2-D Ising Model with Wolff Algorithm

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1 Introduction

In this project, we use the Wolff Algorithm to simulate the 2-D Ising Model. In the project folder, there are three files.

`Documentation.pdf` is the file you are reading now.

`main.py` is the main function of this project, and you are suppose to run this project through `main.py`.

`Ising_2d_wolff.py` contains the `Grid` class, in which we implement the Wolff Algorithm.

2 Results of the Relationship between Binder Cumulant and Temperature.

2.1 Results

You can see the simulation results in Figure 1. Consider the size of the grid is $n \times n$, then we take n as $10, 12, 14, 16$ separately. To simplify the problem, we take $J = 1, k_B = 1$. And the temperature ranges from 1 to 3.

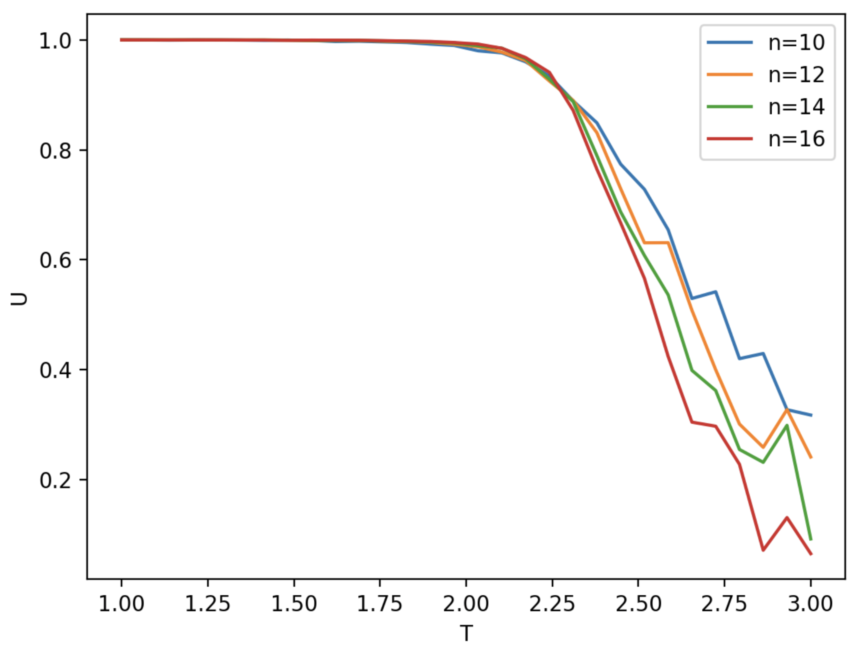

Figure 1: Binder Cumulant - Temperature Relationship

From Figure 1 we can know that these curves have a cross point. In this cross point, $T \approx 2.27$, so we take the phase transition point $T_c = 2.27$.

2.2 Analysis

From Figure 1, it can be seen that near the transition point, the bigger n is, the steeper the curve is, which aligns well with the theory. And when $n \to \infty$, the curve that cross the transition point should be parallel to the y-axis.

3 Proof of the Scaling Function

3.1 Results

For different size of grid ($n = 10, 12, 14, 16$), we can draw the relationship between ML^a and tL^b. You can see the results in Figure 2. a and b are the scaling factor in 2-D Ising Model. In this problem, we take $a = 0.125$ and $b = 1$.

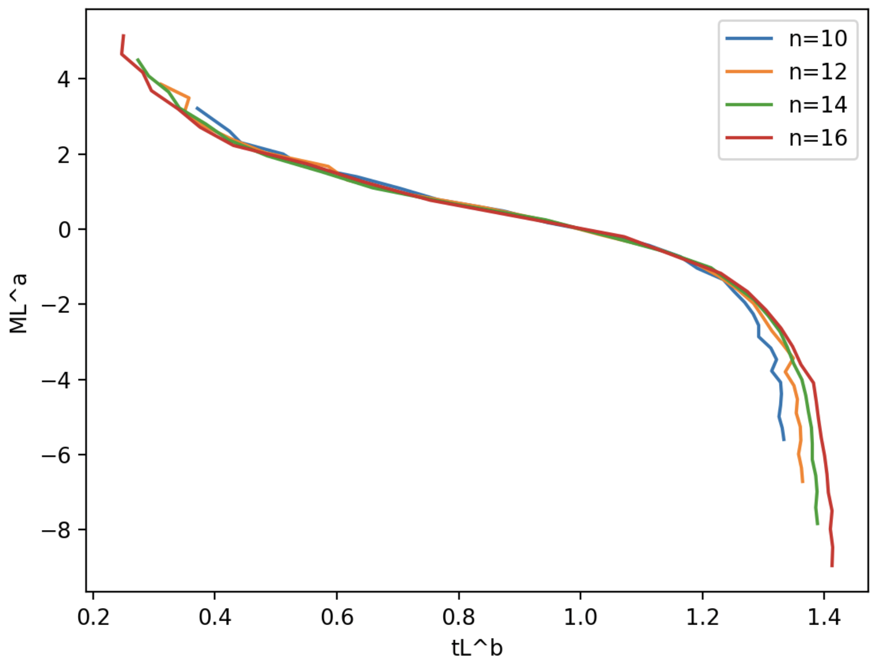

Figure 2: Scaling Function

From this picture we can see that though different grids have different size, they all obey the same scaling function.