VQGO

& its problem

Task: Optimize θ with

~~Find $U(U_s, \theta)$ s.t.~~

cost fn: $\| U(U_s, \theta) - U_T \|$

Equivalently, if we can generate random samples: $\{|i\rangle\}$ with Harr metric.

Then: task is: ~~Find~~ optimize θ ~~s.t.~~ with cost fn

$$1 - \sum_{\{|i\rangle\}} | \langle i | U^{\dagger}(U_s, \theta) U_T | i \rangle |^2 .$$

With VQGO we can do this task pretty good; cost fn exponentially decreases with optimization step.

But, we can not prepare rample samples perfectly. Introduce ansatz $A(U, \phi)$ s.t.

if ϕ is random sample on $[0, 2\pi)^{\otimes N}$

then $A(U_T, \phi)|0\rangle$ is random sample of $\{|i\rangle\}$

Since $\| A(U_s, \phi) - A(U_T, \phi) \| \approx \| U_s - U_T \|$

it's impossible to optimize θ with

$$1 - \sum_{\{\phi\}} | \langle 0 | A^{\dagger}(U_s, \phi) U^{\dagger}(U_s, \theta) U_T A(U_T, \phi) | 0 \rangle |^2$$

Improved VQGO

However, we can measure the prepared state $A(U_s, \phi)|0\rangle$ for a few shots, get an estimation:

$$|est\rangle \approx A(U_s, \phi)|0\rangle + \delta|d\psi\rangle$$

Then, with confidence α, update simulated state

In the code I set $\delta = 0$, $\alpha = 0.1$ for now

$$|sim\rangle = \sqrt{1-\alpha}\, A(U_T, \phi)|0\rangle + \sqrt{\alpha}\, |est\rangle$$

Finally, we can optimize θ with cost fn:

$$1 - \sum_{\phi} |\langle 0| A^{\dagger}(U_s, \phi)\, U^{\dagger}(U_s, \theta)\, U_T |sim\rangle|^2$$

QSI

We can further improve our protocol by:

For after every few step of Improved VQGO, check with optimized θ':

$$\text{IF } \left[1 - \sum_{\phi} |\langle 0| A^{\dagger}(U(U_s, \theta'), \phi)\, U^{\dagger}(U_s, \theta')\, U_T\, A(U_T, \phi)|0\rangle|^2\right]$$
$$< \left[1 - \sum_{\phi} |\langle 0| A^{\dagger}(U_s, \phi)\, U_s\, U_T\, A(U_T, \phi|0\rangle|^2\right]:$$

then update $A(U_s, \phi)$ to $A(U(U_s, \theta'), \phi)$