CH5170 : Process Optimization

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CH18B067

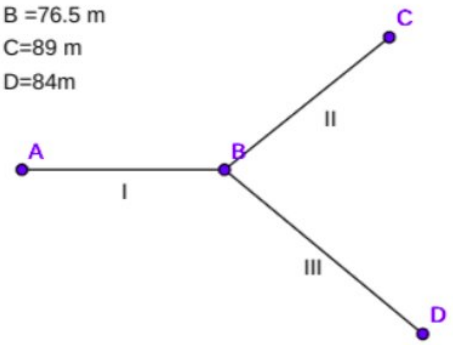
Given data:

Click

Roll number CH18B067

Enter your roll number in the box below and press the button titled Click

Minimum pressure at node B =76.5 m
Minimum pressure at node C=89 m
Minimum pressure at node D=84m



Formulae used:

$$H_A - H_B = \Delta H = 4.457 \times 10^8 \frac{LQ^{1.85}}{D^{4.87}},$$

$$c = 1.2654 D^{1.327},$$

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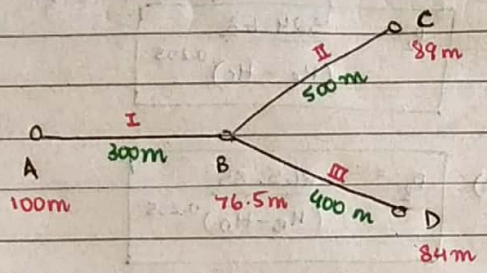
Q1.



$$\Delta H = 4.457 \times 10^8 \cdot \frac{L \cdot Q^{1.85}}{D^{4.87}}$$

$L_1 = 300\,m$ $\quad Q_1 = 9\,m^3/min$

$L_2 = 500\,m$ $\quad Q_2 = 3\,m^3/min$

$L_3 = 400\,m$ $\quad Q_3 = 2\,m^3/min$

1) a) $H_A - H_B = 4.457 \times 10^8 \frac{L_1 Q_1^{1.85}}{D_1^{4.87}}$

$\Rightarrow 100 - H_B = 4.457 \times 10^8 \cdot \frac{300 \times 9^{1.85}}{D_1^{4.87}}$

$\Rightarrow \boxed{100 - H_B = \frac{7.7895 \times 10^{12}}{D_1^{4.87}}}$

b) $H_B - H_C = 4.457 \times 10^8 \frac{L_2 Q_2^{1.85}}{D_2^{4.87}}$

$\Rightarrow H_B - H_C = 4.457 \times 10^8 \times \frac{500 \times 3^{1.85}}{D_2^{4.87}}$

$\Rightarrow \boxed{H_B - H_C = \frac{1.7 \times 10^{12}}{D_2^{4.87}}}$

c) $H_B - H_D = 4.457 \times 10^8 \frac{L_3 Q_3^{1.85}}{D_3^{4.87}}$

$\Rightarrow H_B - H_D = 4.457 \times 10^8 \cdot \frac{400 \times 2^{1.85}}{D_3^{4.87}}$

$\Rightarrow \boxed{H_B - H_D = \frac{6.427 \times 10^{11}}{D_3^{4.87}}}$

3) $\boxed{\text{cost} = 1.2654 \left(L_1 D_1^{1.327} + L_2 D_2^{1.327} + L_3 D_3^{1.327} \right)}$

3) a) $D_1^{4.87} = \frac{100 - H_B}{7.7895 \times 10^{12}}$

$D_1^{4.87} = \frac{7.7895 \times 10^{12}}{100 - H_B}$

$$\boxed{D_1 = \frac{443.74}{(100 - H_B)^{0.205}}}$$

b) $D_2 = \left(\frac{1.7 \times 10^{12}}{H_B - H_C}\right)^{\frac{1}{4.87}}$

$$\boxed{D_2 = \frac{324.63}{(H_B - H_C)^{0.205}}}$$

c) $$\boxed{D_3 = \frac{265.85}{(H_B - H_D)^{0.205}}}$$

5) $$\text{cost} = 1.2654 \times \left(300 \times \left(\frac{443.74}{(100 - H_B)^{0.205}}\right)^{1.327} + 500 \times \left(\frac{324.63}{(H_B - H_C)^{0.205}}\right)^{1.327} + 400 \times \left(\frac{265.85}{(H_B - H_D)^{0.205}}\right)^{1.327}\right)$$

$$\Rightarrow \text{cost} = \frac{1.236 \times 10^6}{(100 - H_B)^{0.272}} + \frac{1.36 \times 10^6}{(H_B - H_C)^{0.272}} + \frac{0.835 \times 10^6}{(H_B - H_D)^{0.272}}$$

5) $$\min_{H_B, H_C, H_D} \frac{1.236 \times 10^6}{(100 - H_B)^{0.272}} + \frac{1.36 \times 10^6}{(H_B - H_C)^{0.272}} + \frac{0.835 \times 10^6}{(H_B - H_D)^{0.272}}$$

s.t $H_B \geq 76.5\,m$

$H_C \geq 89\,m$

$H_D \geq 84\,m$

6) At optimum, $H_C = 89m$, $H_D = 84m$ while $H_B \geq 76.5\,m$ becomes redundant since as long as $H_C > 89m$ we can keep decreasing it to decrease D_2 thereby decreasing cost. However at $H_C = 89m$ we cannot decrease it any further, hence we stop. similarly, we stop at $H_D = 84m$.

$$\min_{H_B} \text{cost} = \frac{1.236 \times 10^6}{(100 - H_B)^{0.272}} + \frac{1.36 \times 10^6}{(H_B - 89)^{0.272}} + \frac{0.835 \times 10^6}{(H_B - 84)^{0.272}}$$

on solving, we get minimum cost $= 2.06746 \times 10^6$

$H_B = 95.2538\,m$

$\therefore D_1 = 322.46\,mm$ $\quad D_3 = 161.85\,mm$.

$D_2 = 222.935\,mm$

Continuous Pipe Problem

Using Wolfram Alpha to solve the continuous pipe problem

minimize	$\frac{1.236\times10^6}{(100-x)^{0.272}}+\frac{0.835\times10^6}{(x-84)^{0.272}}+\frac{1.36\times10^6}{(x-89)^{0.272}}$

Solution:

$$\min\left\{\frac{1.236\times10^6}{(100-x)^{0.272}}+\frac{0.835\times10^6}{(x-84)^{0.272}}+\frac{1.36\times10^6}{(x-89)^{0.272}}\right\}\approx 2.06746\times10^6 \text{ at } x\approx 95.2538$$

Where 'x' is H_b

8) For the discrete pipe problem:

AB

D_1 = 300 mm cost = ₹2451/m

D_2 = 350 mm cost = ₹3008/m

BC

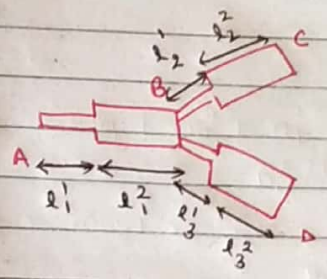
D_1 = 200 m cost = ₹1431/m

D_2 = 250 m cost = ₹1924/m

BD

D_1 = 150 m cost = ₹977/m

D_2 = 200 m cost = ₹1431/m



$l_1^1 + l_1^2 = 300$ m $l_2^1 + l_2^2 = 500$ m $l_3^1 + l_3^2 = 400$ m

$H_B \geq 76.5$ m

$H_C \geq 89$ m

$H_D \geq 84$ m

$$100 - H_B = 4.457 \times 10^8 \left(\frac{l_1^1}{300^{4.87}} + \frac{l_1^2}{250^{4.87}} \right) \times 9^{1.85}$$

$$H_B - H_C = 4.457 \times 10^8 \left(\frac{l_2^1}{200^{4.87}} + \frac{l_2^2}{250^{4.87}} \right) \times 3^{1.85}$$

$$H_B - H_D = 4.457 \times 10^8 \left(\frac{l_3^1}{250^{4.87}} + \frac{l_3^2}{200^{4.87}} \right) \times 2^{1.85}$$

Objective: To minimise cost

$$\text{cost} = 2451 l_1^1 + 3008 l_1^2 + 1431 l_2^1 + 1924 l_2^2 + 977 l_3^1 + 1431 l_3^2$$

$l_1^1, l_1^2, l_2^1, l_2^2, l_3^1, l_3^2$

On solving,

optimal cost = ₹ 2.156344×10^6

$\ell_1^1 = 0\ m$

$\ell_1^2 = 300\ m$

$\ell_2^1 = 303.206\ m$

$\ell_2^2 = 196.794\ m$

$\ell_3^1 = 388.492\ m$

$\ell_3^2 = 111.508\ m$

Discrete Pipe Problem

Using online-optimizer.appspot.com to get solution to discrete pipe problem:

```
var l11 >= 0;
var l12 >= 0;
var l21 >= 0;
var l22 >= 0;
var l31 >= 0;
var l32 >= 0;
var Hb >= 76.5;
var Hc >= 89;
var Hd >= 84;

minimize z:     3008*l11 + 2451*l12 + 1924*l21 + 1431*(l22+l31) + 977*l32;

subject to c11:   l11 + l12  =  300;
subject to c12:   l21 + l22  =  500;
subject to c13:   l31 + l32  =  400;
subject to c14:   100 - Hb   = 4.457*10^8 *(9^1.85)*(l11/(350^4.87)+l12/(300^4.87));
subject to c15:   Hb - Hc  = 4.457*10^8 *(3^1.85)*(l21/(250^4.87)+l22/(200^4.87));
subject to c16:   Hb - Hd    = 4.457*10^8 *(2^1.85)*(l31/(200^4.87)+l32/(150^4.87));

end;
```

Solution:

l11	Real	300
l12	Real	0
l21	Real	196.7940467
l22	Real	303.2059533
l31	Real	111.5077919
l32	Real	288.4922081
Hb	Real	96.8238489
Hc	Real	89
Hd	Real	84