CS 4501: Optimization - Assignment 5

Your name and email

Part I

The goal of this part is to implement mirror descent with $w : \mathbb{R}^n_+ \to \mathbb{R}$ defined as

$$w(x) = \sum_{i=1}^{n} x_i \ln x_i,$$

where $\mathbb{R}_+ \doteq (0, \infty)$

Task 1 (1pt): Compute $\nabla w(x)$

(SZ: Complete the computation)

$$\frac{\partial w(x)}{\partial x_i} =$$

Task 2 (2pts): Compute $w^*(y)$

$$w^*(y) \doteq \max_{x \in \mathbb{R}^n_+} y^\top x - w(x).$$

(SZ: Complete the computation)

Task 3 (1pt): Compute $\nabla w^*(y)$

(SZ: Complete the computation)

$$\frac{\partial w^*(y)}{\partial y_i} =$$

Task 4 (2pt): Implement Mirror Descent w.r.t. w

Define

$$f(x) = \|x\|_2^2.$$

Then implement

$$x_{k+1} = \nabla w^* \left(\nabla w(x_k) - \alpha \nabla f(x_k) \right).$$

Part II

The goal of this part is to implement mirror descent with $w : \Delta_n \to \mathbb{R}$ defined as

$$w(x) = \sum_{i=1}^{n} x_i \ln x_i,$$

where

$$\Delta_n \doteq \left\{ x \in \mathbb{R}^n \mid \sum_{i=1}^{n} x_i = 1, x_i > 0 \right\}.$$

Task 5 (2pt): Compute $\nabla w(x)$ and $\nabla w^*(y)$

(SZ: You can use results from Assignment 4 directly.)

Proof. (SZ: Complete the computation) □

Task 6 (2pt): Implement Mirror Descent w.r.t. w

Define

$$f(x) = \|x\|_2^2.$$

Then implement

$$x_{k+1} = \nabla w^* \left(\nabla w(x_k) - \alpha \nabla f(x_k) \right).$$