

# Functions

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## Announcements

# Expressions

## Types of expressions

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$$18 + 69$$

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$$|-1869|$$

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## Call Expressions in Python

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All expressions can use function call notation  
(Demo)

## Anatomy of a Call Expression

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add ( 2 , 3 )

## Anatomy of a Call Expression

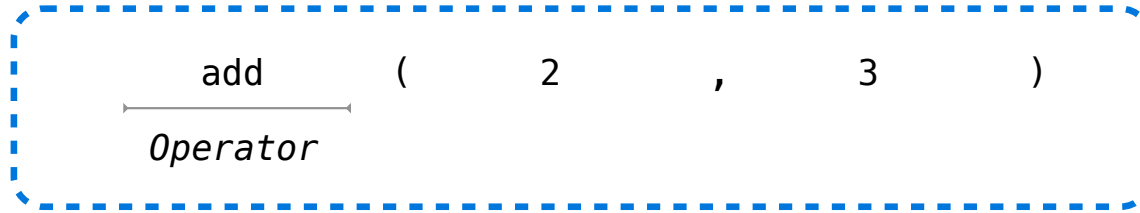
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add ( 2 , 3 )

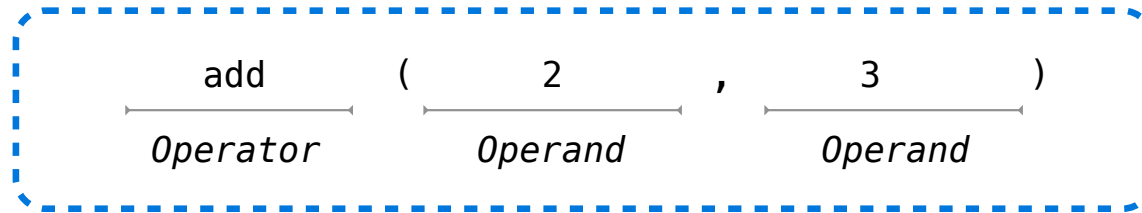
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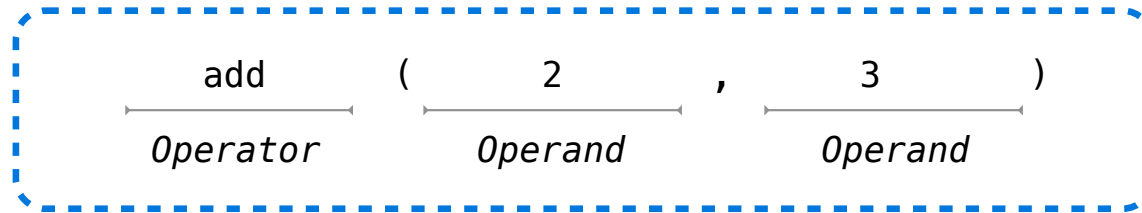
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## Anatomy of a Call Expression

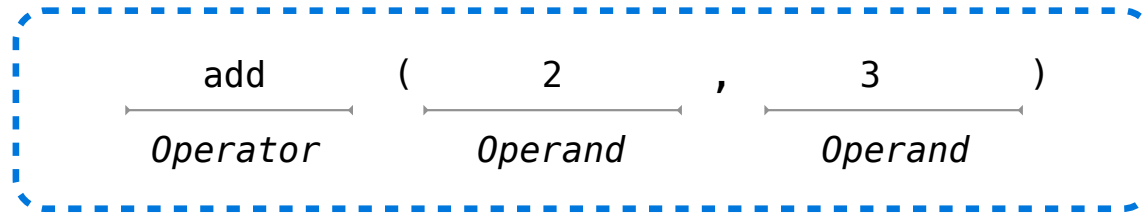
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Operators and operands are also expressions

## Anatomy of a Call Expression

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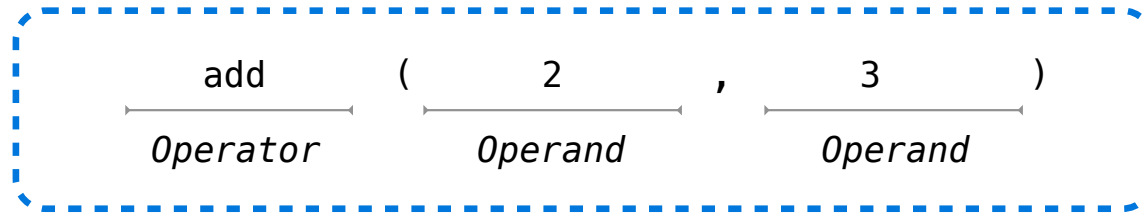


Operators and operands are also expressions

So they evaluate to values

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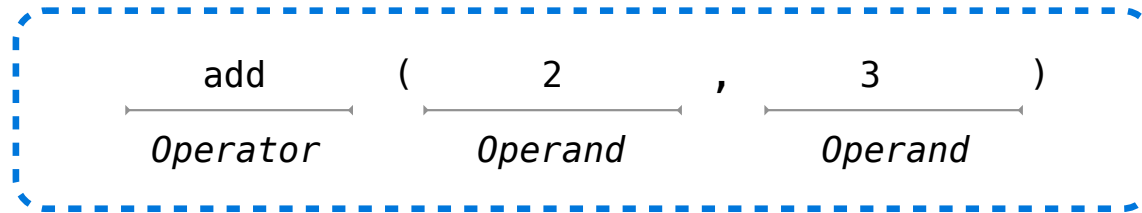
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So they evaluate to values

**Evaluation procedure for call expressions:**

## Anatomy of a Call Expression

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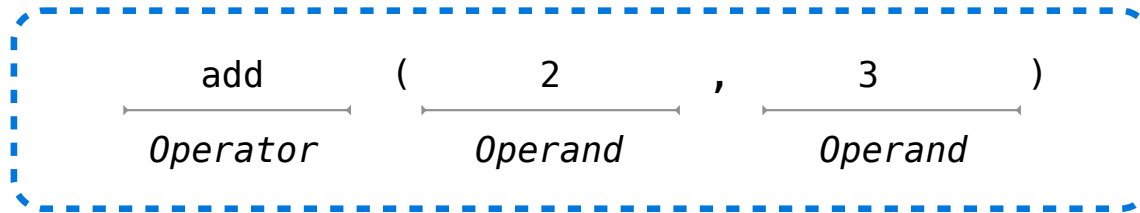
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### Evaluation procedure for call expressions:

1. Evaluate the operator and then the operand subexpressions

## Anatomy of a Call Expression

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Operators and operands are also expressions

So they evaluate to values

### Evaluation procedure for call expressions:

1. Evaluate the operator and then the operand subexpressions
2. **Apply** the **function** that is the value of the operator to the **arguments** that are the values of the operands

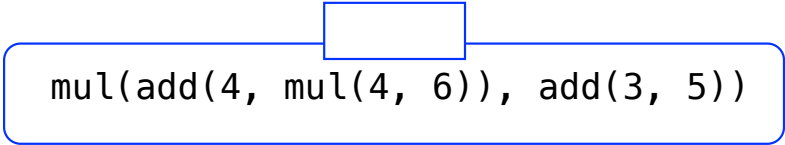
## Evaluating Nested Expressions

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```
mul(add(4, mul(4, 6)), add(3, 5))
```

## Evaluating Nested Expressions

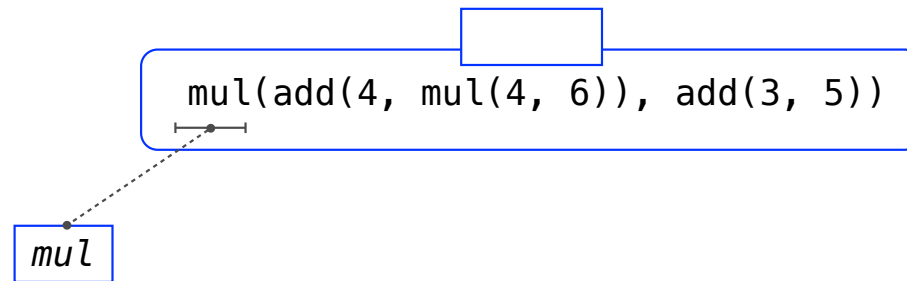
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`mul(add(4, mul(4, 6)), add(3, 5))`

## Evaluating Nested Expressions

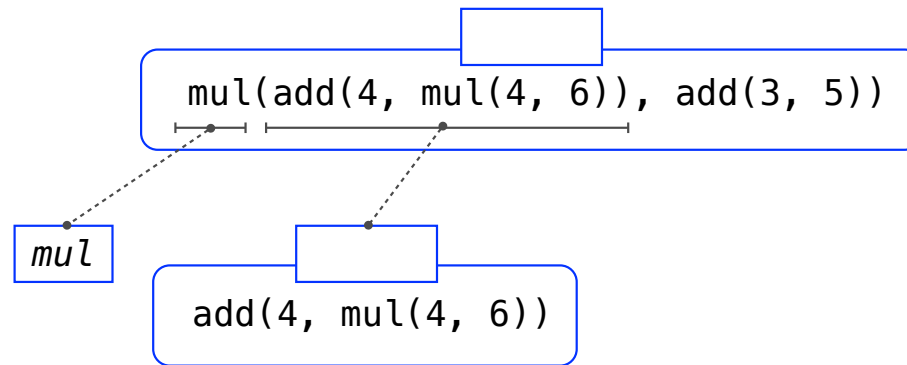
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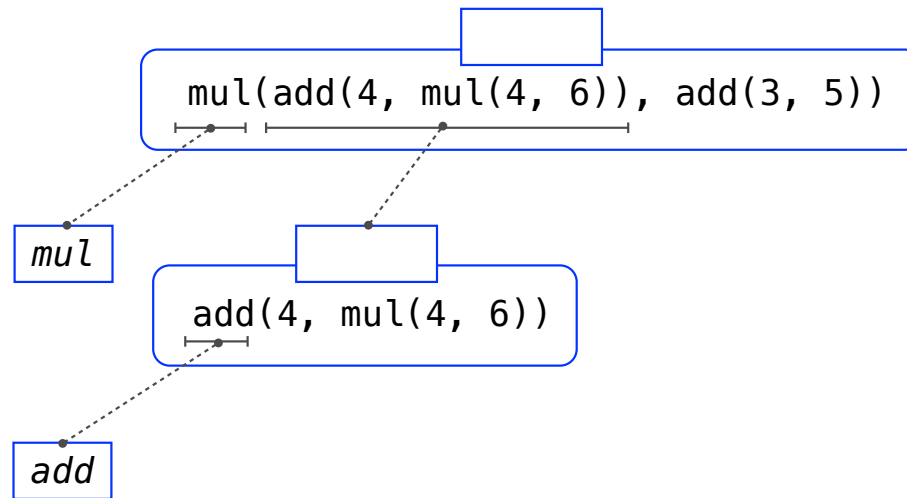
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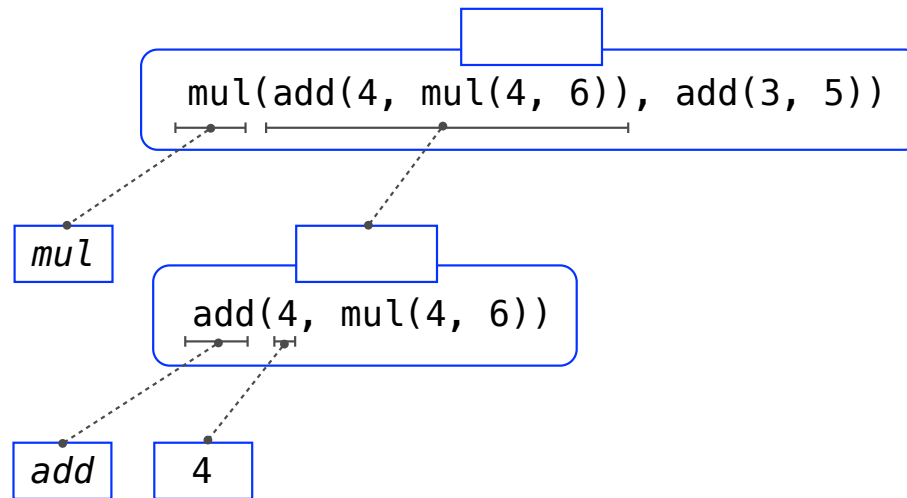
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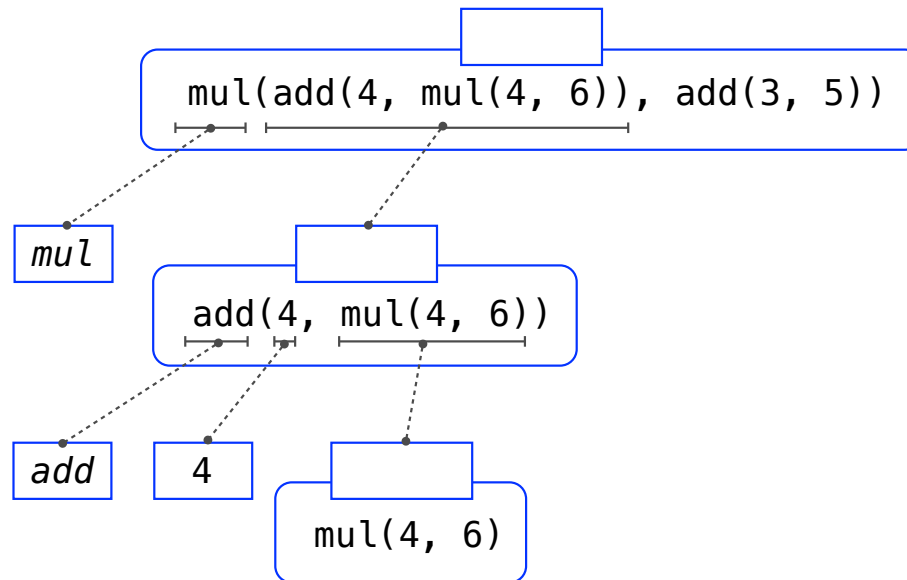
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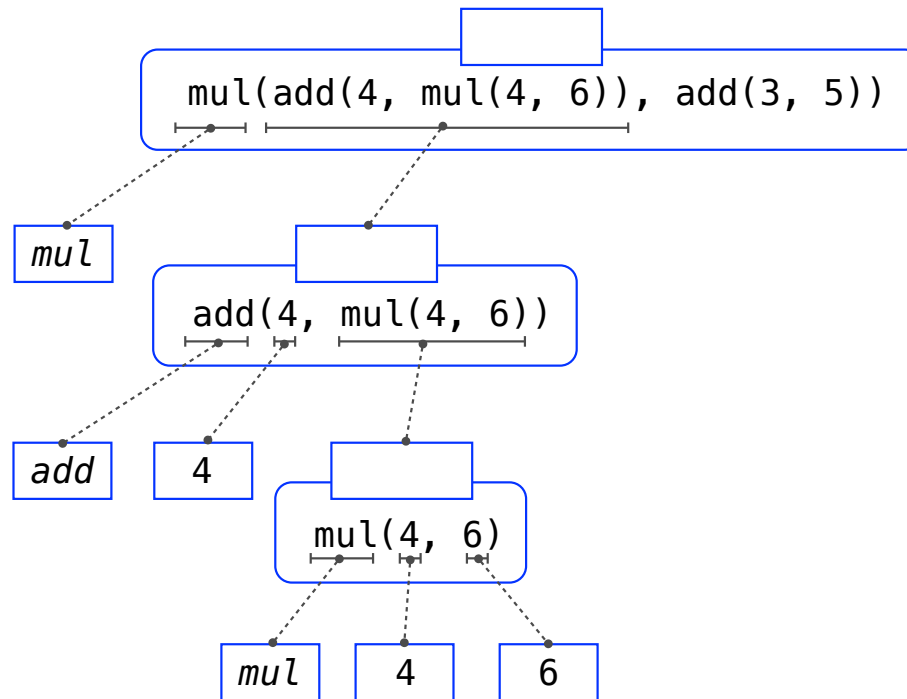


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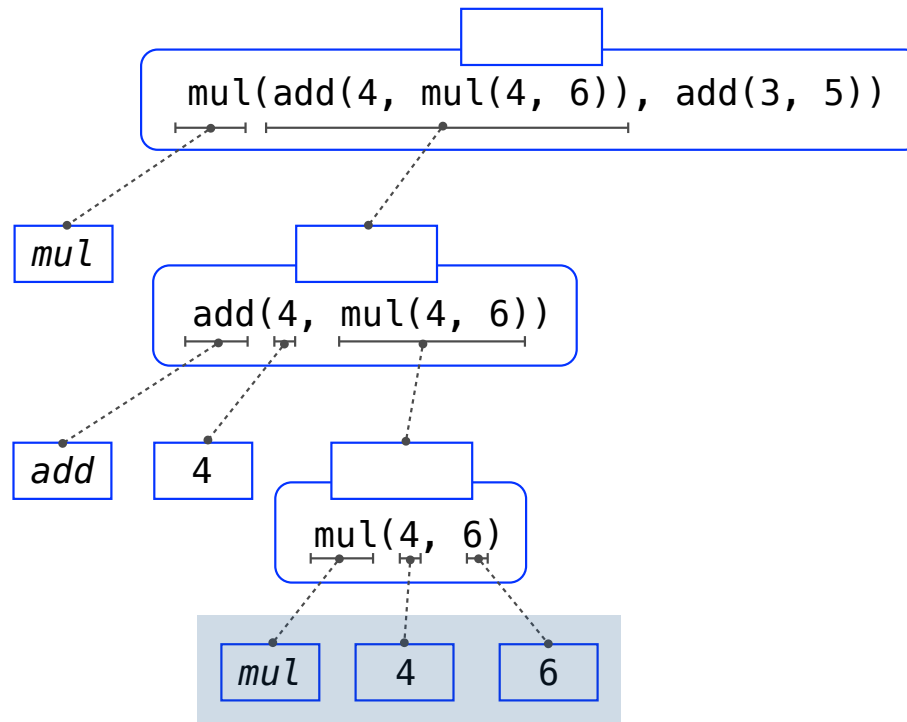
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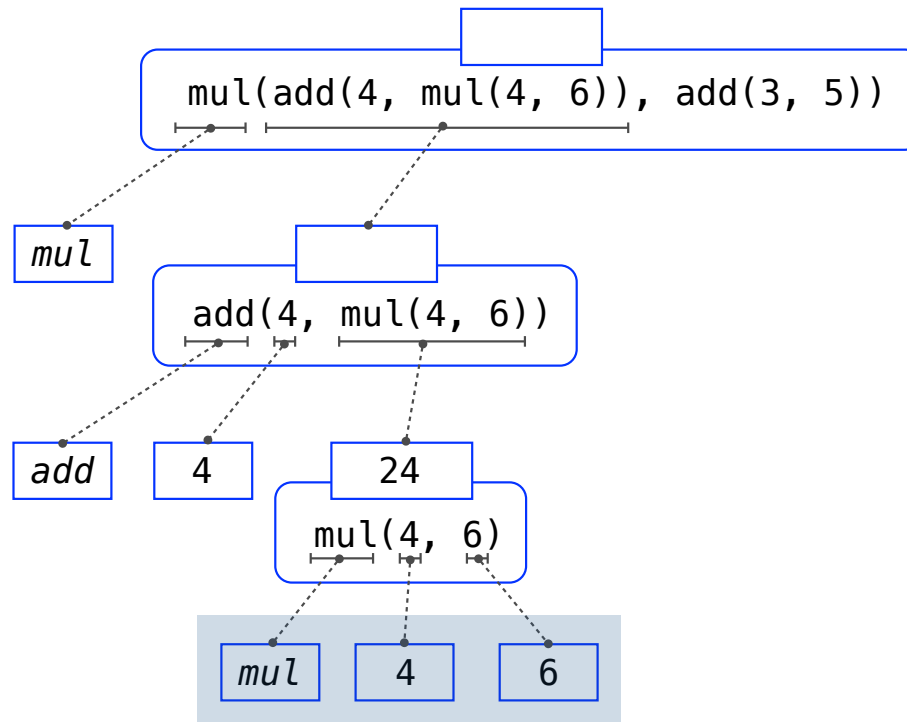
## Evaluating Nested Expressions



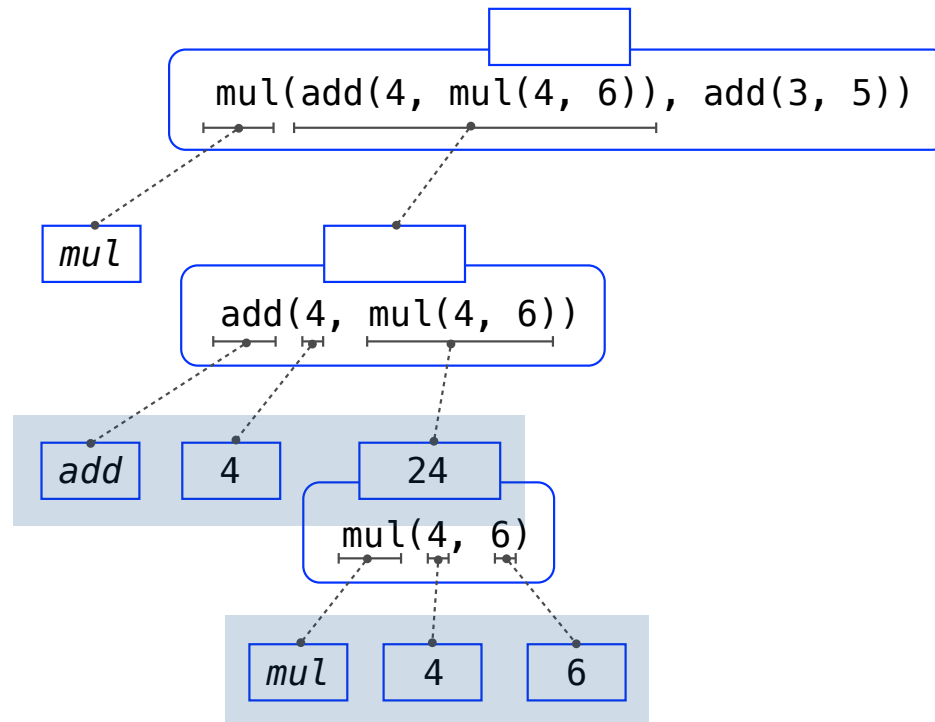
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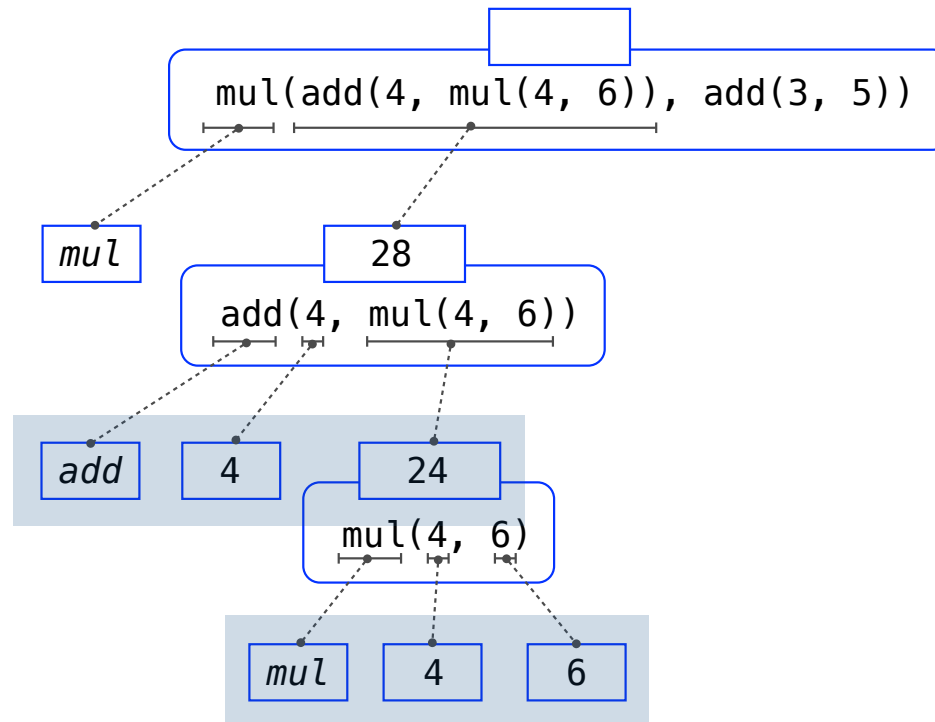


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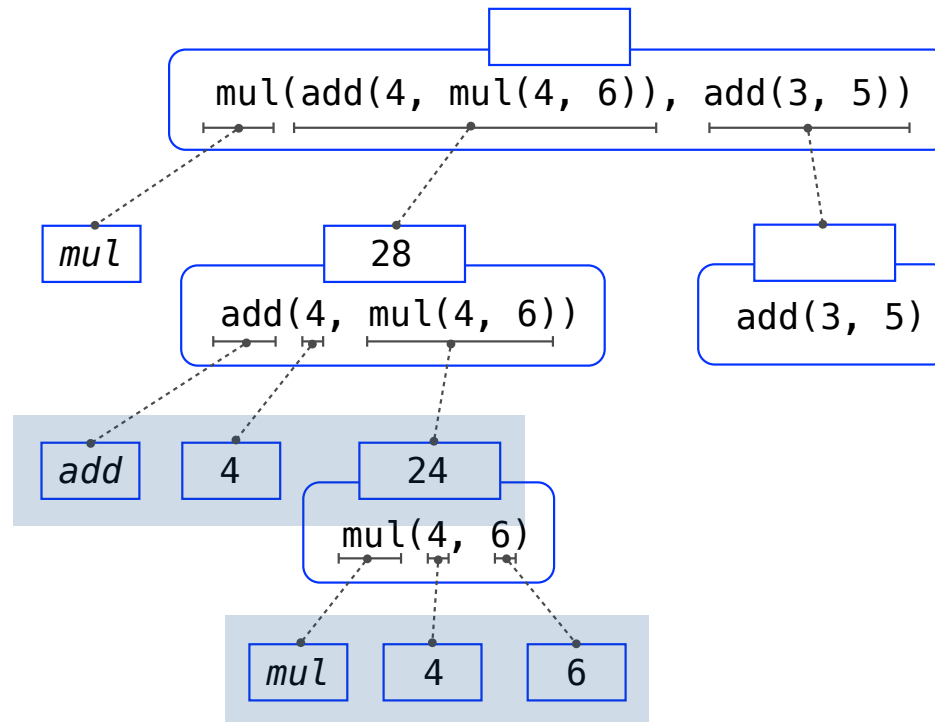




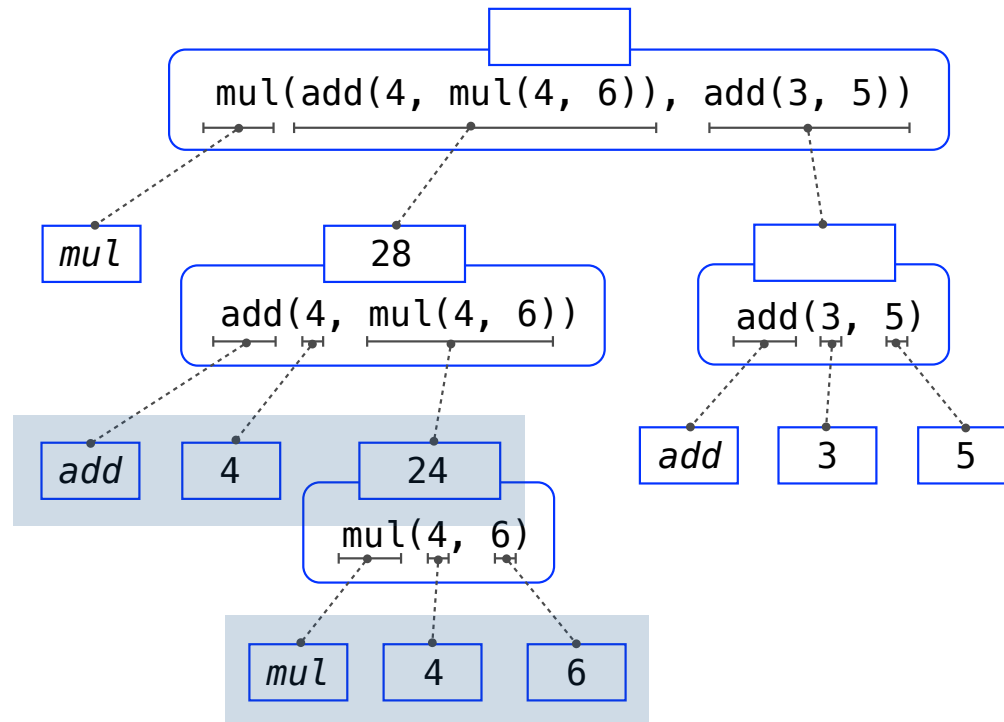
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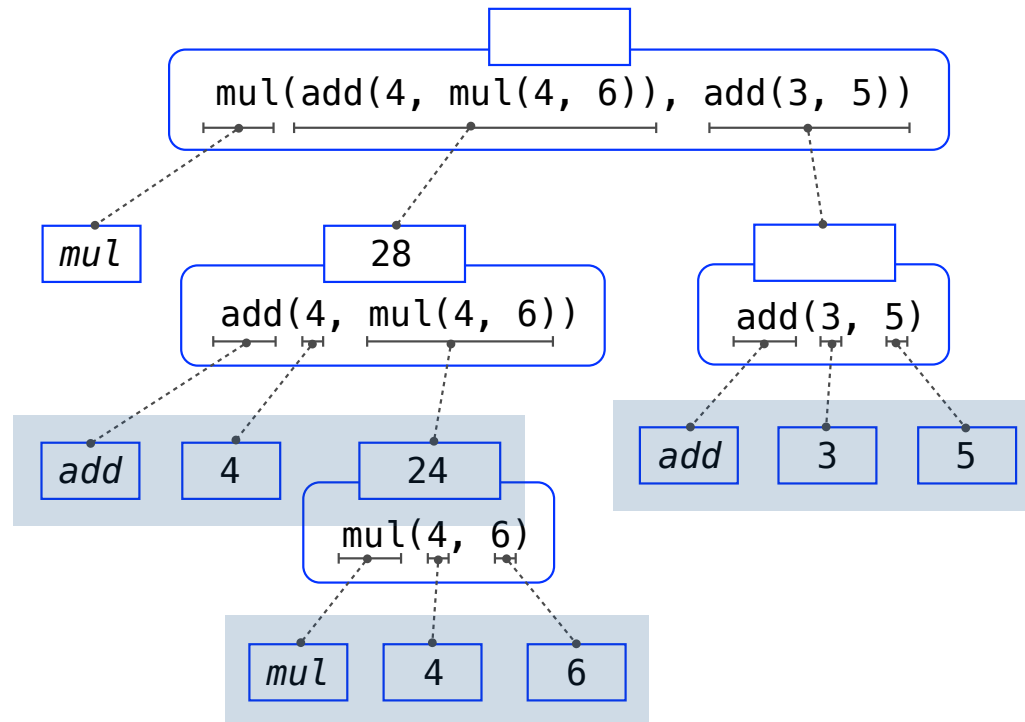
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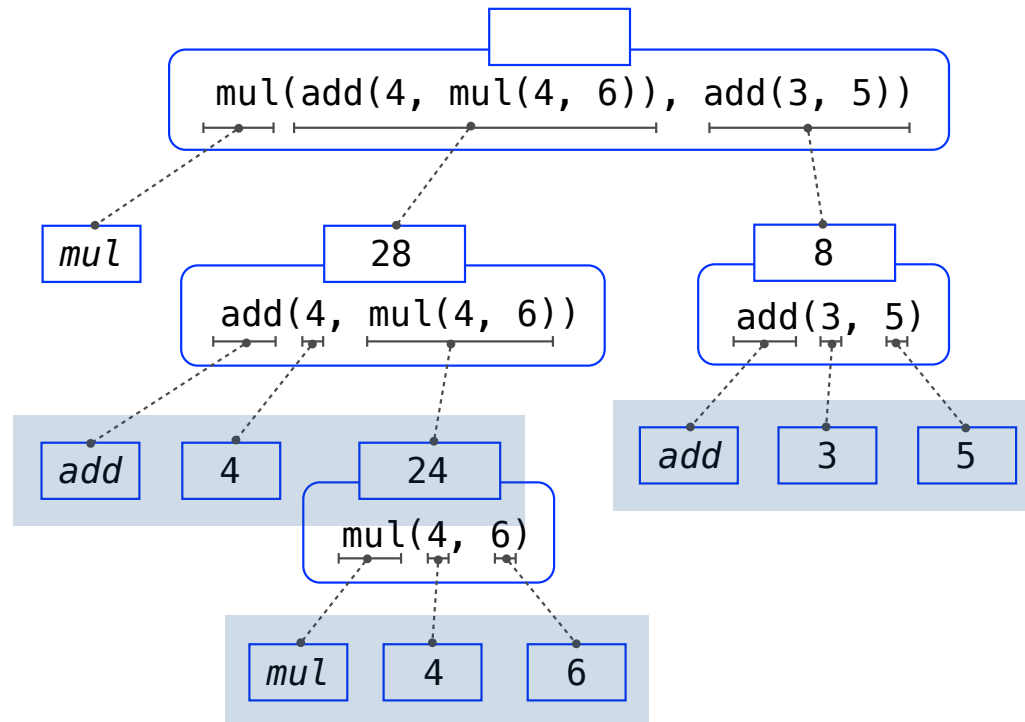
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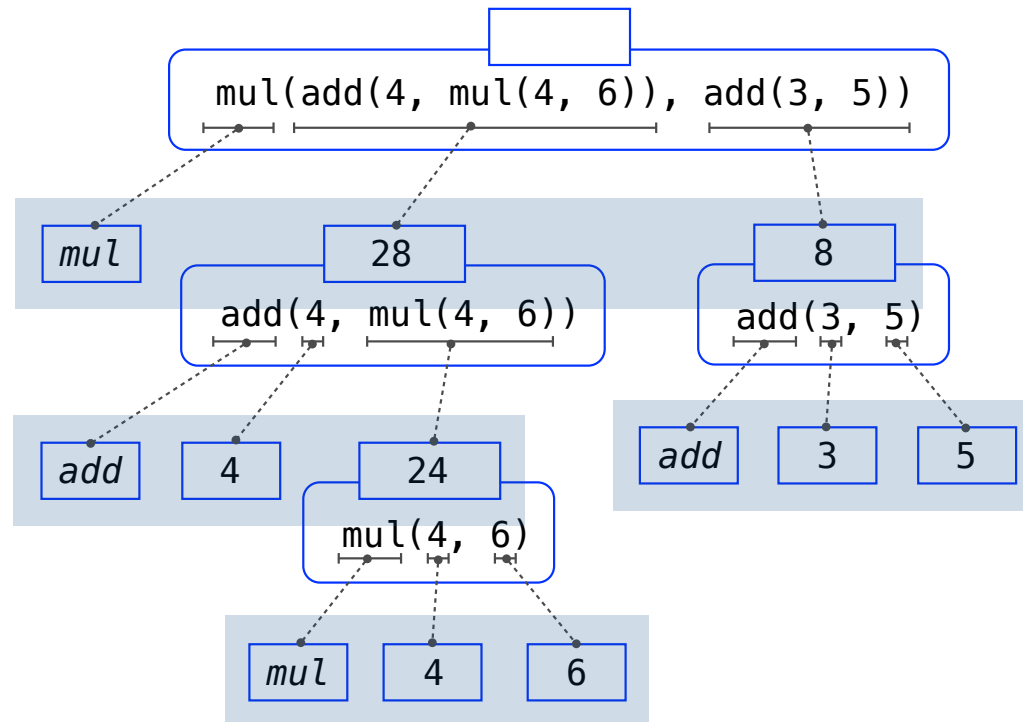
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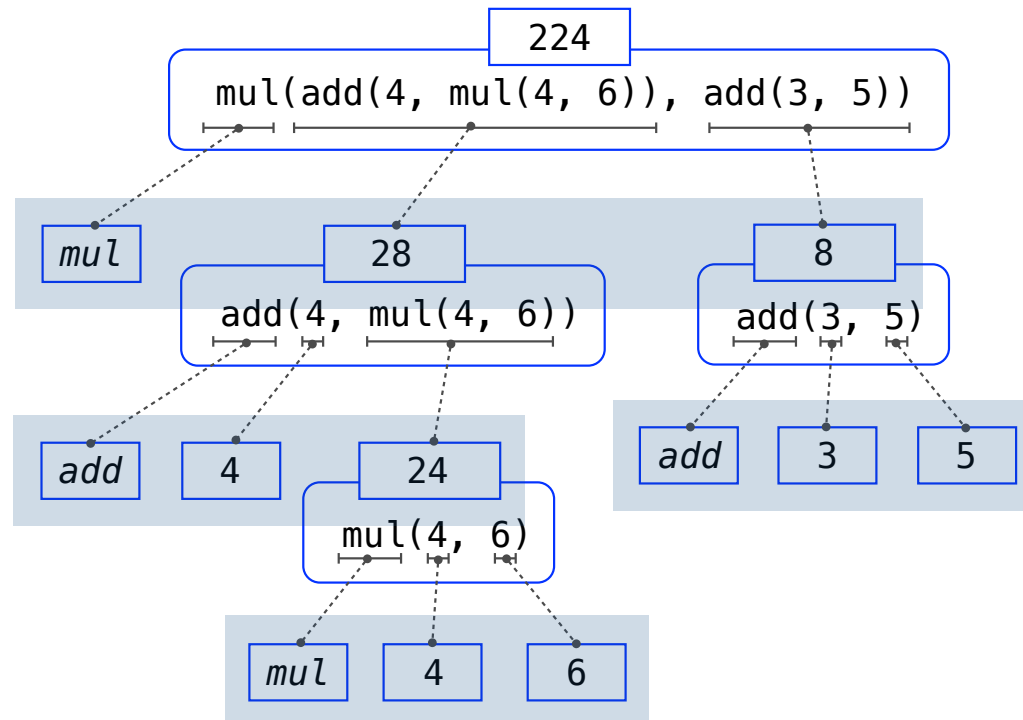
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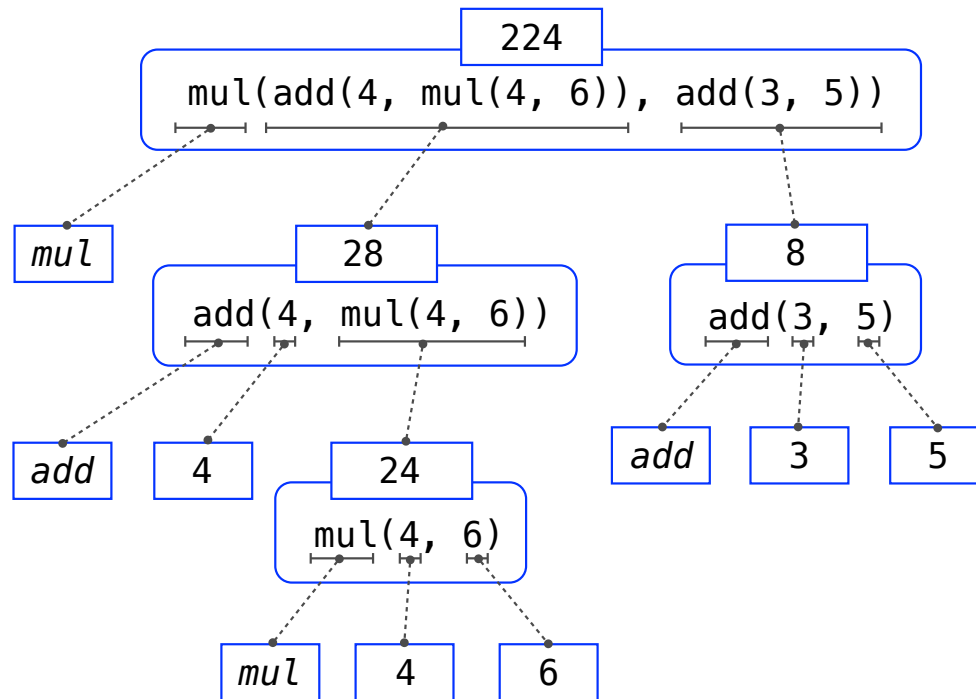
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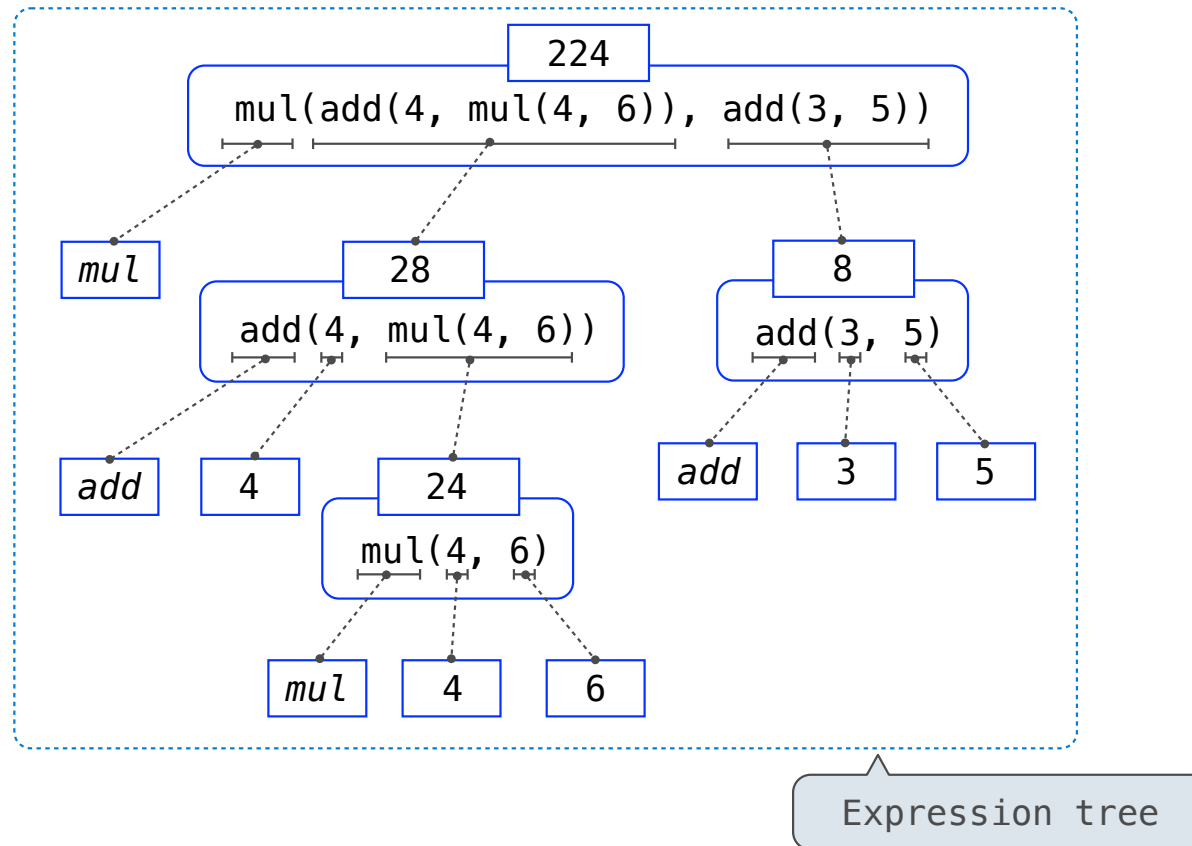


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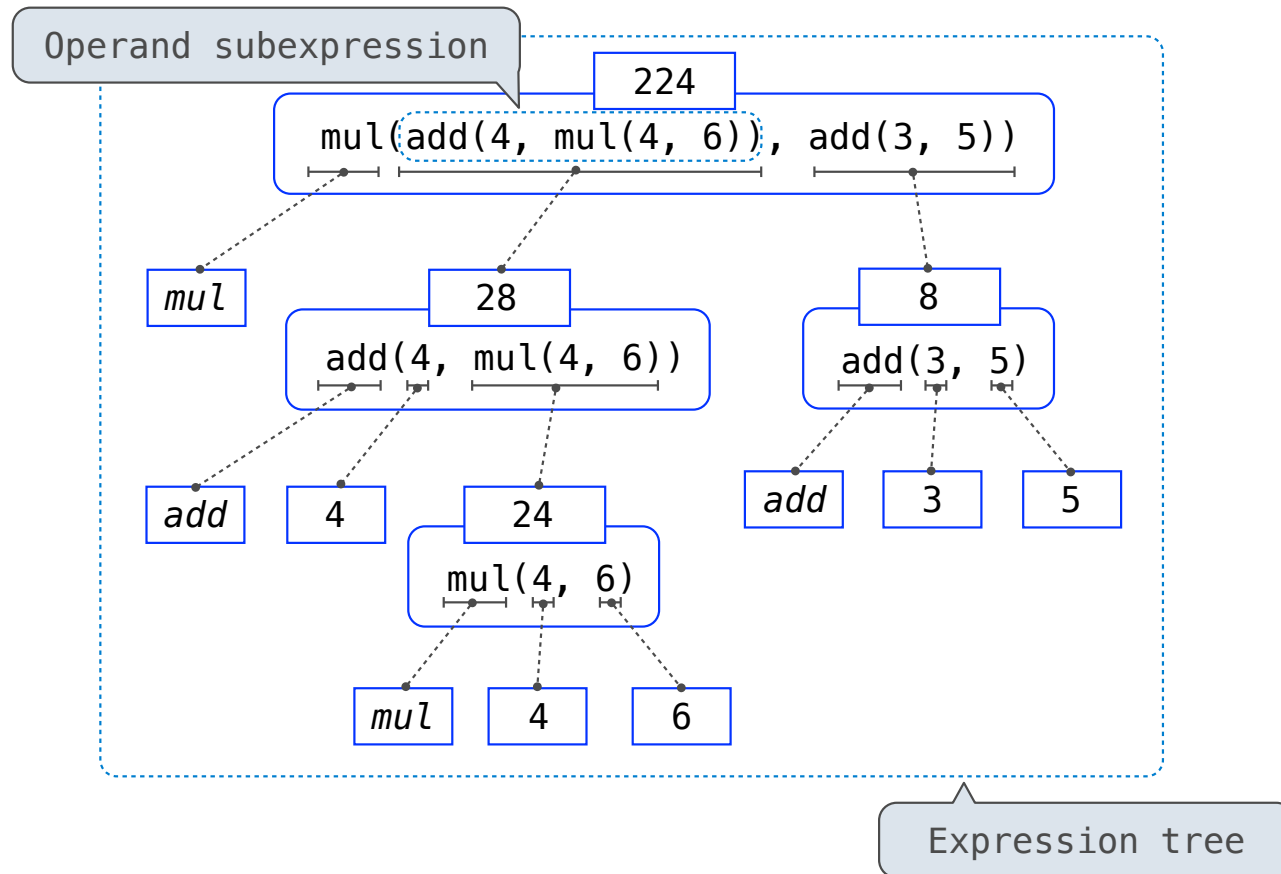




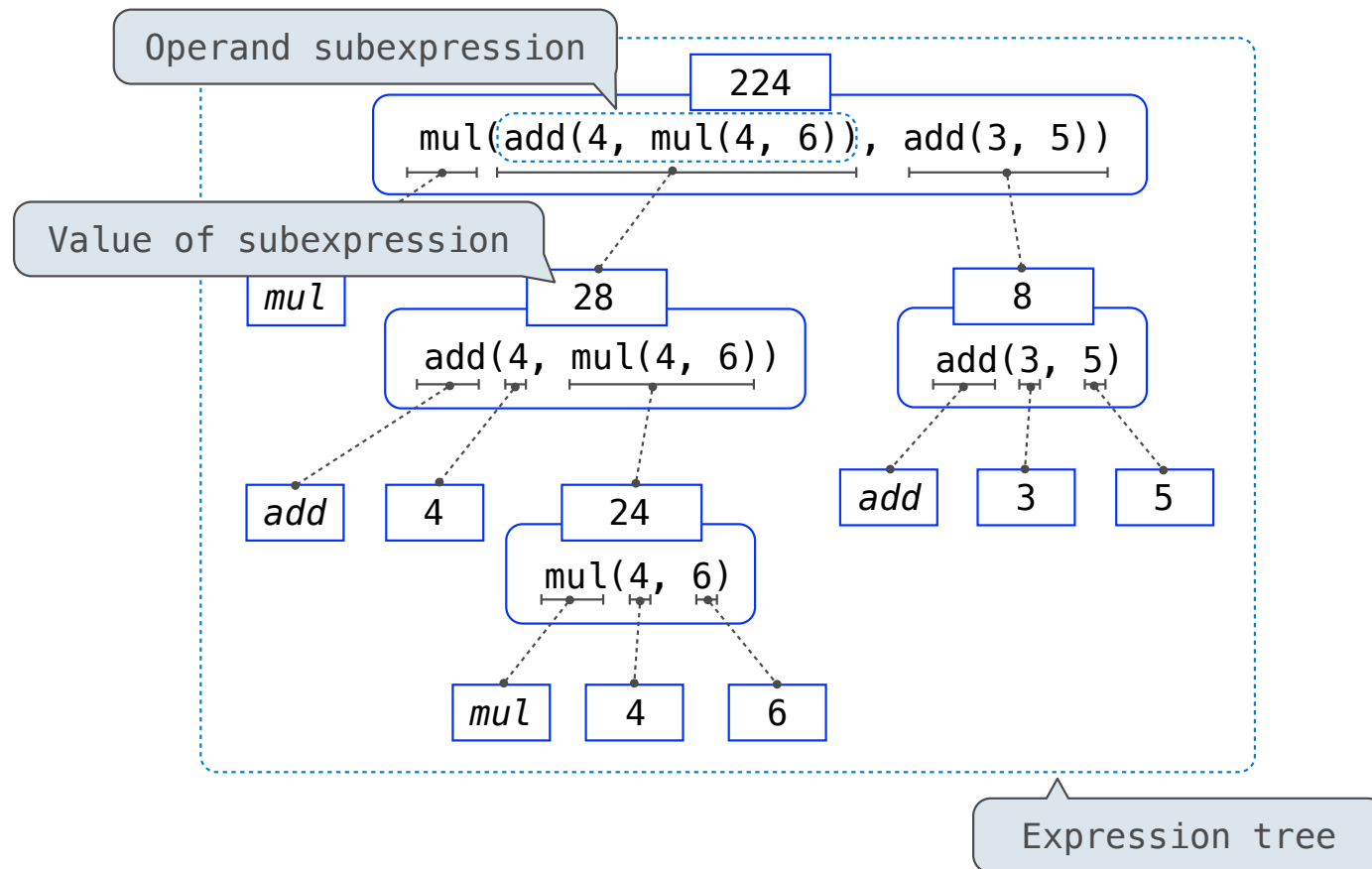
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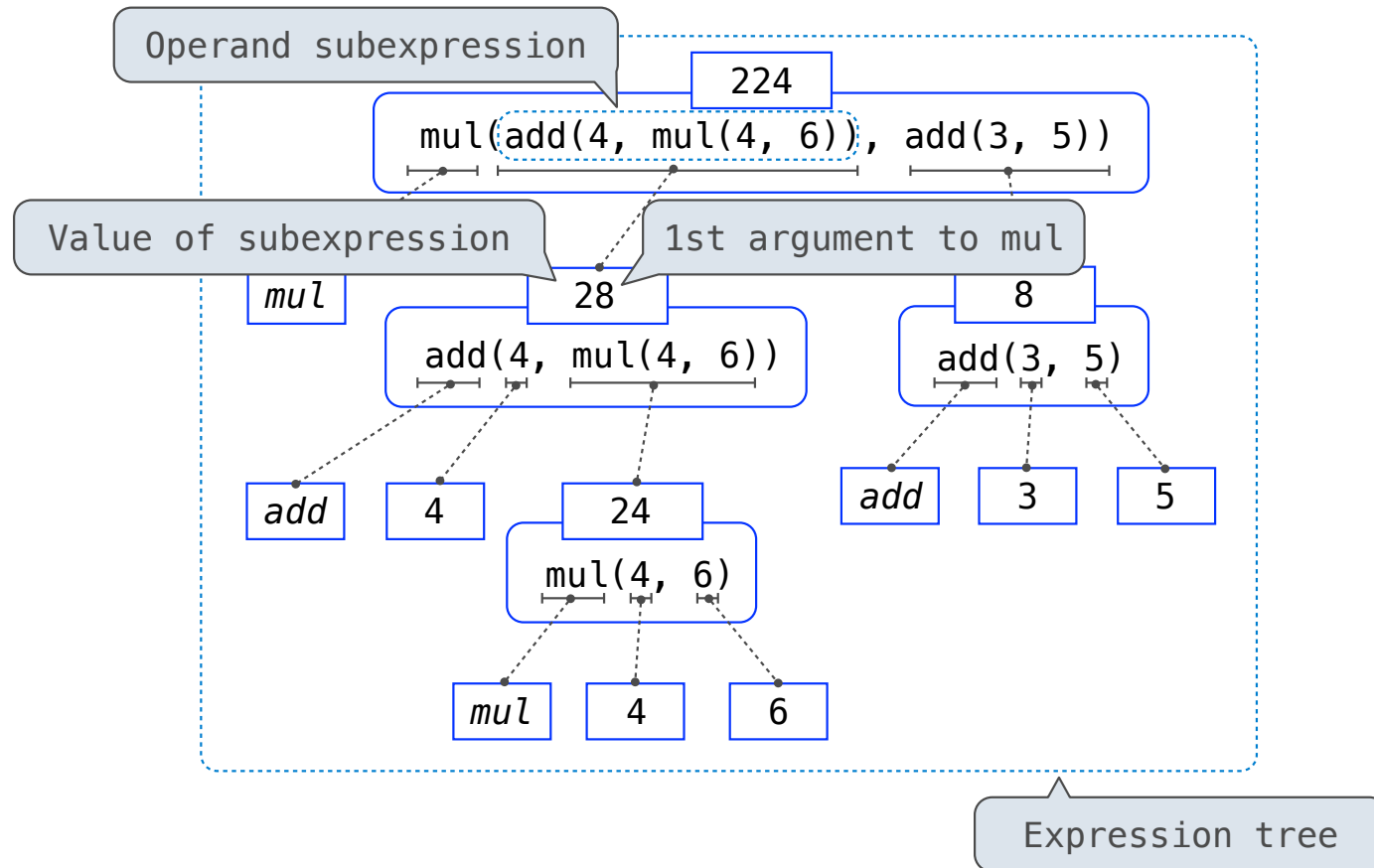
## Evaluating Nested Expressions



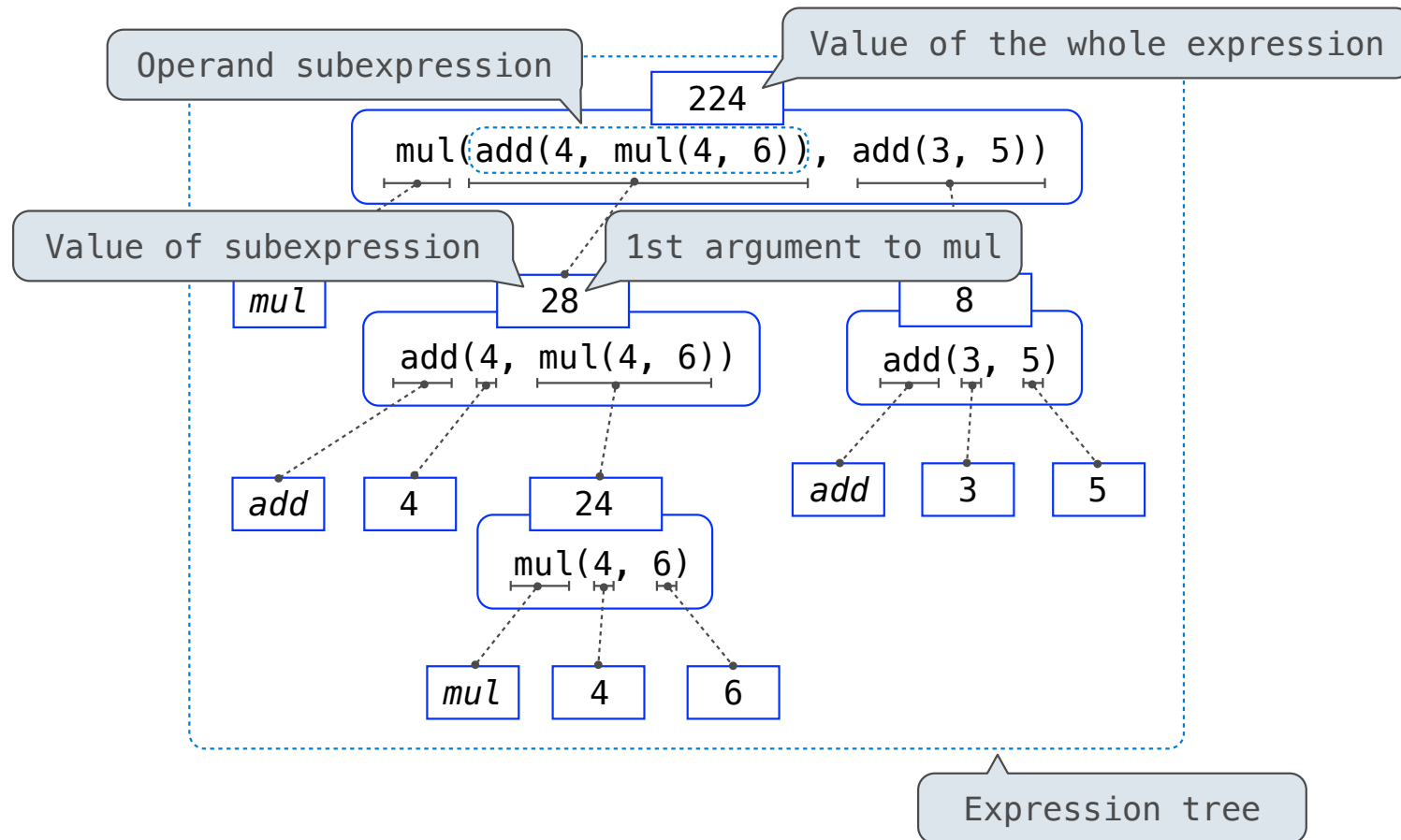
## Evaluating Nested Expressions



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## Evaluating Nested Expressions



# Names, Assignment, and User-Defined Functions

(Demo)

## Types of Expressions

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**Primitive expressions:**



## Types of Expressions

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**Primitive expressions:**

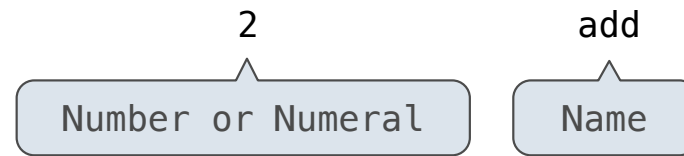
2

Number or Numeral

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**Call expressions:**

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**Call expressions:**

`max ( 2 , 3 )`

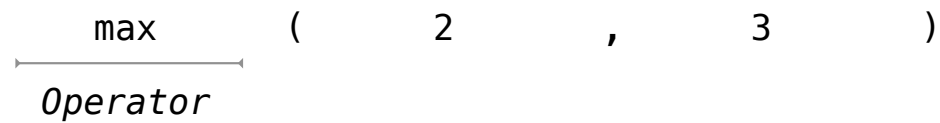
## Types of Expressions

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**Primitive expressions:**



**Call expressions:**



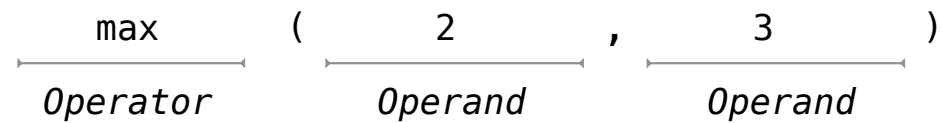
## Types of Expressions

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**Primitive expressions:**



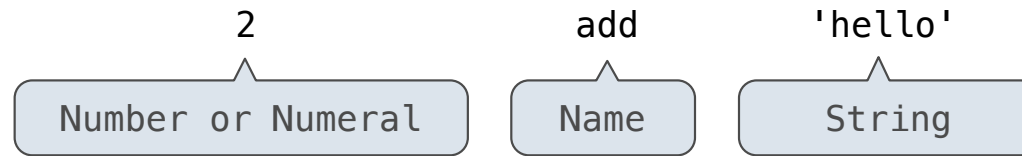
**Call expressions:**



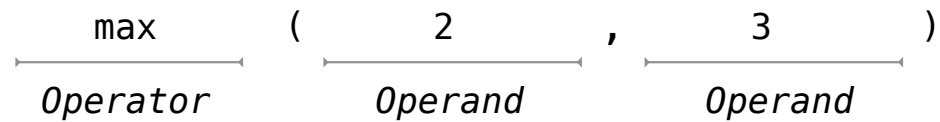
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**Primitive expressions:**



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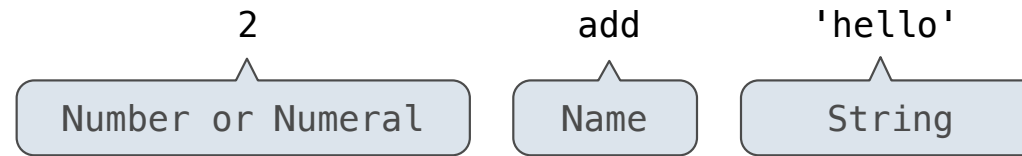
`max(min(pow(3, 5), -4), min(1, -2))`



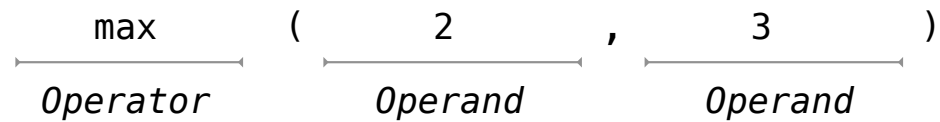
## Types of Expressions

---

### Primitive expressions:



### Call expressions:

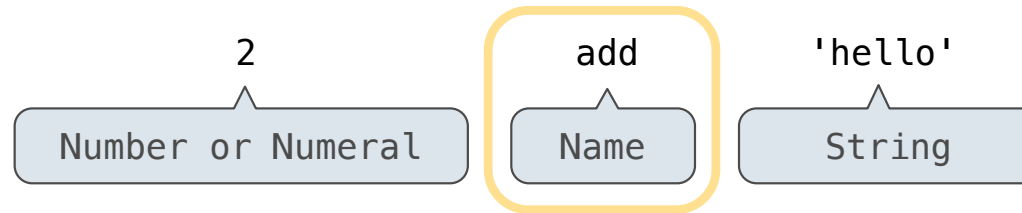


An operand can also  
be a call expression

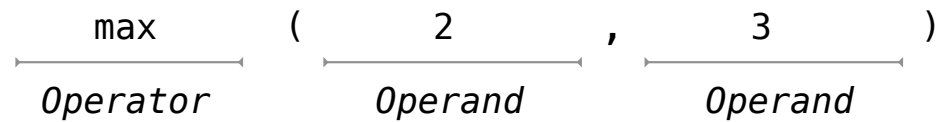
max(min(pow(3, 5), -4), min(1, -2))

## Types of Expressions

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```
>>> g, h = min, max
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>>> max = g
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```
>>> max(f(2, g(h(1, 5), 3)), 4)
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**???**

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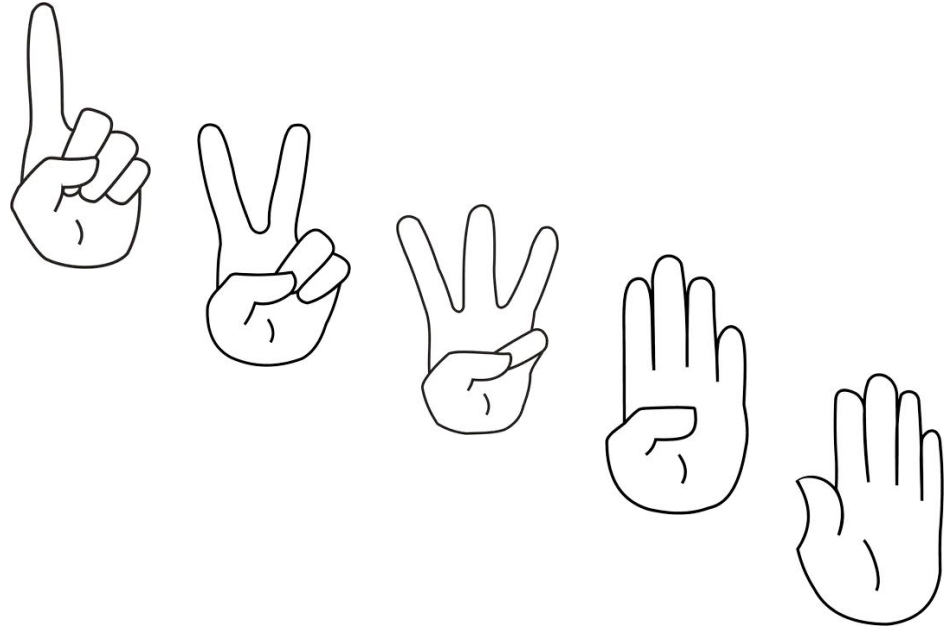
```
>>> f = max
```

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>>> max(f(2, g(h(1, 5), 3)), 4)
```

???



## Environment Diagrams

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Environment diagrams visualize the interpreter's process.

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→ 2 tau = 2 * pi
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Global frame  
  pi | 3.1416
```

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**Code (left):**

```
Global frame
  pi 3.1416
```

**Frames (right):**



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Global frame	
pi	3.1416

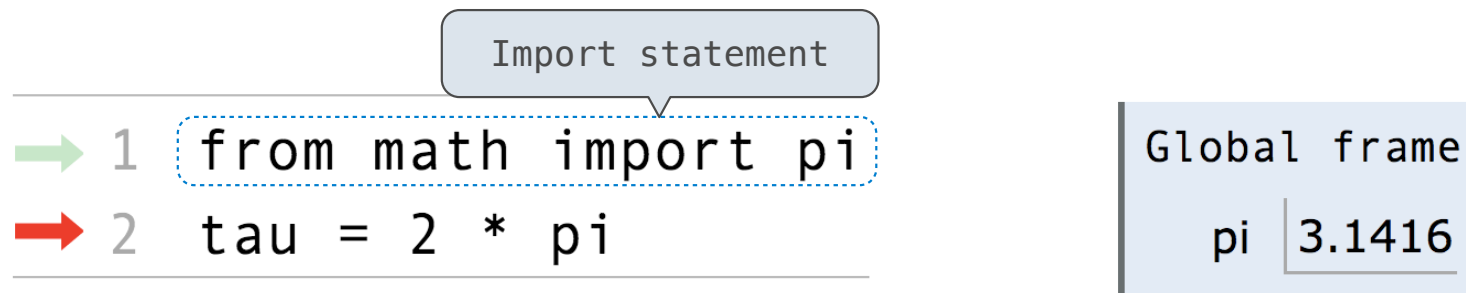
**Code (left):**

Statements and expressions

**Frames (right):**

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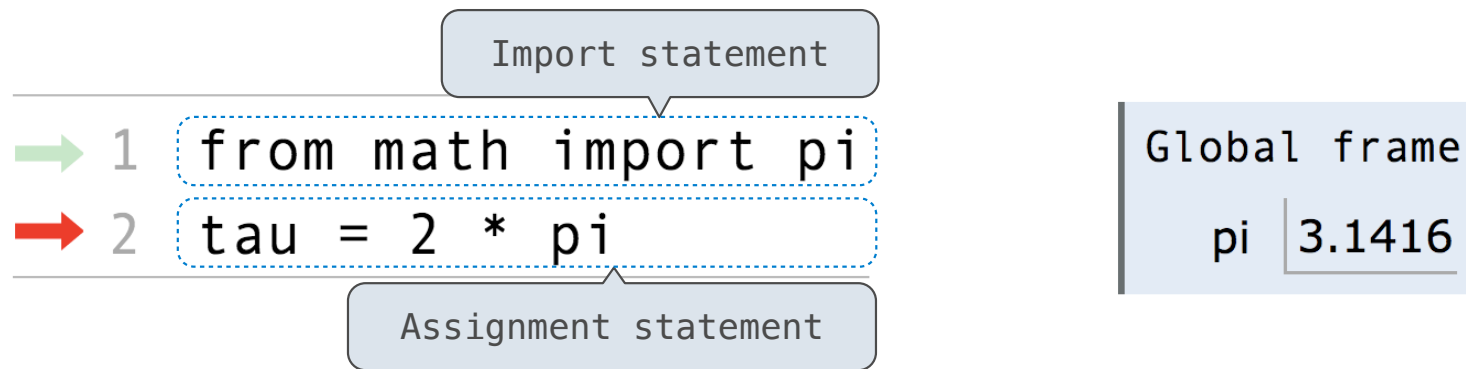
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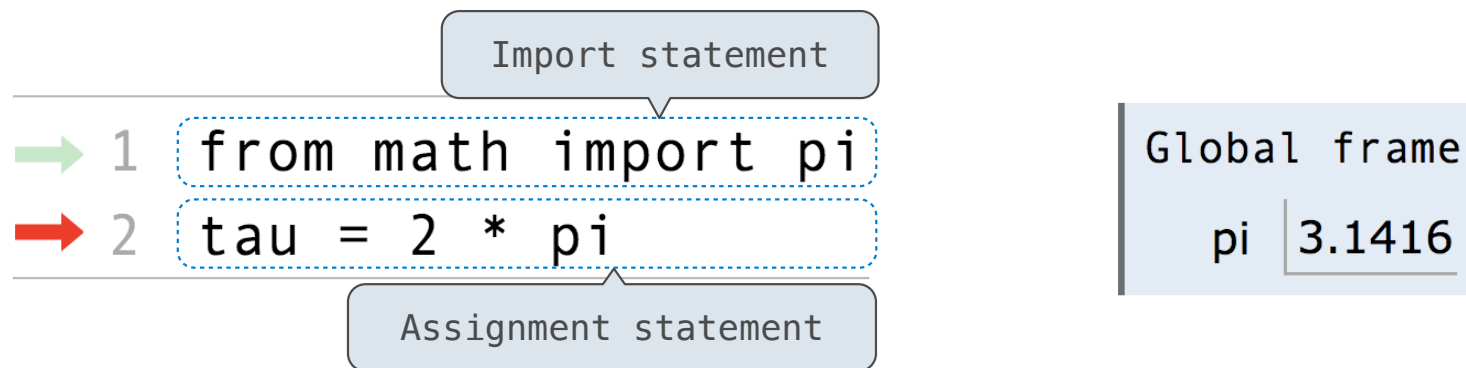
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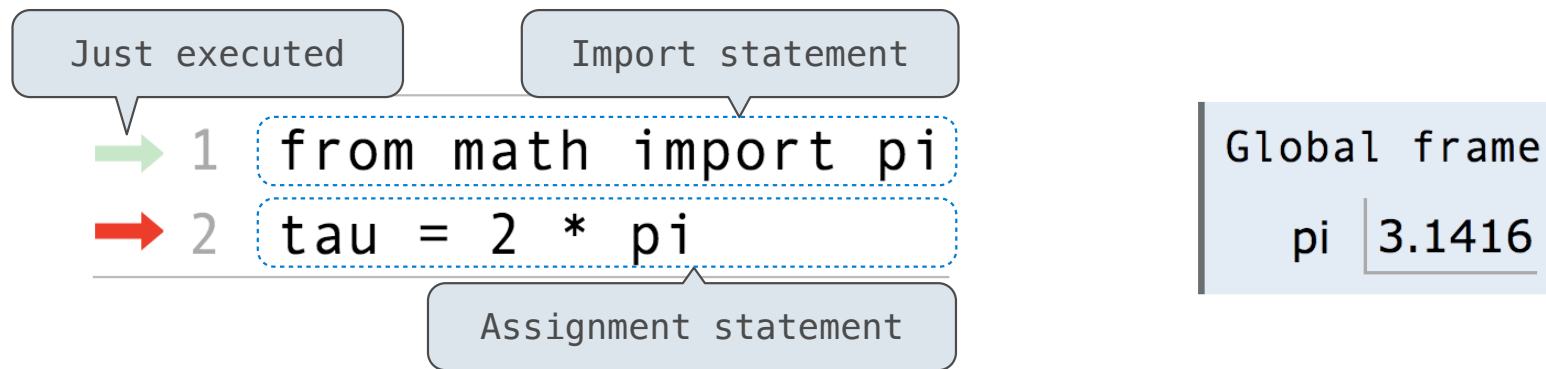
Statements and expressions

Arrows indicate evaluation order

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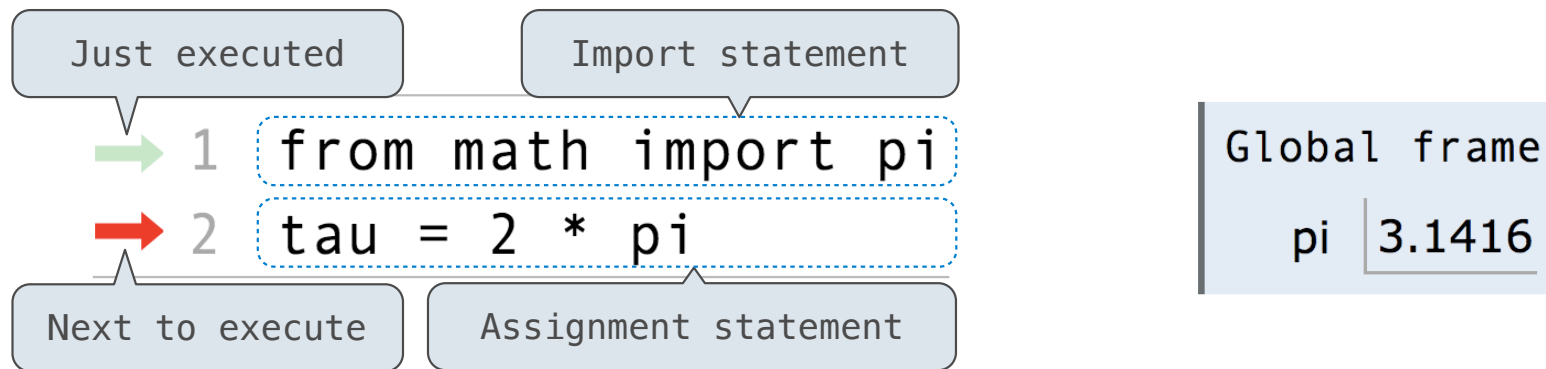
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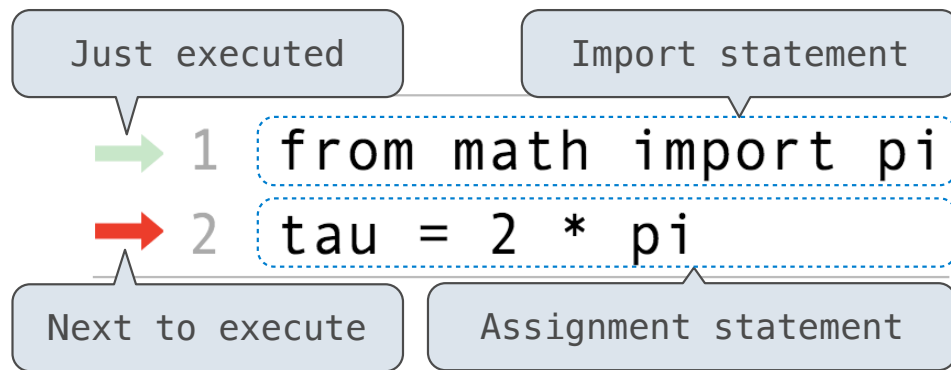
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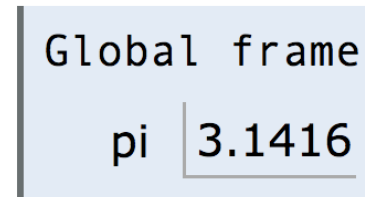
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Statements and expressions

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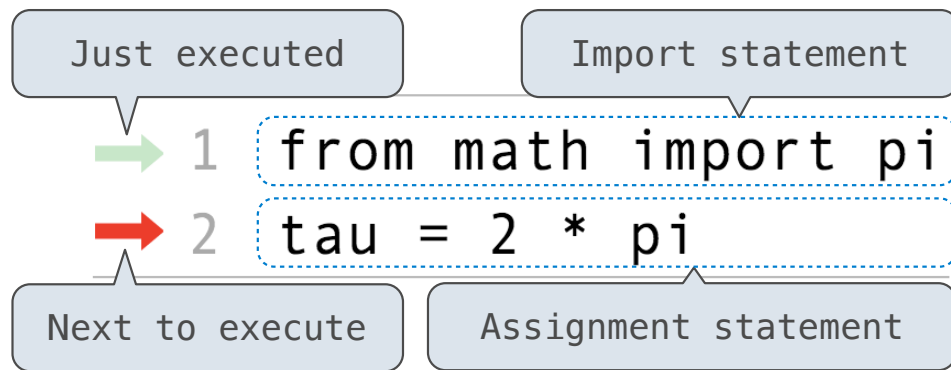


### Frames (right):

Each name is bound to a value

## Environment Diagrams

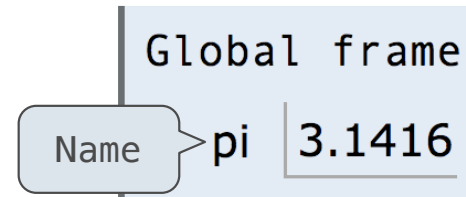
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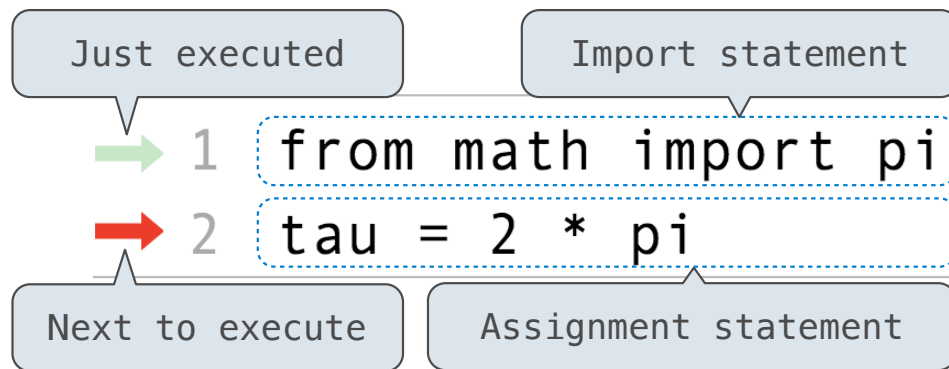
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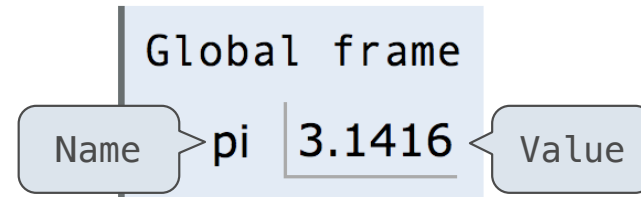
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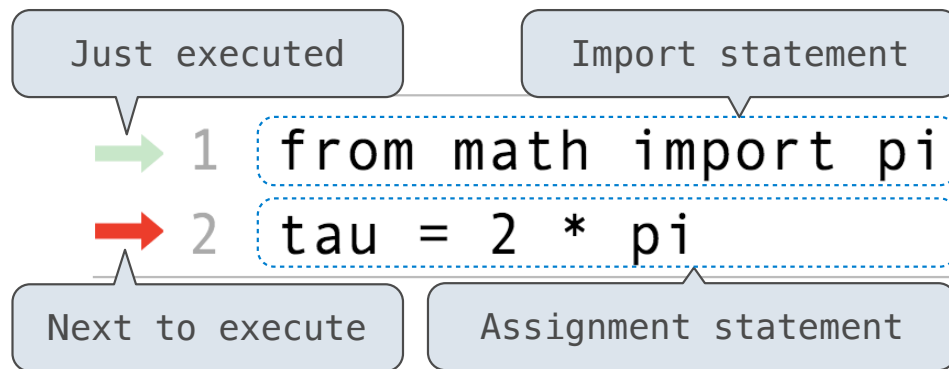


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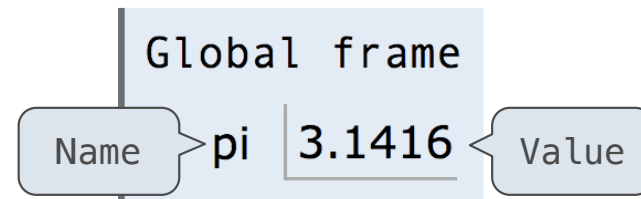
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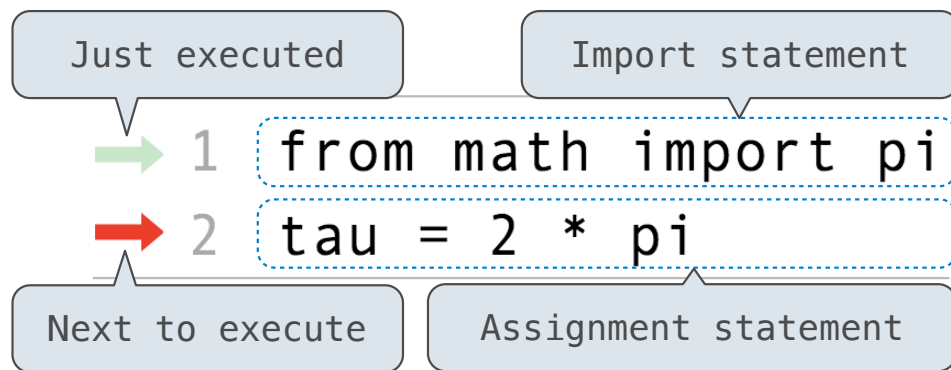
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Within a frame, a name cannot be repeated

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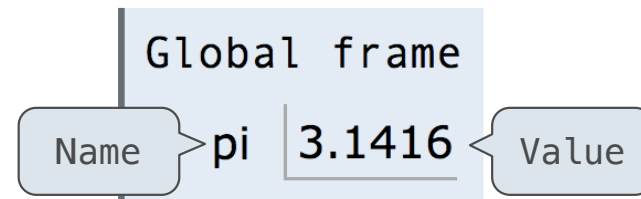
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### Code (left):

Statements and expressions

Arrows indicate evaluation order



### Frames (right):

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(Demo)

## Assignment Statements

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---

---

```
1 a = 1
→ 2 b = 2
→ 3 b, a = a + b, b
```

---

## Assignment Statements

```

1  a = 1
→ 2  b = 2
→ 3  b, a = a + b, b

```

Global frame

a	1
---	---

b	2
---	---

## Assignment Statements

Just executed

```

1  a = 1
>→ 2  b = 2
→ 3  b, a = a + b, b

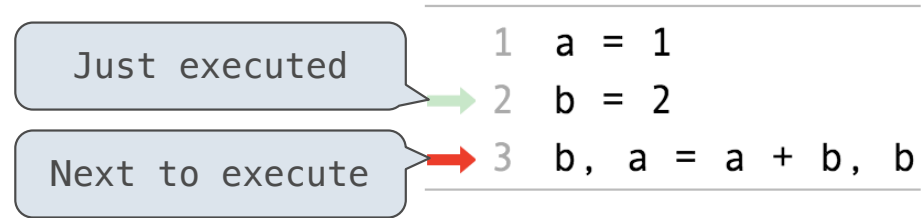
```

Global frame

a	1
---	---

b 2

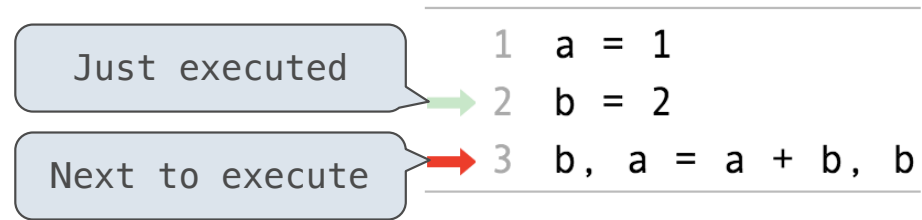
## Assignment Statements



```
Global frame
a | 1
b | 2
```



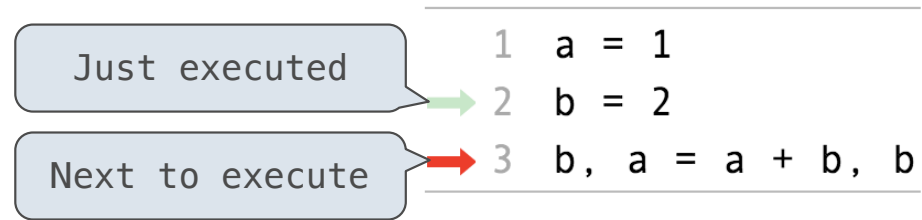
## Assignment Statements



```
Global frame
a | 1
b | 2
```

**Execution rule for assignment statements:**

## Assignment Statements



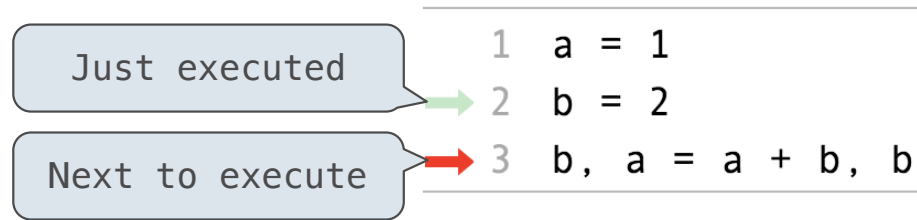
Global frame

a	1
b	2

### Execution rule for assignment statements:

1. Evaluate all expressions to the right of = from left to right.

## Assignment Statements

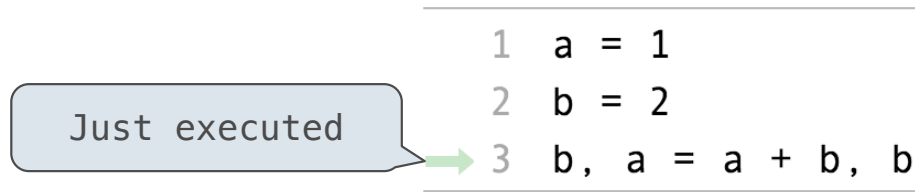
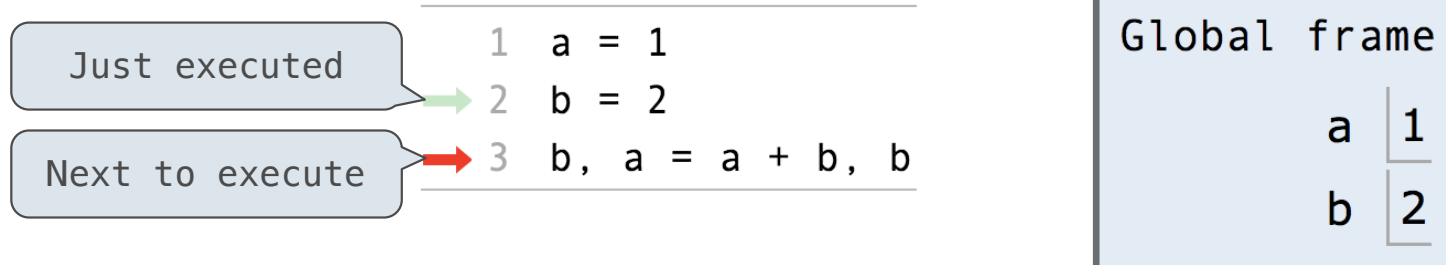


```
Global frame
a | 1
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```

### Execution rule for assignment statements:

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2. Bind all names to the left of = to those resulting values in the current frame.

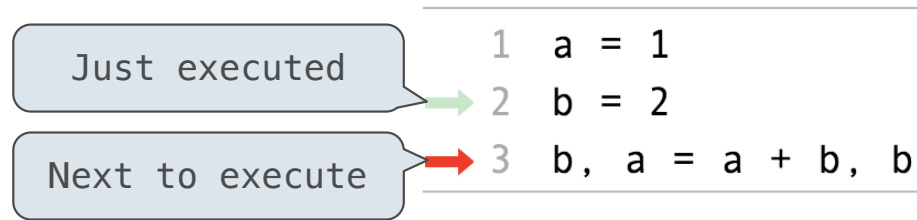
## Assignment Statements



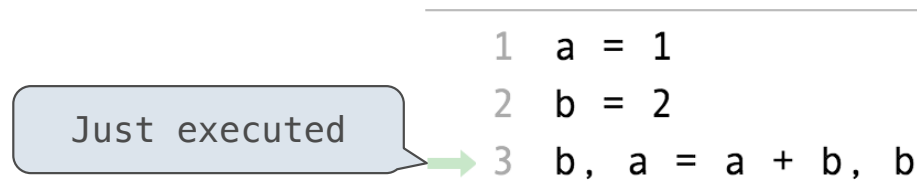
### Execution rule for assignment statements:

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## Assignment Statements



```
Global frame
a | 1
b | 2
```



```
Global frame
a | 2
b | 3
```

### Execution rule for assignment statements:

1. Evaluate all expressions to the right of = from left to right.
2. Bind all names to the left of = to those resulting values in the current frame.

## Discussion Question 1 Solution

---

(Demo)

## Discussion Question 1 Solution

( Demo )

```

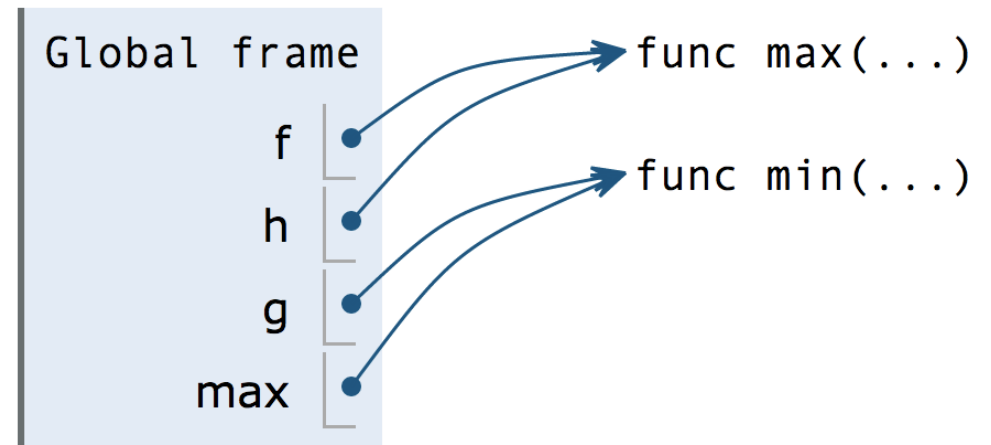
1  f = min
2  f = max
3  g, h = min, max
→ 4  max = g
→ 5  max(f(2, g(h(1, 5), 3)), 4)

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( Demo )

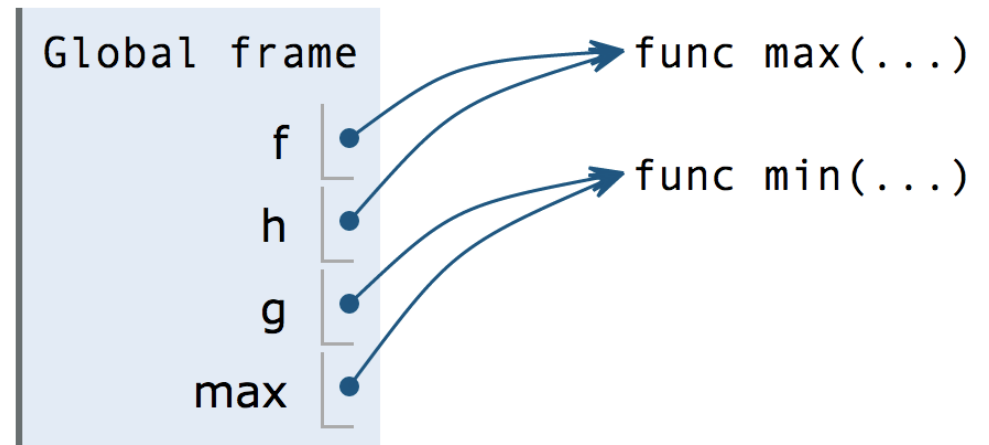




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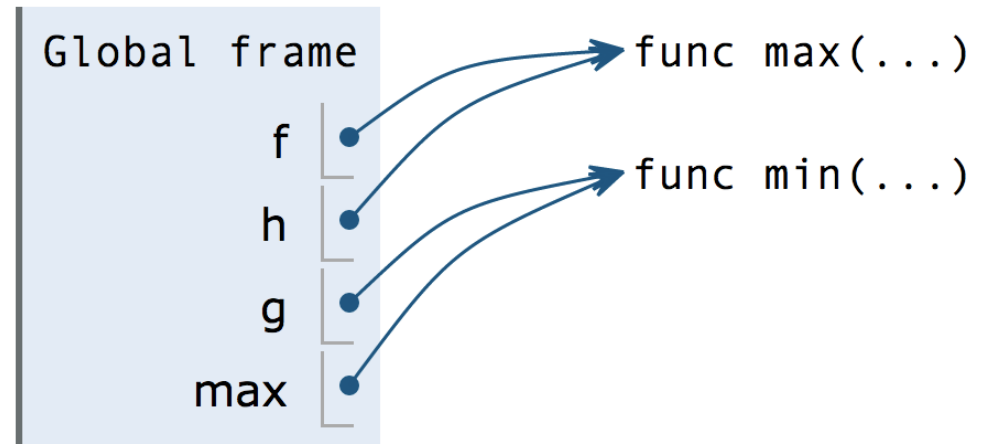
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```

```
func min(...)
```

( Demo )

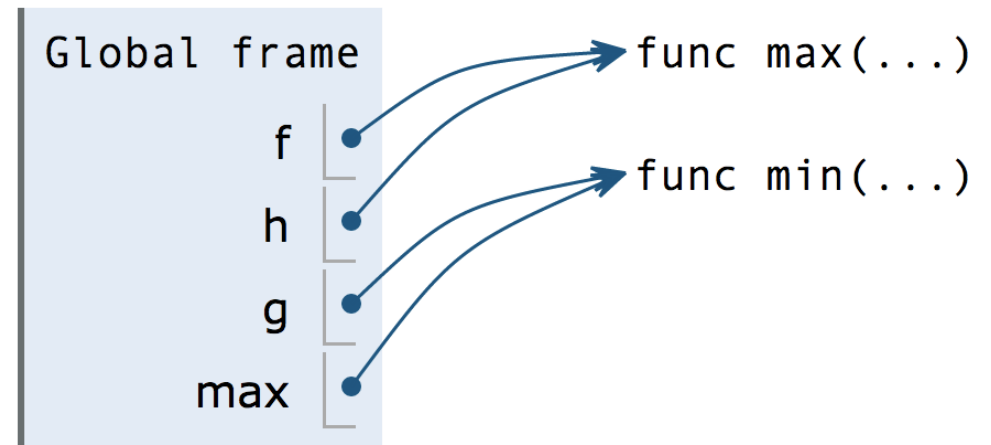


## Discussion Question 1 Solution

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1 f = min
2 f = max
3 g, h = min, max
4 max = g
5 max(f(2, g(h(1, 5), 3)), 4)
```

Diagram illustrating the evaluation of the expression `max(f(2, g(h(1, 5), 3)), 4)`. The code snippets show the definitions of `f`, `g`, `h`, and `max`. The diagram highlights the evaluation flow: the innermost expression `h(1, 5)` is evaluated first, then `g(h(1, 5), 3)`, then `f(2, g(h(1, 5), 3))`, and finally the outer `max` function. Blue boxes and arrows indicate the sequence of evaluations and the return values being passed between function calls.

( Demo )



## Discussion Question 1 Solution

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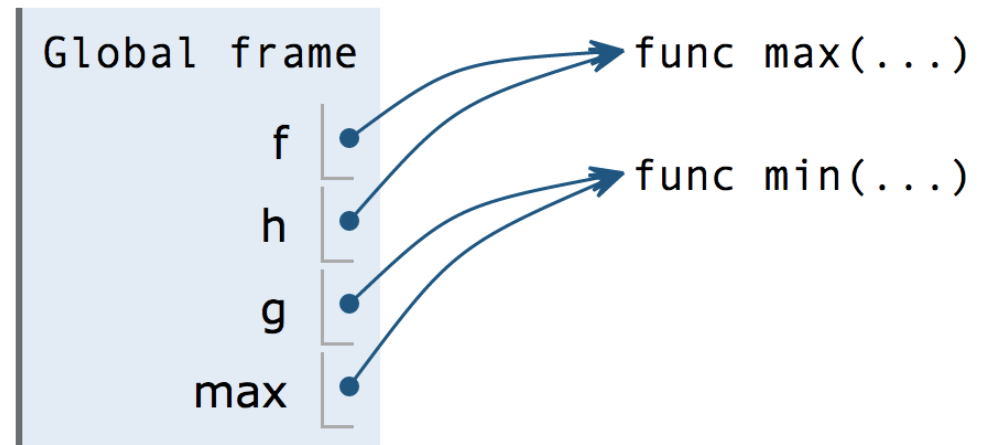
```

```
func min(...)
```

$$f(2, \underbrace{g(h(1, 5), 3)})$$

func max(...)	2
---------------	---

( Demo )

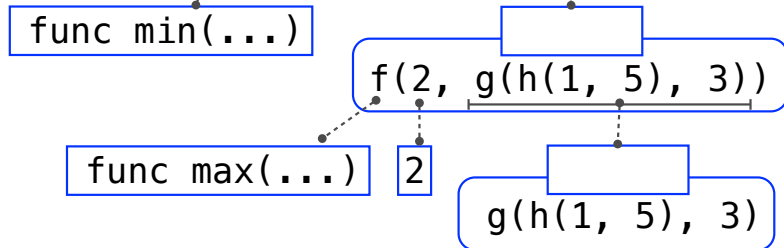


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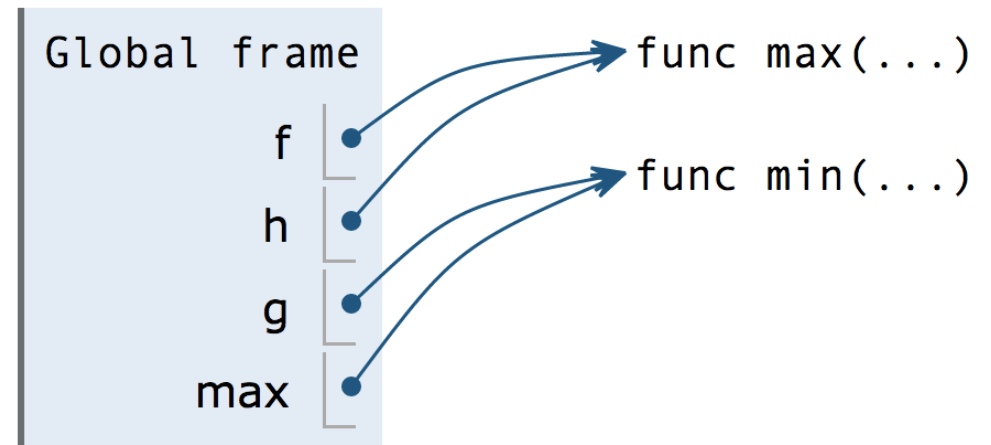
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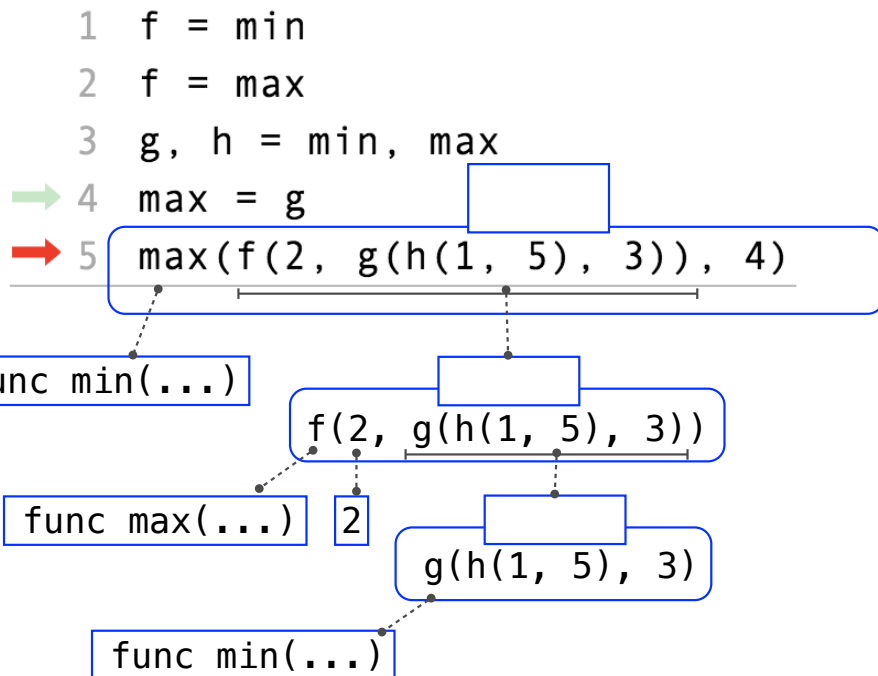
```



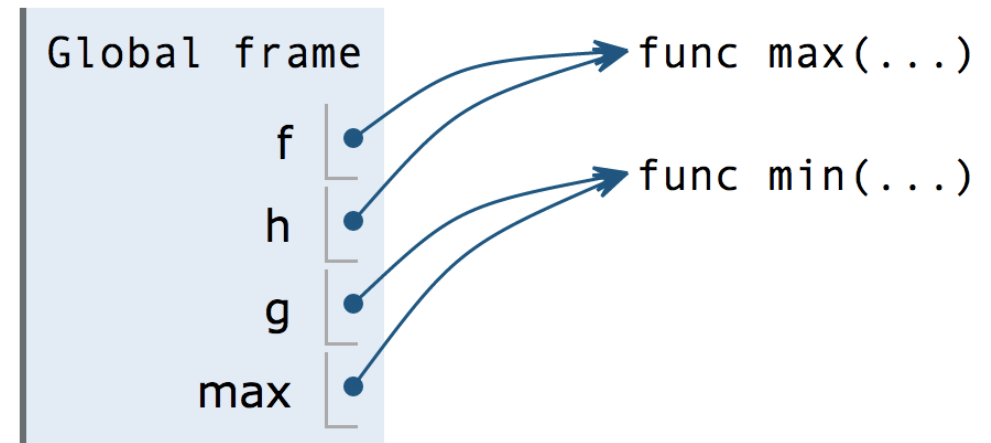
( Demo )



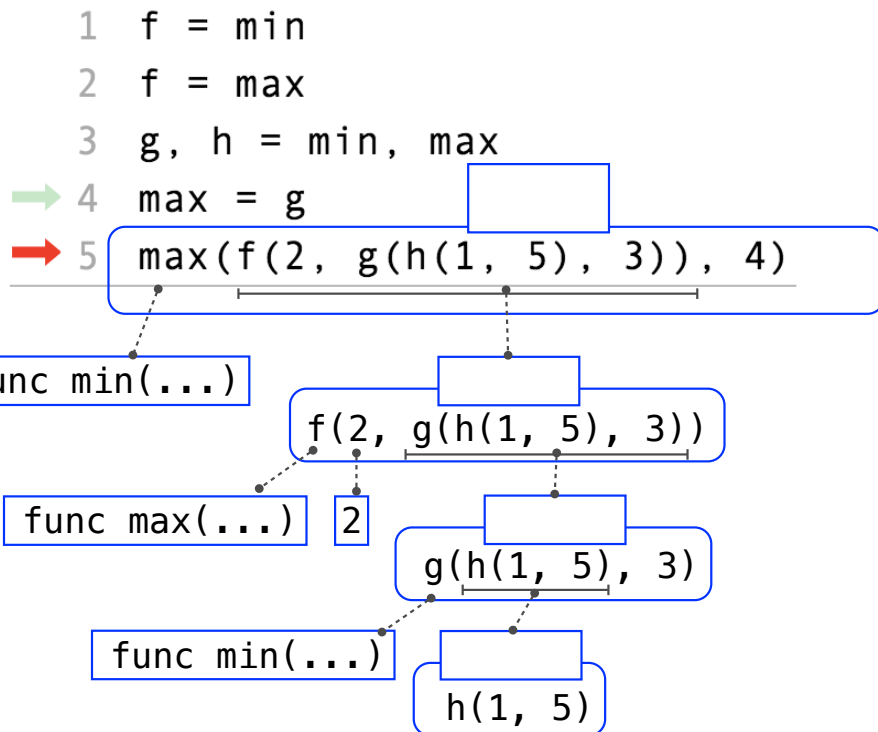
## Discussion Question 1 Solution



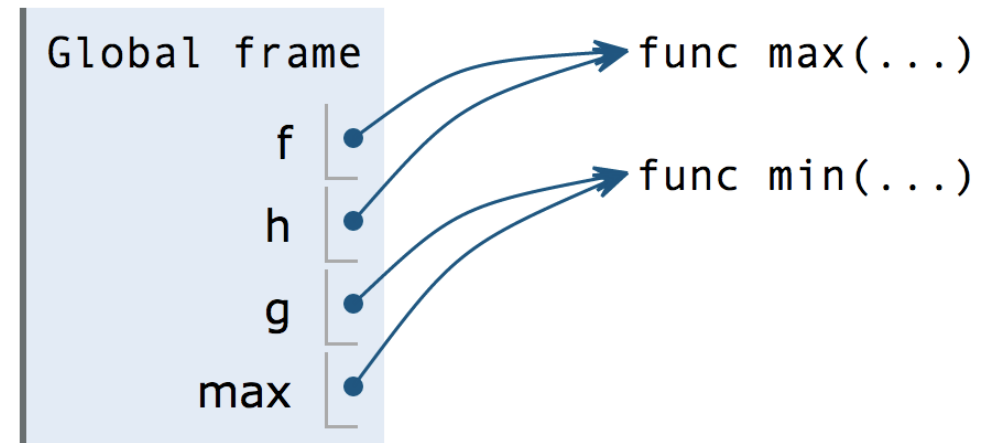
(Demo)



## Discussion Question 1 Solution



(Demo)



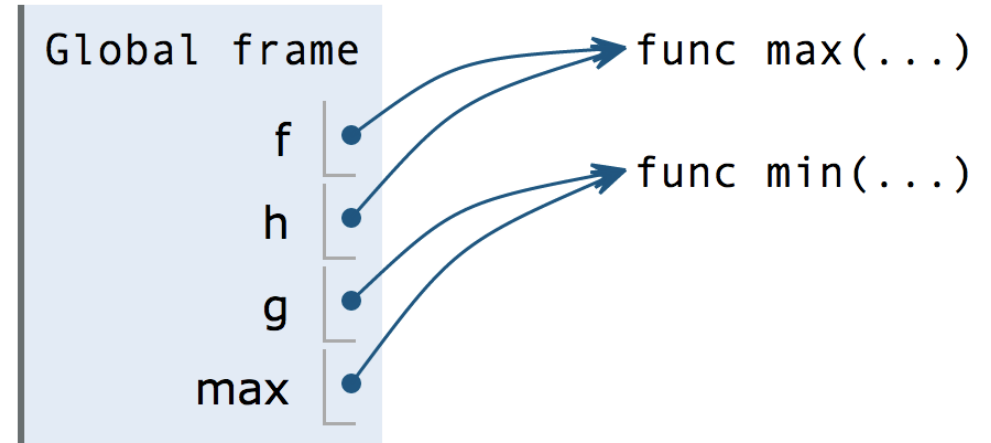
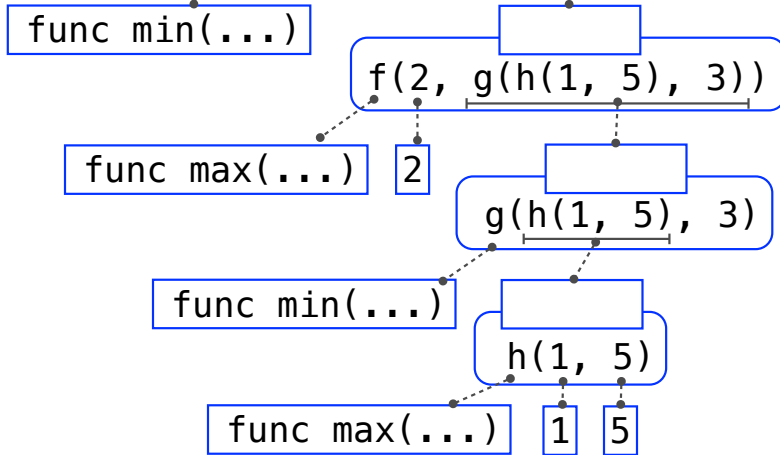
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( Demo )

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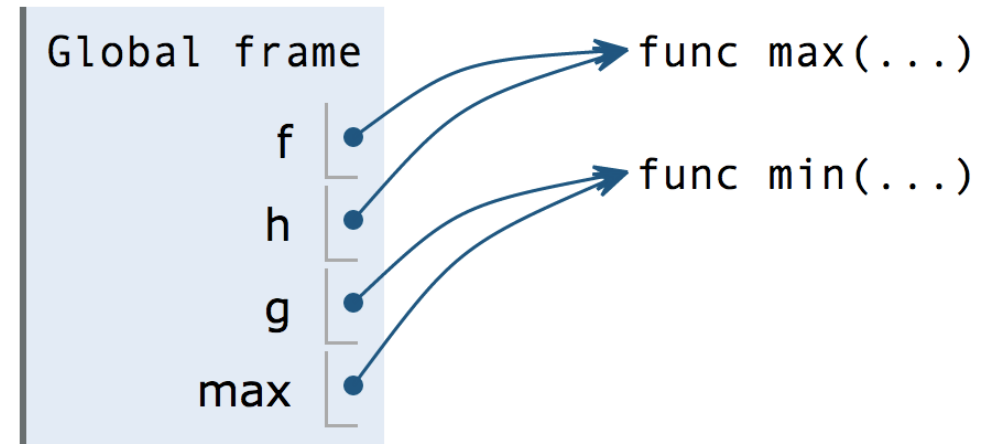
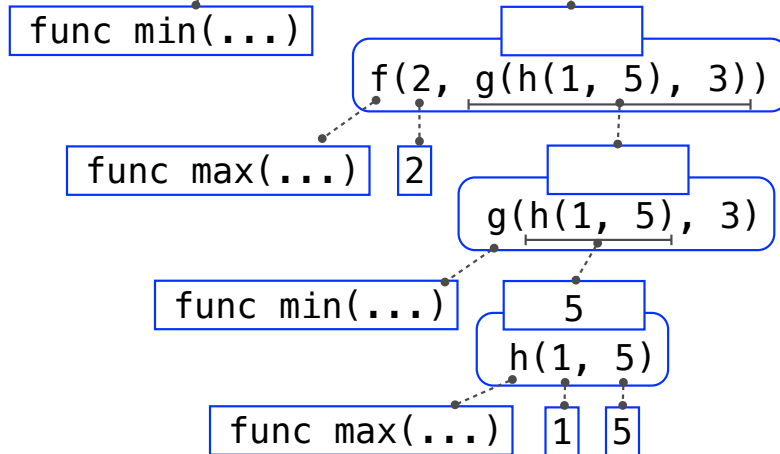
## Discussion Question 1 Solution

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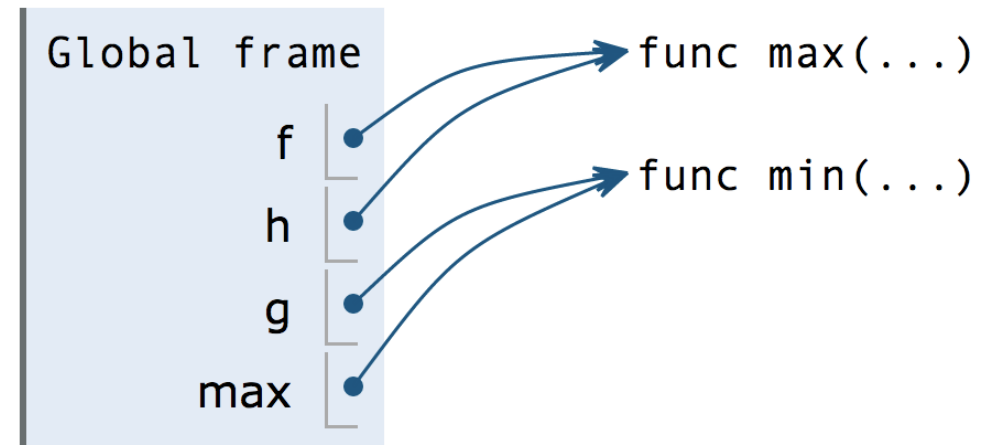
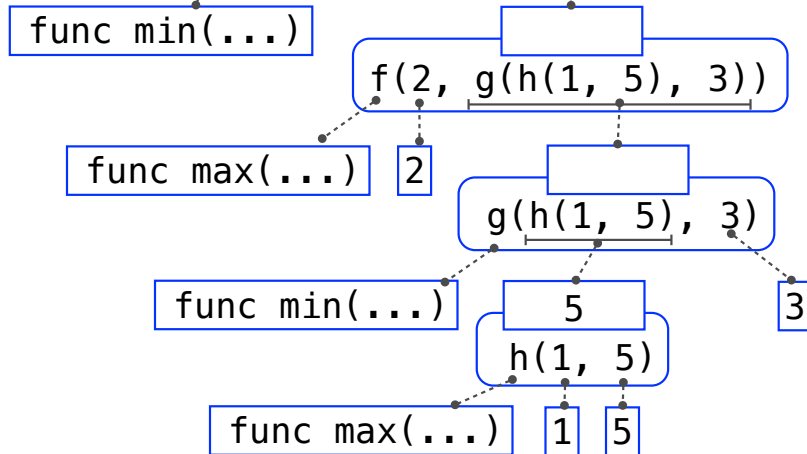
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( Demo )

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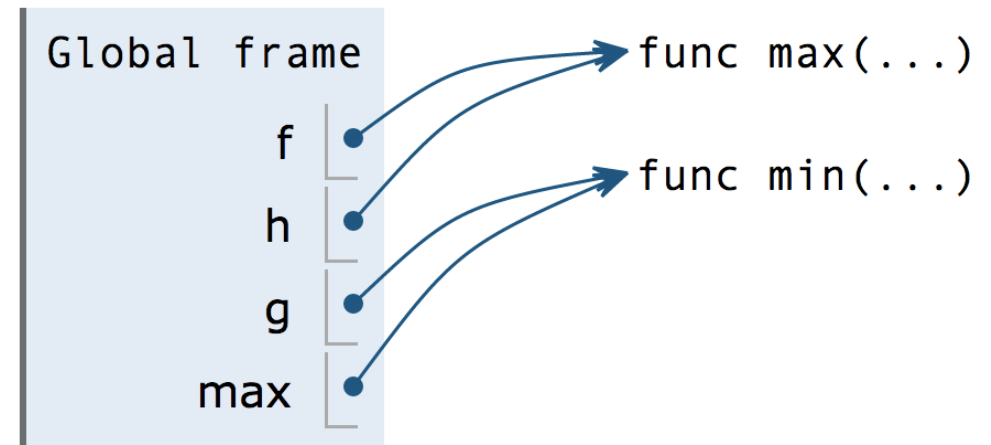
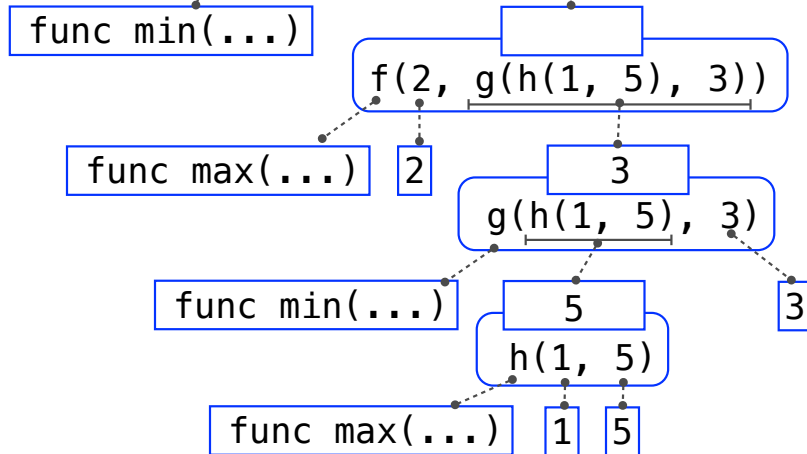
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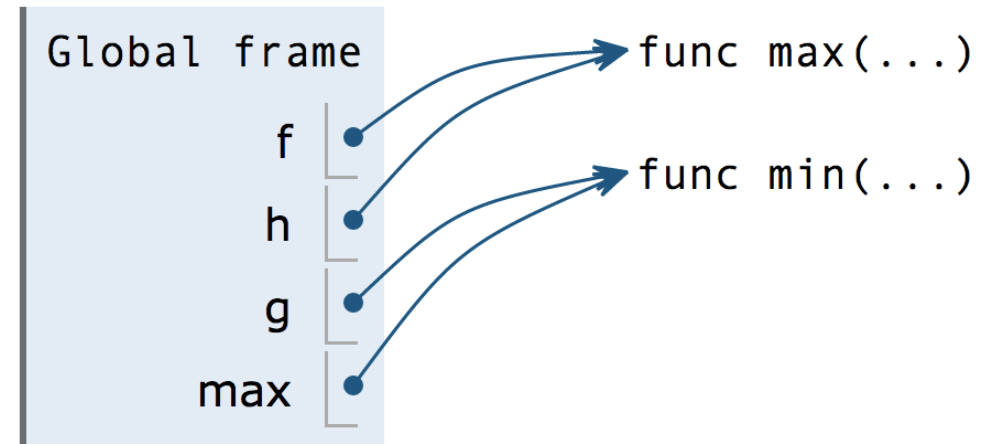
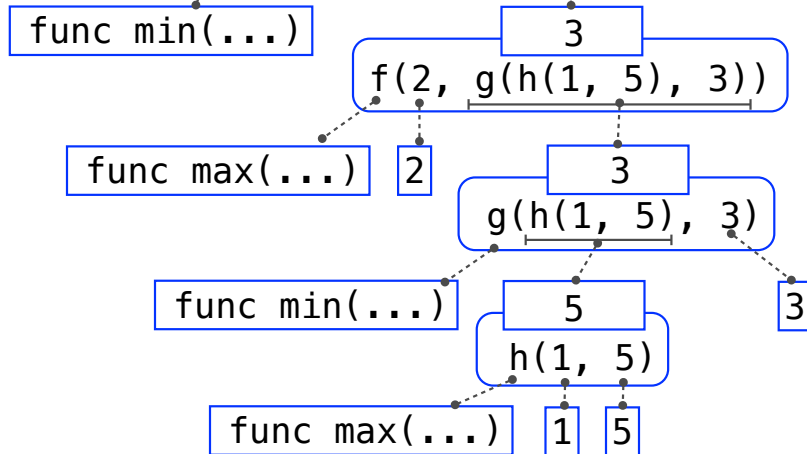
## Discussion Question 1 Solution

( Demo )

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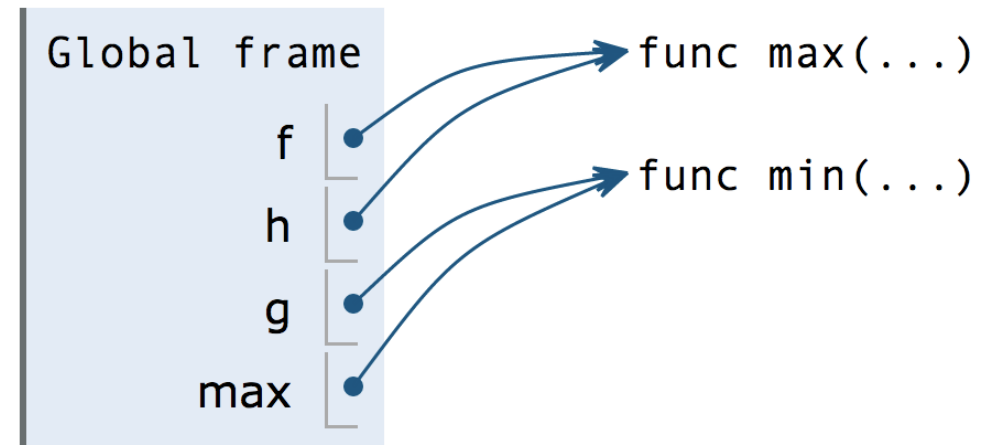
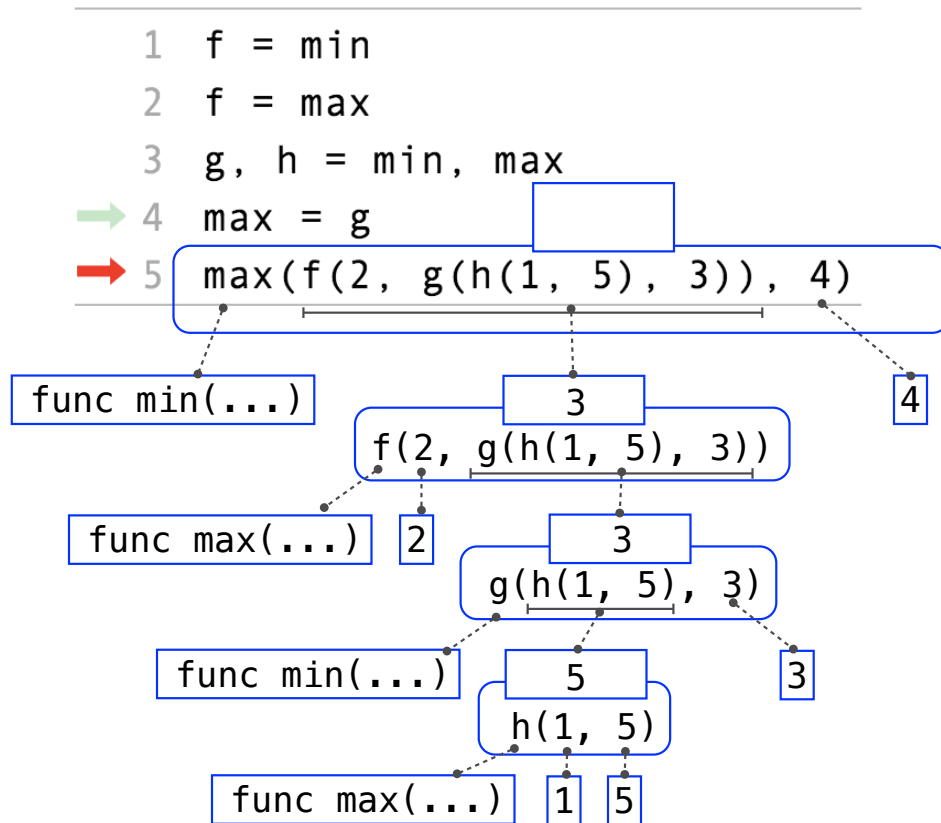
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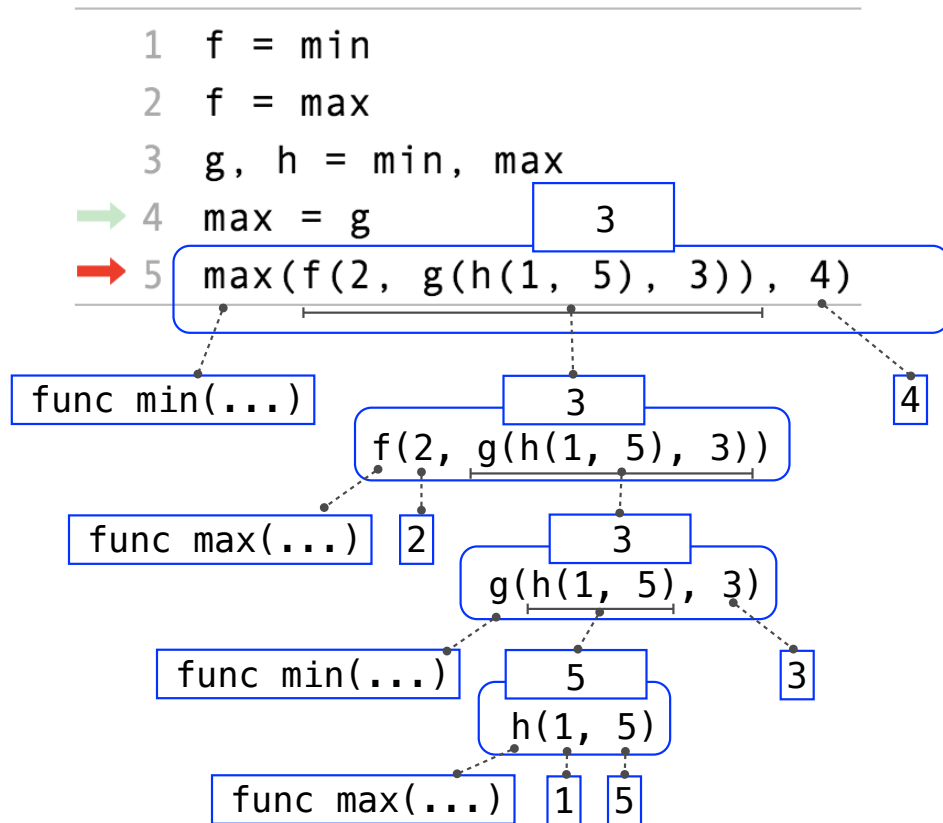


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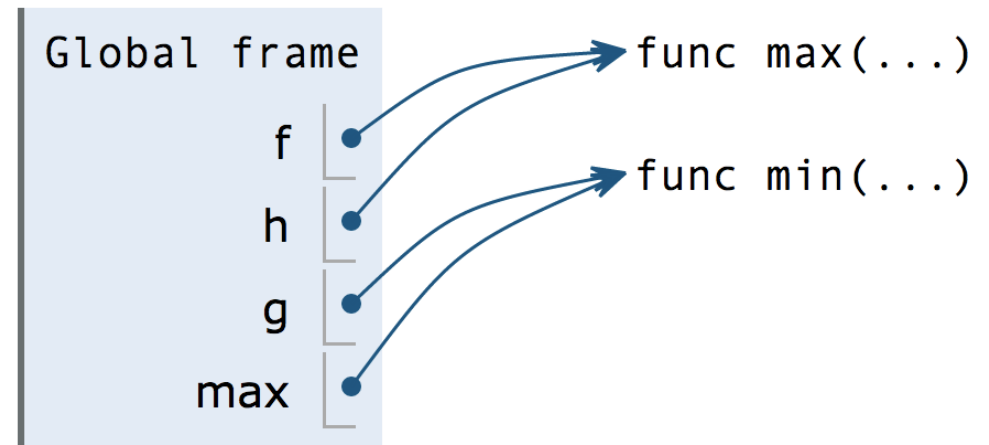
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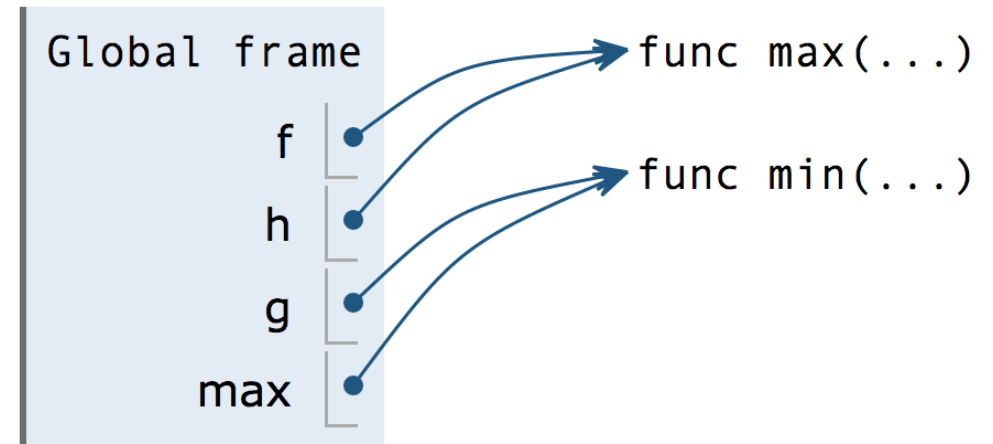
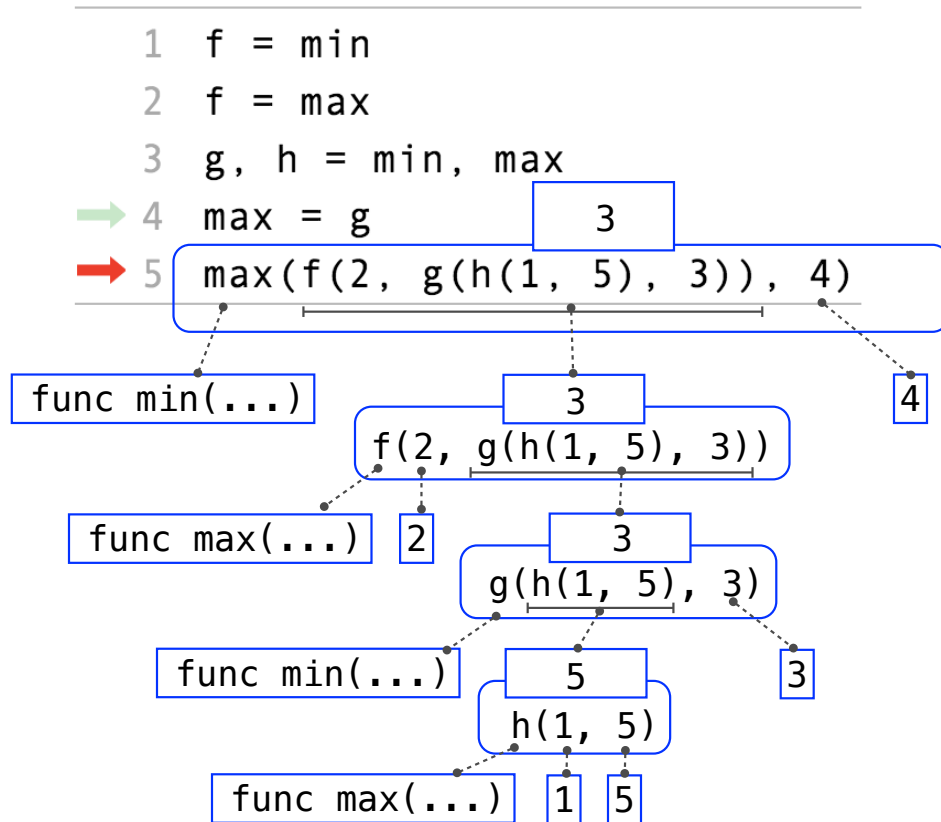


(Demo)



## Discussion Question 1 Solution

( Demo )



# 3

## Defining Functions



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Assignment is a simple means of abstraction: binds names to values

Function definition is a more powerful means of abstraction: binds names to expressions

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>>> def <name>(<formal parameters>):  
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## Calling User-Defined Functions

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**Procedure for calling/applying user-defined functions (version 1):**

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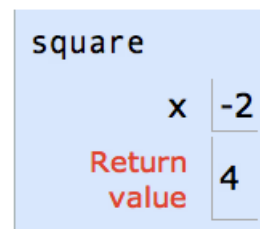
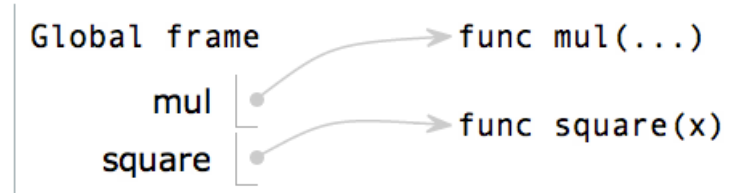
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## Calling User-Defined Functions

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```

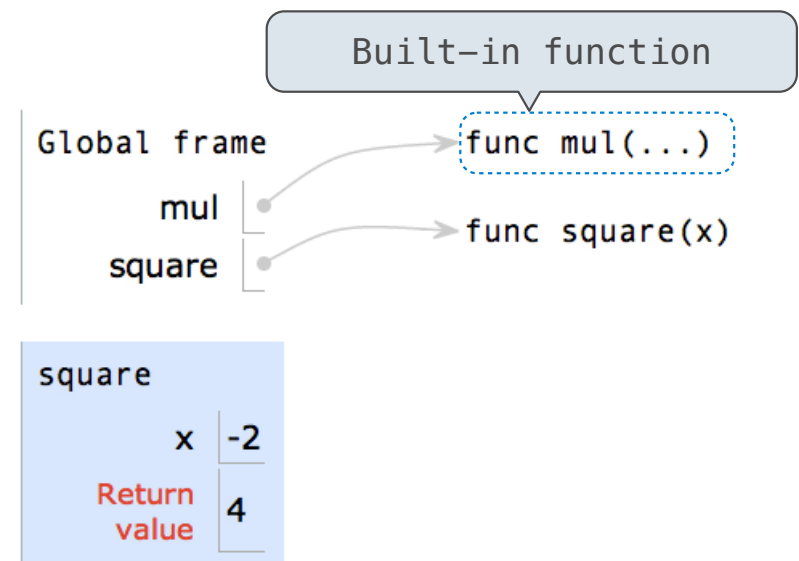


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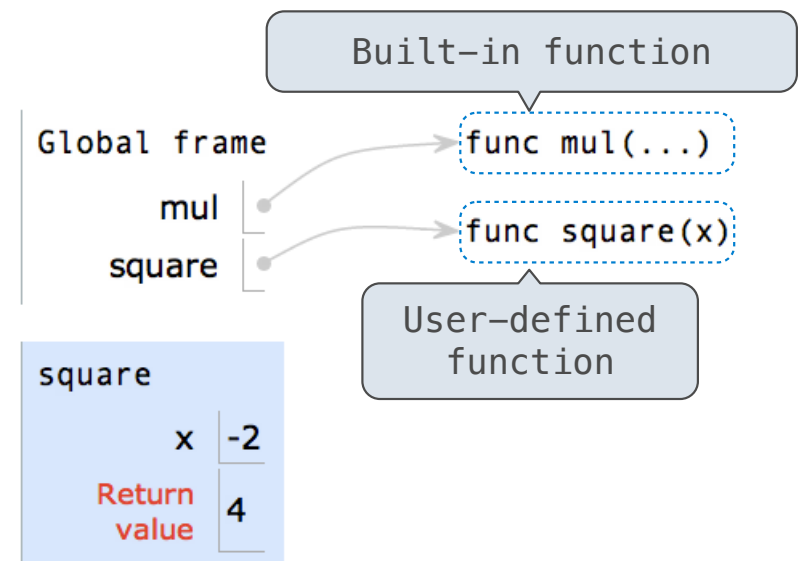


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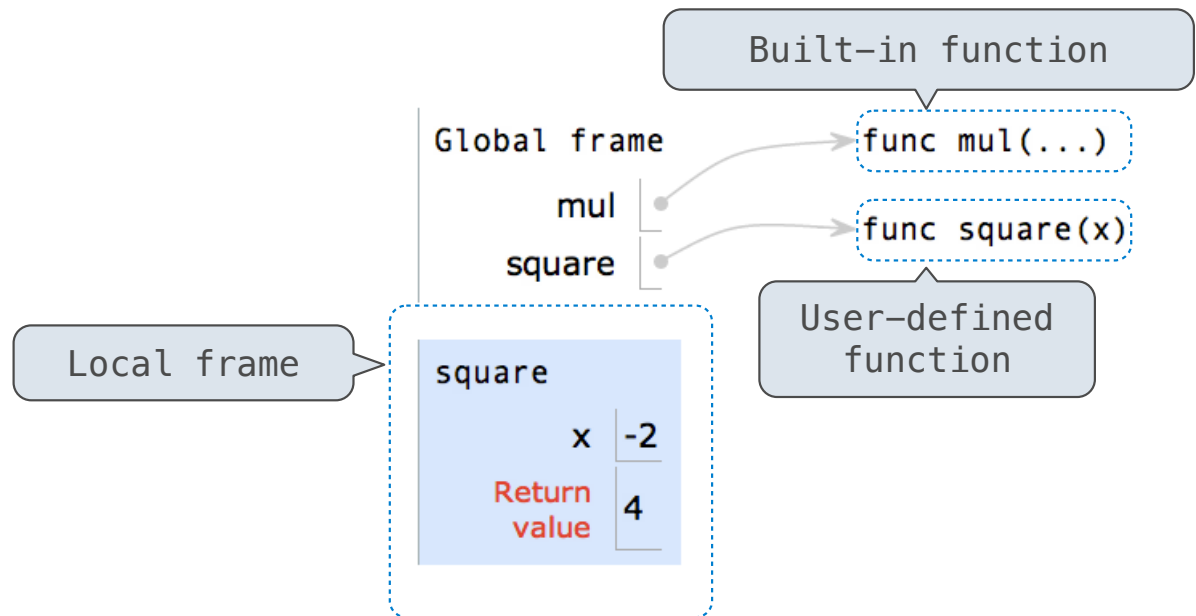


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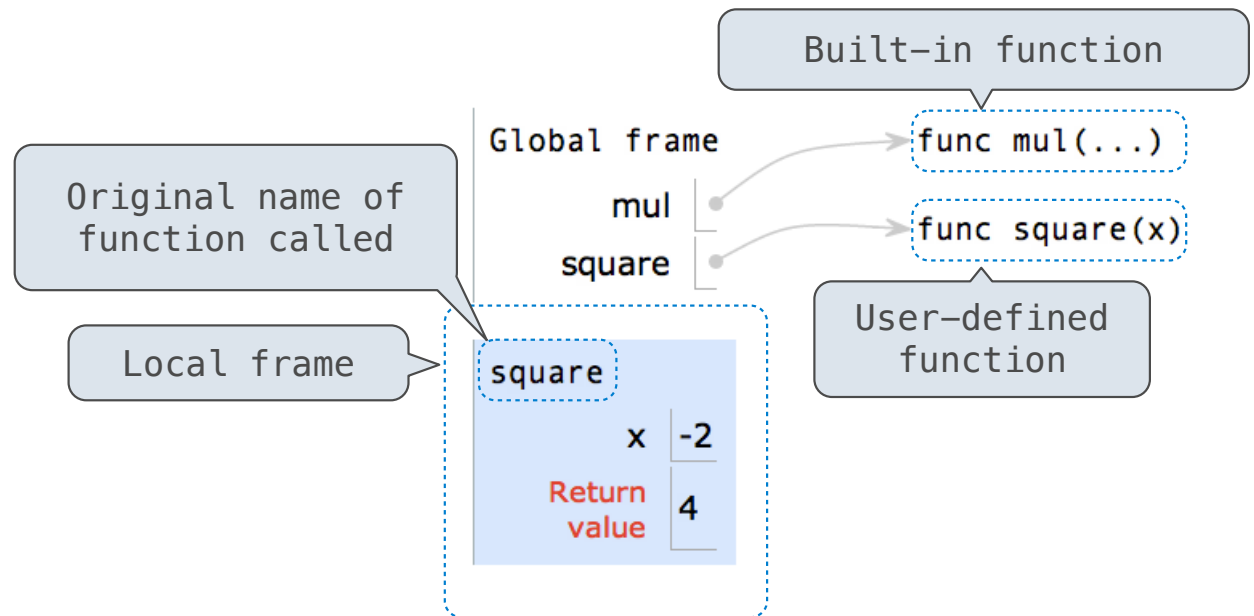


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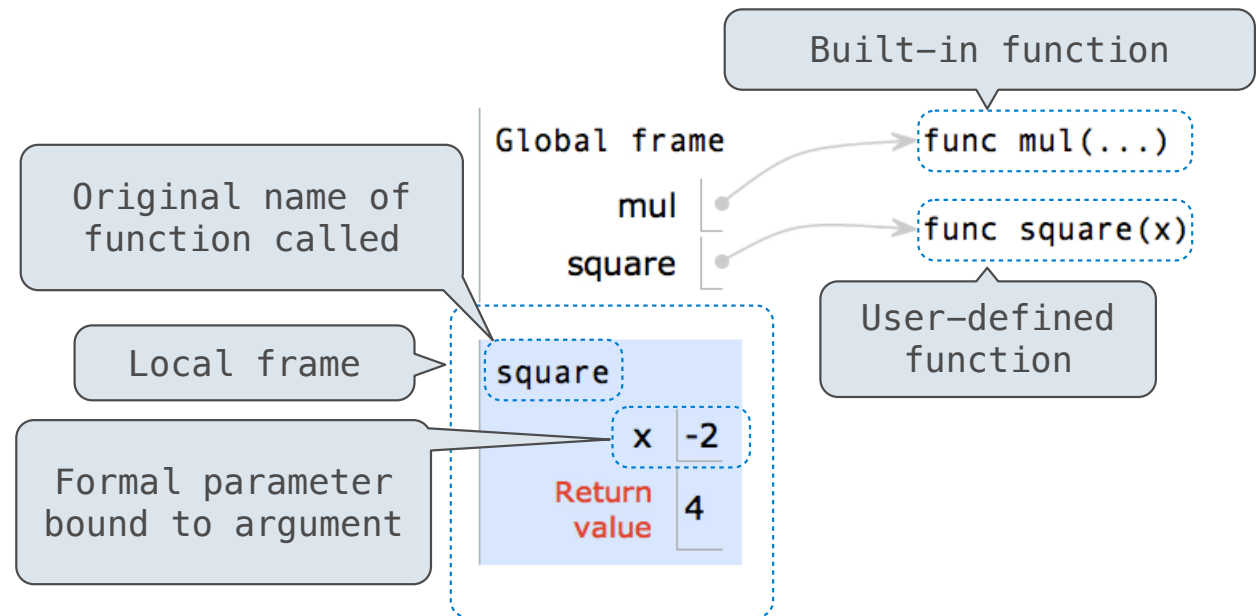


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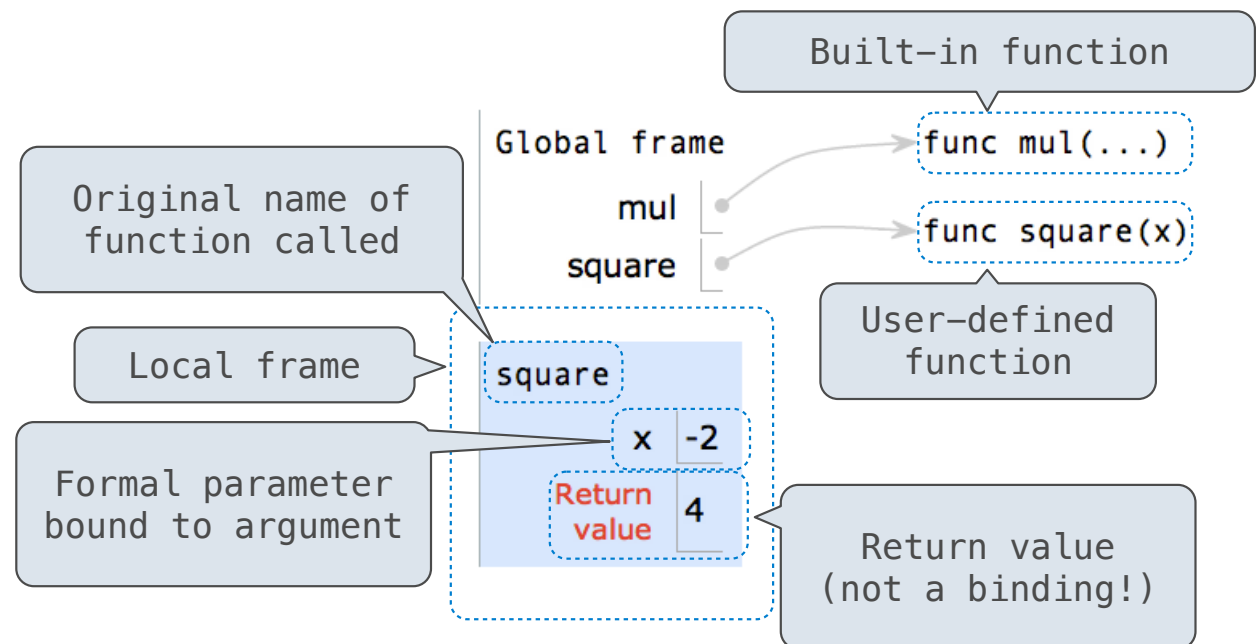


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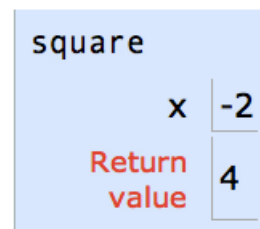


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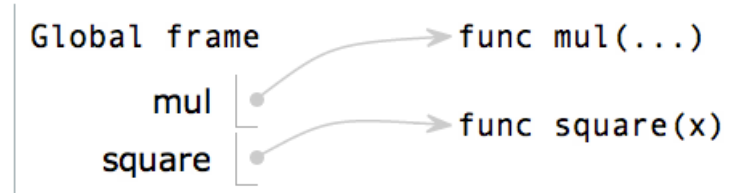
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A function's signature has all the information needed to create a local frame



square	
x	-2
Return value	4

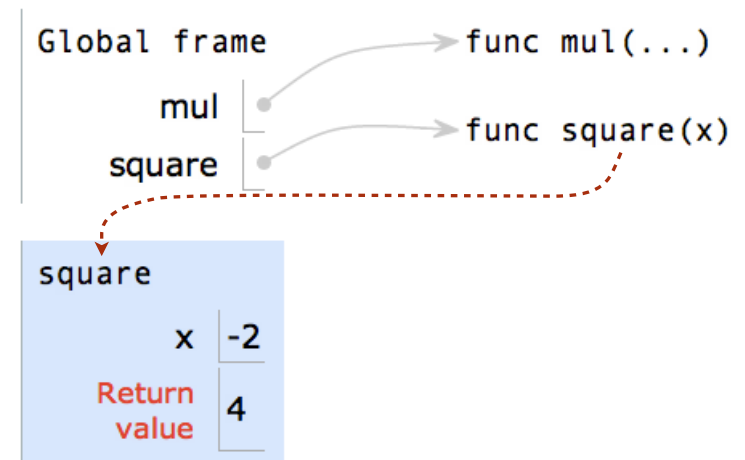
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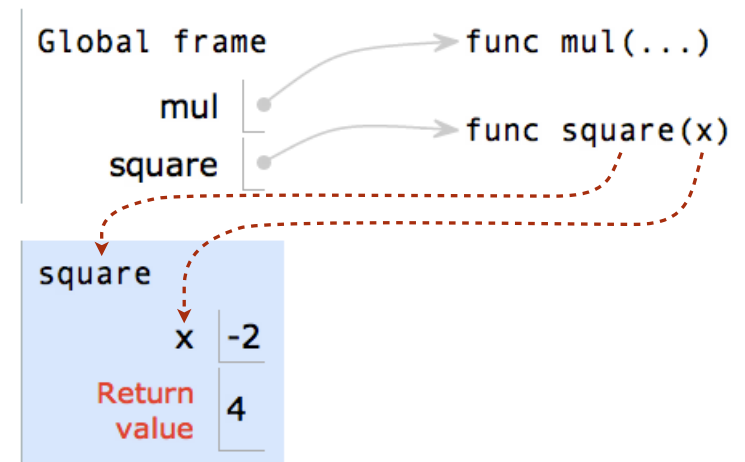
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## Looking Up Names In Environments

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Every expression is evaluated in the context of an environment.

So far, the current environment is either:

- The global frame alone, or
- A local frame, followed by the global frame.

*Most important two things I'll say all day:*

An environment is a sequence of frames.

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(Demo)