University of Hyderabad

Classical Mechanics (IMSc VIIth Semester)

FINAL EXAMINATION

Total Marks - 45 Date 2.12.22

The paper is divided into 2 parts. You must answer all the questions from Part A and any one question from part B. In part B please answer all the sub-parts of the questions together.

Part A (6X 5 = 30 marks)

Answer all the questions

1) Two equal mass points of mass "**m**" are connected by a string passing through a hole in a smooth table. One mass rests on the table surface and the other hangs suspended. Assuming the hanging mass moves only in a vertical line, write the Lagrange's equations for the system. Obtain the Hamiltonian and the Hamilton's equations of motion.

2) A particle of mass "m" is rolling down an 2-dimensional inclined plane, under the influence of gravity which is acting downwards. Choose a coordinate system, write the Lagrangian and identify the conserved quantity if any.

3) Show that the motion of a particle in the potential field $V(r)=-\frac{k}{r}+\frac{h}{r^2}$ is the same as that of the motion under the Kepler potential alone, when expressed in terms of a coordinate system rotating or precessing around the center of force.

4) Find the Lagrangian for a double pendulum.

5) Find the time period of a relativistic harmonic oscillator with the Lagrangian given by $L=\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}-m_0c^2-\frac{1}{2}kx^2$

6) Given a Hamiltonian $H(q,p,t) = \frac{p^2}{2} + V(q-\omega t)$. Use a canonical transformation to remove the time dependence from the Hamiltonian and obtain the new Hamiltonian

Please Turn Over

PART B
(Answer *any* one question 15 marks)

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7)

a) A central potential is given by $V=-k\frac{e^{-r/a}}{r}$. Sketch the potential for the following cases : (i) a small angular momentum (ii) a large angular momentum

b) For which of the cases {(i) or(ii)} above can you get a bound stable orbit ? Explain your answer in detail

c) The equilibrium configuration of a molecule is represented by three atoms of equal mass at the vertices of a 45º right triangle connected by springs of equal force constant. Obtain the secular determinant for the modes of vibration in the plane.

4+4+7

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8)

a) Find the generating function $F_2(q,P)$ and the transformed Hamiltonian K(Q,P), if $F_1(q,Q)=\frac{1}{2}\sqrt{km}\,q^2\cot Q$ and $H(p,q)=\frac{p^2}{2m}+\frac{kq^2}{2}$

b) Assume a magnetic field $\vec{B}=b\frac{\vec{r}}{r^3}$, and a particle moving in the field of that magnetic monopole as well as a central force field derived from a potential $v(r)=\frac{-k}{r}$ Show that the angular momentum is not conserved and find the new conserved vector.

7+8
