1. (a) Consider induced transitions between the singlet and triplet Hyperfine levels of $n = 1$ level of the hydrogen atom, coupled to a circularly polarised magnetic field, $\vec{B}(t) = B_0(\hat{x}\cos(\omega t) + \hat{y}\sin(\omega t))$. What is the perturbed Hamiltonian, ignoring the coupling of the proton spin to the magnetic field. Examine the transition probabilities for transitions from the singlet initial state to each of the triplet final states, and find out which of them can occur, both for the emission and absorption. [4]

2. Consider a time-dependent Hamiltonian, where the Hamiltonian changes linearly in time in a time interval $T = t_1 - t_0$. The Hamiltonian given by

$$H = \frac{2\beta}{\hbar}S^z + \frac{4\delta}{\hbar(t_1^2 - t_0^2)}S^x\theta(t - t_0)\theta(t_1 - t)t + 2\delta S^x\theta(t - t_1).$$

What is the average Hamiltonian in the interval. Find the criterion for using the sudden approximation for the time evolution of the state $|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$. [2,4]

3. Consider elastic scattering ($|\vec{k}| = |\vec{k}'|$) of plane waves off a potential given by $V(\vec{r}) = V_0 e^{-\mu r}/r$. Find the scattering amplitude $f(\vec{k}', \vec{k})$ using the Born approximation. Find the functional dependence of the scattering cross section on the angle between the initial and final wave vectors, both for $k < \mu$ and $k > \mu$. [4,2]

4. The scattering amplitude can be expanded as a partial wave series over angular momenta, given as,

$$f(\theta) = \sum_{l=0} (2l+1) f_l P_l(\cos\theta),$$

where the partial wave apmlitude f_l is related to the phase shift δ_l as

$$f_l = \frac{e^{i\delta_l}\sin\delta_l}{k}.$$

Find the total scattering cross section by integrating over all angles. Find Im $f(0)$, and show that the optical theorem is satisfied.
(You can use $\int d\Omega\ P_l(\cos\theta)P_m(\cos\theta) = \delta_{l,m}4\pi/(2l+1)$).

[4]