School of Physics
University of Hyderabad
End Semester Examination
Electromagnetic Theory – I (PH452)

Date:2/6/2022 | Total Marks: 50 | Time: 3 hours

(Questions 1-5 carry two marks each)

1. The electric field of a plane Electromagnetic wave is $\vec{E} = E_0 \exp[i(\hat{x}\, k\cos\alpha + \hat{y}\, k\sin\alpha - \omega t)]$. If $\hat{x}$ and $\hat{y}$ are Cartesian unit vectors. The wave vector $\vec{k}$ of the Electromagnetic wave is

   (a) $\hat{z}$ k (b) $\hat{x}$ ksinα + $\hat{y}$kcosα (c) $\hat{x}$ kcosα + $\hat{y}$ksinα (d) - $\hat{z}$ k

2. Three infinitely long wires are placed equally apart on the circumference of a circle of radius a, perpendicular to its plane. Two of the wires carry current I each, in the same direction, while the third carries the current $2I$ along the direction opposite to the other two. The magnitude of the magnetic induction $\vec{B}$ at a distance r from the centre of the circle, for $r > a$, will be

   (a) 0 (b) $\frac{2\mu_0}{\pi}\frac{I}{r}$ (c) $\frac{2\mu_0}{\pi}\frac{I}{r}$ (d) $\frac{2\mu_0}{\pi}\frac{Ia}{r}$

3. For a vector potential $\vec{A}$ The divergence of $\vec{A}$ is $\nabla . A = -\frac{\mu_0}{4\pi}\frac{Q}{r^2}$ Where Q is a constant of approximate dimension. The corresponding scalar potential ϕ(r, t) that makes $\vec{A}$ and ϕ Lorentz gauge invariant is

   (a) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{Qt}{r}$ (c) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{Qt}{r^2}$

4. A square loop of wire with sides of length a, lies in the first quadrant of the XY plane with one corner at the origin. In this region, there is a nonuniform time-dependent magnetic field $B(x,y) = ky^2t^2\hat{z}$, find the induced emf.

   (a) $-\frac{1}{2}kta^5$ (b) $-\frac{1}{2}kta^5$ (c) $-\frac{2}{3}kta^4$ (d) $-\frac{3}{2}k^2ta^2$

5. If the system is time reversed, i.e $t' = -t$. what happens to charge density ρ, current density j, electric field E, Magnetic field B.

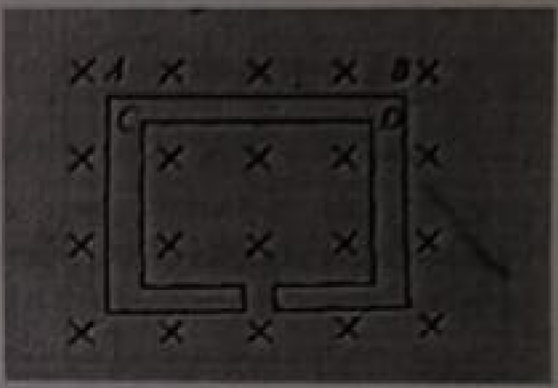
   (a) $\rho' = \rho$, $j' = j$, $E' = E$, $B' = -B$ (b) $\rho' = \rho$, $j' = -j$, $E' = E$, $B' = B$

   (c) $\rho' = \rho$, $j' = -j$, $E' = -E$, $B' = B$ (d) $\rho' = -\rho$, $j' = -j$, $E' = -E$, $B' = B$

(Questions 6 and 7 carry 3 marks each)

6. A current I flow in a thin wire shaped regular polygon of n sides which can be inscribed in a circle of radius R. calculate the magnetic field of induction ($\boldsymbol{B}$) at the centre of polygon due to one side of the polygon.

7. A rectangular loop with a sliding conductor of length l is located in uniform magnetic field perpendicular to the plane of the loop. The magnetic induction is $\boldsymbol{B}$. The conductor has resistance R. The sides AB and CD have resistance R_1 and R_2 respectively. Find the current through the conductor during its motion to the right with a constant velocity v.



(Questions 8 and 9 carry five marks each)

8. Sea water at frequency $\nu = 4 \times 10^8$ Hz has permittivity $\varepsilon = 81\varepsilon_0$, permeability $\mu = \mu_0$ and resistivity $\rho = 0.23\Omega$m. what is the ratio of conduction current to displacement current? [Hint : Consider a parallel plate capacitor immersed in sea water and driven by voltage $v_0 \cos(2\pi\nu t)$]

9. (a) The waveguide dimensions are 2.5 x 1 cm, and the frequency is 8.6 GHz. Find the following
(i) possible mode
(ii) cut off frequency
(iii) Guide wavelength

(b) A rectangular metal waveguide filled with a dielectric material of relative permittivity $\varepsilon_r = 4$ has the inside dimensions 3 x 1.2 cm. Find the cut off frequency for the dominant mode.

(Questions 10 to 12 carry six marks each)

10. A fat wire, with radius a, carries a constant current I, uniformly distributed over its cross-section. A narrow gap in the wire, of width $w \ll a$, forms a parallel-plate capacitor
(a) Find the electric and magnetic field in the gap, as functions of the distance s from the axis and the time t. (Assume the charge is zero at $t = 0$)
(b) Find the energy density u_{em} and Poynting vector S in the gap. Note especially the direction of S.
(c) Determine the total energy in the gap, as a function of time. Calculate the total power flowing into the gap, by integrating the Poynting vector over the appropriate surface. Check that the power input is equal to the rate of increase of energy in the gap. (

$$\frac{dw}{dt} = -\frac{d}{dt}\int_v \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)d\tau - \frac{1}{\mu_0}\oint_s (\boldsymbol{E} \times \boldsymbol{B})\, da$$

$\frac{\mu_0 R \sigma}{3}(\omega \times r)$

$\frac{\mu_0 R \sigma \omega}{3} r \sin\theta$

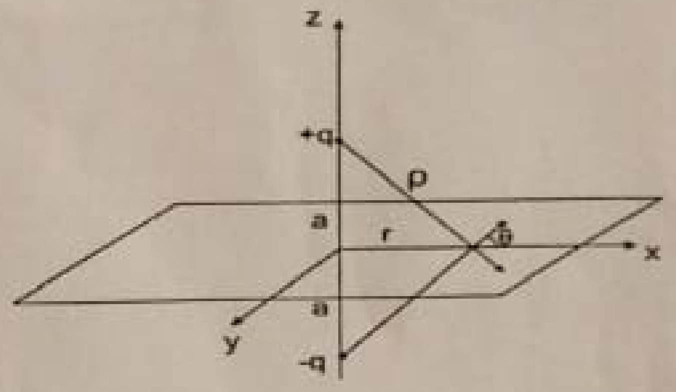
- in the case W = 0, because there is no charge in the gap). [If you are worried about the fringing fields, do it for a volume of radius b < a well inside the gap.]

11. Consider an infinite parallel plate capacitor, with the lower plate (at $z = -d/2$) carrying the charge density $-\sigma$, and the upper plate (at $z = d/2$) carrying the charge density σ

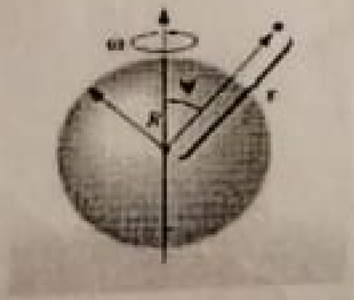
(a) Determine all nine elements of the stress tensor, in the region between the plates. Display your answer as a 3×3 matrix

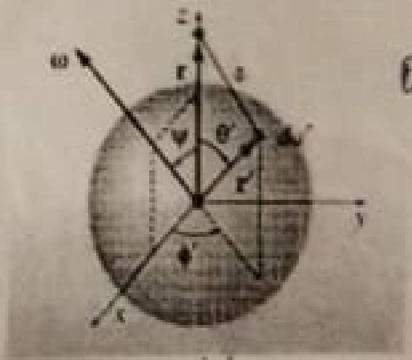
(b) Determine the force per unit area on the top plate

(c) What is the momentum per unit area, per unit time, crossing the XY plane (or any other plane parallel to that one, between the plates)?



12. A spherical shell of radius R carries a uniform surface charge σ, is set spinning at angular velocity ω. Find the vector potential produces at point r.





$E_1 - \frac{E_2^2}{2}$.

13. Derive the differential version of Poynting's theorem and compare the same with continuity equation. (6 marks)