

# Amazon Air Fleet Route Optimization Analysis

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## WGU D605 Task 2 Performance Assessment

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## Executive Summary

This performance assessment presents a comprehensive optimization solution for Amazon Air's fleet routing and scheduling challenges. The analysis employs Mixed Integer Linear Programming (MILP) methodology to minimize operational costs while ensuring efficient service delivery across Amazon's distribution network. The solution addresses the complex combinatorial optimization problem of aircraft assignment, route planning, and resource allocation within Amazon's air logistics operations.

The optimization model successfully reduces total operational costs through systematic aircraft deployment, optimal route selection, and efficient capacity utilization. Implementation using Python's PuLP optimization library demonstrates the practical viability of the mathematical formulation, while comprehensive risk assessment ensures realistic deployment expectations.

## A. Optimization Problem Identification

Amazon Air faces a critical fleet route optimization and scheduling problem that directly impacts operational efficiency and cost management. The company operates a diverse fleet of cargo aircraft serving multiple distribution centers and destinations across the United States, requiring systematic coordination to minimize costs while meeting service level requirements.

The core optimization challenge involves determining the optimal assignment of aircraft to routes while satisfying multiple operational constraints. Amazon Air must decide which aircraft should serve which routes, when departures should occur, and how to manage capacity allocation across the network. This decision-making process becomes increasingly complex due to varying aircraft capacities, different fuel costs, handling requirements at different airports, and strict delivery time windows demanded by Amazon's logistics commitments.

The optimization problem encompasses several key components:

**Aircraft Assignment:** Amazon Air operates multiple aircraft types with varying cargo capacities, fuel efficiency characteristics, and operational costs. The challenge lies in selecting the most appropriate aircraft for each route segment while considering capacity constraints and cost implications.

**Route Planning:** The network includes major distribution hubs (Cincinnati CVG, Ontario ONT, Seattle BFI) and numerous destination airports. Each route has associated distance, flight time, and cost parameters that must be optimized collectively rather than individually.

**Capacity Management:** Each destination has specific demand requirements that must be satisfied exactly once, while ensuring that aircraft capacity constraints are never exceeded. This creates a complex balancing act between service requirements and resource limitations.

**Cost Optimization:** The objective focuses on minimizing total operational costs, including flight costs, fuel expenses, handling fees, and penalty costs for potential delays. These cost components interact in complex ways, requiring sophisticated mathematical modeling.

**Operational Constraints:** Real-world limitations include fleet availability, time window restrictions, fuel requirements, and regulatory compliance factors that constrain the solution space.

This optimization problem is classified as a Vehicle Routing Problem (VRP) with additional constraints, specifically falling under the category of Capacitated Vehicle Routing Problem with Time Windows (CVRPTW). The problem's complexity stems from its combinatorial nature, where the number of potential solutions grows exponentially with fleet size and destination count.

## B. Mathematical Representation of Optimization Components

### B1. Objective Function Expression

The optimization objective is to minimize the total operational cost of Amazon Air's fleet operations. The mathematical formulation of the objective function is:

$$\text{Minimize: } Z = \sum_{ijk} (C_{ijk} \times x_{ijk}) + \sum_k (f_k \times y_k) + \sum_{ik} (h_{ik} \times s_{ik}) + \sum_{jk} (p_j \times t_{jk})$$

Where:

- $C_{ijk}$  represents the cost of operating aircraft  $k$  from hub  $i$  to destination  $j$
- $x_{ijk}$  is a binary decision variable (1 if aircraft  $k$  flies from hub  $i$  to destination  $j$ , 0 otherwise)
- $f_k$  represents the fuel cost associated with utilizing aircraft  $k$
- $y_k$  is a binary decision variable (1 if aircraft  $k$  is used in the solution, 0 otherwise)
- $h_{ik}$  represents the handling cost for aircraft  $k$  at location  $i$
- $s_{ik}$  is a binary decision variable (1 if aircraft  $k$  makes a stop at location  $i$ , 0 otherwise)
- $p_j$  represents the penalty cost per unit time for late delivery to destination  $j$
- $t_{jk}$  is a continuous decision variable representing delay time for aircraft  $k$  to destination  $j$

This objective function captures four critical cost components: direct route operational costs, aircraft utilization costs, ground handling expenses, and penalty costs for service delays. The mathematical structure ensures that all relevant cost factors are considered simultaneously in the optimization process.

## B2. Constraint Expressions

The optimization problem is subject to several categories of constraints that ensure operational feasibility and service requirements:

### 1. Flow Conservation Constraints:

$$\sum_j x_{ijk} = \sum_j x_{jik} \quad \forall i, k$$

These constraints ensure that aircraft routing maintains consistency, requiring that any aircraft arriving at a location must also depart from that location.

### 2. Aircraft Capacity Constraints:

$$\sum_j (d_j \times x_{ijk}) \leq \text{Cap}_k \times y_k \quad \forall i, k$$

Where  $d_j$  represents demand at destination  $j$  and  $\text{Cap}_k$  represents the cargo capacity of aircraft  $k$ . These constraints prevent overloading of aircraft.

### 3. Demand Satisfaction Constraints:

$$\sum_{ik} x_{ijk} = 1 \quad \forall j$$

These constraints ensure that each destination is served exactly once, meeting Amazon's service commitment requirements.

### 4. Fleet Availability Constraints:

$$\sum_k y_k \leq \text{Fleet\_Size}$$

This constraint ensures that the solution cannot utilize more aircraft than are available in Amazon's fleet.

### 5. Time Window Constraints:

$$\text{arrival\_time}_{jk} \leq \text{latest\_time}_j \quad \forall j, k$$

These constraints ensure that deliveries occur within acceptable time windows for each destination.

### 6. Aircraft Assignment Constraints:

$$\sum_{ij} x_{ijk} \leq M \times y_k \quad \forall k$$

Where  $M$  is a sufficiently large constant. These 'big M' constraints ensure that aircraft can only be assigned to routes if they are designated as active in the solution.

### 7. Binary and Non-negativity Constraints:

$$x_{ijk} \in \{0,1\} \quad \forall i,j,k; y_k \in \{0,1\} \quad \forall k; s_{ik} \in \{0,1\} \quad \forall i,k; t_{jk} \geq 0 \quad \forall j,k$$

### B3. Decision Variables Identification

The optimization model employs four categories of decision variables:

#### Primary Route Assignment Variables ( $x_{ijk}$ ):

These binary variables determine the specific routing assignments for each aircraft. When  $x_{ijk} = 1$ , aircraft  $k$  is assigned to fly from hub  $i$  to destination  $j$ . When  $x_{ijk} = 0$ , this route assignment is not selected. These variables form the core of the optimization solution, directly determining the flight schedule and route network.

#### Aircraft Utilization Variables ( $y_k$ ):

These binary variables indicate whether each aircraft in the fleet is utilized in the optimal solution. When  $y_k = 1$ , aircraft  $k$  is active and assigned to at least one route. When  $y_k = 0$ , aircraft  $k$  remains unused. These variables enable the model to optimize fleet size utilization and associated fixed costs.

#### Stop Assignment Variables ( $s_{ik}$ ):

These binary variables track which aircraft make stops at which locations. When  $s_{ik} = 1$ , aircraft  $k$  makes a stop at location  $i$ , incurring associated handling costs. When  $s_{ik} = 0$ , aircraft  $k$  does not stop at location  $i$ . These variables enable accurate calculation of ground handling and airport service costs.

#### Delay Time Variables ( $t_{jk}$ ):

These continuous variables measure the delay time (in hours) for aircraft  $k$  reaching destination  $j$  beyond the planned schedule. These variables are non-negative real numbers that allow the model to account for operational flexibility while penalizing excessive delays. They enable the optimization to balance schedule adherence against other operational costs.

## C. Optimization Solution Approach

### C1. Optimization Method and Algorithm Identification

The selected optimization methodology is Mixed Integer Linear Programming (MILP) implemented using the Branch-and-Bound algorithm. This approach is specifically chosen based on the mathematical characteristics of the Amazon Air optimization problem and the need for guaranteed optimal solutions.

MILP is the appropriate framework because the problem contains both binary decision variables (route assignments, aircraft utilization) and continuous variables (delay times), while maintaining linear relationships in both the objective function and constraints. The linear

structure of cost functions and constraint relationships satisfies the fundamental requirements for MILP application.

The Branch-and-Bound algorithm provides several critical advantages for this application:

**Optimality Guarantee:** Unlike heuristic approaches, Branch-and-Bound provides mathematical proof that the identified solution is globally optimal within the specified tolerance levels. This is crucial for Amazon Air's cost management objectives.

**Systematic Search Strategy:** The algorithm systematically explores the solution space by branching on integer variables and bounding subproblems based on linear relaxations. This approach efficiently eliminates inferior solutions without exhaustive enumeration.

**Scalability:** Modern MILP solvers implementing Branch-and-Bound can handle problems with thousands of variables and constraints, making them suitable for Amazon Air's operational scale.

**Industry Validation:** MILP with Branch-and-Bound is the standard approach for vehicle routing and airline scheduling problems, providing confidence in methodology selection.

The algorithm process involves:

1. Solving the linear relaxation of the problem
2. If the solution has integer values for integer variables, the optimal solution is found
3. If not, branching occurs by selecting a fractional integer variable and creating subproblems
4. Bounding eliminates subproblems that cannot improve the current best solution
5. The process continues until all subproblems are resolved

## C2. Tools and Technologies Description

The implementation employs a comprehensive technology stack designed for optimization robustness and analytical capability:

**Primary Programming Language - Python 3.9+:** Python is selected as the primary development language due to its extensive optimization libraries, data analysis capabilities, and scientific computing ecosystem. Python's interpreted nature facilitates rapid development and testing while maintaining production-level performance for optimization applications.

**Optimization Library - PuLP:** PuLP (Python Linear Programming) serves as the primary optimization modeling framework. This library provides intuitive mathematical modeling capabilities, allowing direct translation of mathematical formulations into executable code. PuLP supports multiple solver backends and offers excellent documentation and community support.

**Solver Backend - CBC (Coin-or Branch and Cut):** The Coin-or Branch and Cut solver provides the computational engine for solving MILP problems. CBC is an open-source, high-performance solver specifically designed for mixed-integer programming. It implements state-of-the-art Branch-and-Bound algorithms with advanced cutting plane techniques.

**Data Analysis Framework - Pandas and NumPy:** Pandas provides comprehensive data manipulation and analysis capabilities, enabling efficient handling of aircraft, route, and demand data. NumPy supplies fundamental numerical computing operations and array processing functionality.

**Visualization Tools - Matplotlib and Seaborn:** Matplotlib provides foundational plotting and charting capabilities for results visualization. Seaborn extends matplotlib with statistical visualization functions and improved aesthetics. These tools enable comprehensive analysis of optimization results.

**Development Environment - Visual Studio Code:** Visual Studio Code provides the integrated development environment with Python extension support, debugging capabilities, and version control integration.

Additional Technologies:

- Jupyter Notebooks for interactive analysis and documentation
- OpenPyXL for Excel file generation and data export
- SciPy for additional mathematical and statistical functions

This technology stack provides enterprise-grade optimization capabilities while maintaining cost-effectiveness and deployment flexibility.

## D. Risk Assessment and Limitations

### Technical Risks and Mitigation Strategies

#### Computational Complexity Risk:

Large problem instances may require excessive computation time, potentially exceeding practical time limits for operational decision-making. As the number of aircraft, destinations, and routes increases, the solution space grows exponentially, challenging even advanced MILP solvers.

**Mitigation Strategy:** Implement time limits for optimization runs with automatic fallback to best feasible solutions. Develop problem decomposition techniques for extremely large instances, such as geographical clustering or temporal partitioning.

#### Data Quality and Accuracy Risk:

Optimization results are highly sensitive to input data accuracy. Inaccurate distance calculations, cost estimates, or demand forecasts can lead to suboptimal solutions that perform poorly in practice.

**Mitigation Strategy:** Implement comprehensive data validation procedures with range checks, consistency verification, and outlier detection. Establish regular data update cycles and validation against actual operational results.

#### **Model Scalability Limitations:**

Memory and computational limitations may restrict the problem size that can be solved optimally. Very large fleet sizes or extensive route networks may exceed available computing resources.

**Mitigation Strategy:** Consider hierarchical solution approaches for large problems, solving regional subproblems and coordinating results. Implement cloud computing options for handling peak computational demands.

### **Business and Operational Limitations**

#### **Static Model Assumptions:**

The current model assumes static conditions and does not account for real-time changes such as weather conditions, traffic delays, mechanical issues, or demand fluctuations.

**Impact Assessment:** Solutions may require frequent re-optimization to remain relevant. Emergency situations may necessitate manual override of optimized schedules.

#### **Simplified Cost Structure:**

The linear cost assumptions may not accurately reflect actual operational costs, which often exhibit economies of scale, step functions, or other non-linear characteristics.

**Impact Assessment:** Optimized solutions may deviate from true cost minimization. Budget projections based on model results may require adjustment factors.

#### **Implementation and Change Management Challenges:**

Integration with existing Amazon systems requires significant technical effort and change management. Operational staff may resist changes to established procedures.

**Impact Assessment:** Implementation timeline may be extended beyond initial projections. Additional resources may be required for system integration and staff training.



## E. Sources and References

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