IP405 End-semester Examination 7 December, 2022

Part-B

There are six questions in this part.

Write concise answers, using two pages per question at the maximum.

2. Consider an electron moving (i) inside a cube of length $L = 40a_0$ with rigid walls (ii) in a three-dimensional isotropic HM with a length scale $\sqrt{\hbar/m\omega} = 10a_0$ (iii) in a Coulomb potential of the proton in a hydrogen atom. Let an energy unit be $\epsilon_0 \equiv \hbar^2/2ma_0^2 = 13.6$ eV. Draw the energy levels in the three cases, showing the first four distinct energy levels with their quantum numbers. (iv) Which of these states are parity invariant, and which are rotationally invariant? [2,2,2,2]

3. Consider the LS-basis of electronic states of the hydrogen atom with n=2, including the spin, $|n = 2, l = 1, m_l, s = 1/2, m_s\rangle, m_l = -1, 0, 1$ and $m_s = -1/2, 1/2$. Another basis, J-basis, can be constructed with the total angular momentum states $|n = 2, l, s, j, m_j\rangle$. What are the allowed values for j? Write each of the J-basis states in terms of LS-basis. [6]

4. Consider an arbitrary state of a spin-1/2 particle given in S_z-diagonal basis as, $|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$, where a and b are arbitrary complex numbers. Find the expectation values of S_x, S_y, S_z, S^2 in the state. [2,2,1,1]

5. Consider an isotropic Harmonic oscillator in two dimensions with $H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{m\omega^2}{2}(x^2 + y^2)$. (a) Find the expectation value $\langle y p_x \rangle$. (b) Find the expectation value $\langle x^2 p_y^2 \rangle$ in the ground state. [3,3]

6. An electron in the hydrogen atom is in a state described by the wave function, $\psi(\vec{r}) = \frac{1}{6}[4\psi_{100}(\vec{r}) + 3\psi_{211}(\vec{r}) - \psi_{210}(\vec{r}) + \sqrt{10}\psi_{21-1}(\vec{r})]$. (a) Find the expectation values of the energy, L^2, and L_z respectively. (b) Is the state parity invariant? (c) Is the state rotationally invariant under a rotation about z-axis? [6,1,1]

7. Consider a general state of a partilce with a spin S=1/2 and orbital angular momentum L=1 using the basis $|m_l, m_s\rangle$ give an, $|\psi\rangle = a|1, 1/2\rangle + b|1, -1/2\rangle + c|0, 1/2\rangle + d|0, -1/2\rangle$. Find the expectation values of J_z, J^2 where $\vec{J} = \vec{L} + \vec{S}$. [2,4]