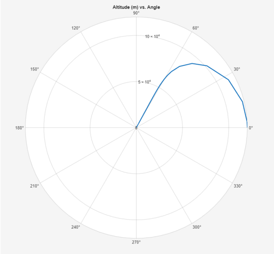
(a) Mass 100x smaller



(b) Mass 10x smaller



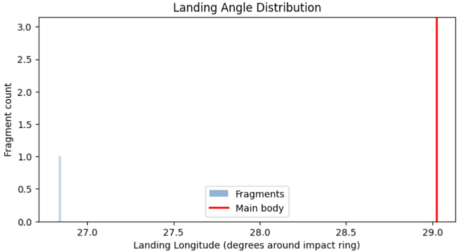
(c) Mass 10x larger

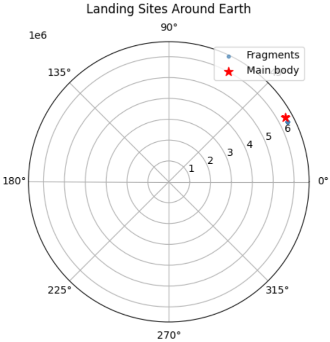


(d) Mass 100x larger

Determining Particle Properties from Distribution

- Able to determine original cosmic distribution from landing distribution
- Fragment trails give direction in which the particle fell from, and the distance between fragments gives us speed and angle.





Conclusion

- Entry dynamics and ablation significantly modify initial mass distribution.
- Fragmentation broadens surface deposition distribution.
- Surface diffusion further smooths the distribution.
- **Yes:** With forward modeling, observed deposits *can* be used to infer historical incoming flux.

Thank You!

Presented by

Shang Nelson, Amber Deamer, Connor McMillan

Citations

- Kim, S., Chen, J., Cheng, T., Gindulyte, A., He, J., He, S., Li, Q., Shoemaker, B. A., Thiessen, P. A., Yu, B., Zaslavsky, L., Zhang, J., & Bolton, E. E. (2025). *PubChem 2025 update. Nucleic Acids Res.*, 53(D1), D1516–D1525. https://doi.org/10.1093/nar/gkae1059
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