Validation against Abramzon-Sirignano model

A semi-theoretical model for fast evaporation of moving droplets

1. Droplet dynamics

Governing equations:

$$\frac{dU}{dt} = \frac{3C_D}{2R_s}\left(\frac{\rho_\infty}{\rho_L}\right)|U_\infty - U|(U_\infty - U)$$

$$\frac{dR_s}{dt} = -\frac{\dot{m}}{4\pi\rho_L R_s^2}$$

$$\dot{m} = 2\pi\bar{\rho}_g\bar{D}_g R_s Sh^* \ln(1 + B_M)$$

$$B_M = \frac{Y_{Fs} - Y_{F\infty}}{1 - Y_{Fs}} = f(T_s)$$

where the drag coefficient C_D is usually expressed as a function of the Reynolds number, i.e. $C_D = C_D(\mathrm{Re})$, Sh^* the dimensionless Sherwood number, and B_M the Spalding mass transfer number.

The transient droplet temperature is calculated using two extreme models bounding the possible range of real conditions:

1) In the conduction limit which assumes that heat is transferred within liquid solely by thermal conduction, the energy equation inside the liquid droplet is described by

$$\frac{\partial T}{\partial t} = \alpha_L \nabla^2 T$$

In a spherical coordinate system, the energy equation becomes

$$\frac{\partial T}{\partial t} = \alpha_L\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right)$$

where the boundary conditions at $r = 0$ and $r = R_s$ are given by

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$$

$$\left(-k\left.\frac{\partial T}{\partial r}\right|\right)_{r=R_s} = -\frac{Q_L}{4\pi R_s^2}$$

2) In the rapid mixing limit which postulates that the temperature within the droplet is spatially uniform, the temperature difference is given by

$$\Delta T = \frac{Q_L \Delta t}{C_L \rho_L V_L} = \frac{3Q_L\Delta t}{4\pi R_s^3 C_L \rho_L}$$

2. Numerical methodology

1) Solve the surface temperature of the droplet

(a) Conduction limit model

$$\frac{T_i^{k+1}-T_i^k}{\Delta t}=\alpha_L\left(\frac{T_{i+1}^k+T_{i-1}^k-2T_i^k}{\Delta r^2}+\frac{2}{r_i}\frac{\left(T_{i+1}^k-T_{i-1}^k\right)}{2\Delta r}\right)$$

$$\left(1+\frac{2\alpha_L\Delta t}{\Delta r^2}\right)T_i^{k+1}=\left(\frac{\alpha_L\Delta t}{\Delta r^2}+\frac{\alpha_L\Delta t}{r_i\Delta r}\right)T_{i+1}^{k+1}+\left(\frac{\alpha_L\Delta t}{\Delta r^2}-\frac{\alpha_L\Delta t}{r_i\Delta r}\right)T_{i-1}^{k+1}+T_i^k$$

Boundary conditions:

$$T_2-T_1=0$$

$$-k\cdot\frac{T_N-T_{N-1}}{\Delta r}=-\frac{Q_L}{4\pi R_s^2}$$

Solution: Tridiagonal matrix algorithm (TDMA)

A special form of Gaussian elimination method to solve tridiagonal systems of algebraic equations.

The general form of the discretization equations can be written as

$$a_iT_i=b_iT_{i+1}+c_iT_{i-1}+d_i$$

To account for the boundary conditions, we have

$$a_1=1, b_1=1, c_1=0, d_1=0$$

$$a_N=1, b_N=0, c_N=1, d_N=\frac{Q_L\Delta r}{4\pi kR_s^2}$$

(b) Rapid mixing limit model

$$T^{k+1}-T^k=\frac{3Q_L\Delta t}{4\pi R_s^3C_L\rho_L}$$

2) Solve droplet radius -- Euler explicit scheme

$$\frac{R_s^{k+1}-R_s^k}{\Delta t}=-\frac{\dot{m}}{4\pi\rho_L\left(R_s^k\right)^2}$$

3) Solve momentum equation

$$\frac{U^{k+1}-U^k}{\Delta t}=\frac{3C_D}{2R_s^k}\left(\frac{\rho_\infty}{\rho_L}\right)|U_\infty-U^k|(U_\infty-U^k)$$

3. Comparison with CFD results

- A water droplet of 1 mm in radius and 343 K in temperature is injected into the quiescent air at 363 K and 1 atm. The initial velocity of the water droplet is 10 mm/s.

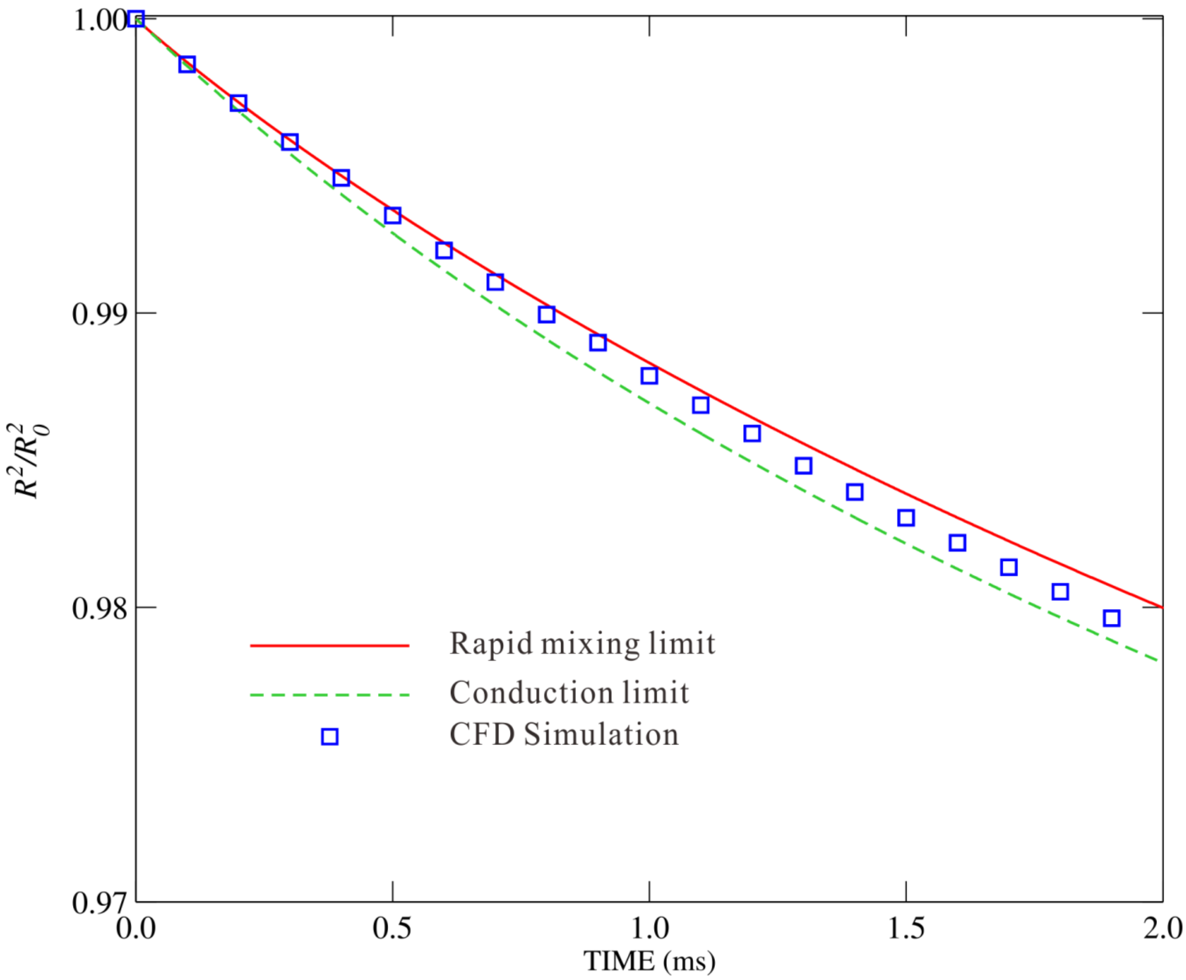

Fig. 1 Dimensionless radius of an evaporating water droplet predicted by rapid mixing limit (RML) model, conduction limit (CL) model and CFD simulation.