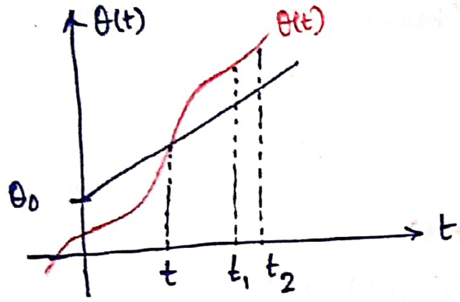
* Angle Modulation

$$\varphi(t) = \underbrace{A}_{\text{Amplitude}} \cos \underbrace{\theta(t)}_{\text{Angle Part.}}$$

$\theta(t)$ = Genaralized Angle. as a fu^n. of time.

$= W_c t + \theta_0$



$$\varphi(t) = A\cos(W_c t + \theta_0) \quad \text{for } t_1 < t < t_2.$$

* Instantaneous freq^n: $\omega_i = \frac{d\theta(t)}{dt}$ (Definition)

$$\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha)\,d\alpha.$$

* Equation of PM and FM

PM

$$\theta(t) = W_c t + \theta_0 + k_p m(t)$$

$$= W_c t + k_p m(t) \quad (\because \theta_0 = 0) \ \langle\text{Assumption}\rangle$$

$$\boxed{\varphi_{PM}(t) = A\cos[W_c t + k_p m(t)]}$$

FM

$$\omega_i(t) = W_c + k_f m(t)$$

$$\frac{d\theta(t)}{dt} = W_c + k_f m(t)$$

$$\theta(t) = \int_{-\infty}^{t} W_c\,d\alpha + \int_{-\infty}^{t} k_f m(\alpha)\,d\alpha$$

$$= W_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha.$$

$$\therefore \boxed{\varphi_{FM}(t) = A\cos\left[W_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right]}$$

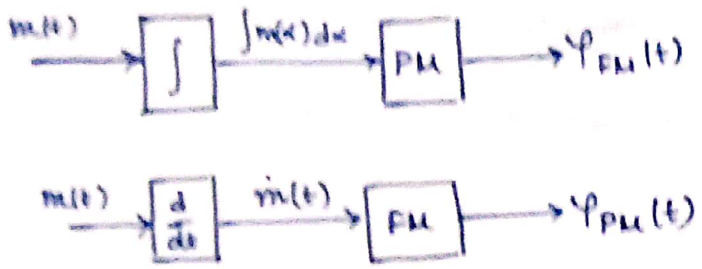
For tone modulation, $m(t) = A_m \cos W_m t$.

$$\varphi_{FM}(t) = A\cos\left(W_c t + k_f \int_{-\infty}^{t} A_m \cos(W_m \alpha)\,d\alpha.\right)$$

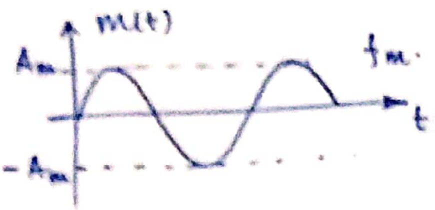
$$= A\cos\left(W_c t + \frac{k_f A_m}{W_m}\sin W_m t\right) = A\cos(W_c t + m_f \sin W_m t)$$

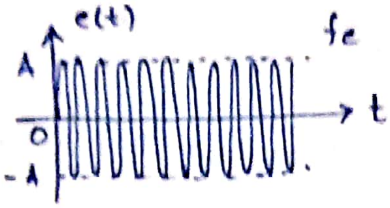
modulation index.

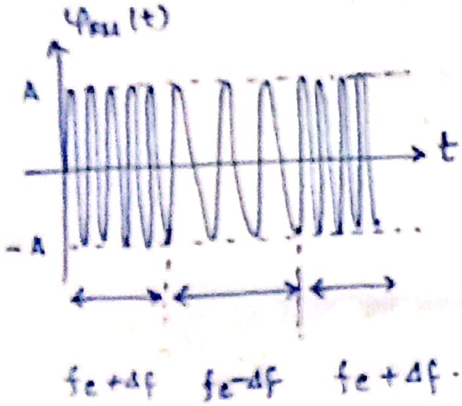
* Rel^n bet^n FM and PM

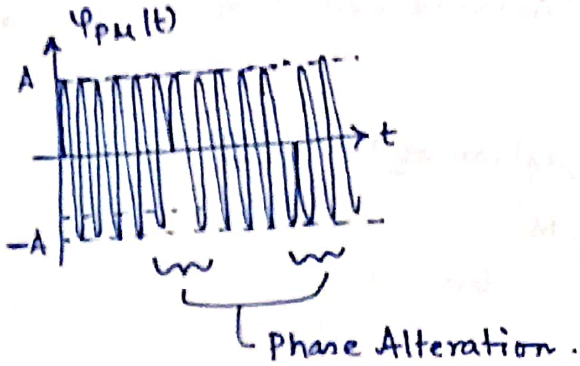


* Waveform of FM and PM









$$f_i = f_c + \frac{k_f}{2\pi} m(t)$$

$$(f_i)_{min} = f_c + \frac{k_f}{2\pi} [m(t)]_{min}$$

$$(f_i)_{max} = f_c + \frac{k_f}{2\pi} [m(t)]_{max}$$

$$f_i = f_c + \frac{k_p}{2\pi} \dot{m}(t)$$

$$(f_i)_{min} = f_c + \frac{k_p}{2\pi} [\dot{m}(t)]_{min}$$

$$(f_i)_{max} = f_c + \frac{k_p}{2\pi} [\dot{m}(t)]_{max}.$$

$m(t) = A_m \sin \omega_m t$.

$m(t)|_{max} = +A_m$; $m(t)|_{min} = -A_m$

$$f_i|_{max} = f_c + \frac{k_f A_m}{2\pi} = f_c + \Delta f$$

$$f_i|_{min} = f_c - \frac{k_f A_m}{2\pi} = f_c - \Delta f$$

* BW of Angle Modulated Signals

Assume,
$$a(t) = \int_{-\infty}^{t} m(\alpha)\,d\alpha.$$

$$\hat{\varphi}_{FM}(t) = A\, e^{j(\omega_c t + k_f a(t))} = \underbrace{A\cos(\omega_c t + k_f a(t))}_{\varphi_{FM}(t)} + jA\sin(\omega_c t + k_f a(t))$$

$$= A e^{j\omega_c t} e^{jk_f a(t)}$$

$\Delta f = (k_f A_m)/2\pi$.

k_f = Deviation Sensitivity $= \frac{(\Delta f) 2\pi}{A_m}$

Now

$$\varphi_{FM}(t) = Re[\hat{\varphi}_{FM}(t)]$$

$$\hat{\varphi}_{FM}(t) = A e^{jk_f a(t)} e^{j\omega_c t}$$
$$= A\left[1 + jk_f a(t) - \frac{k_f^2 a^2(t)}{2!} + \dots + j^n \frac{k_f^n}{n!} a^n(t) + \dots\right] e^{j\omega_c t}$$

$$\varphi_{FM}(t) = Re\left[\hat{\varphi}_{FM}(t)\right]$$
$$= A\left[\cos\omega_c t - k_f a(t)\sin\omega_c t - \frac{k_f^2}{2!} a^2(t)\cos\omega_c t + \frac{k_f^3}{3!} a^3(t)\sin\omega_c t + \dots\right] \quad \dots (1)$$

Modulated wave consists of an unmodulated carrier plus various amplitude modulated terms: $a(t)\sin\omega_c t$, $a^2(t)\cos\omega_c t$, $a^3(t)\sin\omega_c t$, ...

Recall, $a(t) = \int_{-\infty}^{t} m(\alpha)\,d\alpha$.

Thus, if $M(\omega)$ is band-limited to B, hence $A(\omega)$ will be also band limited to B. Similarly $a^2(t)$ spectrum will be band-limited to 2B. Thus, spectrum of $a^n(t)$ is band limited to nB. Thus overall spectrum of the angle modulated signal is not band limited, rather it has infinite BW.

(*) Narrow Band Angle Mod^n.

Angle mod^n. is nonlinear.

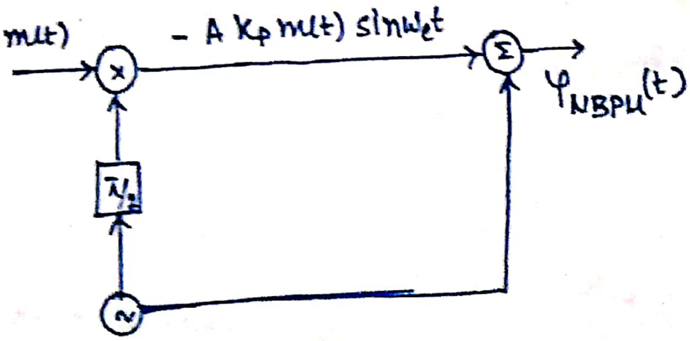
Now if $|k_f a(t)| \ll 1$, then only first two terms of eq^n (1) is an good approximation and we can write,

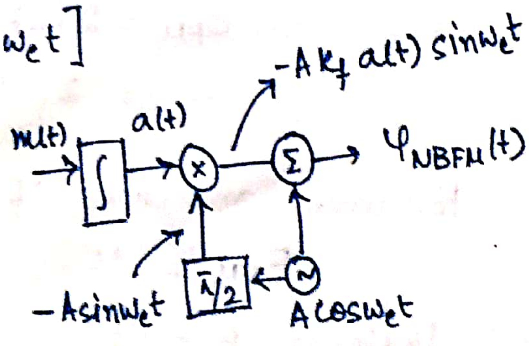
$$\varphi_{FM}(t) \cong A\left[\cos\omega_c t - k_f a(t)\sin\omega_c t\right]$$

Spectrum of $a(t)$: B

Spectrum of FM/PM : 2B.

$$\varphi_{PM}(t) \cong A\left[\cos\omega_c t - k_p m(t)\sin\omega_c t\right]$$



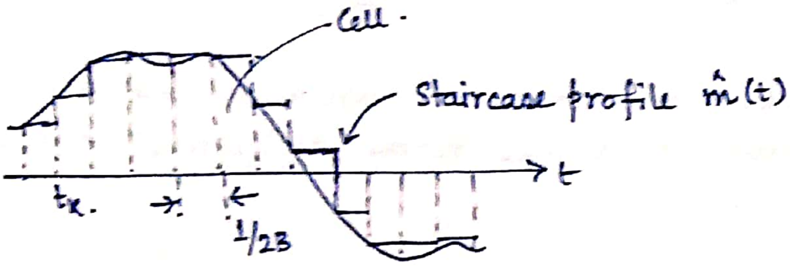


* Wide Band FM

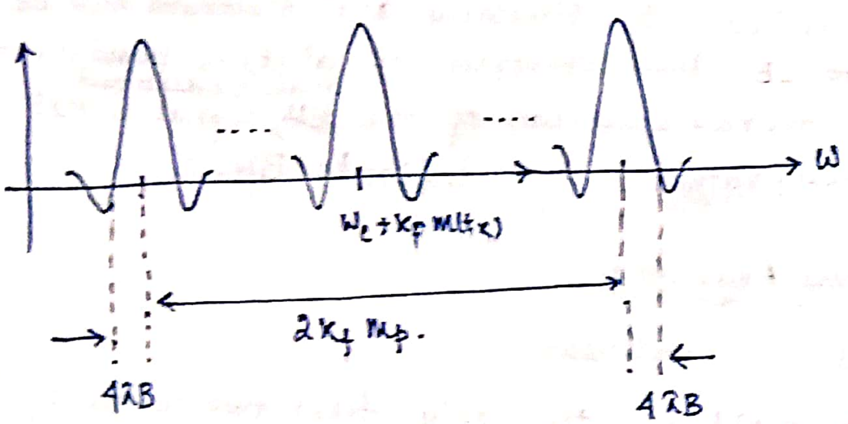
Now assume, K_f is chosen such that,

$$|K_f a(t)| << 1 \text{ not satisfied}$$

or, $|K_f a(t)| > 1$.



$$\Delta f = \frac{K_f m_p}{2\pi}$$ } m_p: Peak Amplitudes of $m(t)$



$$BW = \frac{1}{2\pi}\left(2K_f m_p + 4\pi B + 4\pi B\right)$$

$$= 2(\Delta f + 2B)$$

$$\simeq 4B \quad (\text{ignoring } \Delta f)$$

Better BW estimation,

$$B_{FM} = 2(\Delta f + B)$$

$$= 2\left(\frac{K_f m_p}{2\pi} + B\right)$$

For wide band case, $\Delta f >> B$.

$$B_{FM} \simeq 2\Delta f.$$

Defining, $\beta = \frac{\Delta f}{B}$

$$B_{FM} = 2(B\beta + B) = 2B(\beta + 1).$$

↳ Mod^n. Index. → Special case for Tone Modulated signal.

* Phase Modulation and Bandwidth

$$\omega_i = \omega_c + k_p \dot{m}(t)$$
$$= \omega_c + k_p[\dot{m}_p]_{max}$$
$$= \omega_c + \underbrace{\Delta\omega}$$

$$|\dot{m}_q(t)|_{min} = m_p'$$

where,

$$\left.\begin{aligned} \dot{m}(t)|_{min} &= m_p' \\ \dot{m}(t)|_{max} &= m_p' \end{aligned}\right\}$$

$$B_{PM} = 2(\Delta f + B)$$
$$= 2\left(\frac{k_p m_p'}{2\pi} + B\right)$$

$$\Delta\omega = k_p m_p'$$
$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{k_p m_p'}{2\pi}$$

* FM Bandwidth : Carson's Relation.

Assume, $m(t) = \alpha \cos\omega_m t$

$$a(t) = \frac{\alpha}{\omega_m} \sin\omega_m t$$

$$a(t) = \int_{-\infty}^{t} m(\alpha)\, d\alpha$$
$$= \int_{-\infty}^{t} \alpha \cos\omega_m t = \frac{\alpha}{\omega_m} \sin\omega_m t.$$

$$\hat{\varphi}_{FM}(t) = A\left[e^{\left(j\omega_c t + \frac{k_f \alpha}{\omega_m}\sin\omega_m t\right)}\right]$$

$$\Delta\omega = k_f m_p = \alpha k_f. \longrightarrow m_p = \max(\alpha \cos\omega_m t) = \alpha.$$

Assume the BW of $m(t)$ is B (Hz).

$$\beta = \frac{\Delta f}{f_m} = \frac{\Delta\omega}{\omega_m} = \frac{\alpha k_f}{\omega_m}.$$

Hence,

$$\hat{\varphi}_{FM}(t) = A e^{j(\omega_c t + k_f a(t))}$$
$$= A e^{j\left(\omega_c t + \frac{k_f \alpha}{\omega_m}\sin\omega_m t\right)}$$
$$= A e^{j\omega_c t} \underbrace{e^{j\beta \sin\omega_m t}}_{\text{Periodic Signal. with period } = \frac{2\pi}{\omega_m}.}$$

$$e^{j\beta \sin\omega_m t} = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_m t}$$

where,

$$c_n = \frac{\omega_m}{2\pi} \int_{-\pi/\omega_m}^{\pi/\omega_m} e^{j\beta \sin\omega_m t} \cdot e^{-jn\omega_m t}\, dt$$

Alternatively,

$$e^{j\beta \sin\omega_m t} = \sum_{n=-\infty}^{+\infty} J_n(\beta)\, e^{jn\omega_m t}.$$

$$\hat{\varphi}_{FM}(t) = A\sum_{n=-\infty}^{+\infty} J_n(\beta)\, e^{j(\omega_c t + n\omega_m t)}$$

$$\varphi_{FM}(t) = Re\left\{\hat{\varphi}_{FM}(t)\right\}$$

$$= Re\left\{A\sum_{n=-\infty}^{+\infty} J_n(\beta)\left(\cos(\omega_c t + n\omega_m t) + j\sin(\omega_c t + n\omega_m t)\right)\right\}$$

$$= A\sum_{n=-\infty}^{+\infty} J_n(\beta)\cos(\omega_c t + n\omega_m t)$$

Modulated signals has a carrier component and an infinite number of sidebands at frequencies,

$$\omega_c \pm \omega_m ;\ \omega_c \pm 2\omega_m ,\ \dots ,\ \omega_c \pm n\omega_m .$$

Strength of n^{th} signal will be,

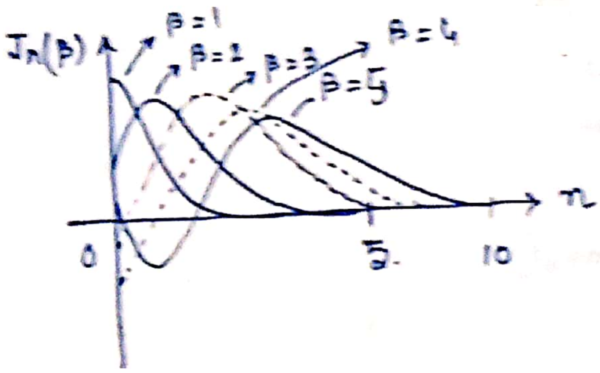
$$\omega = \omega_c + n\omega_m \rightarrow J_n(\beta)$$

BW of FM signal is,

$$B_{FM} = 2n \quad f_m$$

$$\cong 2(\beta+1) f_m = 2(\beta f_m + B) = 2(\Delta f + B).$$

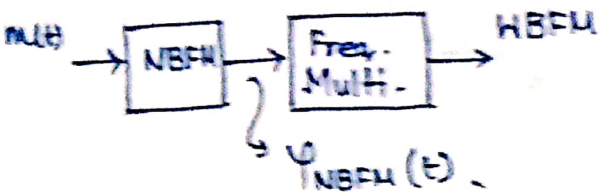
$$\boxed{B_{FM} = 2(\Delta f + B)}$$



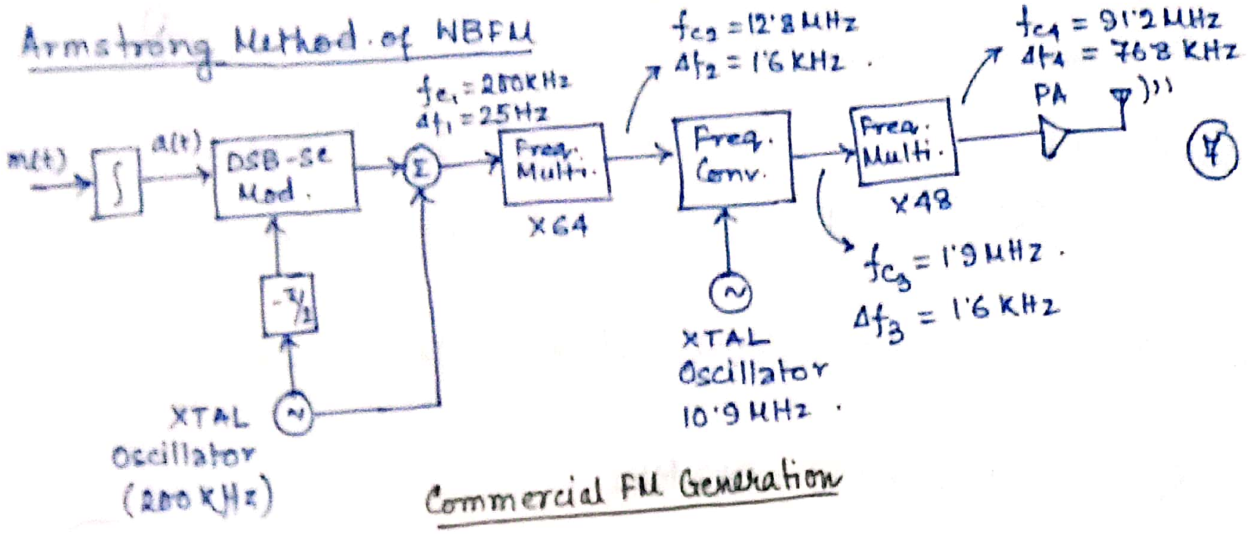
$J_n(\beta) \rightarrow 0$ for sufficiently large 'n'.

Generation of FM Waves

1) Indirect Method



Armstrong Method of NBFM



(7)

Commercial FM Generation

WBFM $\longrightarrow$ Freq^n. Deviation is higher compared to NBFM. Increase in freq^n. deviation also accompanies with increase in carrier signal freq^n. which is not required sometimes. If we want 12 fold increment in Δf ; f_c also increases by 12 times. 12 fold increment in Δf and f_c can be achieved by two 2nd Order and one 3rd order non-linear system/device

NBFM generated by Armstrong's method has some distortions as it contains some amount of amplitude modulation. Amplitude limiting in the freq^n. multipliers removes most of the distortion.

Commercial FM Transmitter

Final output $= 91.2$ MHz. (f_c)
$\Delta f = 75$ KHz.
} (Target)

Starting NBFM,

$f_{c_1} = 200$ KHz. $\rightarrow$ At this freq^n. it is easy to construct a stable x-tal oscillator.
$\Delta f =$ arbitrary
25 Hz
(chosen)

Required cond^n. $\beta \ll 1$. for NBFM

$$\frac{\Delta f_{req}}{\Delta f_{ini}} = \left(\frac{75 \times 10^3}{25}\right) = 3 \times 10^3$$

$\boxed{64 \times 48 = 3072}$ $\rightarrow$ Freq^n Doublers and Tripplers. } Reliability is more.

$64 \to 2^6$

$48 \to 8 \times 6$
$= 2^3 \times 3 \times 2$
$= 2^4 \times 3$

$3072 \times 2 \times 10^3 \simeq 6.14$ MHz.

Earlier Target $= 91.2$ MHz.

So frequency translation is required.

$f_{c_1} = 200$ KHz. $\Delta f_1 = (25 \times 64) = 1'6$ KHz

$f_{c_2} = (200 \times 10^3 \times 64) = 12'8$ MHz. $\Delta f_2 = 1'6$ KHz.

$f_{c_3} = (f_{c_2} - f_c') = (12'8 - 10'9) = 1'9$ MHz. $\Delta f_3 = 1'6$ KHz

$f_{c4} = (1'9 \text{ MHz} \times 48) = 91'2$ MHz.

$\Delta f_4 = (1'6 \text{ KHz} \times 48) = 76'8$ KHz.

The scheme has the advantage of frequency stability but suffers from inherent noise caused by the excessive multiplication and distortion at lower modulating freq^n. where $\Delta f / f_m$ is not small.

2) <u>Direct Generation: VCO method</u>

In VCO, the frequency is controlled by an external voltage. The oscillation frequency varies linearly with the control voltage.

Frequency of the oscillator can be given by,

$$(\sqrt{LC})^{-1} = \omega_0$$

If the capacitance C is varied by the modulating signal, then we can write,

$$C = C_0 - K m(t)$$

Then,

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{L(C_0 - Km(t))}}$$

$$= \frac{1}{\sqrt{LC_0(1 - Km(t)/C_0)}}$$

$$= \frac{1}{\sqrt{LC_0}\left(1 - \frac{Km(t)}{C_0}\right)^{1/2}} = \frac{1}{\sqrt{LC_0}}\left(1 - \frac{Km(t)}{C_0}\right)^{-1/2}$$

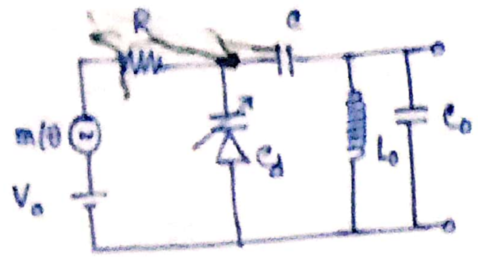
$$\simeq \frac{1}{\sqrt{LC_0}}\left[1 + \frac{Km(t)}{2C_0}\right] \quad \left(\because \frac{Km(t)}{C_0} \ll 1\right)$$

$$= \frac{1}{\sqrt{LC_0}} + \frac{1}{\sqrt{LC_0}} \times \frac{Km(t)}{2C_0}$$

$$= \omega_c + k_f m(t)$$

where, $\omega_c = \frac{1}{\sqrt{LC_0}}$, $k_f = \frac{K}{2C_0\sqrt{LC_0}}$

Circuit Level Implementation



Problems of Direct Method FM Generation

i) In direct method, it is difficult to obtain stability in carrier frequency. It is due to the fact, carrier generation is directly affected by the modulating signal m(t). In this case, m(t) ~~is~~ directly controls the generated signal frequency.

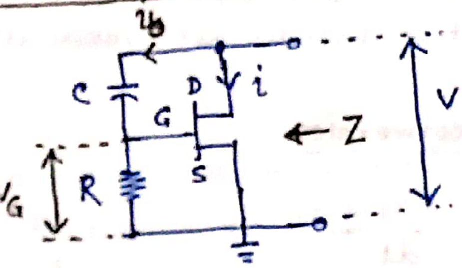
Recall,

$$C = C_o - k\,m(t).$$

$$\omega_c = \frac{1}{\sqrt{LC_o}} \quad \text{where,} \quad C_o = C + k\,m(t).$$

ii) Non linearity produces a frequency variation due to harmonics of the modulating signal. Thus FM signal gets distorted.

Reactance Modulator : Direct Generation Method



$$v_g = i_b R = \frac{Rv}{R - jX_c}$$

FET drain current is,

$$i = g_m v_g = \frac{g_m R v}{R - jX_c}$$

$$Z = \frac{v}{i} = \frac{\cancel{v}}{\frac{g_m R \cancel{v}}{R - jX_c}} = \frac{R - jX_c}{g_m R} = \frac{1}{g_m}\left(1 - \frac{jX_c}{R}\right)$$

For, $X_c \gg R$

$$Z = -\frac{jX_c}{g_m R}$$

$$\therefore X_{eq} = \frac{X_c}{g_m R} = \frac{1}{(2\pi f)\, g_m R C} = \frac{1}{2\pi f C_{eq}} \quad \text{where, } C_{eq} = g_m R C.$$

Since,

$$C_{eq} = g_m R C$$

$$\therefore X_{eq} = f_1(g_m, R, C)$$

Again, $g_m = f_2(v)$

Hence,

$$X_{eq} = f_1(f_2(v), R, C)$$

Hence, reactance of the circuit and thus the output signal frequency can be controlled by voltage. Hence the o/p signal will be a FM signal and as the controlling parameter of the circuit is reactance and thus the name of the circuit is reactance modulator.

* FM Demodulation Technique

Recall, $W_i = W_c + k_f m(t)$ for FM signal.

Hence, a freq^n. selective network with a transfer function of the form;

$$|H(w)| = aw + b$$

over the FM signal band would result the output proportional to the W_i. Most simplest circuit with the required characteristics will be an ideal differentiator.

i) FM to AM conversion

$$\dot{\varphi}_{FM}(t) = \frac{d\varphi_{FM}(t)}{dt}$$

$$= \frac{d}{dt}\left\{ A\cos\left(W_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right)\right\}$$

$$= \underbrace{A\,(W_c t + k_f\, m(t))}_{\substack{\text{AM Part} \\ \text{(Envelope Part)}}}\ \underbrace{\sin\left(W_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right)}_{\text{FM Part}}$$

Since,

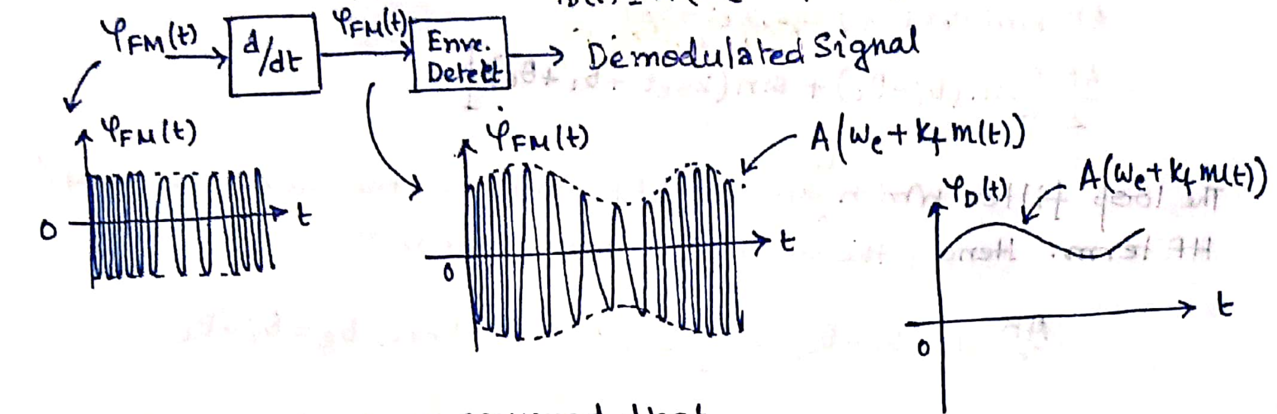
$$\Delta W = k_f m_p \quad \text{for FM signal}$$

and,

as $\Delta W < W_c$ hence, $W_c + k_f m(t) > 0$ for all time t.

Hence $m(t)$ can be detected by the help of envelope detector.

$\varphi_D(t) = A(\omega_c + k_f m(t))$



In this case we have assumed that the carrier signal amplitude 'A' is constant.
If 'A' is not constant, then there will be additional terms in the RHS of $\dot{\varphi}_{FM}(t)$. Hence, the o/p of the envelope detector will be proportional to,

$$\varphi_D(t) = A(t)\left[\omega_c + k_f m(t)\right]$$

Hence, the o/p of envelope detector will be,

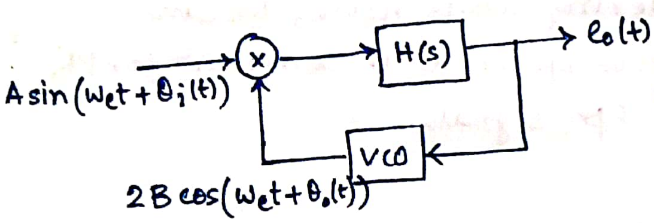
$$\varphi_D(t) \propto m(t)\, A(t).$$

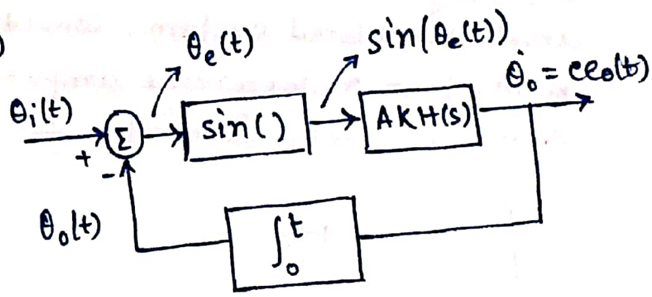
To remove the error in the o/p of envelope detector we can use a hard limiter which will remove the variation of amplitude in the carrier signal / FM signal.

ii) Phase-Locked Loop (PLL)

In practice, the received signal SNR is generally low. Since, PLL offers superior performance in low SNR and low cost, ∴ practical FM demodulation circuit utilizes PLL.

PLL block diagram





Assume the i/p signal is given by,

$$A\sin(\omega_c t + \theta_i(t))$$

Let the VCO o/p at initial condn. becomes,

$$B\cos(\omega_c t + \theta_o(t)).$$

The o/p of the multiplier becomes,

$$AB \sin(\omega_c t+\theta_i)\cos(\omega_c t+\theta_o)$$
$$= \frac{AB}{2}\left[\sin(\theta_i-\theta_o)+\sin(2\omega_c t+\theta_i+\theta_o)\right]$$

The loop filter which is also a LPF, suppress the sum frequency / HF term. Hence, the o/p of the loop filter is

$$\frac{AB}{2}\sin(\theta_i-\theta_o) = \frac{AB}{2}\sin(\theta_e). \quad \text{where, } \theta_e = \theta_i - \theta_o.$$

Now the instantaneous freq^n. of the vco is given by,

$$\dot{\theta}_{vco}(t) = \omega_c + \dot{\theta}_o(t) = \omega_c + c e_o(t).$$

Considering the i/p FM signal is given by

$$\varphi_{FM}(t) = A\cos(\omega_c t + \theta_i(t)).$$

$$\text{where, } \theta_i = k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha.$$

Hence,

$$\theta_e(t) = \theta_i(t) - \theta_o(t)$$

$$\text{or, } \theta_o(t) = \theta_i(t) - \theta_e(t)$$
$$= k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha - \theta_e(t)$$

Assuming small error of θ_e,

$$e_o(t) \simeq \frac{1}{c}\dot{\theta}_o(t) \simeq \frac{k_f\, m(t)}{c}.$$

* Pre-Emphasis and De-emphasis in FM brodcasting.

In FM signal the channel noise acts as interference in an angle modulated system. Considering white noise, we can write, that interference amplitude spectrum is constant for PM and increases linearly for FM signals.