1. Consider the energy levels in ls-basis $\{|2, l, m_l, m_s\rangle\}$ of the hydrogen atom. (a) Construct the total angular momentum j-basis states $\{|2, j, m_j, l, s\rangle\}$ in terns of ls-basis states. (b) Show the energy level shifts in the presence of an external magnetic field $\vec{B} = B_0\hat{z}$, along with the spin-orbit coupling $K\vec{L}\cdot\vec{S}/\hbar^2$, for $\mu_B B_0 << K$. (c) Show the energy level structure for $\mu_B B_0 >> K$. [2,2,2]

2. Consider two spin-1/2 particles with the Hamiltonian given as, $H = K\vec{S}_1\cdot\vec{S}_2 + \Delta S_1^z S_2^z - h(S_1^z + S_2^z)$. Consider S^z-diagonal basis states $|\uparrow\rangle, |\downarrow\rangle$. Using the variational ansatz $|\psi\rangle = a|\uparrow\downarrow\rangle + b|\downarrow\uparrow\rangle$, find the expectation value of the Hamiltonian $E_\psi \equiv \langle\psi|H|\psi\rangle$. Find an upper bound on the ground state energy. [2,2]

3. Consider electric dipole transitions in a hydrogen atom in the presence of an em field, between $|2, l, m\rangle$ and $|1, 0, 0\rangle$. (a) Show all possible transitions when it is linearly polarised along $\hat{x}$ direction. (b) Show the transitions when the polarisation is along $\hat{z}$. (c) Estimate the decay time for $n = 2$ level. [3,2,1]

4. Consider a one-dimensional harmonic oscillator in its ground state for time $t < 0$. For $t \geq 0$, it is subjected to a perturbation $V = F_0 x e^{-t/\tau}$. Find the probability of of finding it in the first excited state as function of t. What is the probability for other excited states? [2,1,1]