Double-Sideband Suppressed Carrier (DSB-SC) Modulation

Communication that uses modulation to shift the frequency spectrum of a signal is also known as carrier modulation. In this category, one of the basic parameters like amplitude, frequency and phase of a sinusoidal carrier of high frequency (W_c) is varied in proportion to the baseband signal/message signal $m(t)$.

In amplitude modulation, the amplitude of the carrier $A_c \cos(W_c t + \theta_c)$ is varied in proportion to the baseband signal $m(t)$. In this case, the frequency(W_c) and the phase (θ_c) are constant.

Mathematical Analysis

Assume that, $m(t)$ and $c(t)$ are the message signal and carrier signal, respectively which can be expressed as,

$$m(t) = A_m \cos W_m t$$
$$c(t) = A_c \cos(W_c t + \theta_c)$$

To achieve, frequency translation, we have to multiply these two signals. Hence the modulated signal can be found by,

$$\begin{aligned}
x_{DSB-SC}(t) &= m(t)\, c(t) \\
&= (A_m \cos(W_m t))(A_c \cos(W_c t + \theta_c)) \\
&= \frac{A_m A_c}{2}\left[2 \cos(W_m t) \cos(W_c t + \theta)\right] \\
&= \frac{A_m A_c}{2}\left[\cos(W_c t + \theta_c - W_m t) + \cos(W_c t + \theta_c + W_m t)\right] \\
&= \frac{A_m A_c}{2}\left[\cos[(W_c - W_m)t + \theta_c] + \cos[(W_c + W_m)t + \theta_c]\right]
\end{aligned}$$

It is important to note that, since $m(t)$ is represented by a single frequency sinusoidal function, some time this is also known as tone modulation. For generalized message signal, the expression modifies to,

$$\begin{aligned}
x_{DSB-SC}(t) &= m(t)\, A_c \cos(W_c t + \theta_c) \\
&= A_c\, m(t) \cos(W_c t + \theta_c)
\end{aligned}$$

From the concept of the Fourier Transformation, the spectrum of the modulated signal can be represented as,

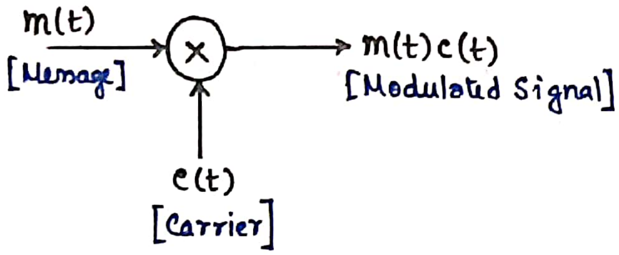
$$X_{DSB-SC}(W) \leftrightarrow \frac{1}{2}\left[M(W + W_c) + M(W - W_c)\right]$$

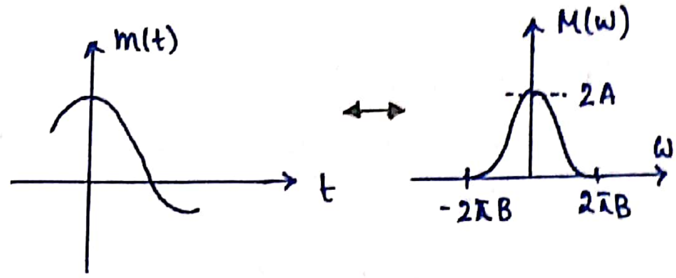
Where we have assumed,

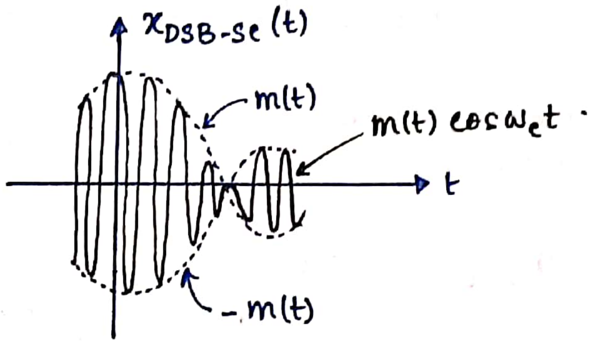
$$\theta_c = 0$$

$$m(t) \leftrightarrow M(\omega)$$

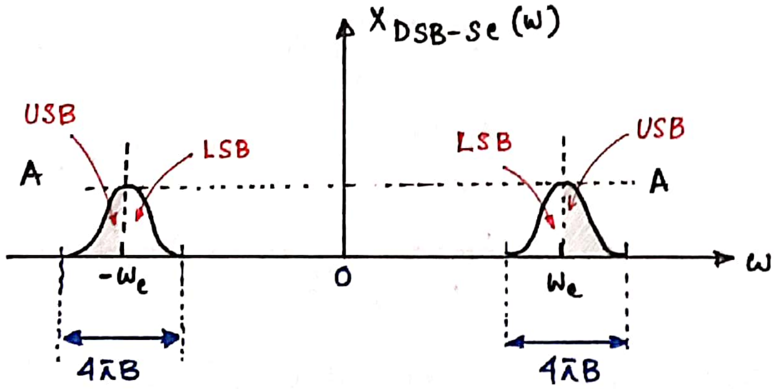
Now, recall that $M(\omega - \omega_c)$ is shifted version of $M(\omega)$ to the right by ω_c and $M(\omega + \omega_c)$ is the shifted version of $M(\omega)$ to the left by ω_c. Also, the bandwidth of the signal doubles in the modulated signal. Thus if the bandwidth was earlier B (Hz) then the modulated signal bandwidth becomes $2B$ (Hz).







From the frequency domain spectra of the modulated signal, we observe that the modulated signal spectrum at ω_c is composed of two parts: a portion which lies above ω_c and this is termed to be as the upper side band (USB); another portion which lies below ω_c and this is termed as the lower side band (LSB). Similar, USB and LSB are also present in the spectrum centered at $-\omega_c$.



Note that, this signal has no discrete component of the carrier frequency ω_c. For this reason it will be referred as suppressed carrier modulation scheme. Also, the modulation scheme has two side bands in the frequency domain and thus it will be termed as double side band modulation scheme. Hence, the overall name of this modulation scheme is double-sideband suppressed carrier or in short DSB-SC.

The relationship of B to W_c is of interest. Figure of frequency domain spectra of the modulated signal shows that if $W_c \geq 2\pi B$, then only overlap of the spectra of the two components centered at W_c and $-W_c$ can be avoided. If $W_c < 2\pi B$, then two spectra overlap and information of the $m(t)$ is lost in the process. In such scenario, it is impossible to get back the information from the modulated signal $m(t)\cos W_c t$.

* **Demodulation of DSB-SC signal**

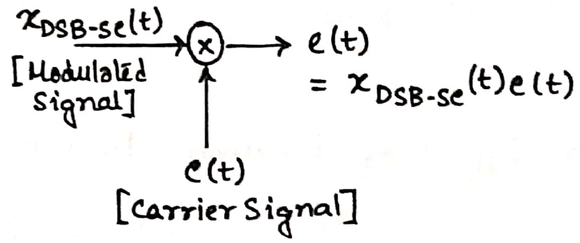
For the demodulation of DSB-SC signal, it is necessary to retranslate the spectrum to its original position. The process of recovering the message signal from the modulated signal is referred to as demodulation or detection.

Observe that, if the modulated signal spectrum is shifted to the left and to the right by 'W_c' we can obtain the spectrum of the original signal and some unwanted spectrum at $\pm 2W_c$. The unwanted spectrum can be easily be suppressed by the help of a low pass filter (LPF). Thus the demodulation of DSB-SC signal is almost identical to modulation scheme, where modulated signal is once again multiplied with the help of the carrier signal.

Mathematical Analysis

Demodulated signal can be obtained from the modulated signal as,

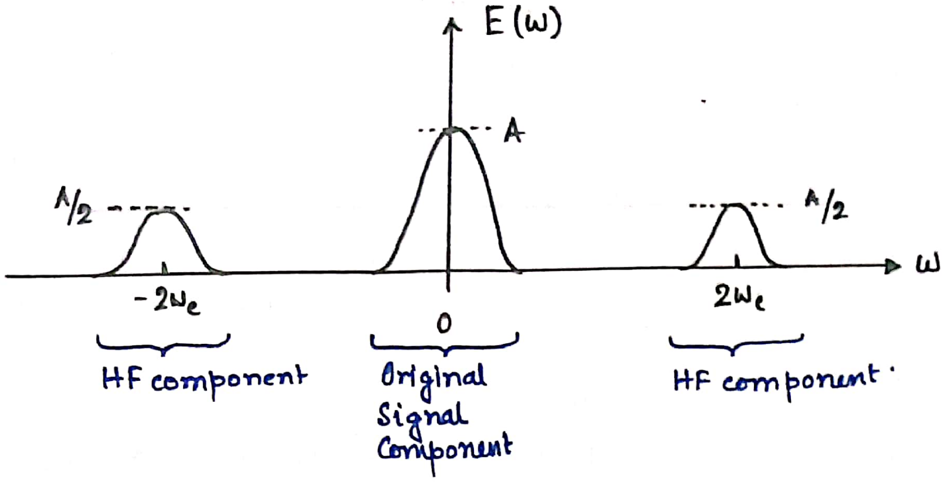
$$
\begin{aligned}
e(t) &= x_{DSB\text{-}SC}(t)\, c(t) \\
&= [m(t)\cos W_c t](\cos W_c t) \\
&= m(t)\cos^2 W_c t \\
&= \frac{1}{2}[m(t) + m(t)\cos 2W_c t] \\
&= \frac{m(t)}{2} + \underbrace{\frac{m(t)}{2}\cos 2W_c t}_{\text{HF component}}
\end{aligned}
$$



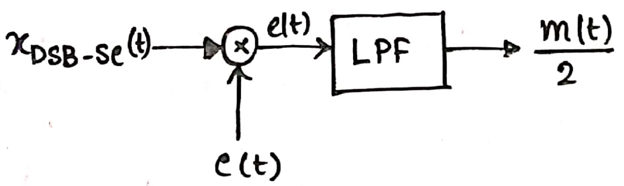
Thus, the frequency domain spectra of the frequency retranslated signal can mathematically be represented as,

$$E(W) = \frac{1}{2}M(W) + \frac{1}{4}[M(W+2W_c) + M(W-2W_c)]$$

The frequency domain spectra of such signal can be shown as,



Thus applying a simple low pass filter can remove the HF unwanted components, and we can retrive the original signal. Hence, the complete block diagram of the demodulator will be-



It is to be noted that, this method of recovering the base band signal is called synchronous detection or coherent detection. This is due to the fact, we use a carrier of exactly same frequency and phase as the carrier used for modulation. Thus for demodulation we need to generate a local carrier at the receiver end in frequency and phase coherence (synchronism) with the carrier used at the modulator.

⊛ Effect of Frequency Deviation in the receiver

We now try to estimate what will happen if there is some frequency deviation present in the carrier at the receiver end.

Let us assume that, the frequency of the carrier that is used to demodulate the DSB-SC signal has an offset of $\Delta\omega$. Then after the product we have,

$$e(t) = [m(t) \cos \omega_c t] (\cos(\omega_c + \Delta\omega)t)$$

$$= m(t) \cos(\omega_c t) \cos[(\omega_c + \Delta\omega)t]$$

$$= \frac{m(t)}{2} [\cos(\omega_c t - (\omega_c + \Delta\omega)t)] + \frac{m(t)}{2} [\cos(\omega_c t + (\omega_c + \Delta\omega)t)]$$

$$= \frac{m(t)}{2} \cos(\Delta\omega t) + \underbrace{\frac{m(t)}{2} \cos[2\omega_c t + \Delta\omega t]}_{\text{HF component}}$$

Similar to the coherent demodulator, the LPF will remove the second term leaving the output as,

$$e_o(t) = m(t)\cos(\Delta\omega t)$$

The output of the demodulator is not merely an attenuated replica but is also distorted. Generally, $\Delta\omega$ is small and thus the signal $m(t)$ by a LF sinusoid. This causes the amplitude of the desired signal $m(t)$ to vary from maximum to zero periodically at twice the period of the beat frequency ($\Delta\omega$). This beating effect is catastrophic even for very small value of $\Delta\omega$.

Effect of Phase Mismatch

Now consider, the local carrier signal has a phase offset of δ in the DSB-SC demodulator. In such case, the demodulator output prior to the LPF can be estimated as,

$$
\begin{aligned}
e(t) &= [m(t)\cos\omega_c t]\cos(\omega_c t+\delta)\\
&= \frac{m(t)}{2}[2\cos(\omega_c t)\cos(\omega_c t+\delta)]\\
&= \frac{m(t)}{2}\cos(\omega_c t-\omega_c t-\delta) + \frac{m(t)}{2}\cos(\omega_c t+\omega_c t+\delta)\\
&= \frac{m(t)}{2}\cos(-\delta) + \frac{m(t)}{2}\cos(2\omega_c t+\delta)\\
&= \frac{m(t)}{2}\cos(\delta) + \underbrace{\frac{m(t)}{2}\cos(2\omega_c t+\delta)}_{\text{HF component}}
\end{aligned}
$$

Similar to the DSB-SC demodulator in ideal case, the second term of the $e(t)$ will be filtered out by the LPF. Hence, the output of the demodulator becomes,

$$e_o(t) = \frac{m(t)}{2}\cos\delta$$

The output is proportional to $m(t)$ if δ = constant. The output is maximum if $\delta = 0$ and minimum (zero) if $\delta = \pi/2$. Thus the phase error of the carrier signal induces attenuation in the output signal without causing any distortion as long as δ = constant. However, in real life δ varies randomly with time and thus the demodulated signal in the receiver will become randomly varying $m(t)$.

If both phase and frequency mismatch present in the local carrier signal, then the expression of the output signal $e_o(t)$ becomes,

$$e_o(t) = \cos[\Delta W t + \delta]\, m(t).$$

$$(= m(t) \quad \text{iff} \quad \Delta W = \delta = 0)$$

Thus, we can say that neither frequency nor phase mismatch is desirable at the receiver end for synchronous demodulation. Also, the output of the coherent receiver becoming null at $\delta = \pi/2$ is also referred to as quadrature null effect.