Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Introduction to Partial Differential Equations
COMPLETE SOLUTION SET

1. A partial differential equation has
   - (A) one independent variable
   - (B) two or more independent variables
   - (C) more than one dependent variable
   - (D) equal number of dependent and independent variables

Solution
The correct answer is (B).

If a differential equation has only one independent variable then it is called ordinary differential equation. A partial differential equation has two or more independent variables.

2. A solution to the partial differential equation.

$$\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$$

is

(A) $\cos(3x - y)$

(B) $x^2 + y^2$

(C) $\sin(3x - y)$

(D) $e^{-3\pi x}\sin(\pi y)$

Solution

The correct answer is (D).

We will solve this by substituting the given choices. The choice which satisfies the partial differential equation is the correct answer.

Let's start with option (A)

$$u = \cos(3x - y)$$

$$\frac{\partial u}{\partial x} = -3\sin(3x - y)\,;\frac{\partial u}{\partial y} = \sin(3x - y)$$

$$\frac{\partial^2 u}{\partial x^2} = -9\cos(3x - y)\,;\frac{\partial^2 u}{\partial y^2} = -\cos(3x - y)$$

Substituting in partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$$

$$-9\cos(3x - y) = -9\cos(3x - y)$$

Let's start with option (B)

$$u = x^2 + y^2$$

$$\frac{\partial u}{\partial x} = 2x\,;\frac{\partial u}{\partial y} = 2y$$

$$\frac{\partial^2 u}{\partial x^2} = 2\,;\frac{\partial^2 u}{\partial y^2} = 2$$

Substituting in partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$$

$$2 \neq 9 \times 2$$

Let's start with option (C)

$$u = \sin(3x - y)$$

$$\frac{\partial u}{\partial x} = 3\cos(3x - y)\,; \frac{\partial u}{\partial y} = -\cos(3x - y)$$

$$\frac{\partial^2 u}{\partial x^2} = -9\sin(3x - y)\,; \frac{\partial^2 u}{\partial y^2} = -\sin(3x - y)$$

Substituting in partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$$

$$-9\sin(3x - y) = -9\sin(3x - y)$$

Let’s start with option (D)

$$u = e^{-3\pi x}\sin(\pi y)$$

$$\frac{\partial u}{\partial x} = -3\pi e^{-3\pi x}\sin(\pi y)\,; \frac{\partial u}{\partial y} = \pi e^{-3\pi x}\cos(\pi y)$$

$$\frac{\partial^2 u}{\partial x^2} = 9\pi^2 e^{-3\pi x}\sin(\pi y)\,; \frac{\partial^2 u}{\partial y^2} = -\pi^2 e^{-3\pi x}\sin(\pi y)$$

Substituting in partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$$

$$9\pi^2 e^{-3\pi x}\sin(\pi y) \neq -9\pi^2 e^{-3\pi x}\sin(\pi y)$$

3. The partial differential equation

$$5\frac{\partial^2 z}{\partial x^2}+6\frac{\partial^2 z}{\partial y^2}=xy$$

is classified as

(A) elliptic
(B) parabolic
(C) hyperbolic
(D) none of the above

Solution

The correct answer is (A).

A general second order partial differential equation with two independent variables is of the form

$$A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}+D=0$$

where A, B, and C are functions of x and y and D is a function of x, y, u and $\frac{\partial u}{\partial x},\frac{\partial u}{\partial y}$.

The above PDE can be rewritten as

$$5\frac{\partial^2 z}{\partial x^2}+0\frac{\partial^2 z}{\partial x\partial y}+6\frac{\partial^2 z}{\partial y^2}-xy=0$$

Depending on the value of B^2-4AC, the 2nd order linear PDE can be classified into three categories.

1. if $B^2-4AC<0$, it is called elliptic
2. if $B^2-4AC=0$, it is called parabolic
3. if $B^2-4AC>0$, it is called hyperbolic

In the above question,

$$A=5, B=0, C=6,$$

giving

$$\begin{aligned}B^2-4AC&=0-4(5)(6)\\&=-120<0\end{aligned}$$

This classifies the differential equation as elliptic.

4. The partial differential equation

$$xy\frac{\partial z}{\partial x} = 5\frac{\partial^2 z}{\partial y^2}$$

is classified as

(A) elliptic
(B) parabolic
(C) hyperbolic
(D) none of the above

Solution

The correct answer is (B).

A general second order partial differential equation with two independent variables is of the form

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D = 0$$

where A, B, and C are functions of x and y and D is a function of x, y, u and $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$.

The above PDE can be rewritten as

$$0\frac{\partial^2 z}{\partial x^2} + 0\frac{\partial^2 z}{\partial x \partial y} + 5\frac{\partial^2 z}{\partial y^2} - xy\frac{\partial z}{\partial x} = 0$$

Depending on the value of $B^2 - 4AC$, the 2nd order linear PDE can be classified into three categories.

1. if $B^2 - 4AC < 0$, it is called elliptic
2. if $B^2 - 4AC = 0$, it is called parabolic
3. if $B^2 - 4AC > 0$, it is called hyperbolic

In the above question,

$$A = 0, B = 0, C = 5,$$

giving

$$\begin{aligned} B^2 - 4AC &= 0 - 4(0)(5) \\ &= 0 \end{aligned}$$

This classifies the differential equation as parabolic.

5. The partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial y^2} = 0$$

is classified as

(A) elliptic
(B) parabolic
(C) hyperbolic
(D) none of the above

Solution

The correct answer is (C).

A general second order partial differential equation with two independent variables is of the form

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D = 0$$

where A, B, and C are functions of x and y and D is a function of x, y, u and $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$.

The above PDE can be rewritten as

$$1\frac{\partial^2 z}{\partial x^2} + 0\frac{\partial^2 z}{\partial x \partial y} - 5\frac{\partial^2 z}{\partial y^2} = 0$$

Depending on the value of $B^2 - 4AC$, the 2nd order linear PDE can be classified into three categories.

1. if $B^2 - 4AC < 0$, it is called elliptic
2. if $B^2 - 4AC = 0$, it is called parabolic
3. if $B^2 - 4AC > 0$, it is called hyperbolic

In the above question,

$$A = 1, B = 0, C = -5,$$

giving

$$\begin{aligned} B^2 - 4AC &= 0 - 4(1)(-5) \\ &= 20 > 0 \end{aligned}$$

This classifies the differential equation as hyperbolic.

6. The following is true for the following partial differential equation used in nonlinear mechanics known as the Korteweg-de Vries equation.

$$\frac{\partial w}{\partial t} + \frac{\partial^3 w}{\partial x^3} - 6w\frac{\partial w}{\partial x} = 0$$

(A) linear; 3rd order
(A) nonlinear; 3rd order
(B) linear; 1st order
(C) nonlinear; 1st order

Solution

The correct answer is (B).

The partial differential equation is nonlinear because the coefficient of the derivative term $\frac{\partial w}{\partial x}$ is a function of the dependent variable, w. The equation is a 3rd order as that is the highest derivative in the partial differential equation.