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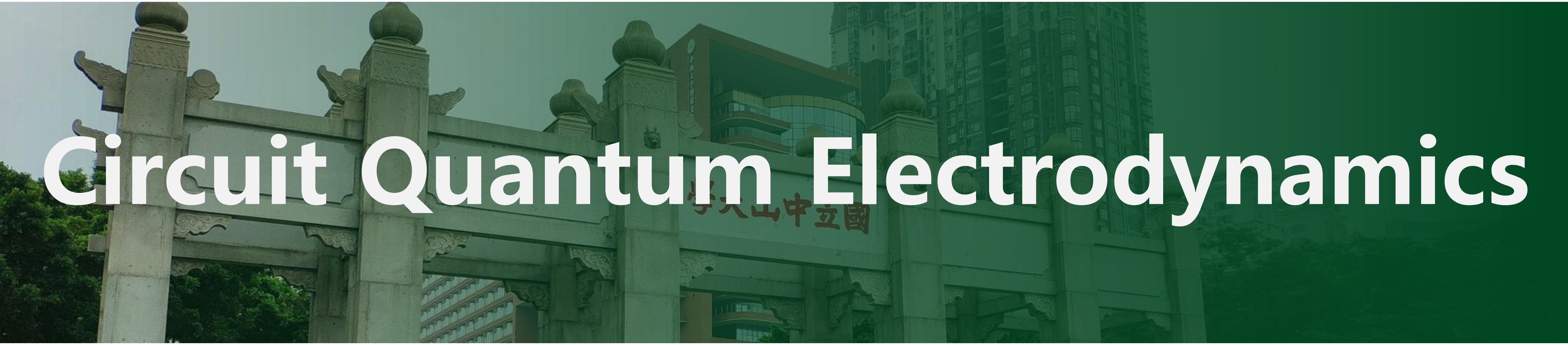
SUN YAT-SEN UNIVERSITY



Circuit Quantum Electrodynamics

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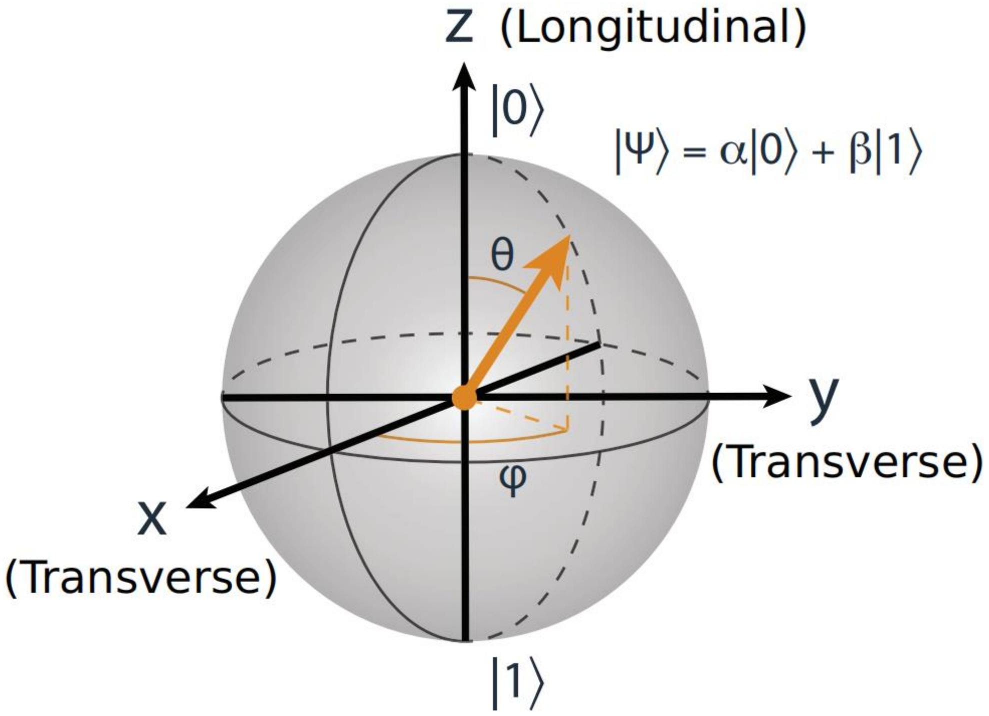
CONTENTS

Part.01

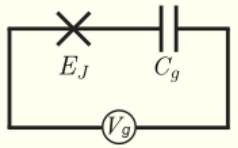
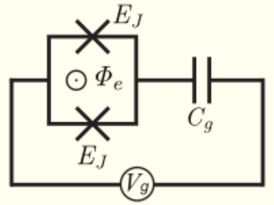
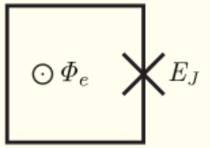
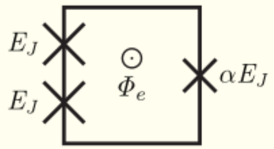
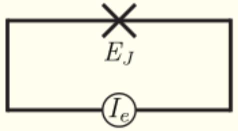
Introduction

- Bloch Sphere



Bloch sphere

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

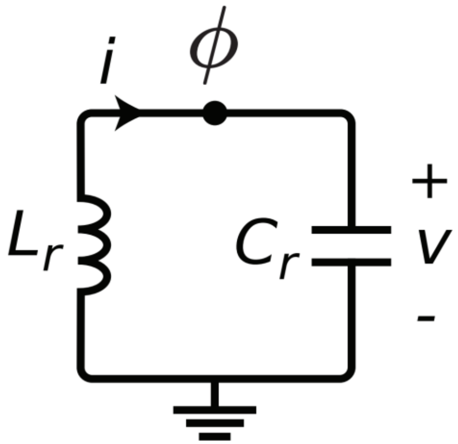
	Circuit	Properties	Dominant noise
Charge qubit		$E_J/E_C < 1$ Controlled by V_g.	Charge fluctuations; mainly 1/f noise.
		$E_J/E_C < 1$ Controlled by both V_g and Φ_e.	
Flux qubit		$E_J/E_C > 1$ Controlled by Φ_e.	Flux fluctuations; mainly 1/f noise.
		$E_J/E_C > 1$ $0.5 < \alpha < 1$ Controlled by Φ_e.	
Phase qubit		$E_J/E_C \gg 1$ Controlled by I_e.	Flux fluctuations; mainly 1/f noise.

Different types of superconducting qubits

Part.02

LC Resonator

·Quantum Harmonic Oscillator



LC circuit

$$V = \frac{d\Phi}{dt}$$

$$E_L = \frac{\Phi^2}{2L}$$

$$E_C = \frac{C\dot{\Phi}^2}{2}$$

$$L = \frac{C\dot{\Phi}^2}{2} - \frac{\Phi^2}{2L}$$

$$p_\Phi = \frac{\partial L}{\partial \dot{\Phi}} = C\dot{\Phi} = Q$$

$$\{\Phi, Q\} = 1$$

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

$$[\hat{\Phi}, \hat{Q}] = i\hbar$$

·Quantization

Harmonic oscillator

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$

$$[\hat{x}, \hat{p}] = i\hbar$$

LC Circuit

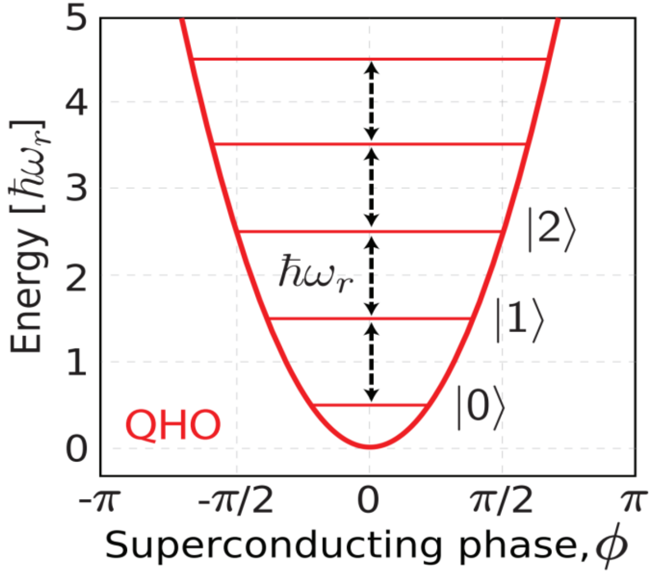
$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

$$[\hat{\Phi}, \hat{Q}] = i\hbar$$

$$a^\dagger = \frac{1}{\sqrt{2\hbar Z_r}}(\hat{\Phi} - iZ_r\hat{Q})$$

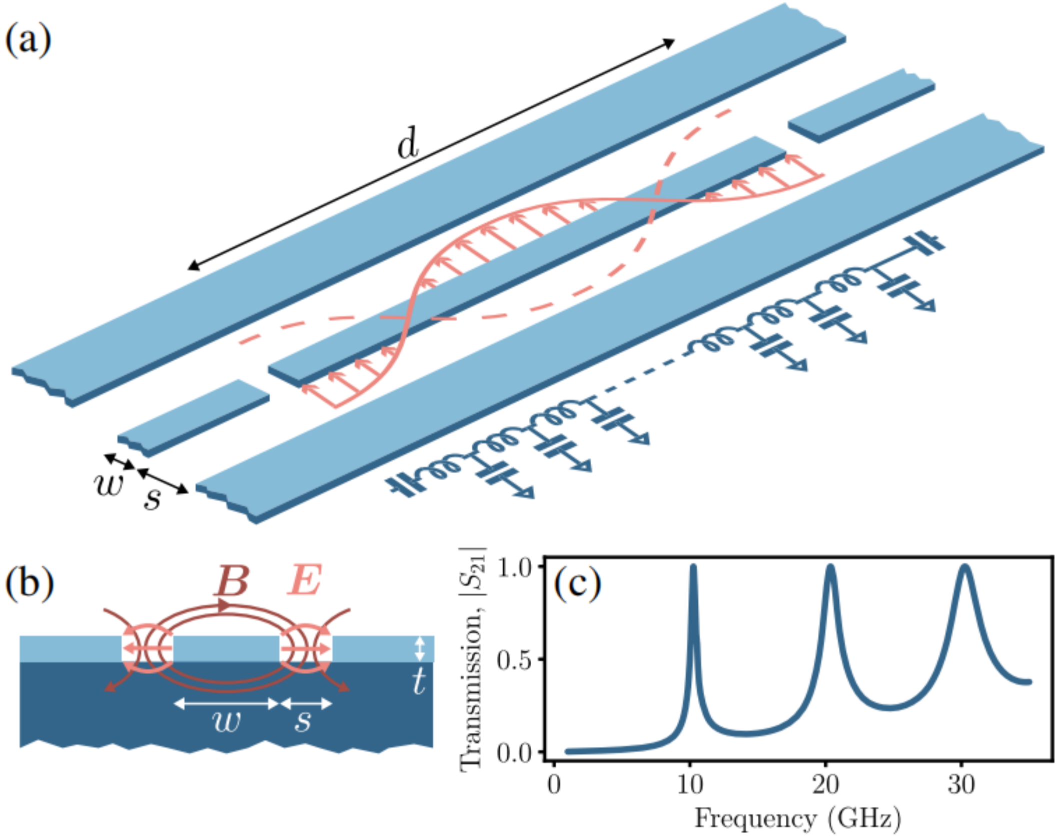
$$a = \frac{1}{\sqrt{2\hbar Z_r}}(\hat{\Phi} + iZ_r\hat{Q})$$

$$\hat{H} = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) = \hbar\omega\left(\hat{n} + \frac{1}{2}\right)$$

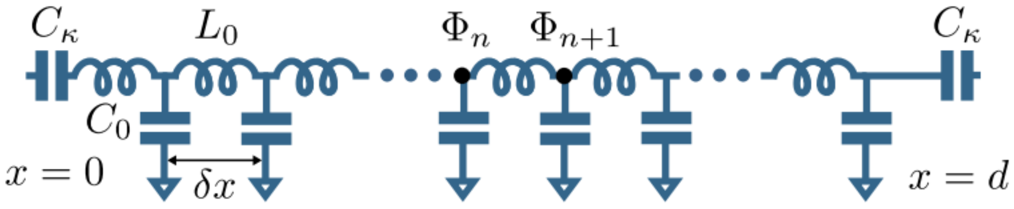


Energy levels

• 2D Resonator



Coplanar waveguide resonator



Telegrapher model

$$C_0 = c_0 \delta x$$
$$L_0 = l_0 \delta x$$

$$H = \sum_{n=0}^{N-1} \left[\frac{Q_n^2}{2C_0} + \frac{(\Phi_{n+1} - \Phi_n)^2}{2L_0} \right]$$

$\delta x \to 0$

$$H = \int_0^d dx \left\{ \frac{1}{2c_0} Q(x)^2 + \frac{1}{2l_0} [\partial_x \Phi(x)]^2 \right\}$$

Hamiltonian equation:

$$v_0^2 \frac{\partial^2 \Phi(x,t)}{\partial x^2} - \frac{\partial^2 \Phi(x,t)}{\partial t^2} = 0$$

$$\Phi(x,t) = \sum_{m=0}^{\infty} u_m(x) \Phi_m(t)$$

$$H = \sum_{m=0}^{\infty} \left[\frac{Q_m^2}{2Cr} + \frac{1}{2} C_r \omega_m^2 \Phi_m^2 \right] = \sum_{m=0}^{\infty} \hbar \omega_m \, \hat{a}_m^\dagger \hat{a}_m$$

Part.03

Fritz London Theory

·Fritz London Theory

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{2m^*}(-i\hbar\nabla + q^*\vec{A})^2\psi + q^*\phi\psi + V\psi$$

$$\vec{J} = \frac{iq^*\hbar}{2m^*}(\psi\nabla\psi^* - \psi^*\nabla\psi) - \frac{q^{*2}}{m^*}\psi^*\psi\vec{A}$$

$$\psi = \sqrt{n(\vec{r},t)}e^{i\theta(\vec{r},t)}$$

$$\vec{J} = \frac{-2ne^2}{m_e}(\frac{\hbar}{2e}\nabla\theta + \vec{A})$$

·London First Equation

$$\frac{\partial\vec{J}}{\partial t} = \frac{-2ne^2}{m_e}(\frac{\hbar}{2e}\nabla\frac{\partial\theta}{\partial t} + \frac{\partial\vec{A}}{\partial t})$$

$$\frac{\partial\theta}{\partial t} = \frac{-\epsilon}{\hbar} = -\frac{1}{\hbar}(\frac{m_e}{2ne^2}\frac{J^2}{2n} - 2e\phi)$$

$$\frac{\partial\vec{J}}{\partial t} = \frac{-2ne^2}{m_e}(\nabla\phi + \frac{\partial\vec{A}}{\partial t}) + ne\nabla(\frac{J^2}{4n^2e^2}) \approx \frac{-2ne^2}{m_e}\vec{E}$$

·London Second Equation

$$\nabla\times\vec{J} = \frac{-2ne^2}{m_e}(\frac{\hbar}{2e}\nabla\times\nabla\theta + \nabla\times\vec{A}) = \frac{-2ne^2}{m_e}\vec{B}$$

·Fritz London Theory

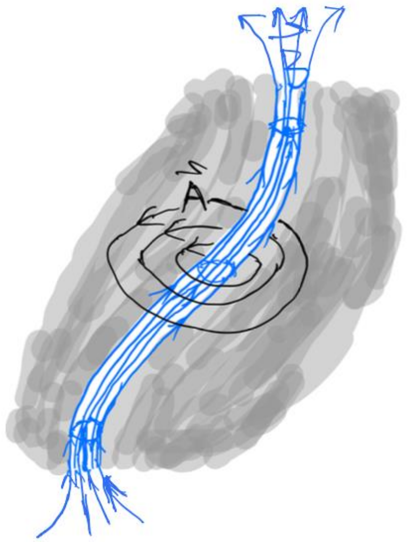
$$i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{2m^*}(-i\hbar\nabla + q^*\vec{A})^2\psi + q^*\phi\psi + V\psi$$

$$\vec{J} = \frac{iq^*\hbar}{2m^*}(\psi\nabla\psi^* - \psi^*\nabla\psi) - \frac{q^{*2}}{m^*}\psi^*\psi\vec{A}$$

$$\psi = \sqrt{n(\vec{r},t)}e^{i\theta(\vec{r},t)}$$

$$\vec{J} = \frac{-2ne^2}{m_e}(\frac{\hbar}{2e}\nabla\theta + \vec{A})$$

·Flux Quantum



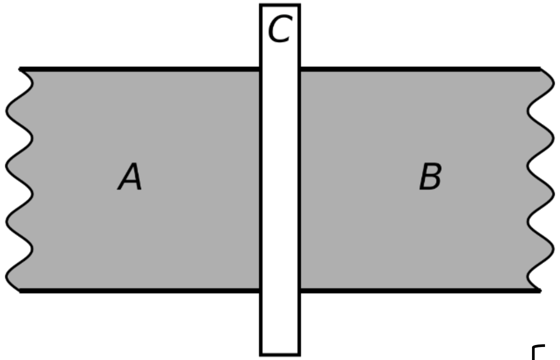
$$0 = \oint \vec{J}\cdot d\vec{l} = \frac{-2ne^2}{m_e}(\oint \frac{\hbar}{2e}\nabla\theta\cdot d\vec{l} + \oint \vec{A}\cdot d\vec{l})$$

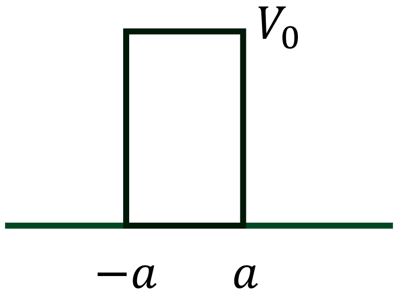
$$\Phi = \oint \vec{A}\cdot d\vec{l} = -\oint \frac{\hbar}{2e}\nabla\theta\cdot d\vec{l} = \frac{\hbar}{2e}2k\pi$$

Part.04

Josephson Junction

•Josephson Junction





$$J_0 = \frac{-n\hbar e}{m_e}\nabla\theta(\pm a, t)$$

$$\begin{cases} \psi(-a,t) = \sqrt{n_1}e^{i\theta_1} \\ \psi(+a,t) = \sqrt{n_2}e^{i\theta_2} \end{cases}$$

$$\epsilon_0\psi = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi$$

$$\psi(x) = C_1\cosh(x/b) + C_2\sinh(x/b)$$

$$b = \sqrt{\frac{\hbar^2}{2m^*(V_0-\varepsilon_0)}}$$

$$\begin{cases} C_1 = \dfrac{\sqrt{n_2^*}e^{i\theta_2} + \sqrt{n_1^*}e^{i\theta_1}}{2\cosh(a/b)} \\ C_2 = \dfrac{\sqrt{n_2^*}e^{i\theta_2} - \sqrt{n_1^*}e^{i\theta_1}}{2\sinh(a/b)} \end{cases}$$

$$J = \frac{e\hbar\sqrt{n_1 n_2}}{m_e b\sinh(\frac{2a}{b})}\sin(\theta_1 - \theta_2)$$

$$\vec{A}' = \vec{A} + \xi \qquad \phi' = \phi - \frac{\partial\xi}{\partial t} \qquad \theta' = \theta - \frac{2e}{\hbar}\xi$$

$$\varphi \equiv \theta_1 - \theta_2 + f = \theta_1 - \theta_2 - \frac{2e}{\hbar}\int_{12}\vec{A}\cdot d\vec{l}$$

$$J = \frac{e\hbar\sqrt{n_1 n_2}}{m_e b\sinh(\frac{2a}{b})}\sin(\varphi)$$

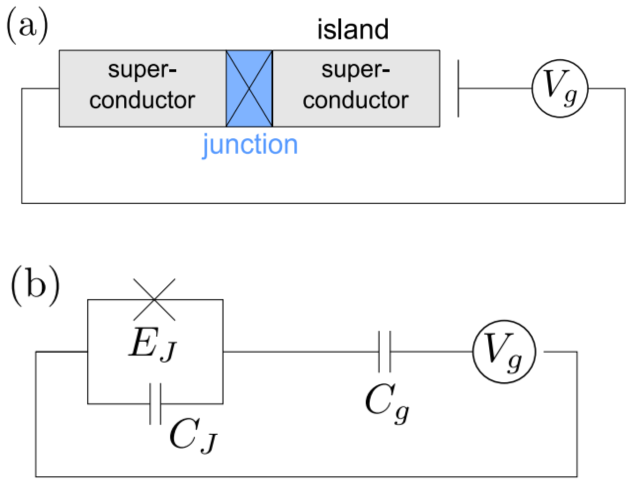
$$\frac{\partial\varphi}{\partial t} = \frac{2e}{\hbar}U$$

$$I = I_C\sin\varphi$$

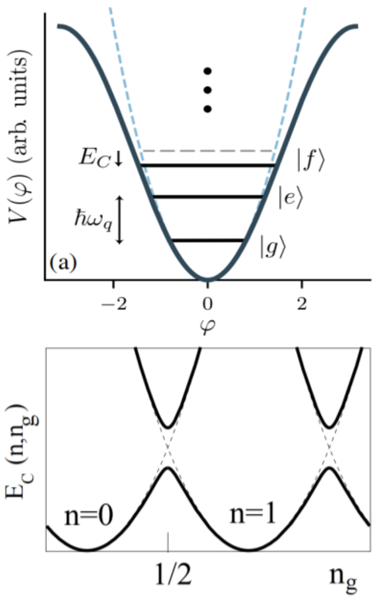
Part.05

Superconducting Circuits

·Charge Qubit



Charge qubit



Energy levels

$$L = \frac{1}{2}C_J(\dot{\Phi})^2 + \frac{1}{2}C_g(\dot{\Phi} - V_g)^2 + E_J\cos(2\pi\frac{\Phi}{\Phi_0})$$

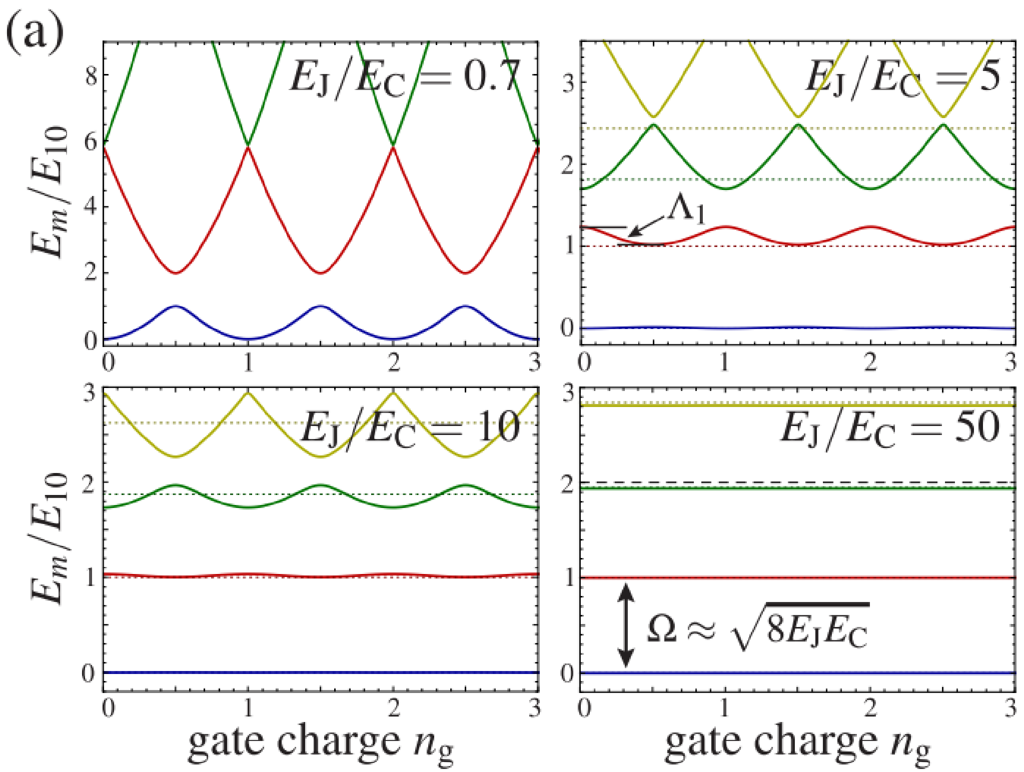
$$p = 2en$$

$$n_g = C_g V_g/2e \qquad H = E_C(n - n_g)^2 - E_J\cos\left(2\pi\frac{\Phi}{\Phi_0}\right)$$

·Transmon Qubit

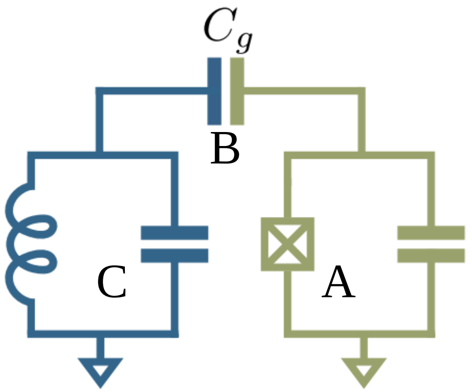
Loss of anharmonicity $\sim (E_J/E_C)^{-1/2}$

$E_J/E_C \ll 1$: charge qubit
$E_J/E_C \sim 1$: quantronium
$E_J/E_C \gg 1$: transmon qubit



Energy spectrum at different energy ratios

·Exchange Interaction



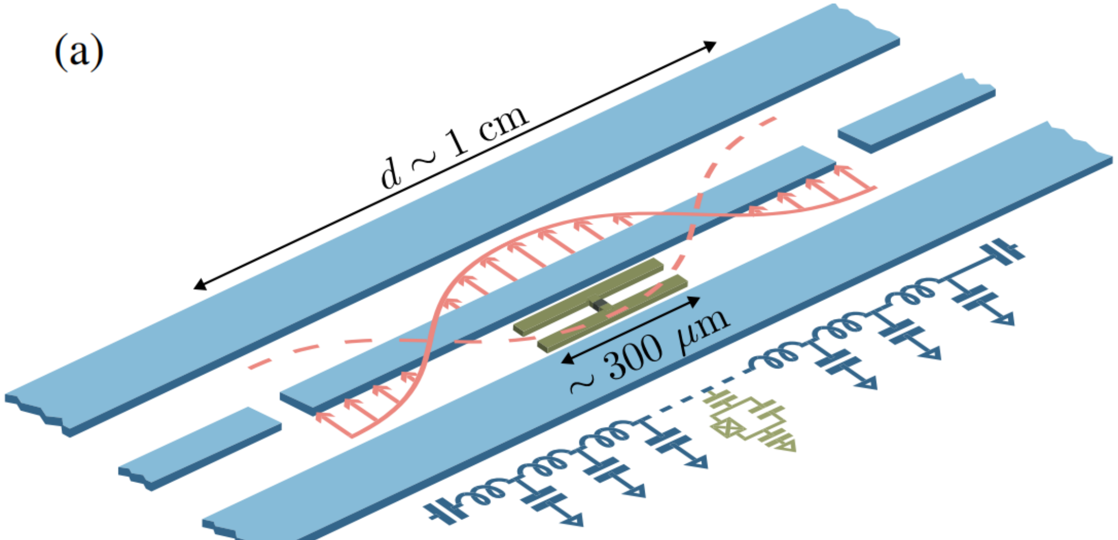
$$\begin{cases} \dot{\Phi}_C + \dot{\Phi}_B - \dot{\Phi}_A = 0 \\ I_A = -I_B \\ I_B = I_C \\ I_C = C\ddot{\Phi}_C + \dfrac{\Phi_C}{L} \\ I_B = C_g\ddot{\Phi}_B \end{cases}$$

Transmon and Oscillator

$$H \approx \frac{1}{2C_\Sigma C}\left[CQ_A^2 + C_\Sigma Q_C^2 + C_g(Q_A+Q_C)^2\right] + V$$

$$= \frac{Q_A^2 + \frac{C_g}{C}(Q_A+Q_C)^2}{2C_\Sigma} + \frac{\Phi^2}{2L} + \frac{Q_C^2}{2C} - E_J\cos(\phi_A)$$

$$\approx \frac{\left(Q_A + \frac{C_g}{C}Q_C\right)^2}{2C_\Sigma} + \frac{\Phi^2}{2L} + \frac{Q_C^2}{2C} - E_J\cos(\phi_A)$$



Transmon Qubit Coupled to 2D Transmission-line Resonator

$$8E_r\hat{n}\cdot\hat{n}_r = -\boxed{\hbar\omega_r\frac{C_g}{C_\Sigma}\sqrt{\frac{\pi Z_r}{R_k}}\left(\frac{E_J}{2E_C}\right)^{\frac{1}{4}}}(a^\dagger - a)(b^\dagger - b)$$

Coupling factor: g

$$\hat{H} = \hbar\omega_r a^\dagger a + \hbar\omega_q b^\dagger b - \frac{E_C}{2}b^\dagger b^\dagger bb - \hbar g(\underbrace{ab + a^\dagger b^\dagger}_{\text{CRT}} + \underbrace{ab^\dagger + a^\dagger b}_{\text{RWA}})$$

Part.00

Appendix

·References

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