PY/IP453 Minor-1 Examination 27 March 2023

Write concise answers, using two pages per question atmost.

1. Consider rank-1spherical tensor operators $\{U_0, U_\pm\}$, and $\{V_0, V_\pm\}$. Using these operators construct rank-2 spherical tensor operators $\{T^2_q\}$, rank-1 spherical tensor operators $\{T^1_q\}$, and rank-0 spherical tensor operator T^0_0. [3,2,1]

2. Consider $\{|n, l, m\rangle\}$, the eigenstates of the hydrogen atom. Find out the values of l and m for which the following matrix elements can be nonzero, stating the reason for being zero or nonzero.
(a) $\langle 3, l, m|x - iy|3, 1, 1\rangle$ (b) $\langle 3, l, m|p_z|3, 1, 1\rangle$ [2,2]

3. Consider an unperturbed two-dimensional Harmonic oscillator with the Hamiltonian given as $H_0 = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{1}{2}m\omega^2(x^2 + y^2)$. When a perturbation $H' = bxp_y$ is included, find the first-order energy shift for (a) the ground state (b) the first excited state. [2,4]

4. Consider the states $\{|2, l, m\rangle\}$ of the hydrogen atom with n=2, ignoring the spin. The hydrogen atom is placed in an external static electric field $\vec{E} = E_0\hat{x}$. Find the first-order corrections to the energy and the eigenstates. [2,2]