IP405 Quantum Mechanics. Minor-2 27 October 2022

Write concise answers clearly, using a maximum of two pages per question

1. Consider the ground state wave function for the one-dimensional harmonic oscillator, given by $\psi_0(x) = A\, e^{-\frac{x^2}{2x_0^2}}$, where $x_0 = \sqrt{\hbar/m\omega}$. Is it an eigenfunction of the momentum operator? Find the expectation value of the energy for this state by evaluating $\langle x^2 \rangle$ and $\langle p^2 \rangle$. [1,2+2]

2. Evaluate the commutators, $[x, H]$, $[p, H]$ for a (a) free particle (b) particle moving under the harmonic oscillator potential. [2,3]

3. Consider an arbitrary state of a particle at time $t = 0$, given by $|\psi(0)\rangle = \alpha_1|1\rangle + \alpha_2|2\rangle + \alpha_3|3\rangle$, where $|i\rangle$ is the energy eigenstate with eigenvalue E_i. Find the expectation value of the energy at $t = 0$. Find the time-evolved state $|\psi(t)\rangle$, and the corresponding energy expectation value. [2,2]

4. Consider a two-dimensional Hilbert space with a basis $\{|1\rangle, |2\rangle\}$. The matrix elements of the operator A are given as $A_{11} = A_{22} = 0.5, A_{12} = A_{21} = 0.1$. Find the eigenvalues and the eigenvectors of A. Write down the matrix representation of A in its eigenbasis. Consider another operator given by $B = |1\rangle\langle 2| + |2\rangle\langle 1|$. Does A commute with B? [2,2,2]