Assignment2

October 15, 2018

1 Assignment 2

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Due Wednesday, Oct. 17 at 11:30 AM

```
In [1]: # Import packages
    import numpy as np
    import pandas as pd
    import statsmodels.api as sm
    from sklearn import preprocessing
    import matplotlib.pyplot as plt
    plt.rc("font", size=14)
    from sklearn.linear_model import LogisticRegression
    # from sklearn.cross_validation import train_test_split
    import seaborn as sns
    sns.set(style="white")
    sns.set(style="white")
    # Turn of Notebook Package Warnings
    import warnings
    import warnings
```

1.0.3 1. Imputing age and gender

(a) In order to impute age (age_i) and gender $(female_i)$ variables into the BestIncome.txt data by using information from the SurveyIncome.txt data, I will fit a linear regression model and a logistic model using SurveyIncome.txt:

For the linear regression model, I will set age as the response variable, total income (tot_inc) and weight (wgt) as the explanatory variables.

$$age = \beta_0 + \beta_1 \times totinc + \beta_2 \times wgt + \epsilon$$

For the logistic regression model, I will set gender as the response variable, total income and weight as the explanatory variables.

$$logit(female) = \beta_0 + \beta_1 \times totinc + \beta_2 \times wgt + \epsilon$$

Then, I will use the linear regression models to predict the age variable in BestIncome.txt using the total income and weight variables (Note that total income can be calculated by adding labor income, lab_inc, and capital income, cap_inc, together) and use logistic regression model to predict the gender variable in BestIncome.txt using total income and weight variables.

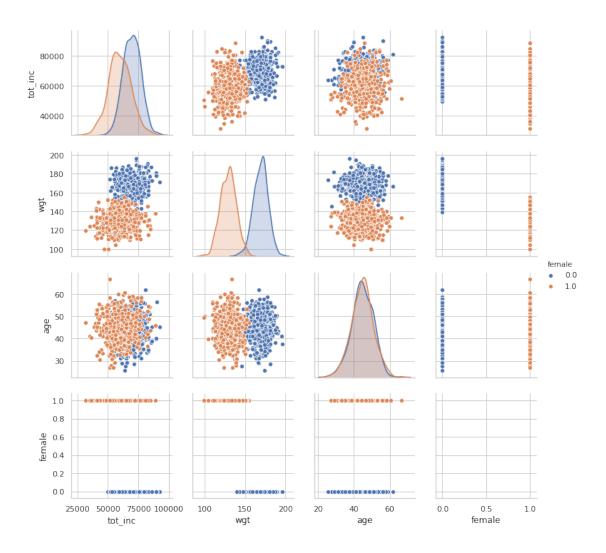
$$age_{pred} = b_0 + b_1 \times totinc + b_2 \times wgt$$

$$P(female) = \frac{\exp(b_0 + b_1 \times totinc + b_2 \times wgt)}{1 + \exp(b_0 + b_1 \times totinc + b_2 \times wgt)}$$

Then, to categorize the predicted observation to female when the predicted value is greater than or equal to 0.5 and to male when the predicted value is less than 0.5.

```
In [2]: # Read Data
       BestIncome = pd.read_csv('BestIncome.txt', header = None, names = ["lab_inc", "cap_inc",
       BestIncome.head()
Out[2]:
               lab_inc
                             cap_inc
                                            hgt
                                                        wgt
         52655.605507
                         9279.509829 64.568138 152.920634
       1 70586.979225
                         9451.016902 65.727648 159.534414
       2 53738.008339
                         8078.132315 66.268796 152.502405
        3 55128.180903 12692.670403 62.910559 149.218189
        4 44482.794867
                         9812.975746 68.678295 152.726358
In [3]: BestIncome.shape
Out[3]: (10000, 4)
In [4]: SurveyIncome = pd.read_csv('SurvIncome.txt', header = None, names = ["tot_inc", "wgt", "
       SurveyIncome.head()
Out [4]:
                                          age female
               tot_inc
                               wgt
       0 63642.513655 134.998269 46.610021
                                                  1.0
       1 49177.380692 134.392957 48.791349
                                                  1.0
       2 67833.339128 126.482992 48.429894
                                                  1.0
       3 62962.266217 128.038121 41.543926
                                                  1.0
        4 58716.952597 126.211980 41.201245
                                                  1.0
In [5]: SurveyIncome.shape
Out[5]: (1000, 4)
In [6]: sns.pairplot(SurveyIncome, hue = "female")
```

Out[6]: <seaborn.axisgrid.PairGrid at 0x120d60400>



As we can see from the percentage calculation result shown below, we can see that the count of male and female are same.

(b) Here is where I'll use my proposed method from part (a) to impute variables.

```
In [8]: # Define response and explanatory variables
       res = 'age'
       expl = ['tot_inc', 'wgt']
       X, y = SurveyIncome[expl], SurveyIncome[res]
In [9]: X.head()
Out [9]:
              tot_inc
                             wgt
       0 63642.513655 134.998269
       1 49177.380692 134.392957
       2 67833.339128 126.482992
       3 62962.266217 128.038121
       4 58716.952597 126.211980
In [10]: y.head()
Out[10]: 0 46.610021
          48.791349
        1
          48.429894
          41.543926
        3
        4 41.201245
        Name: age, dtype: float64
In [11]: X = sm.add_constant(X, prepend = False)
        X.head()
               tot_inc
Out[11]:
                             wgt const
        0 63642.513655 134.998269
                                    1.0
        1 49177.380692 134.392957 1.0
        2 67833.339128 126.482992 1.0
        3 62962.266217 128.038121 1.0
        4 58716.952597 126.211980
                                  1.0
In [12]: m = sm.OLS(y, X)
        res = m.fit()
        print(res.summary())
                         OLS Regression Results
______
                              age R-squared:
Dep. Variable:
                                                                  0.001
                              OLS Adj. R-squared:
Model:
                                                                 -0.001
Method:
                    Least Squares F-statistic:
                                                                 0.6326
                 Mon, 15 Oct 2018 Prob (F-statistic): 14:22:17 Log-Likelihood:
Date:
                                                                  0.531
Time:
                                                                -3199.4
No. Observations:
                              1000 AIC:
                                                                  6405.
Df Residuals:
                              997
                                    BIC:
                                                                  6419.
```

Df Model:	2
Covariance Type:	nonrobust

	coef	std err	t	P> t	[0.025	0.975]
tot_inc	2.52e-05 -0.0067	2.26e-05 0.010	1.114	0.266 0.493	-1.92e-05 -0.026	6.96e-05 0.013
const	44.2097	1.490	29.666	0.000	41.285	47.134
========			=======			========
Omnibus:		2	.460 Durb	in-Watson:		1.921
Prob(Omnib	us):	0	.292 Jarq	ue-Bera (JB	s):	2.322
Skew:		-0	.109 Prob	(JB):		0.313
Kurtosis:		3	.092 Cond	. No.		5.20e+05
========				=======	========	========

Warnings:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 5.2e+05. This might indicate that there are strong multicollinearity or other numerical problems.

As we can see from the output, we will have the following linear regression equation:

$$age = 44.2097 + 0.0000252 \times totinc - 0.0067 \times wgt$$

Then, we can do prediction as follows:

```
In [13]: # Calculate the total income by adding labor income and capital income together
        BestIncome['tot_inc'] = BestIncome['lab_inc'] + BestIncome['cap_inc']
        BestIncome.head()
        X = BestIncome[expl]
        X = sm.add_constant(X, prepend = False)
        X.head()
Out[13]:
                tot_inc
                                wgt
                                     const
        0 61935.115336 152.920634
                                       1.0
        1 80037.996127 159.534414
                                       1.0
        2 61816.140654 152.502405
                                       1.0
        3 67820.851305 149.218189
                                       1.0
        4 54295.770612 152.726358
                                       1.0
In [14]: BestIncome['age'] = res.predict(X)
        BestIncome.head()
Out[14]:
                lab_inc
                              cap_inc
                                             hgt
                                                                   tot_inc
                                                         wgt
                                                                                 age
        0 52655.605507
                          9279.509829
                                       64.568138
                                                  152.920634 61935.115336
                                                                           44.742614
        1 70586.979225
                          9451.016902 65.727648
                                                  159.534414 80037.996127
                                                                           45.154387
        2 53738.008339
                          8078.132315 66.268796
                                                  152.502405
                                                              61816.140654
                                                                           44.742427
        3 55128.180903 12692.670403 62.910559 149.218189 67820.851305 44.915836
        4 44482.794867
                         9812.975746 68.678295 152.726358 54295.770612 44.551391
```

Next, we will impute the gender variable using logistic regression model:

```
In [15]: # Define response and explanatory variables
       res = 'female'
       expl = ['tot_inc', 'wgt']
       X, y = SurveyIncome[expl], SurveyIncome[res]
In [16]: X = sm.add_constant(X, prepend = False)
       X.head()
Out[16]:
             tot_inc
                           wgt const
       0 63642.513655 134.998269 1.0
       1 49177.380692 134.392957 1.0
       2 67833.339128 126.482992 1.0
       3 62962.266217 128.038121 1.0
       4 58716.952597 126.211980 1.0
In [17]: m = sm.Logit(y, X)
       res = m.fit()
       print(res.summary())
Optimization terminated successfully.
       Current function value: 0.036050
       Iterations 11
                      Logit Regression Results
_____
                         female No. Observations:
Dep. Variable:
                                                             1000
Model:
                          Logit Df Residuals:
                                                             997
                            MLE Df Model:
Method:
           Mon, 15 Oct 2018 Pseudo R-squ.:
                       Oct 2018 Pseudo R-squ.:
14:22:18 Log-Likelihood:
Date:
                                                           0.9480
Time:
                                                           -36.050
                          True LL-Null:
converged:
                                                           -693.15
                                                4.232e-286
                                LLR p-value:
```

coef std err z P>|z| [0.025

Possibly complete quasi-separation: A fraction 0.55 of observations can be perfectly predicted. This might indicate that there is complete quasi-separation. In this case some parameters will not be identified.

As we can see from the output, the logistic regression model is

$$logit(female) = 76.7929 - 0.0002 \times totinc - 0.4460 \times wgt$$

```
In [18]: X = BestIncome[expl]
         BestIncome = BestIncome.drop(columns = "tot_inc")
         X = sm.add_constant(X, prepend = False)
         X.head()
Out[18]:
                 tot_inc
                                 wgt
                                      const
         0 61935.115336
                         152.920634
                                        1.0
         1 80037.996127 159.534414
                                        1.0
         2 61816.140654 152.502405
                                        1.0
         3 67820.851305 149.218189
                                        1.0
         4 54295.770612 152.726358
                                        1.0
In [19]: BestIncome['female'] = res.predict(X)
         BestIncome['female'] [BestIncome['female'] >= 0.5] = 1
         BestIncome['female'] [BestIncome['female'] < 0.5] = 0</pre>
         BestIncome.head()
Out[19]:
                                                                           female
                 lab_inc
                               cap_inc
                                              hgt
                                                          wgt
         0 52655.605507
                           9279.509829
                                        64.568138
                                                   152.920634
                                                                              0.0
                                                                44.742614
         1 70586.979225
                                        65.727648
                                                                              0.0
                           9451.016902
                                                   159.534414
                                                                45.154387
         2 53738.008339
                           8078.132315
                                        66.268796
                                                   152.502405
                                                                44.742427
                                                                              0.0
         3 55128.180903 12692.670403
                                        62.910559
                                                   149.218189
                                                                44.915836
                                                                              0.0
         4 44482.794867
                           9812.975746 68.678295
                                                   152.726358 44.551391
                                                                              1.0
```

(c) Here is where I'll report the descriptive statistics for my new imputed variables.

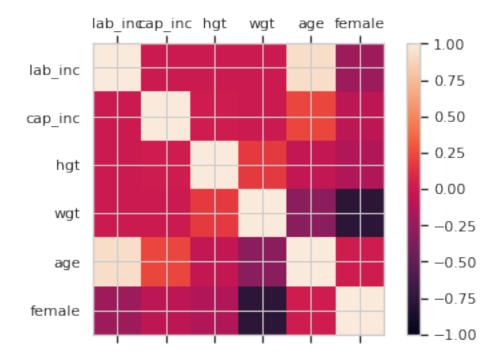
```
Out [20]:
                                      female
                          age
         count
                10000.000000
                               10000.000000
         mean
                    44.890828
                                    0.454600
                     0.219150
                                    0.497959
         std
         min
                    43.976495
                                    0.000000
         25%
                    44.743776
                                    0.00000
         50%
                    44.886944
                                    0.00000
         75%
                    45.038991
                                    1.000000
                    45.703819
                                    1.000000
         max
```

As we can see from the output, for the predicted age variable in BestIncome.txt, the mean is 44.890828, the standard deviation is 0.219150, the minimum is 43.976495, the maximum is 45.703819, the number of observations is 10000. For the predicted female variable in BestIncome.txt, the mean is 0.4546, the standard deviation is 0.497959, the minimum is 0, the maximum is 1, the number of observations is 10000.

(d) Correlation matrix for the now six variables

```
In [21]: # Correlation matrix code and output
         def corr_plot(df):
             import matplotlib.pyplot as plt
             import numpy as np
             import pandas as pd
             names = df.columns
             N = len(names)
             correlations = df.corr()
             fig = plt.figure()
             ax = fig.add_subplot(111)
             cax = ax.matshow(correlations, vmin=-1, vmax=1)
             fig.colorbar(cax)
             ticks = np.arange(0,N,1)
             ax.set_xticks(ticks)
             ax.set_yticks(ticks)
             ax.set_xticklabels(names)
             ax.set_yticklabels(names)
             plt.show()
```

corr_plot(BestIncome)



```
Out[22]: <pandas.io.formats.style.Styler at 0x123b8c828>
```

1.0.4 2. Stationarity and data drift

(a) Estimate by OLS and report coefficients

```
In [23]: # Read in my third data set
       IncomeIntel = pd.read_csv('IncomeIntel.txt', header = None, names = ["grad_year", "gre_
       IncomeIntel.head()
Out [23]:
         grad_year
                    gre_qnt salary_p4
           2001.0 739.737072 67400.475185
       1
           2001.0 721.811673 67600.584142
       2
           2001.0 736.277908 58704.880589
       3
           2001.0 770.498485 64707.290345
           2001.0 735.002861 51737.324165
In [24]: # Run regression model
       X = IncomeIntel["gre_qnt"]
       X = sm.add_constant(X, prepend = False)
       y = IncomeIntel["salary_p4"]
       m = sm.OLS(y, X)
       res = m.fit()
       print(res.summary())
                     OLS Regression Results
______
Dep. Variable:
                     salary_p4 R-squared:
                                                         0.263
Model:
                          OLS Adj. R-squared:
                                                         0.262
                  Least Squares F-statistic:
Method:
                                                         356.3
Date:
               Mon, 15 Oct 2018 Prob (F-statistic):
                                                     3.43e-68
                      14:22:18 Log-Likelihood:
Time:
                                                       -10673.
No. Observations:
                         1000 AIC:
                                                     2.135e+04
Df Residuals:
                          998
                              BIC:
                                                      2.136e+04
Df Model:
                           1
Covariance Type:
                nonrobust
______
                                     P>|t|
                                              [0.025
             coef std err
                                t
______
         -25.7632 1.365 -18.875 0.000 -28.442
8.954e+04 878.764 101.895 0.000 8.78e+04
                                                      -23.085
gre_qnt
                                                      9.13e+04
_____
Omnibus:
                         9.118 Durbin-Watson:
                                                         1.424
Prob(Omnibus):
                         0.010 Jarque-Bera (JB):
                                                         9.100
Skew:
                         0.230 Prob(JB):
                                                        0.0106
                         3.077
                               Cond. No.
Kurtosis:
                                                      1.71e+03
```

Warnings:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 1.71e+03. This might indicate that there are strong multicollinearity or other numerical problems.

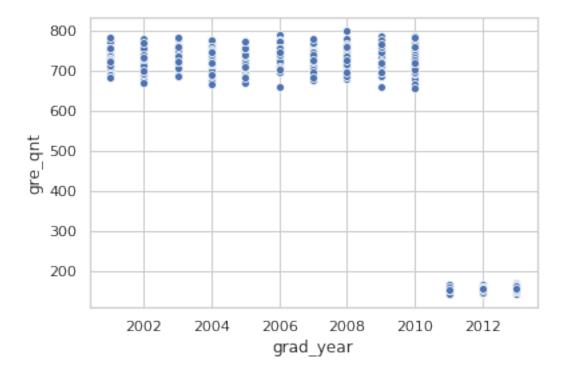
Report coefficients and SE's

$$\beta_0 = 89540$$
 s.e. $(\beta_0) = 878.764$
 $\beta_1 = -25.7632$ s.e. $(\beta_1) = 1.365$

(b) Create a scatterplot of GRE score and graduation year.

```
In [25]: # Code and output of scatterplot
    x = IncomeIntel['grad_year']
    y = IncomeIntel['gre_qnt']
    sns.scatterplot(x,y)
```

Out[25]: <matplotlib.axes._subplots.AxesSubplot at 0x1241db828>

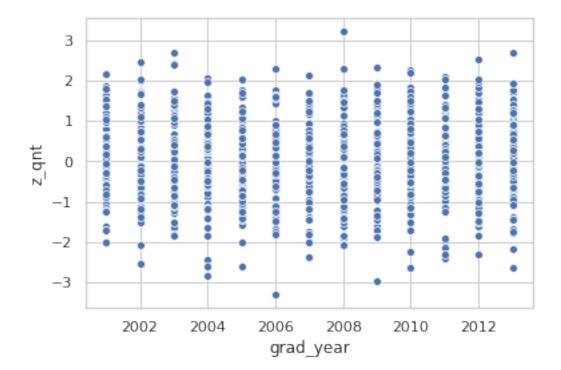


As we can see from the output, the scatterplot has a huge change starting year 2011, which is the year that 800 score scale has changed to 170. Thus, our linear regression model used previously is not reasonable, since the GRE quant scores are not on a same scale. In addition, the difficulty of GRE quant is different among different years. In order to modify this, we have to figure out a way to make GRE quant score on the same scale.

The solution is that I will calculate the Z-score for each year scores, and then concate them. Then, the GRE score will uniquely represent the position of the students' performance.

```
In [26]: # Code to implement solution
         mean_gre = {}
         sd_gre = {}
         for i in range(2001, 2014):
             gre = IncomeIntel['gre_qnt'][IncomeIntel['grad_year'] == i]
             mean_gre[i] = gre.mean()
             sd_gre[i] = gre.std()
         IncomeIntel['z_qnt'] = 0
         for i in range(2001, 2014):
             IncomeIntel['z_qnt'][IncomeIntel['grad_year'] == i] = (IncomeIntel['gre_qnt'][IncomeIntel['gre_qnt']]
         IncomeIntel.head()
                                       salary_p4
Out [26]:
            grad_year
                          gre_qnt
                                                     z_qnt
               2001.0 739.737072 67400.475185 0.406740
         0
         1
               2001.0 721.811673 67600.584142 -0.356635
               2001.0 736.277908 58704.880589 0.259427
         2
         3
               2001.0 770.498485 64707.290345 1.716750
         4
               2001.0 735.002861 51737.324165 0.205128
In [27]: # Code and output of new scatterplot
         x = IncomeIntel['grad_year']
         y = IncomeIntel['z_qnt']
         sns.scatterplot(x,y)
```

Out[27]: <matplotlib.axes._subplots.AxesSubplot at 0x12402ca90>

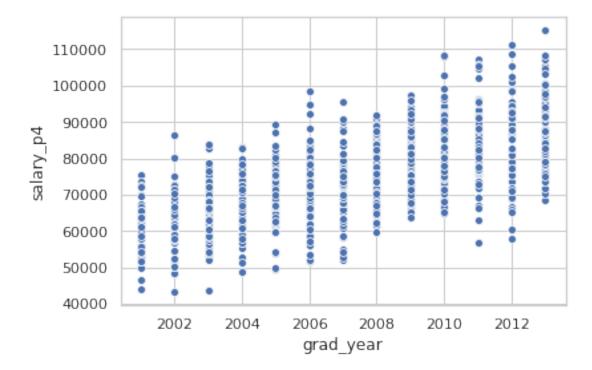


As we can see from the new scatterplot of the z-score of GRE quantitative score, the plot looks much better than the old one. The change of score scale has been solved.

(c) Create a scatterplot of income and graduation year

```
In [28]: # Code and output of scatterplot
    x = IncomeIntel['grad_year']
    y = IncomeIntel['salary_p4']
    sns.scatterplot(x,y)
```

Out[28]: <matplotlib.axes._subplots.AxesSubplot at 0x123b507f0>

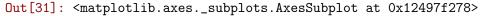


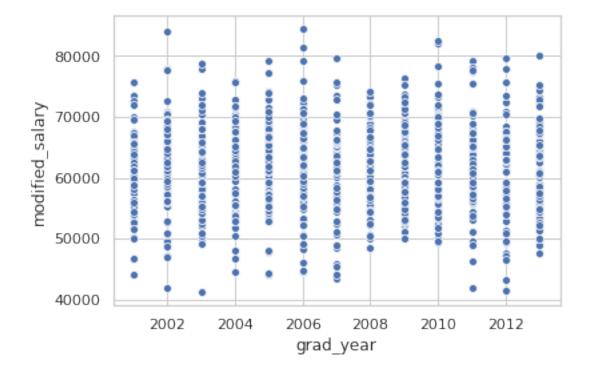
As we can see from the scatterplot above, we can see that there is an increasing pattern in the plot, which implies that the data is not stationary. In order to deal with this, we can estimate the growth rate and eliminate the effect of it.

Specifically, we can first take the average salary of each year and estimate the growth rate by taking the average of each year's growth rate. For each year's growth rate, we can use the following equation:

```
growth \ rate_{year_i} = \frac{average \ salary \ of \ year_i - average \ salary \ of \ year_{i-1}}{average \ salary \ of \ year_{i-1}}
```

```
growth_rate_by_year = ((later_year - former_year) / former_year).mean()
         growth_rate = growth_rate_by_year.mean()
         growth_rate
Out[29]: 0.030835347092883603
In [30]: IncomeIntel['modified_salary'] = 0
        for i in range(2001, 2014):
            IncomeIntel['modified_salary'][IncomeIntel['grad_year'] == i] = IncomeIntel['salary']
         IncomeIntel.head()
Out[30]:
                                                   z_qnt modified_salary
           grad_year
                         gre_qnt
                                     salary_p4
              2001.0 739.737072 67400.475185 0.406740
                                                             67400.475185
         0
              2001.0 721.811673 67600.584142 -0.356635
         1
                                                             67600.584142
         2
              2001.0 736.277908 58704.880589 0.259427
                                                             58704.880589
         3
              2001.0 770.498485 64707.290345 1.716750
                                                             64707.290345
              2001.0 735.002861 51737.324165 0.205128
                                                             51737.324165
In [31]: # Code and output of scatterplot
        x = IncomeIntel['grad_year']
         y = IncomeIntel['modified_salary']
        sns.scatterplot(x,y)
```





As we can see from the scatterplot, the increasing pattern has been removed.

(d) Re-estimate coefficients with updated variables.

OLS Regression Results

Dep. Varia		modi	fied_sa	alarv	R-squ	ared:		0.000
Model:			_	OLS	-	R-squared:		-0.001
Method:		Le	ast Squ	ares	·	tistic:		0.4395
Date:		Mon,	15 Oct	2018	Prob	(F-statistic)):	0.508
Time:			14:2	22:19	Log-L	ikelihood:		-10291.
No. Observ	ations:			1000	AIC:			2.059e+04
Df Residua	ıls:			998	BIC:			2.060e+04
Df Model:				1				
Covariance	Type:		nonro	bust				
========			======		=====			
	coet	f s	td err		t	P> t	[0.025	0.975]
z_qnt	-150.6097	 7 2	27.193	-0	 .663	0.508	 -596.440	295.221
const	6.142e+04	4 2	25.711	272	. 117	0.000	6.1e+04	6.19e+04
Omnibus:	========	=====	·====== ().776	===== Durbi	n-Watson:		2.025
Prob(Omnib	ous):		C	.678	Jarqu	e-Bera (JB):		0.687
Skew:			C	0.059	Prob(JB):		0.709
Kurtosis:			3	3.049	Cond.	No.		1.01

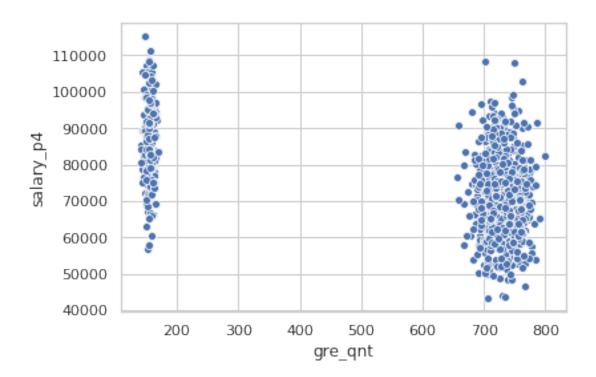
Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

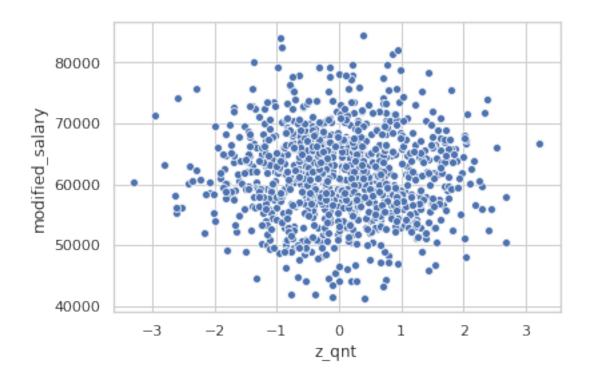
Report new coefficients and SE's

$$\beta_0 = 61420$$
 s.e. $(\beta_0) = 225.711$ $\beta_1 = -150.6097$ s.e. $(\beta_1) = 227.193$

Out[33]: <matplotlib.axes._subplots.AxesSubplot at 0x123eaf780>



Out[34]: <matplotlib.axes._subplots.AxesSubplot at 0x12490f5c0>



Before we changed the variables, we can see from the scatterplot of the old variables version that the scatterplot actually plots the latest years' salaries on the left and old years' salaries on the right. Thus, since the salary is increasing by year, we have a negitive relationship between gre_qnt and salary_p4.

On the other hand, after we modified the gre_qnt to z_qnt and salary_p4 to modified_salary, we can see that the scatterplot looks much better. Thus, the final result of the linear regression will be more informative than the old one. Therefore, we can conclude that the GRE quantitative score is negatively associated with the salary after 4 years of graduation, since the coefficient of β_1 is negative.

1.0.5 3. Assessment of Kossinets and Watts.

See attached PDF.

3. Assessment of Kossinets and Watts (2009)

In this sociology-related paper, the author targeted on the following research question: "On what grounds do individuals selectively make or break some ties over others, and how do these choices shed light on the observation that similar people are more likely to become acquainted than dissimilar people?"

To investigate this research question, the author conducted his analysis based on the population of undergraduate and graduate students, faculty, and staff in a large U.S. university, who used their university e-mail accounts to both send and receive messages during one academic year. The author selected 30,396 observations, who sent and received e-mail using their university e-mail accounts in both the first and the last months of the academic year. The data set was constructed by merging three different databases, including the logs of e-mail interactions, individual attributes, and records of course registration. The available variables were categorized into four groups: personal characteristics (age, gender, home state, formal status, years in school); organizational affiliations (primary department, school, campus, dormitory, academic field); course-related variables (courses taken, courses taught); and e-mailrelated variables (days active, messages sent, messages received, in-degree, out-degree, reciprocated degree). For e-mail interactions data, the author included only messages that were sent to a single recipient other than the sender, which accounted for 82% of all e-mail, in order to ensure that the data present interpersonal communication. As a result of this, the author got 7,156,162 messages exchanged by 30,396 stable e-mail users during 270 days of observation.

As we mentioned previously, the author collected the data and did the data cleaning process. However, there is a potential problem that the process intro-duce in a way that

diminishes the authors' ability to answer the research question. In the paper, the author selected the students who sent and received e-mail in both the first and the last months of the academic year to ensure that the analysis was unaffected by population turnover. Since the author eliminated around 13,000 individuals, which is more than 25% of the 43,553 individuals, the selection process is very influential to the result. This selection process may include many students who did not send and receive e-mail for the rest of months, and exclude many students who just did not send and receive e-mail for the first month but did for the rest months. This population selection can make the result of the analysis be biased. In addition, many students may use forward function in school e-mail system, which will forward all the e-mails to his own e-mail address instead of the university e-mail. Thus, I think the author did not take care of these issues when he cleaned the data.

In this paper, the underlying theoretical construct is "social relationships" and the data are e-mail logs linked to other characteristics of the senders and receivers. One weakness of this match of data source and theoretical construct is that the measure of individual similarity acts, in effect, as an indicator variable for sharing a class, since for students, a number of its components are highly correlated with choice of class. To address this potential systematic bias in the data, the author found that the similarity values and variance tend to be higher when individuals share same class, share same explicit foci, or same implicit foci. Furthermore, the increasing of variance for sharing implicit foci is higher than the other two. Since the author found that implicit foci, when properly calibrated, were qualitatively similar to explicit foci, and also because implicit foci apply to the entire population rather than just to students, the author used implicit foci as his primary measure of shared affiliation, whenever appropriate.