Bonus

Your name and email

April 5, 2024

We say a differentiable function $f : \mathbb{R}^N \to \mathbb{R}$ to be L-smooth if and only if

$$\|\nabla f(x) - \nabla f(y)\| \le L\|x - y\|, \tag{1}$$

where $\|\cdot\|$ denotes the ℓ_2 norm in $\mathbb{R}^N$.

1   log-sum-exp (6 points)

Consider the log-sum-exp function $f : \mathbb{R}^N \to \mathbb{R}$ defined as

$$f(x) = \ln\left(\sum_{i=1}^{N} \exp(x_i)\right). \tag{2}$$

Prove that f is 1-smooth.

2   Remarks

- You can use whatever sufficient conditions (e.g., conditions on Hessian) for proving L-smooth, as long as you clearly document where you get the sufficient condition from.

- Since this is a bonus, I and TAs will not provide any hint other than clarification.

- The LaTeX file for generating this PDF is provided in the same folder. Please use it for this homework.