The relation between the total power and carrier power is also equally important. The total power in the modulated wave will be

$$P_t = P_{carrier} + P_{USB} + P_{LSB}$$

$$= \frac{V_{carrier}^2}{R} + \frac{V_{USB}^2}{R} + \frac{V_{LSB}^2}{R}$$

where, $V_{carrier}$, V_{USB}, and V_{LSB} are the rms voltages of the carrier and sidebands; and R is the resistance. In terms of carrier signal amplitude we can write,

$$P_{carrier} = \frac{V_{carrier}^2}{R} = \frac{(V_c/\sqrt{2})^2}{R} = \frac{V_c^2}{2R} = P_c \text{ (say)}$$

Similarly,

$$P_{USB} = P_{LSB} = \frac{V_{SB}^2}{R} = \frac{((\mu V_c/2)/\sqrt{2})^2}{R} = \frac{\mu^2 V_c^2}{8R}$$

Now putting the values in the total power equation we get,

$$P_t = \frac{V_c^2}{2R} + \frac{\mu^2 V_c^2}{8R} + \frac{\mu^2 V_c^2}{8R}$$

$$= \frac{V_c^2}{2R}\left(1 + \frac{\mu^2}{4} + \frac{\mu^2}{4}\right)$$

$$= \frac{V_c^2}{2R}\left(1 + \frac{\mu^2}{2}\right)$$

$$= P_c\left(1 + \frac{\mu^2}{2}\right)$$

Hence, $$\boxed{P_t = P_c\left(1 + \frac{\mu^2}{2}\right)}$$

It is important to note that for a message signal consisting of several sinusoidal signals or other fundamental signals, the overall/effective modulation index of such AM signal is given by,

$$\mu_{eff} = \sqrt{\mu_1^2 + \mu_2^2 + \cdots + \mu_n^2}$$

For such AM signal, $$P_t = P_c\left(1 + \frac{\mu_{eff}^2}{2}\right)$$

Current Relation in AM

In AM, it is more convenient to measure the AM signal current than the power. In this case, the modulation index may be calculated from the values of un modulated and modulated currents in the AM transmitter.

Let I_c be the rms value of the carrier or un modulated current and I_t be the rms value of the total current of an AM transmitter. Assuming R be the antenna resistance through which the current flow. Now, we know that for a single-tone modulation, the power relation is expressed as,

$$P_t = P_c\left(1 + \frac{\mu^2}{2}\right)$$

or, $$\frac{P_t}{P_c} = 1 + \frac{\mu^2}{2}$$

or, $$\frac{I_t^2 R}{I_c^2 R} = 1 + \frac{\mu^2}{2}$$

or, $$I_t^2 = I_c^2\left(1 + \frac{\mu^2}{2}\right) \quad (\because R \neq 0)$$

or, $$I_t = I_c\sqrt{1 + \frac{\mu^2}{2}}$$

Hence, the current relation between the total current and carrier current is given by,

$$\boxed{I_t = I_c\left(1 + \frac{\mu^2}{2}\right)^{1/2}}$$

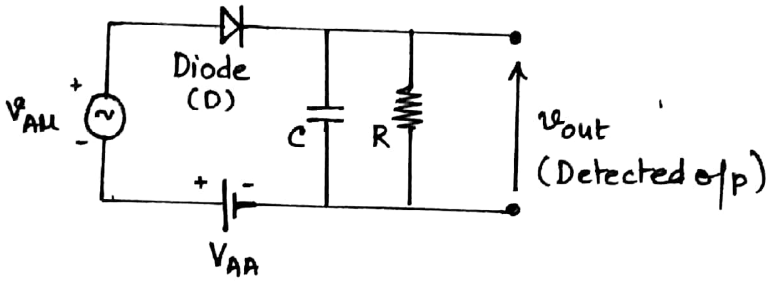
Demodulation of AM Signal

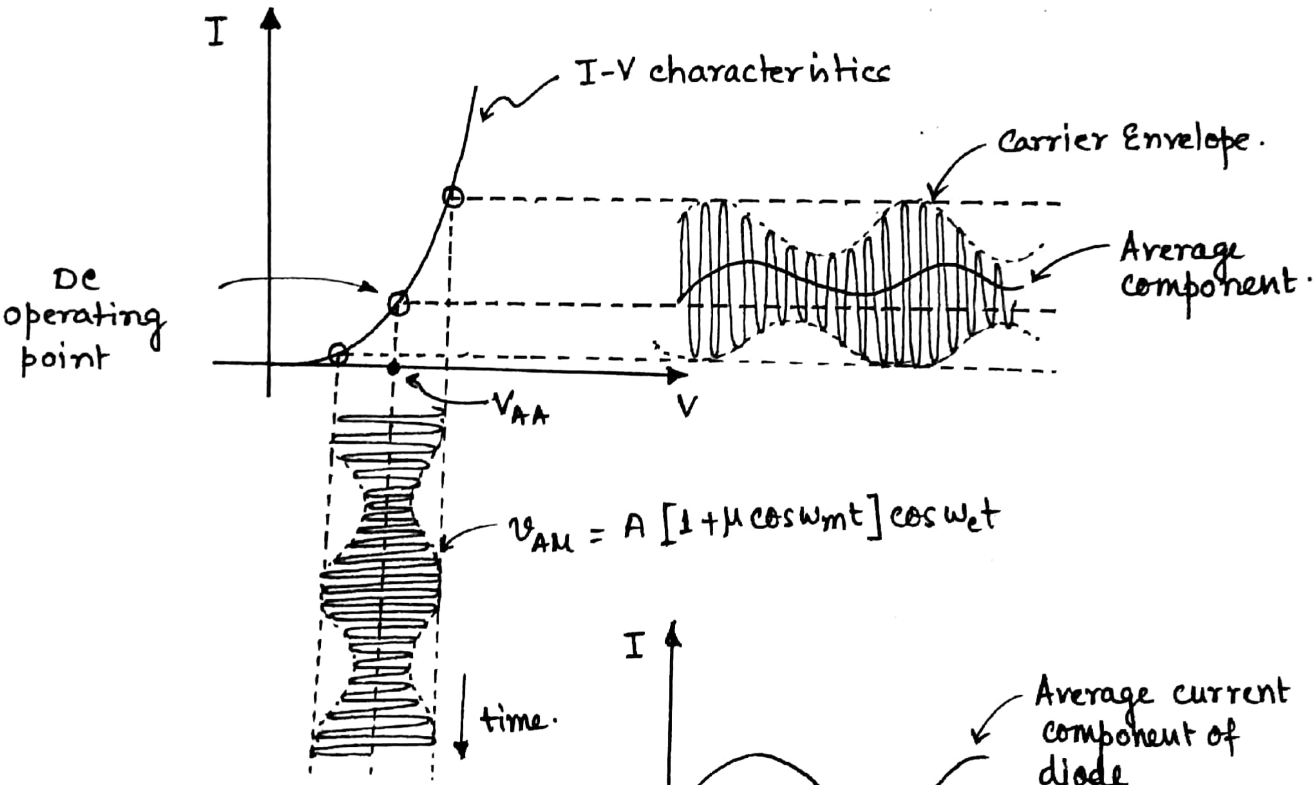
(i) Square-Law Detector

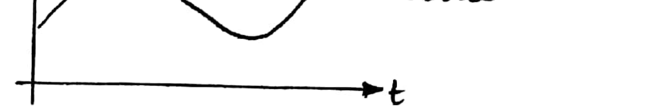
The square-law detector circuits is used for detecting modulated signal of small amplitude (< 1 volt) such that the operating point/region may be restricted to the non-linear portion of the V-I characteristics of the device (PN-diode).

Let us consider the circuit of the square law detector as shown in figure in the next page.

In the circuit, the DC supply voltage (V_{AA}) is used to get the fix operating voltage in the non-linear portion of the V-I characteristics of the diode. Since the operating condition or operation is limited to the non-linear region of the diode characteristics, the lower half portion of the modulated waveform is compressed. Due to this, the average value of the diode current is no longer constant, rather it varies with time as shown in the figure below.







The distorted output diode current is expressed as,

$$i = a v_{AM} + b v_{AM}^2$$

where, $v_{AM}(t) = A\,(1+\mu\cos\omega_m t)\cos\omega_c t$

Replacing the expression of v_{AM} in the previous equation of 'i' we get,

$$i = a[A(1+\mu \cos \omega_m t) \cos \omega_c t] + b[A(1+\mu \cos \omega_m t) \cos \omega_c t]^2$$

If we expand the signal expression, we notice the presence of components at $2\omega_c$, $2(\omega_c \pm \omega_m)$, ω_m and $2\omega_m$ apart from input frequency terms.

Now diode current 'i' is passed through a low pass filter which allows to pass the frequencies below or upto modulating frequency ω_m and rejects other higher frequency components. Thus the modulating signal with frequency ω_m is recovered from the input modulated signal.

(ii) Rectifier Detector

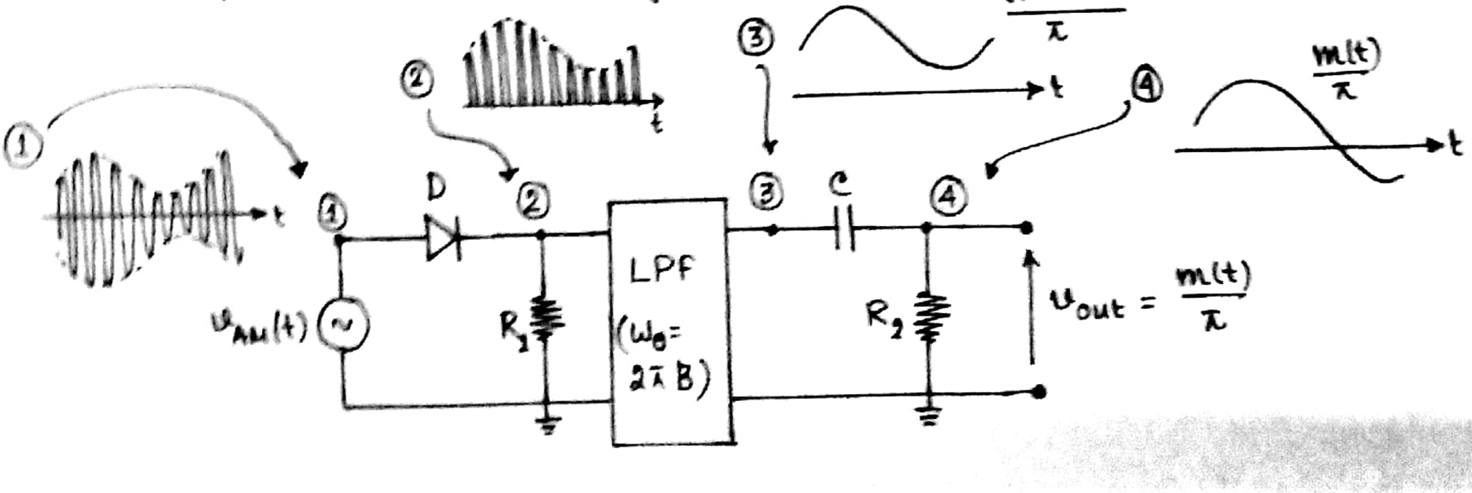
Refer to the circuit diagram. If an AM signal is applied to a diode and a resistor circuit, the negative part of the AM wave will be removed. The o/p across the resistor will be a half-wave rectified version of the AM signal. In short, at the rectifier output, the AM signal is multiplied by $W(t)$. Hence the half-wave rectified o/p is given by,

$$v_R(t) = ([A+m(t)] \cos \omega_c t)\, w(t)$$

$$= [A+m(t)] \cos \omega_c t \left[\frac{1}{2} + \frac{2}{\pi}\left(\cos \omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \dots\right)\right]$$

$$= \frac{1}{\pi}(A+m(t)) + \text{other terms.}$$

When $v_R(t)$ is applied to a LPF of cut-off B (Hz), the o/p is $(A+m(t))/\pi$ and all other terms in $v_R(t)$ of higher frequencies are suppressed. The DC term A/π may easily be blocked by a capacitor 'C' to get the desired output $m(t)/\pi$. It is interesting to note that, due to multiplication of $W(t)$ with modulated signal, rectifier detection is in effect synchronous detection performed without using a local carrier.

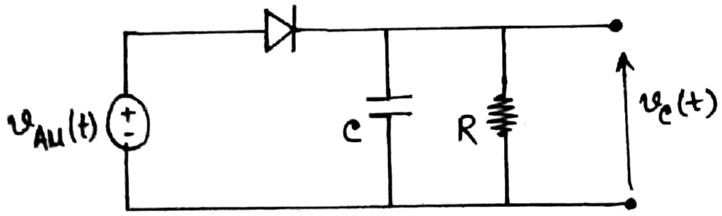


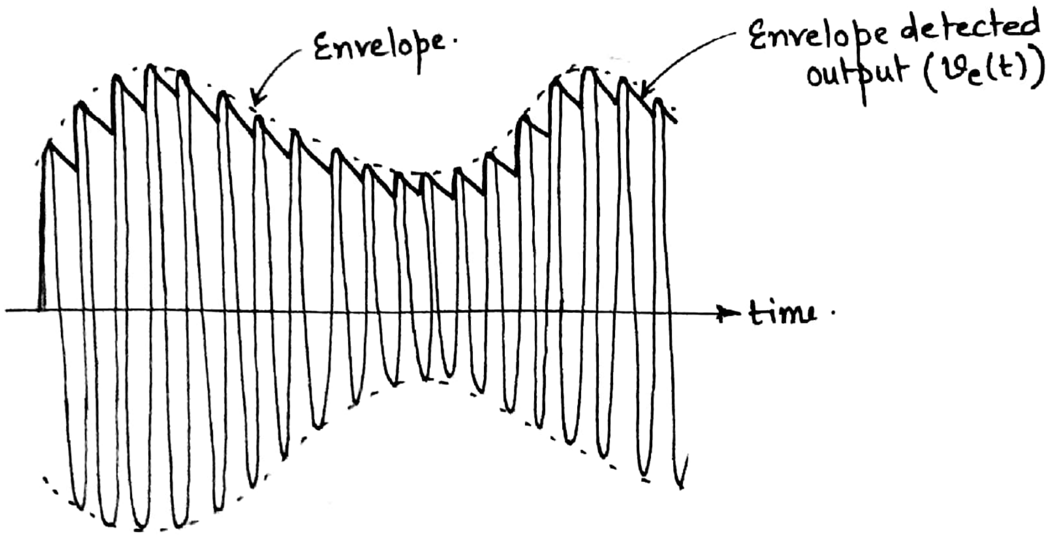
(iii) Envelope Detector/ Linear Diode Detector :-

Consider the circuit of the envelope detector. It is a known fact that a diode operating in the linear regime of its I-V characteristics can extract/ or follow the envelope of an AM wave. This is the main principle of envelope detector. It is the most popular and commercial receiver circuitry to detect AM signal.

On the positive cycle of the input modulated signal, the input grows and may exceed the charged voltage on the capacity $v_c(t)$, turning on the diode and allowing the capacitor C to charge up to the peak voltage of the input signal cycle. As the input signal falls below this peak value, it falls quickly below the capacitor voltage, thus causing the diode to open. The capacitor now discharges through the resistor R at a slower rate, and with time constant = RC.

During each positive cycle, the capacitor charges upto the peak voltage of the input signal and then decays slowly until the next positive cycle as depicted below. Thus the output voltage $v_c(t)$, closely follows the envelope of the input AM signal.





It is important to note that, the slow capacitor discharge via the resistor 'R' allows the capacitor voltage to follow a declining envelope. Capacitor discharge between positive peaks causes a ripple signal of frequency (w_c) in the output. This ripple can be reduced by choosing a larger time constant Rc such that the capacitor discharges between the positive peaks ($Rc \gg w_c^{-1}$). The ripple may be further reduced by a simple Rc high filter followed by a lowpass Rc filter.

Maximum Criteria For Choosing Rc Value

Recall that,

$$v_{AM}(t) = A\,(1+\mu\cos w_m t)\cos w_c t$$
$$= \underbrace{E(t)}_{\text{Envelope}}\cos w_c t.$$

The voltage across the capacitor may be expressed as,

$$v_c(t) = E(t)\,e^{-t/Rc}$$
$$\simeq E(t)\left[1-\frac{t}{Rc}\right] \quad (\because \text{Taylor Series Approximation})$$

The slope of discharge can be calculated as,

$$\frac{dv_c(t)}{dt} = -\frac{E(t)}{Rc}$$

For capacitor to follow the envelope $E(t)$, the magnitude of Rc discharge should be higher the magnitude of the slope of envelope $E(t)$. Mathematically,

$$\left|\frac{dv_c}{dt}\right| \geqslant \left|\frac{dE}{dt}\right|$$

or, $$\frac{E(t)}{Rc} \geqslant |-\mu A w_m \sin w_m t|$$

or, $$\frac{A(1+\mu\cos w_m t)}{Rc} \geqslant \mu A w_m \sin w_m t$$

or, $$\boxed{Rc \leq \frac{1+\mu\cos w_m t}{\mu w_m \sin w_m t}}$$

Limitations of AM

The limitations of the AM signals are -

a) Low transmitted power efficiency.

b) As AM broadcasting radio stations are assigned transmission bandwidth of 10 KHz only, therefore the reception quality is poor.

c) An AM receiver can't distinguish between amplitude variations that contain the desired message signal and that represent noise. Thus, the received signal is generally noisy. Thus AM is quite vulnerable by static noises or other forms of electrical noises.

d) Due to low transmitted power efficiency, the signal can't be transmitted over long distance without increasing transmitter power substantially. But it is to be noted that long-range broadcast communication is possible only for short-wave AM due to wave propagation characteristics.