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Addition and subtraction of matrices

par Claude Foumtum Notions of sums for matrices in linear algebra In mathematics, matrix addition is the operation of adding two matrices by adding the corresponding entries together. However, there are other operations which could also be considered addition for matrices, such as the direct sum and the Kronecker sum. Entrywise sum Two matrices must have an equal number of rows and columns to be added.[1] In which case, the sum of two matrices A and B will be a matrix which has the same number of rows and columns as A and B. The sum of A and B, denoted A + B,[2] is computed by adding corresponding elements of A and B:[3][4] A + B = [a 11 a 12 ⋯ a 1 n a 21 a 22 ⋯ a 2 n ⋮ ⋮ ⋱ ⋮ a m 1 a m 2 ⋯ a m n] + [b 11 b 12 ⋯ b 1 n b 21 b 22 ⋯ b 2 n ⋮ ⋮ ⋱ ⋮ b m 1 b m 2 ⋯ b m n] = [a 11 + b 11 a 12 + b 12 ⋯ a 1 n + b 1 n a 21 + b 21 a 22 + b 22 ⋯ a 2 n + b 2 n ⋮ ⋮ ⋱ ⋮ a m 1 + b m 1 a m 2 + b m 2 ⋯ a m n + b m n] $\displaystyle {\begin{aligned}\mathbf {A} +\mathbf {B} &={\begin{bmatrix}a_{11}&a_{12}&\cdots &a_{1n}\\a_{21}&a_{22}&\cdots &a_{2n}\\\vdots &\vdots &\ddots &\vdots \\a_{m1}&a_{m2}&\cdots &a_{mn}\end{bmatrix}}+{\begin{bmatrix}b_{11}&b_{12}&\cdots &b_{1n}\\b_{21}&b_{22}&\cdots &b_{2n}\\\vdots &\vdots &\ddots &\vdots \\b_{m1}&b_{m2}&\cdots &b_{mn}\end{bmatrix}}\\&={\begin{bmatrix}a_{11}+b_{11}&a_{12}+b_{12}&\cdots &a_{1n}+b_{1n}\\a_{21}+b_{21}&a_{22}+b_{22}&\cdots &a_{2n}+b_{2n}\\\vdots &\vdots &\ddots &\vdots \\a_{m1}+b_{m1}&a_{m2}+b_{m2}&\cdots &a_{mn}+b_{mn}\end{bmatrix}}\\\end{aligned}}\,\!$ Or more concisely (assuming that A + B = C):[5][6] c i j = a i j + b i j $\displaystyle c_{ij}=a_{ij}+b_{ij}$ For example: [1 3 1 0 1 2] + [0 0 7 5 2 1] = [1 + 0 3 + 0 1 + 7 0 + 5 1 + 2 2 + 1] = [1 3 8 5 3 3] $\displaystyle {\begin{bmatrix}1&3\\1&0\\1&2\end{bmatrix}}+{\begin{bmatrix}0&0\\7&5\\2&1\end{bmatrix}}={\begin{bmatrix}1+0&3+0\\1+7&0+5\\1+2&2+1\end{bmatrix}}=$ Similarly, it is also possible to subtract one matrix from another, as long as they have the same dimensions. The difference of A and B, denoted A − B,[2] is computed by subtracting elements of B from corresponding elements of A, and has the same dimensions as A and B. For example: [1 3 1 0 1 2] − [0 0 7 5 2 1] = [1 − 0 3 − 0 1 − 7 0 − 5 1 − 2 2 − 1] = [1 3 − 6 − 5 − 1 1] $\displaystyle {\begin{bmatrix}1&3\\1&0\\1&2\end{bmatrix}}-{\begin{bmatrix}0&0\\7&5\\2&1\end{bmatrix}}={\begin{bmatrix}1-0&3-0\\1-7&0-5\\1-2&2-1\end{bmatrix}}={\begin{bmatrix}1&3\\-6&-5\\-1&1\end{bmatrix}}$ Direct sum Another operation, which is used less often, is the direct sum (denoted by ⊕). Note the Kronecker sum is also denoted ⊕; the context should make the usage clear. The direct sum of any pair of matrices A of size m × n and B of size p × q is a matrix of size (m + p) × (n + q) defined as:[7][3] A ⊕ B = [A 0 0 B] = [a 11 ⋯ a 1 n 0 ⋯ 0 ⋮ ⋱ ⋮ ⋮ ⋱ ⋮ a m 1 ⋯ a m n 0 ⋯ 0 0 ⋯ 0 b 11 ⋯ b 1 q ⋮ ⋱ ⋮ ⋮ ⋱ ⋮ 0 ⋯ 0 b p 1 ⋯ b p q] $\displaystyle \mathbf {A} \oplus \mathbf {B} ={\begin{bmatrix}\mathbf {A} &{\boldsymbol {0}}\\{\boldsymbol {0}}&\mathbf {B} \end{bmatrix}}={\begin{bmatrix}a_{11}&\cdots &a_{1n}&0&\cdots &0\\\vdots &\ddots &\vdots &\vdots &\ddots &\vdots \\a_{m1}&\cdots &a_{mn}&0&\cdots &0\\0&\cdots &0&b_{11}&\cdots &b_{1q}\\\vdots &\ddots &\vdots &\vdots &\ddots &\vdots \\0&\cdots &0&b_{p1}&\cdots &b_{pq}\end{bmatrix}}$ For instance, [1 3 2 2 3 1] ⊕ [1 6 0 1] = [1 3 2 0 0 2 3 1 0 0 0 0 0 1 6 0 0 0 0 1] $\displaystyle {\begin{bmatrix}1&3&2\\2&3&1\end{bmatrix}}\oplus {\begin{bmatrix}1&6\\0&1\end{bmatrix}}=$ The direct sum of matrices is a special type of block matrix. In particular, the direct sum of square matrices is a block diagonal matrix. The adjacency matrix of the union of disjoint graphs (or multigraphs) is the direct sum of their adjacency matrices. Any element in the direct sum of two vector spaces of matrices can be represented as a direct sum of two matrices. In general, the direct sum of n matrices is:[3] ⊕ i = 1 n A i = diag (A 1 , A 2 , A 3 , ... , A n) = [A 1 0 ⋯ 0 0 A 2 ⋯ 0 ⋮ ⋮ ⋱ ⋮ 0 0 ⋯ A n] $\displaystyle \bigoplus _{i=1}^{n}\mathbf {A} _{i}=\operatorname {diag} (\mathbf {A} _{1},\mathbf {A} _{2},\mathbf {A} _{3},\ldots ,\mathbf {A} _{n})={\begin{bmatrix}\mathbf {A} _{1}&{\boldsymbol {0}}&\cdots &{\boldsymbol {0}}\\{\boldsymbol {0}}&\mathbf {A} _{2}&\cdots &{\boldsymbol {0}}\\\vdots &\vdots &\ddots &\vdots \\{\boldsymbol {0}}&{\boldsymbol {0}}&\cdots &\mathbf {A} _{n}\end{bmatrix}}\,\!$ where the zeros are actually blocks of zeros (i.e., zero matrices). Kronecker sum Main article: Kronecker sum The Kronecker sum is different from the direct sum, but is also denoted ⊕. It is defined using the Kronecker product ⊗ and normal matrix addition. If A is n-by-n, B is m-by-m and I k $\displaystyle \mathbf {I} _{k}$ denotes the k-by-k identity matrix then the Kronecker sum is defined by: A ⊕ B = A ⊗ I m + I n ⊗ B . $\displaystyle \mathbf {A} \oplus \mathbf {B} =\mathbf {A} \otimes \mathbf {I} _{m}+\mathbf {I} _{n}\otimes \mathbf {B} .$ See also Matrix multiplication Vector addition Notes ^ Elementary Linear Algebra by Rorres Anton 10e p53 ^ a b "Comprehensive List of Algebra Symbols". Math Vault. 2020-03-25. Retrieved 2020-09-07. ^ a b c Lipschutz & Lipson. sfn error: no target: CITEREFLipschutzLipson (help) ^ Riley, K.F.; Hobson, M.P.; Bence, S.J. (2010). Mathematical methods for physics and engineering. Cambridge University Press. ISBN 978-0-521-86153-3. ^ Weisstein, Eric W. "Matrix Addition". mathworld.wolfram.com. Retrieved 2020-09-07. ^ "Finding the Sum and Difference of Two Matrices | College Algebra". courses.lumenlearning.com. Retrieved 2020-09-07. ^ Weisstein, Eric W. "Matrix Direct Sum". MathWorld. References Lipschutz, S.; Lipson, M. (2009). Linear Algebra. Schaum's Outline Series. ISBN 978-0-07-154352-1. External links Direct sum of matrices at PlanetMath. Abstract nonsense: Direct Sum of Linear Transformations and Direct Sum of Matrices Mathematics Source Library: Arithmetic Matrix Operations Matrix Algebra and R Retrieved from "