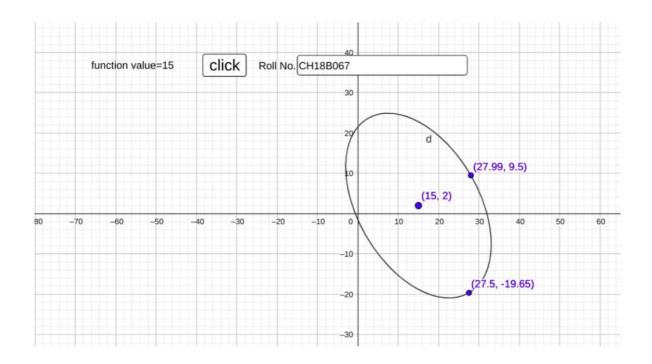
CH5170: Process Optimization HW2

Shania Mitra CH18B067



From the given graph, x*= control = (15,2) point on major aris = (27.5, 19.65)		Apple The Tolk Control
point on major anis = (27.5, -19.65)) From the given graph,	
point on major aris = (27.5, -19.65) a = semi-major aris = (27.9, 9.5) a = semi-major aris = (27.9, 9.5) = 24.99.3 Value of the function at the contour = 15 semi-min a river a that, HV1 = 21V, where H = Hessian matrix of flax V1 = signification to major and to V1 N = V1 = NV A = Light value corresponding to V1 N = V1 = NV A = (27.9) (27.9) V2 = (-12.5) (27.9) (27.9) (27.9) (27.5) W4 = (-12.5) (27.9) (27.9) (27.5) W5 = (-12.5) (27.9) (27.9) (27.5) W6 there that (27.9) (27.9) (27.9) (27.9) (27.5) W6 there that (27.9) (27.9) (27.9) (27.9) (27.5) W6 there that (27.9) (27.9) (27.9) (27.5) W6 there that (27.9) (27.9) (27.9) (27.5) W6 there that (27.9) (27.9) (27.9) (27.9) (27.5) W6 there that (27.9) (27.9) (27.9) (27.9) (27.9) (27.9) W6 there that (27.9) (27.9) (27.9) (27.9) (27.9) (27.9) W6 there that (27.9) (27.9) (27.9) (27.9) (27.9) (27.9) (27.9) W6 there that (27.9) (27.9) (27.9) (27.9) (27.9) (27.9) (27.9) W6 there that (27.9) (27.9) (27.9) (27.9) (27.9) (27.9) (27.9) W6 there there is a semi-major axis (27.9) (Marie Marie
point on major sais = (27.5, -19.65)	x = centre = (15,2)	
Point on major anis = (27.5, -19.65)		
Value of the function at the contous = 15 L- semi-min a We know that, HV = 21 V, when H= Hensian matrix of f(x) V1 = signification to matrix of f(x) V2 = signification to matrix of f(x) V3 = major value consequencing to V1 N= VAV N= (-12.5) 20 H= (-12.5) 21.65 N2 = (-12.5) 21.65 N= (-12.5) 21.65 N= (-12.5) N= (-1	Point on major aris = (27.5, -19.65)	os a = seni-myon
Value of the function at the contous = 15 b- somi-min a 14. 9997. 14. 9997. 14. 9997. 14. 9997. 14. 1990. 14. 1990. 15. 1990. 16. 1990. 16. 1990. 16. 1990. 16. 1990. 16. 1990. 16. 1990. 16. 1990. 16. 1990. 16. 1990. 16. 1990. 16. 1990. 16. 1990. 16. 1990. 16. 1990. 16. 1990. 16. 1990. 16. 1990. 16. 1990. 16. 1990. 17. 1990. 18. 1990. 1	eoint on mind anis = (27.99, 9.5)	aus .
14. 9997 14. 9997	Value A Atra Limetra, al the contous = 15_	
woo know that, $HV_1 = \lambda_1 V$, where $K = \text{Hessian matrix of } f(x)$ $V_1 = \text{sign value consequenting to } M_1 = \text{light value consequenting to } V_1$ $V_1 = \{12, 5\}$ $V_2 = \{12, 99\}$ $V_3 = \{12, 99\}$ $V_4 = \{12, 5\}$ $V_4 = \{12, 5\}$ $V_4 = \{12, 5\}$ $V_5 = \{12, 99\}$ $V_6 = \{12, 5\}$ $V_7 = \{12, 5\}$ $V_8 = \{12, 99\}$ $V_9 $	mary more framework was now	
HV ₁ = $\lambda_1 V$, where $N = \text{lignism}$ mother of $f(\alpha)$ V ₁ = eigenvector corresponding to Magnetic value of V_1 $\lambda_1 = \text{lignism}$ are corresponding to V_1 $\lambda_1 = \text{lignism}$ and $\lambda_2 = \text{lignism}$ are corresponding to V_1 $\lambda_1 = \text{lignism}$ and $\lambda_2 = \text{lignism}$ are corresponding to V_1 $\lambda_1 = \text{lignism}$ and $\lambda_2 = \text{lignism}$ are corresponding to V_1 $\lambda_1 = \text{lignism}$ and $\lambda_2 = \text{lignism}$ are corresponding to V_1 $\lambda_1 = \text{lignism}$ and $\lambda_2 = \text{lignism}$ are corresponding to V_1 $\lambda_1 = \text{lignism}$ and $\lambda_2 = \text{lignism}$ are corresponding to V_1 $\lambda_1 = \text{lignism}$ and $\lambda_2 = \text{lignism}$ are corresponding to V_1 $\lambda_1 = \text{lignism}$ and $\lambda_2 = \text{lignism}$ are corresponding to V_1 $\lambda_1 = \text{lignism}$ and $\lambda_2 = \text{lignism}$ are corresponding to V_1 $\lambda_1 = \text{lignism}$ and λ_2	An	* 14.9997.
Similarly $Hv_2 = \lambda v_2$. Similarly $Hv_2 = \lambda v_2$. $\lambda_1 = \frac{116}{12} \sqrt{2000}$ consequence to v_1 $\lambda_2 = \frac{1}{12} \sqrt{200}$ $\lambda_3 = \frac{1}{12} \sqrt{2000}$ consequence $v_4 = v_1$ $\lambda_1 = \frac{1}{12} \sqrt{200}$ $\lambda_2 = \frac{1}{12} \sqrt{200}$ $\lambda_3 = \frac{1}{12} \sqrt{200}$ $\lambda_4 = \frac{1}{12} \sqrt$	and with the same	1 0 C(x)
Similarly $HV_2 > \lambda V_2$. $\lambda_1 = ugin value corresponding to V_1 V_2 = V \times V \times V_2 V_3 = V \times V \times V_4 V_4 = \begin{pmatrix} -12.5 \\ 21.65 \end{pmatrix} V_2 = \begin{pmatrix} -12.5 \\ 21.65 \end{pmatrix} V_4 = \begin{pmatrix} -12.5 \\ 21.65 \end{pmatrix} V_5 = \begin{pmatrix} -12.5 \\ 21.65 \end{pmatrix} V_6 = \begin{pmatrix} -12.5 \\ 21.65 \end{pmatrix} V_7 = \begin{pmatrix} -12.5 \\ 2$	$HV_1 = 1_1V_1$ usual $H = HIMAN VIV$	unandino to monos ar
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	V,= eightpelist of	suspending to Vi
$V_{1} = \begin{pmatrix} -12.5 \\ 21.65 \end{pmatrix}$ $V_{2} = \begin{pmatrix} -12.99 \\ +1.5 \end{pmatrix} \begin{pmatrix} -12.5 \\ 21.65 \end{pmatrix} \begin{pmatrix} -12.5 \\ $	Similarly Hv, = Avz. A1 = 11gen value	one formating and
$V_{1} = \begin{pmatrix} -12.5 \\ 21.65 \end{pmatrix}$ $V_{2} = \begin{pmatrix} -12.99 \\ +1.5 \end{pmatrix} \begin{pmatrix} -12.5 \\ 21.65 \end{pmatrix} \begin{pmatrix} -12.5 \\ $	사용 (1. 45명) 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.	
$V_{1} = \begin{pmatrix} -12.5 \\ 21.65 \end{pmatrix}$ $V_{2} = \begin{pmatrix} -12.99 \\ 21.65 \end{pmatrix} \begin{pmatrix} -12.5 \\$	N where V= [V1 V2]	1747 A
$V_{1} = \begin{pmatrix} -12.5 \\ 21.65 \end{pmatrix}$ $V_{2} = \begin{pmatrix} -12.99 \\ 21.65 \end{pmatrix}$ $V_{3} = \begin{pmatrix} -12.5 \\ 21.65 \end{pmatrix} \begin{pmatrix} -12.5 \\ 21.65 \end{pmatrix}$	M= VNV-1 A=(A, 0)	
$V_{2} = \begin{cases} 12.99 \\ +1.5 \end{cases} = \begin{cases} 12.99 \\ 21.65 \end{cases} = \begin{cases} 12.99 \\ 21.95 \end{cases}$	lo And	
$V_{2} = \begin{bmatrix} \lambda_{12} & qq \\ +1.5 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0$	$y_1 = (-12.5)$	
$V_{2} = \begin{bmatrix} \lambda_{12} & qq \\ +1.5 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{2} & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_{1} & 0$	21.65	
$H = \begin{bmatrix} -12.5 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} $		
$H = \begin{bmatrix} -12.5 \\ 21.65 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 21.65 \end{bmatrix} \begin{bmatrix} $	V2 = (212.99) (12.99)	
$H = \begin{bmatrix} -12.5 & 12.99 & 21.0 & -12.5 & 12.99 \\ 21.65 & 7.5 & 0 & 22 & 21.65 & 7.5 \end{bmatrix}$ we know that $A_1 = \begin{bmatrix} 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 &$		
$H = \begin{bmatrix} -12.5 & 12.99 & 21.0 & -12.5 & 12.99 \\ 21.65 & 7.5 & 0 & 22 & 21.65 & 7.5 \end{bmatrix}$ $we know that A_1 = \begin{bmatrix} A_1 & 0 & -12.5 & 12.99 \\ 0 & 21.65 & 7.5 \end{bmatrix} we know that A_1 = \begin{bmatrix} A_1 & 0 & -12.5 & 12.99 \\ 0 & 21.65 & 7.5 \end{bmatrix} A_2 = \begin{bmatrix} A_1 & 0 & -12.5 & 12.99 \\ 0 & 21.65 & 7.5 \end{bmatrix} A_1 = \begin{bmatrix} A_1 & 0 & -12.5 & 12.99 \\ 0 & 21.65 & 7.5 \end{bmatrix} A_2 = \begin{bmatrix} A_1 & 0 & -12.5 & 12.99 \\ 0 & 21.65 & 7.5 \end{bmatrix}$		
H=, $\begin{bmatrix} -12.5 & 12.99 & \lambda_1 & 0 & -12.5 & 12.99 \\ 21.65 & 7.5 & 0 & \lambda_2 & 21.65 & 7.5 \end{bmatrix}$ we know that $\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} \text{Noni-winer aris} \\ \text{Nemi-major anis} \end{bmatrix}^2$ The Mark 1 is $\lambda_1 = 0.36$.	0 4- (Stas) [2, 0]	A. A. C.
$H = \left(\frac{-12.5}{21.65} \right) \left(\frac{12.99}{21.65} \right) \left(\frac{12.5}{21.65} \right) \left(\frac{12.99}{21.65} \right) - \frac{1}{21.65} $ we know that $\left(\frac{21}{22} \right) = \left(\frac{\text{Somi-winer aris}}{\text{Semi-major anis}} \right)^{2}$ $\left(\frac{21}{21.65} \right) = \left(\frac{\text{Somi-winer aris}}{\text{Semi-major anis}} \right)^{2}$	00	
we know that $ \frac{\lambda_1}{\lambda_1} = \frac{\lambda_1}{\lambda_2} =$		
we know that $ \frac{\lambda_1}{\lambda_2} = \frac{\lambda_1}{\lambda_2} = \frac{\lambda_2}{\lambda_2} = \frac{\lambda_1}{\lambda_2} = \frac{\lambda_2}{\lambda_2} = \frac{\lambda_1}{\lambda_2} = \frac{\lambda_1}{\lambda_2} = \frac{\lambda_1}{\lambda_2} = \frac{\lambda_2}{\lambda_2} = \frac{\lambda_1}{\lambda_2} =$	[-12 = 7 12.99] [21 0] [-1	2.5 12.99] -1
we know that $ \frac{\langle \lambda_1 \rangle}{\langle \lambda_2 \rangle} = \left(\frac{\text{somi-minor aris}}{\text{semi-major axis}} \right)^2 $ $ \frac{\langle \lambda_1 \rangle}{\langle \lambda_2 \rangle} = 0.36. $		11 / 12 mm
$\frac{(\lambda_1)}{(\lambda_2)^2} = \frac{(\text{somi-minor axis})^2}{(\text{semi-major axis})^2}$ $\frac{(\lambda_1)}{(\lambda_2)^2} = \frac{(\text{somi-major axis})^2}{(\text{semi-major axis})^2}$	(21.63)	
$\frac{(\lambda_1)}{(\lambda_2)^2} = \frac{(\text{somi-minor axis})^2}{(\text{semi-major axis})^2}$ $\frac{(\lambda_1)}{(\lambda_2)^2} = \frac{(\text{somi-major axis})^2}{(\text{semi-major axis})^2}$		
(32) (semi-major axis) $(32)(32) = 0.36$		1.\2
01.17 USA = 0.36.		
	1 Senti-major as	<u>us /</u>
- 0.36		
	- 0.36	

\$0 H= [0.84 0.277]	7 201.01 (ENST .EF-) = 0 3.
Lo.2771 0.52	(3320.0-) = 0
11*	1682000
$a = -Hx^*$	N N
= (-13.1542) /2	x [1652.0 12.0] = 51 12.01
L -5/14/0] %59.0	62.0 15F5 0
Prom derivation in cla	was known that
- H > 0- 11	
$f(x^{+}) = convoice$	85.50.0 S100.0
f(x=) = 15-0.3= = 14.5	
	20 A 538, 501 1 4 2.11 = 3
(14) (10 t) = 1 fx - x t) THE COLOR OF SHEET
Also,	and the same of th
(1x 7 18A 2 0 m = 3.35	0.0-10.0-1
on companing (and	②
(14) (S100.C	7800.9 (de 1)() 1.7
c= f(x*) === 2 x*	1800.0 [(h N] L.)-
	= 14.5 + 103.852 \2
750.	inis poid in the ellipse iquation
Substituting a the major	
77.6 1007	Comment of the Commen
f(a) = C+ a 1 + 2 a 2 2 1	[_18.1542 -5.1920] [27.5] Az
	230 Regard to cart incl 719:65
1980 0 F 22.5 - 19.65) [0.84 . 0, 237) [27.5
1 - 22 - 1 - 12 - 102	0.2731014 0.52 19.65 72
(3980.6)	L V L COMPENSA
1 1/5 on solving vusting NIKTUAS	discher of value of sen
2 = 0.0044	
10-90 1 The state of the state	

00 Q = [-13. 1542] x 0.0044 [1550.0 NB.0] 111
-5.1970 C2.0 1050 0
a: [-0.0585]
[-0.0 231]
111ly H = [0.84 Q. 2771] * 0.0044 70113-
(0.2771 0.52)
H= (0.0037 / -0.6012) (10)
0.0023 30.01 - (4x)
C= 14.5 + 103.862 }
C= W.O (Usuio : ALATIAR) (I am
C= 14.9616
0 00 PHTC/ + VTO + 1 (0))
€0 f(x) = 14.9616 + [-0.0585 -0.023][x1]
(a) Long (1) Sing (1)
+1[N, x2] [0.0037 0.0012][N]
0.0012 0.0023
8 CE-101-50 102-500 22 102-500 22 102-500 22 102-500 22 102-500 22 102-500 22 102-500 22 102-500 22 102-500 22
(3°2) Vf major aris = H (x-xx)
marias, on a successful marias, on the successful
A 7f major anign = 1- 0.02
1 (2 - 1 (2 -) (2 -) (2 -) (2 -) (2 -) (2 -)
De Perention of steepest agrent = 7f = (0.00)
[O.D346]
direction of steepest descent = - VS = (-0.02)
0.6346)
direction of nochange = direction 1 to of = (0.0346)
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Scanned with CamScanner

Q3, 1xx = 1/2f) (7f) 10 200 10 10 10 10 10 10 10 10 10 10 10 10 1
(VfT)H(Vf) major axis
DAMIC NATUAL
d* = 624.9725
: Mariene distance along which we can travel
before function starts increasing = Har PH
Will /- V
24.976 unile
2 + 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.
94) Vanig (2.2) west all interner 2.41 (+1)
The = (-0.0485) PAITAN SIND
$\nabla f _{(2,2)} = \begin{pmatrix} -0.0485 \\ -0.0160 \end{pmatrix}$
Direction of stoepest avent = 7 2,2 = (-0.0485) - (51)
Direction of observent = = = = = = = = = = = = = = = = = = =
Direction of no change = direction 1 to Vf/2,2
= (0.0160 -0.0485)
(-0.04857
Z° = $(\nabla f)^{\top} (\nabla f)$
1212 (YG) 1 212
Voing MATLAR,
$d^{2} _{2,2} = 231.1157$
nanimum distance = d^P _2
2 11.80 mits

85) For negative definite hessian, all rigen values can be replaced
H= VAV-1 O AITHU Signil
$Now \Lambda = -\Lambda$
-H = -H 22. FA
a lower we did with motion of the the
But 14 = > + constance of strates northern washing
ea a = +a . 9 1121/21.
MINIT IT S II S
C = f(x*) + 1 x* H-new x*
Drung (C.C.) since (V.C.
11.3 remains the same
C= 14.0384.
16110 C = 1
f(x) = 14.0384 + 10.0585 0.02317[x,)
f(x) = (4.0384 + [0.0585 0.0231)[24] 20
421-121 127 (-0.087 -0.0012) (CXI)
-0.0012 -0.0023 \ 22 ·
and the tell programs.
(0210.00 \0 Meg + = 1 1 1 Non100
(< 300.0-)
F (1) 7 (7) - 100
6.0 (77) N T (77) 1 32

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x_{star} = [15; 2];
 major_axis = [27.5; -19.65];
 minor_axis = [27.99; 9.5];
 c0 = 15;
 vl = x_star - major_axis
 v2 = x_star - minor_axis
 a = sqrt(sum(v1.^2))
 b = sqrt(sum(v2.^2))
 lambda_ratio = b^2/a^2 %lambda1/lambda2 lambda1 -> v1
 V = [v1 \ v2];
 H = V*diag([lambda_ratio 1])*inv(V); %x lambda2
 a = -H*x star;
 f_x star = c0 - 0.5;
 c = f_x_{star} + 0.5*x_{star}'*H*x_{star}; %x lambda2
 x = major_axis;
 lambda\_sum = x\_star'*H*x\_star + 2*a'*x + x'*H*x;
 lambda2 = 1/lambda_sum;
 c = c*lambda2
 a = a*lambda2
 H = H*lambda2
 % x = [16;22]
 % x = [12;24]
 diff = x - x_star;
 f_x1 = f_x_{star} + 0.5*diff'*H*diff
 % Q2, 3
 grad_f_major_axis= H*(major_axis - x_star)
 dist = a
 %% Q4
 x2 = [2;2];
 grad_f_2_2 = H*(x2-x_star);
 alpha_2_2 = grad_f_2_2'*grad_f_2_2'(grad_f_2_2'*H*grad_f_2_2)
 H nd = -H
 a_nd = -a
 c_nd = f_x_star + 0.5*x_star'*H_nd*x_star;
v1 =
  -12.5000
  21.6500
v2 =
  -12.9900
  -7.5000
a =
  24.9994
b =
   14,9997
lambda_ratio =
    0.3600
```

```
lambda2 =
   0.0044
a =
   -0.0585
  -0.0231
H =
  0.0037 0.0012
0.0012 0.0023
c =
  14.9616
f_x1 =
  15
grad_f_major_axis =
   0.0200
   -0.0346
alpha_major_axis =
 624.9725
dist_major_axis =
  24.9994
```

```
grad_f_major_axis =
   0.0200
   -0.0346
alpha_major_axis =
 624.9725
dist_major_axis =
   24.9994
grad_f_2_2 =
  -0.0485
  -0.0160
alpha_2_2 =
 231.1157
dist_2_2 =
  11.8119
H_nd =
  -0.0037 -0.0012
  -0.0012 -0.0023
a_nd =
   0.0585
   0.0231
```

c nd =

14.0384