Problem Chosen	2025 MCM/ICM Summary Sheet	Team Control Number
A		2528299

The accurate estimation of archaeological site ages and populations has been a common problem for historical and archaeological institutions to understand past civilizations. Common practices such as pottery analysis and carbon dating have a difficult time in telling the age of a civilization, whereas structural aspects such as staircases provide a more intuitive look into the age of a civilization. This paper aims to analyze the wear on staircases in order to determine the age of civilizations.

Our model identifies two main contributors to the wear on staircases: foot traffic and flowing water erosion. Through developing equations to understand the effect of these contributions based on material and environmental parameters, we are able to tell the age of a civilization from the depth of a depression that has formed in a step of a staircase. To account for human induced forces, we use the Archard Wear Equation and Fourier Series, while water erosion is determined from hydrodynamics and material decay.

This study uses material properties that can be determined experimentally given the material. In this study, we look primarily at granite steps, as granite is a common material for staircases. Through sensitivity analysis, we see the importance of certain parameters such as the erosion of material per year and highlights its robustness in other scenarios. Looking at multiple models for erosion, we determined the most feasible model, due to how much an erosion model relied on parameters determined experimentally.

Given that the archeologists on site can take measurements of the stairs, they would be able to determine an estimate of the original height of a stair, then measure the farther deviation from that original height and use a graph generated by our model to determine how much time had to pass for the stair to be worn to where it is now. Furthermore, this model could be used to determine the habits of those using the stairs, for example, whether people preferred walking up or down and if they tended to travel on them individually or side-by-side.

Drawbacks of this model are that it assumes many measurements of the stairs and the material it consists of are available, and that it relies on a simplified view of erosion. The benefits of this model are that it may be applied to a general material, provided values of certain physical properties are known, and that interpreting the results is simple.

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Background

Dating archaeological sites has been a point of interest for archaeologists since the first ancient sites were discovered. However, this has been difficult for multiple reasons, such as unidentifiable decay, extended construction of sites, and new parts of buildings being added while civilization existed in the area. However, some parts of societies seem to remain permanent and unchanged during their time. One of these structures are stairs leading to structures.

While common archaeological practices in dating civilizations follow pottery or radiocarbon dating, stairs provide a unique insight into dating civilizations tend to remain unchanged during the existence of a civilization. However, getting data out of worn stairs can be tedious and inaccurate. There are specific aspects, however, that help with dating stairs, especially those in use for a long time. Some of these aspects are depression and crevasses caused by foot traffic and erosion over long times as seen in **figure 1**.

Figure 1. Worn steps leading up to the Great Wall of China (*Instagram*, n.d.).

Problem and Our Work

We have been tasked to determine primary properties about certain staircases, and civilizations by extension, given the worn condition that they have been found in. We are to create a model to help determine these properties. Our model should account for any destructive forces that have caused its worn state. These properties can help us identify population sizes and the age of the population, along with insight about historical customs of the civilizations.

We analyze what causes the erosion of the stairs to their current state and identify 2 major components: the foot traffic of the population, and the erosion caused by water flowing.

We identified ways that these components interact with one another and develop a primary structure for our model. We then determine and research ways that these components can be described mathematically.

We determine and derive equations for each component according to past research and findings and our expectations.

Primary Assumptions

1. Every step distributes an equal dynamic force
2. All steps follow a uniform rectangular prism shape
3. Flow of water and environmental factors are constant over multiple years
4. Stairs are exposed to the environment
5. There is precisely 1 heat cycle per year

Terminology and Symbols

Table 1.
Symbols and Definitions

Symbol	Definition
k_{ad}	Wear coefficient of material of stairs
k_s	Coefficient of sliding friction
γ_T	Tensile strength of the material
λ	Decay rate of stone
$h(t)$	Height of the depression in step
$E(h(t))$	Effect of the water erosion on step
$MLP(t)$	Effect of foot traffic on step
E_r	Coefficient of water erosion
H	Hardness of material (MPa)
A	Cross-sectional area of step
A_{asc}	Cross-sectional area of ascending step
A_{des}	Cross-sectional area of descending step
E_0	Erosion of material per year
F_N	Normal force on surface of step
s	Sliding distance of step
g	Gravitationnel constant (9.81 $\frac{m}{s^2}$)
$p(t)$	Civilization population function
r	Frequency rate of usage of stairs
$\overline{m}$	Average mass of population
ϕ	Phase shift of step vibration
r_{asc}	Fourier coefficient of ascending step
r_{des}	Fourier coefficient of descending step
ϕ_{asc}	Ascending step Fourier phase angle
ϕ_{des}	Descending step Fourier phase angle
I	Number of contributing steps for Fourier Series

Our Model

For our model, we represent the 2 major causes of erosion on the stair step, that of the foot traffic of civilization, and of flowing water from the environment. Here, we assume that they are additive causes but also have some interaction between one another. Our model

$$\frac{dh(t)}{dt} = E_w\big(h(t)\big) + MPL(t) + E_w\big(h(t)\big)MLP(t) \tag{1}$$

describes each factor controlled by a separate equation.

Effect of Erosion of Water

To determine the effect of water erosion on the steps, we analyzed the way water will erode the stairs and increase the depression in stone. We noticed that the deeper the depression, the more water will flow through the depression, thus we determined that the rate at which the step will erode is equal to the depth of the depression and the current rate. From this, the equation

$$\frac{dE(h(t))}{dh} = E_r h(t) E(h(t)) \tag{2}$$

From (2), we determine our primary equation for the effect of erosion on the steps,

$$E(h(t)) = E_0 e^{E_r \frac{h(t)^2}{2}} \tag{3}$$

Archard Wear Equation

To determine the effect that foot traffic has on the stairs we are going to use the magnitude of all the normal forces applied to the stair block. Therefore, assuming we only have gravity being applied, then all of our normal forces can be modeled as so:

$$|F_N| = mg \tag{4}$$

Hence, we imply that all the forces of a given person will transfer to the step. Later on, we will consider a different case where the forces of the footfall are dynamic. Now we will determine the amount of material lost per step given the force found before, hence we can use the Archard Wear Equation. However, the general form of this equation:

$$Q = \frac{k_{ad} \times |F_N| \times s}{H} \tag{5}$$

doesn't account for aspects that are at play when climbing stairs. However, an alternate form that considers the friction and cross-sectional area of contact rather than a sliding value allows for a smoother and more accurate representation of the situation (Choudhry et Al., 2024). This equation:

$$Q = \frac{k_{ad} \times k_s \times |F_N| \times s}{H} \tag{6}$$

accounts for the context in which the person is not sliding but stepping and causing friction.

Effects due to Foot Traffic

Given the amount of material lost per step from the Archard Wear Equation, we are able to determine the total amount of material lost each year from foot traffic. We consider the total amount of people in the population, and the amount of usage that the stairs will get based on this. Along with this, we consider the integrity of the material itself, as the amount of material lost will be affected by the strength and integrity of the material.

The integrity of the material is affected primarily by changing cycles of heat and cold working to deteriorate the material. This deterioration can be described as

$$I_N = I_0 e^{-\lambda N} \tag{7}$$

where N is the number of heats cycles the material has been subjected to (Saba et al., 2018). We have assumed that there is 1 heat cycler per year.

Combining (6) and (7) with a population model and parameters we determine, we get

$$MLP(t) = p(t) \times Q \times I_N \times \gamma_T \times r \tag{8}$$

This function determines the amount of material lost in a year given to the population at that year.

Combining our Models

Finally, combining (3) and (8), we get a full model for the change in height of a depression in steps over t years,

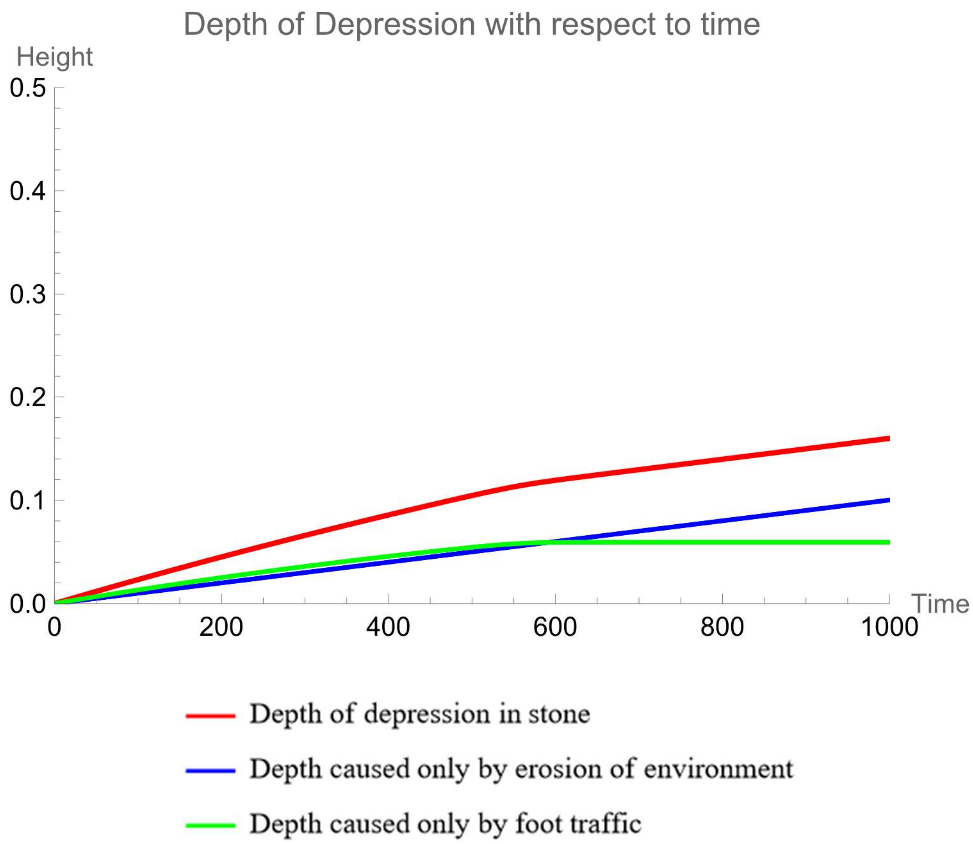
$$\frac{dh(t)}{dt} = E_0 e^{E_r \frac{h(t)^2}{2}} + p(t)QI_N\gamma_T r + E_0 e^{E_r \frac{h(t)^2}{2}} p(t)QI_N\gamma_T r \tag{9}$$

Applying the Model

Given a specific material that can be determined from the archaeological site, we can create models that determine a height of depression vs time model. From this prediction, we can analyze a given set of stairs and compare them to the predicted height, giving us an estimate of the age of the stairs. To better determine our age, we can take multiple steps from our set of stairs and measure each one independently. From there, we can average the estimated ages to create a more precise prediction.

One example that might be common is granite as a building material. Given the abundance of granite, and the many convenient material qualities, a model can be developed based on granite's properties, seen in **Figure 2.**

Figure 2.
Model of depth of depression in stone over time with static forces



Values Used

Table 2.
Parameters for estimating the age of granite stairs

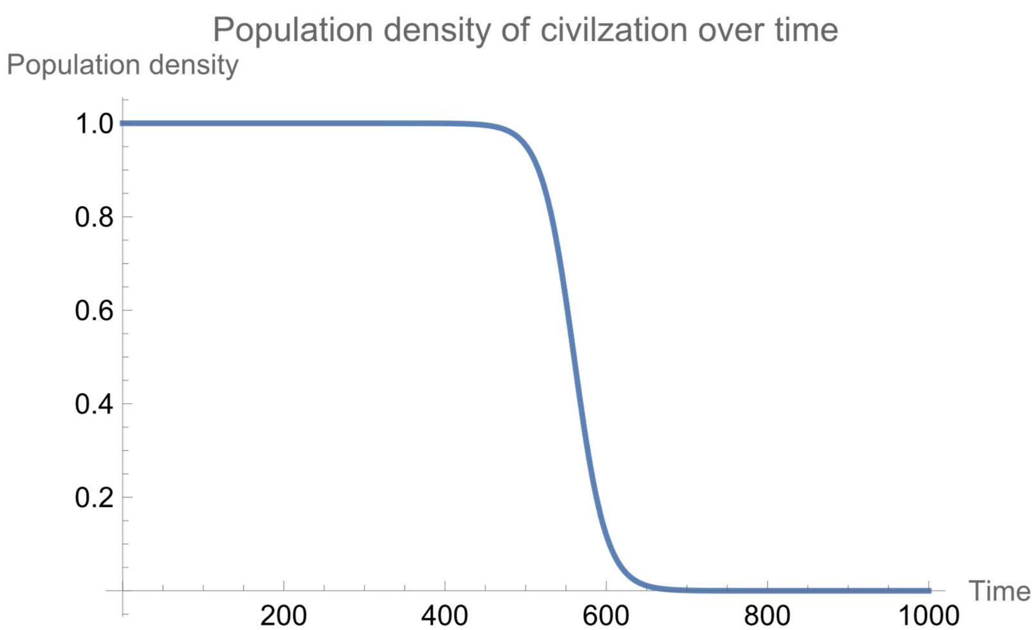
Parameter	Value
E_0	0.0001
E_r	1
γ_T	0.1
r	$300{,}000\ \frac{steps}{year}$
k_{ad}	10^{-6}
k_s	0.5
A	$0.023\ cm^2$
H	$0.3 * 10^6$
$\overline{m}$	$65\ kg$
I_0	0.1%
λ	0.001
g	$9.81 \frac{g}{m^2}$

Along with these values, our population was determined to use a logistic curve as described

$$p(t) = \frac{1}{1 + e^{\frac{t-560}{20}}} \tag{10}$$

This represents what portion of the population is still present and is multiplied by a population estimate to determine the population number. This curve produces a common shape to describe populations and is altered to produce results that coincide with common lengths of civilization existence seen in **Figure 3**.

Figure 3.
Model of population density over time.



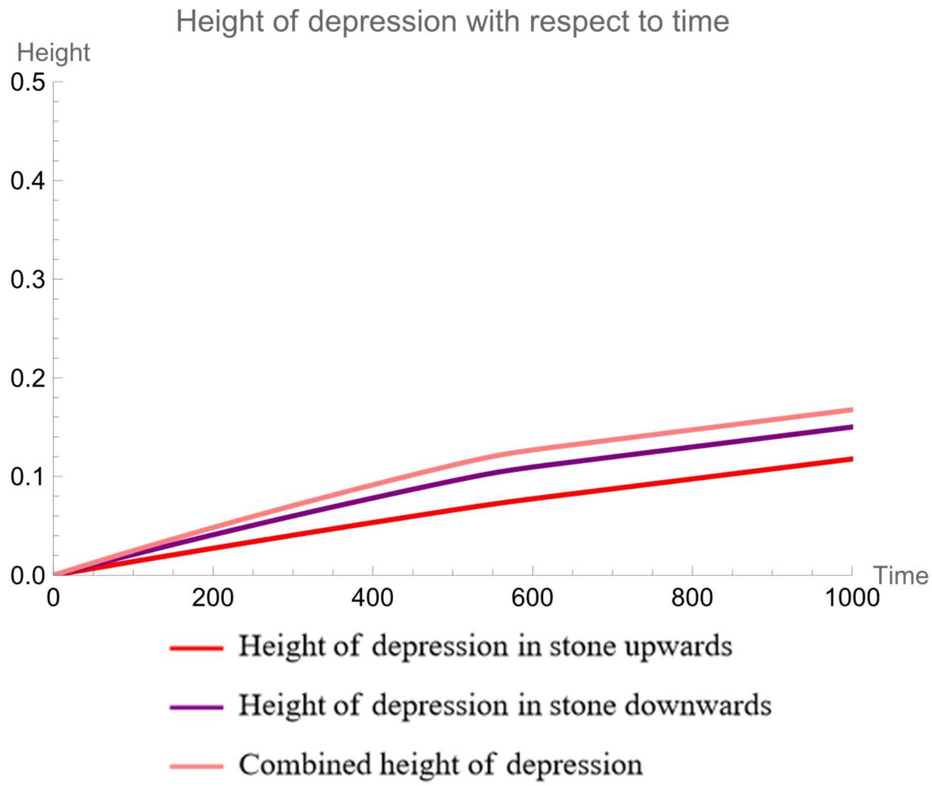
Alternate Force Model

Using footfall induced vibration analysis, we can generate a general formula that models the vertical node responses from dynamic footfall loads (force). Andrade et al. (2021) proposed the following Fourier series model for the magnitude of force due to gravity:

$$f(t) = P\sum_{i=1}^{N} \alpha_i \sin\left(2\pi f_{steps}t + \phi_i\right) \tag{10}$$

In which α_i corresponds to the Fourier coefficients, P is the weight of the person, f_{steps} is the frequency of steps done by a person, and ϕ_i is the phase angle. One benefit of this approach is that it allows us to implement into equation (6) a dynamic force that can account for ascending the stairs and for descending it. With equation (10), we can help identify the flow of traffic by analyzing the wear and position of force on the step.

Figure 4.
Model of height of depression with respect to time with dynamic forces



Values Used

Table 3.
Values assigned for given coefficients.

Parameters	Values
E_0	0.0001
E_r	1
γ_T	0.1
r	$300{,}000\ \frac{steps}{year}$
k_{ad}	10^{-6}
k_s	0.5
A_{asc}	$0.023\ cm^2$
A_{des}	$0.01\ cm^2$
H	$0.3 * 10^6$
$\overline{m}$	$65\ kg$
ϕ_{asc}	$\{9.66°, 2.15°, -142°, 84.5°\}$
ϕ_{des}	$\{20°, -60.3°, -84.5°, -125°\}$
I	4

α_{asc}	$\{0.37, 0.21, 0.1, 0.03\}$
α_{des}	$\{0.6, 0.13, 0.05, 0.03\}$
f_{step}	$2\ Hz$
g	$9.81\ \frac{m}{s^2}$
I_0	0.1%
λ	0.001

Alternate Erosion Model

An alternative model for erosion was considered but was deemed less desirable due to more parameters requiring experimental data, which is not something we had the ability to find or collect. The model comes from Momber (2003) and puts forth a proportionality relationship between the rate of erosion and property of the material. This relationship can be expressed as

$$E_{rate} \propto \frac{d_M E_M \rho_M}{K_{Ic}^m} \tag{12}$$

where E_{rate} is the erosion rate, d_M is the grain size, E_M is Young's Modulus based on the material, ρ_M is the density of the material, K_{Ic} is the fracture toughness, and m is the fracture toughness exponent. We used this relationship to find an equation for how much erosion occurred as a function of time through integrating with respect to time and introducing the proportionality constant. However, due to how we desired to use the erosion rate, we needed to find values for these constants, but with no access to experimental data, we would have had to estimate these values almost arbitrability.

Since granite was often used in buildings due to its abundance and sedimentary properties making it easy to transform, we made the decision to reference granite for the values in the relationship.

This erosion sub-model was then tested for various constant values determined experimentally. This sub-model requires the assumption that erosion affects the stairs at a constant rate over time, when we believe the effects of erosion will magnify over time due to funneling of water flow. Ultimately, the erosion model previously mentioned was chosen in favor over this due to less dependency on experimental data and we feel it captured more realistic behavior.

Other Uses of Model

In addition to determining the age of stairs by comparing the depression height and the initial height of a step, we can determine the traveling behavior of those using the steps. For instance, it can be determined if those traversing the stairs preferred going up or down. Since we have considered the difference in force applied during ascension and descension, we see in Figure 4 that descension contributed more to the depth of the depression. Sets of stairs in the same archaeological area can be compared to each other to determine which were often traveled upwards vs downwards or if it was about equal. Our model might then suggest that some stairs in the same area are older than others. While this may be true it would be practically unnoticeable on the time scale we considered.

We can also determine if the stairs were used by people one at a time or if two or more people used the stairs simultaneously by analyzing where the depressions are positioned on the steps and by the number of them present. The differences may be subtle since after enough time the water flow and erosion may cause two adjacent depressions to level out, making the difference less significant. However, assuming the erosion does not obscure the data to an unusable degree, the position of the warping and number of divots present can help us draw conclusions about how the stairs were used.

These two applications can be used together to determine if the civilization preferred to walk on their left or their right side of pathways. If the right side of stairs is more worn than the left, we may be able to claim that upwards travel stayed to the right while downwards travelers stayed to the left. It's also possible there was no preferred side of travel, but this can also be the claim if the researchers of the stairs consider the difference between two sides of stairs insignificant.

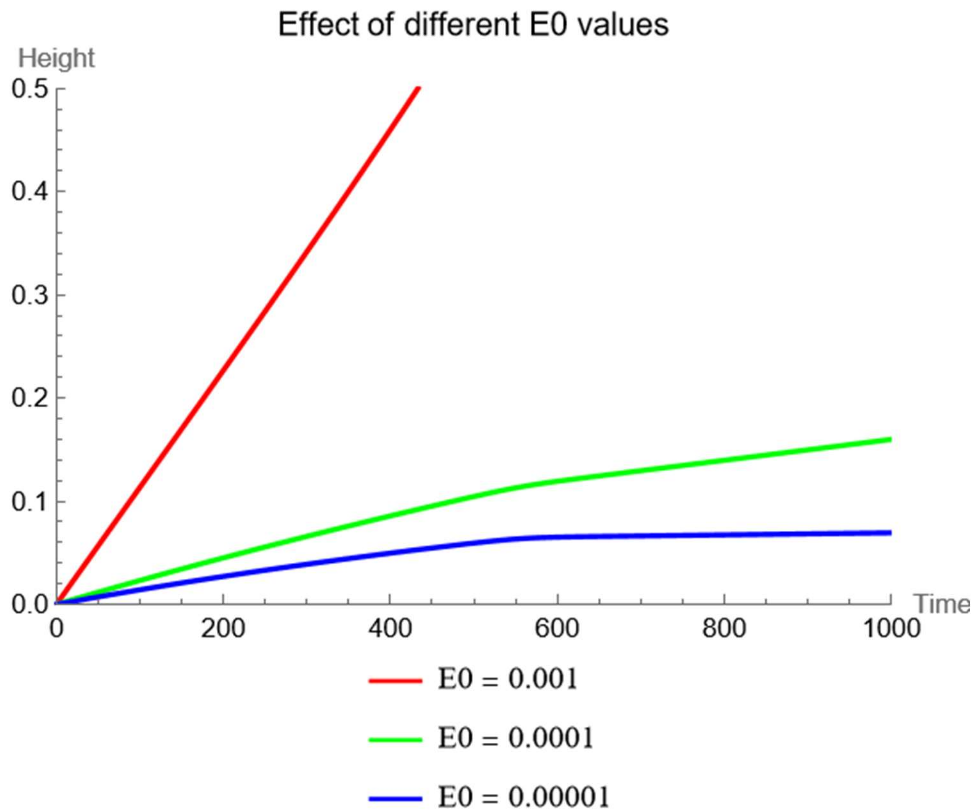
Sensitivity Analysis

Throughout our study, we have maintained the values for many of our parameters, however, some parameters will affect the outcome of the model vastly. Through testing these parameters with different values, while keeping all other parameters the same, we are able to identify the sensitivity of them.

Erosion Per Year ($\boldsymbol{E_0}$)

As we vary the erosion per year, our total erosion, and by extension the height of the depression, varies greatly.

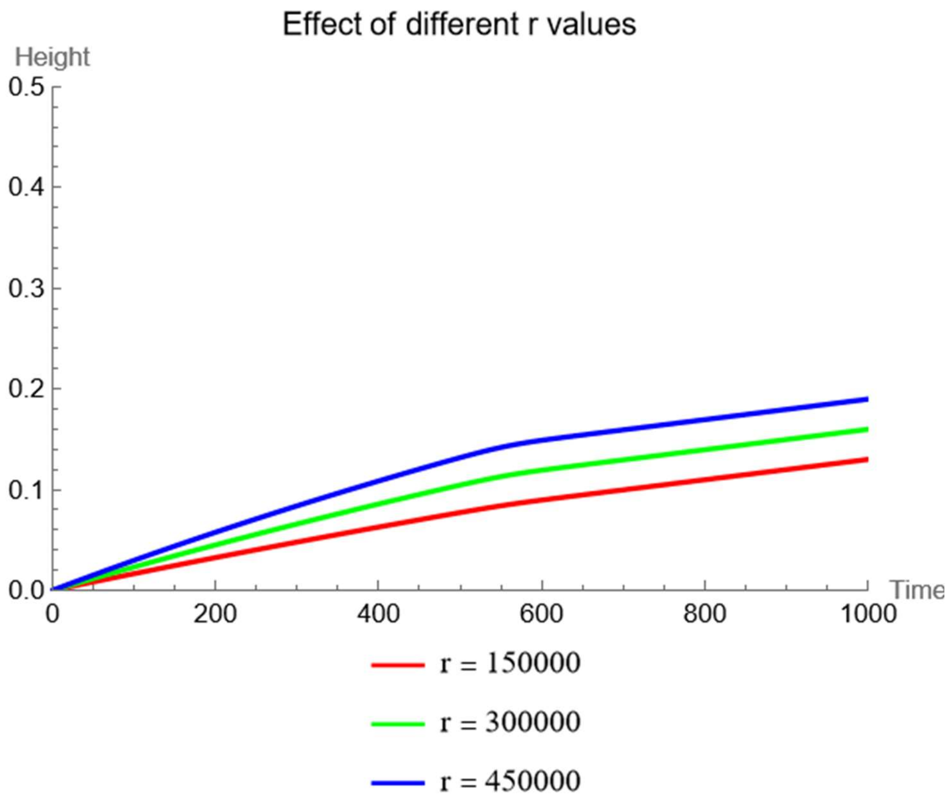
Figure 5.
Model of different eroded materials per year rates



Frequency Usage

Another significant parameter in our model is the usage of the stairs. This affects the total decay caused by the population versus that caused by erosion. This also helps show that the majority of the damage and decay is due to erosion over time.

Figure 6.
Model of effect of different frequency of use rates



Strengths & Weaknesses of Model

Strengths

- Our model provides a comprehensive estimate of the age of a civilization and includes multiple factors such as wear due to erosion and the loss of integrity due to heat cycles
- Our model allows for incorporation of many materials and estimations based on them given measured parameters
- Practical and understandable method for determining age by measurement, readily applicable to archaeological sites
- Highly robust due to the minimal number of sensitive variables which are easily estimated from material properties

Weaknesses

- A large number of parameters and some are difficult to identify or measure
- Knowledge of population dynamics is necessary for accurate estimation of age.
- Simplistic and idealized erosion model for flowing water.
- Limited applicability to more complex staircases that do not follow and aren't aligned with assumptions we made

Next Steps

- Given real-life data, we would be able to assess the error in our model and create more accurate models.
- Considering more factors such as humidity, temperature gradients, and chemical reactions would allow for more accurate estimates, especially in areas of extreme climate.

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Appendix A

Mathematica Code

```
Clear[g, L, FAs, FDs, EVal, d, AAsc, ADes, H, E0, I0, c1,
  gammaTensile, gammaErosion, lambda, f, EFun, IFun, G, iterations, freq,
  rAscending, rDescending, PhaseAngleAscending, PhaseAngleDescending];
g[t_] := 1 / (1 + E^((1 / 20) (t - 560)));
(* Population model of generalized population *)
L[F_, EVal_, d_, H_, A_] := (F * EVal * d) / (H * A);
(* material lost based on force applied ( Archard's Law ) *)
(* Old Force Function *)
(* F[w_,g_]:=w*g; force of step *)

(* New Fourier Force Function *)
G = 65 * 9.81; (* Body Weight of person *)
iterations = 4; (* Steps accounted for *)
freq = 2; (* Step frequency *)
rAscending = {0.37, 0.21, 0.1, 0.03}; (* Fourier coefficients for ascending series *)
rDescending = {0.6, 0.13, 0.05, 0.03}; (* Fourier coefficients for descending series *)

PhaseAngleAscending = {9.66, 2.15, -142, 84.5};
(* Phase angles for fourier series ascending *)
PhaseAngleDescending = {20, -60.3, -84.5, -125};
(* Phase angles for fourier series descending *)

FAs[t_] := G * Sum[rAscending[[h]] *
     Sin[2 * Pi * freq * h * t * PhaseAngleAscending[[h]] * Degree], {h, iterations}];
FDs[t_] := G * Sum[rDescending[[h]] *
     Sin[2 * Pi * freq * h * t * PhaseAngleDescending[[h]] * Degree], {h, iterations}];

FAStored = FAs[0.8];
FDStored = FDs[0.4];

EVal = 10^(-6); (* Wear Coefficient, Typical Values range from 10^-2 to 10^-8 *)
d = 0.5; (* Friction Coefficient (Average) *)
AAsc = 0.023; (* Cross-sectional area of step *)
ADes = 0.01; (* Cross-sectional area of step *)
H = 0.3 * 10^6; (* Hardness rating measured as 3 times the minimum yield strength *)
E0 = 0.0001; (* Initial depth of crevasse *) (* SENSITIVE *)
I0 = 0.10; (* Initial porosity of stone *)
c1 = 300000; (* Passes per year *)
gammaTensile = .1; (* intrinsic stone tensile strength factor *)
gammaErosion = 1; (* intrinsic stone erosion rate *)
lambda = 0.001; (* decay rate of stone *)

MainGraphHeight = 0.5; (* Visual graph height - totals *)
SecondaryGraphHeight = 0.001; (* Visual graph height - Changes *)

fCombined[time_] :=
```

```
  g[time] * c1 * (1 / 2) (L[FAStored, EVal, d, H, AAsc] + L[FDStored, EVal, d, H, ADes]);
fUp[time_] := g[time] * c1 * (1 / 2) * (L[FAStored, EVal, d, H, AAsc]);
(* amount of material lost per year based on population and usage *)
fDown[time_] := g[time] * c1 * (1 / 2) * (L[FDStored, EVal, d, H, ADes]);
(* amount of material lost per year based on population and usage *)
FFunCombined[t_] := fCombined[t] * IFun[t] * gammaTensile;
FFunUp[t_] := fUp[t] * IFun[t] * gammaTensile;
(* total decay and material lost due to foot traffic *)
FFunDown[t_] := fDown[t] * IFun[t] * gammaTensile;
(* total decay and material lost due to foot traffic *)
EFun[height_] := E0 * E^(height^2 / 2) * gammaErosion; (* material lost due to erosion *)
IFun[iteration_] := I0 * E^(-lambda * iteration);
(* stone integrity decay due to temperature changes *)
dhdtUp[t_, h_] := EFun[h] + FFunUp[t]; (* total change in stone *)
dhdtDown[t_, h_] := EFun[h] + FFunDown[t];
dhdtCombined[t_, h_] := EFun[h] + FFunCombined[t];

 (* initialize depth of crevasse *)
hDown = 0;
hUp =  0;
hCombined = 0;

hWithoutFootsteps = 0;
hWithoutErosionUp = 0;
hWithoutErosionDown = 0;

(* Initialize data arrays *)
dPointsDown = {};
dPointsUp = {};
dPointsCombined = {};

dPointsWithoutFootsteps = {};
dPointsWithoutErosionUp = {};
dPointsWithoutErosionDown = {};

dhdtPointsDown = {};
dhdtPointsUp = {};
dhdtCombinedPoints = {};

dhdtPointsFootstepsUp = {};
dhdtPointsFootstepsDown = {};
dhdtPointsErosion = {};

(* Loop over 1000 years *)
For[i = 1, i ≤ 1000, i = i + 1,

  (* Apply the change in depth each year *)
```

```
 hDown = hDown + dhdtDown[i, hDown];
 hUp = hUp + dhdtUp[i, hUp];
 hCombined = hCombined + dhdtCombined[i, hCombined];

 hWithoutFootsteps = hWithoutFootsteps + EFun[hWithoutFootsteps];
 hWithoutErosionUp = hWithoutErosionUp + FFunUp[i];
 hWithoutErosionDown = hWithoutErosionDown + FFunDown[i];

 (* add each point to the graph based on height lost from step *)
 AppendTo[dPointsDown, {i, hDown}];
 AppendTo[dPointsUp, {i, hUp}];
 AppendTo[dPointsCombined, {i, hCombined}];

 AppendTo[dPointsWithoutFootsteps, {i, hWithoutFootsteps}];
 AppendTo[dPointsWithoutErosionUp, {i, hWithoutErosionUp}];
 AppendTo[dPointsWithoutErosionDown, {i, hWithoutErosionDown}];

 (* Add only the change to difference graph *)
 AppendTo[dhdtPointsUp, {i, dhdtUp[i, hUp]}];
 AppendTo[dhdtPointsDown, {i, dhdtDown[i, hDown]}];
 AppendTo[dhdtCombinedPoints, {i, dhdtCombined[i, hCombined]}];

 AppendTo[dhdtPointsErosion, {i, EFun[hWithoutFootsteps]}];
 AppendTo[dhdtPointsFootstepsDown, {i, FFunDown[i]}];
 AppendTo[dhdtPointsFootstepsUp, {i, FFunUp[i]}];
]

Plot[g[t], {t, 0, 1000}, PlotLabel → "Population density of civilzation over time",
 AxesLabel → {"Time", "Population density"}]

(* Plotting data *)

plotDhdtUp = ListPlot[dhdtPointsUp,
   PlotStyle → Red, PlotRange → {{0, 1000}, {0, SecondaryGraphHeight}},
   AxesLabel → {"Time in Years", "Change in Depth of Crevasse"},
   PlotLabel → "Change in Depth of Crevasses over Time"];
plotDhdtDown = ListPlot[dhdtPointsDown,
   PlotStyle → Purple, PlotRange → {{0, 1000}, {0, SecondaryGraphHeight}}];
plotCombinedDhdt = ListPlot[dhdtCombinedPoints,
   PlotStyle → Pink, PlotRange → {{0, 1000}, {0, SecondaryGraphHeight}}];

plotFootstepsUp = ListPlot[dhdtPointsFootstepsUp,
   PlotStyle → Green, PlotRange → {{0, 1000}, {0, SecondaryGraphHeight}}];
plotFootstepsDown = ListPlot[dhdtPointsFootstepsDown,
   PlotStyle → Orange, PlotRange → {{0, 1000}, {0, SecondaryGraphHeight}}];
plotErosion = ListPlot[dhdtPointsErosion,
```

```
    PlotStyle → Blue, PlotRange → {{0, 1000}, {0, SecondaryGraphHeight}}];

plotUp = ListPlot[dPointsUp, PlotStyle → Red, Joined → True,
    PlotRange → {{0, 1000}, {0, MainGraphHeight}}, AxesLabel → {"Time", "Height"},
    PlotLabel → "Height of depression with respect to time"];
plotDown = ListPlot[dPointsDown, PlotStyle → Purple,
    Joined → True, PlotRange → {{0, 1000}, {0, MainGraphHeight}}];
plotCombined = ListPlot[dPointsCombined, PlotStyle → Pink,
    Joined → True, PlotRange → {{0, 1000}, {0, MainGraphHeight}}];

plotWithoutFootsteps = ListPlot[dPointsWithoutFootsteps,
    PlotStyle → Blue, PlotRange → {{0, 1000}, {0, MainGraphHeight}}];
plotWithoutErosionUp = ListPlot[dPointsWithoutErosionUp,
    PlotStyle → Green, PlotRange → {{0, 1000}, {0, MainGraphHeight}}];
plotWithoutErosionDown = ListPlot[dPointsWithoutErosionDown,
    PlotStyle → Orange, PlotRange → {{0, 1000}, {0, MainGraphHeight}}];

combinedMainPlots = Show[plotUp, plotDown, plotCombined,
    plotWithoutFootsteps, plotWithoutErosionUp, plotWithoutErosionDown];
Legended[combinedMainPlots, LineLegend[{Red, Purple, Pink, Blue, Green, Orange},
   {"Height of depression in stone upwards", "Height of depression in stone downwards",
    "Combined height of depression", "Depth caused only by erosion of environment",
    "Depth caused only by foot traffic upwards",
    "Depth caused only by foot traffic downwards"} ]]

dhdtPlots = Show[plotDhdtUp, plotDhdtDown,
    plotCombinedDhdt, plotFootstepsUp, plotErosion, plotFootstepsDown];
Legended[dhdtPlots, LineLegend[{Red, Purple, Pink, Blue, Green, Orange},
   {"The change in h[t] with downwards descent preference",
    "The change in h[t] with upwards ascent preference",
    "Combined change in h[t]", "The effect of erosion",
    "The effect due to footsteps upwards", "The effect due to footsteps downwards"}]]
```