Amplitude Modulation

Previously, we have discussed DSB-SC signal as it was easy to study and understand. However analytical simplicity does not always equate to simplicity in practical implementation. We have already estimated the signal demodulation technique requires a precisely synchronized carrier signal at the receiver end. This requirement is not easy to achieve in practice.

To remove this problem, a possible/potential solution is to transmit the carrier signal along with the modulated signal. This will ensure that there is no requirement of carrier signal generation at the receiver end. For such cases, the transmitter requires much higher power for transmitting both modulated and carrier signal. In point-to-point communication, where there is one dedicated transmitter for one receiver, complexity in the receiver system can be justified provided that the transmitter is less expensive. On the other hand where broadcasting is required i.e. one transmitter is used to transmit message to a large number of receivers, it is recomended that one costly transmitter is more economic compared to complex and expensive receivers.

In the case of DSB-SC signal, the second option to transmit carrier signal along with modulated signal leads to a new variant of modulation technique which is known as double-sideband-full carrier or DSB-FC. It is also commonly known as amplitude modulation or AM. Mathematically it can be expressed as,

$$\begin{aligned}
\varphi_{AM}(t) &= A\cos\omega_c t + m(t)\cos\omega_c t \\
&= A\left(1+\frac{A_m}{A}\cos\omega_m t\right)\cos\omega_c t \qquad (\text{Assume}: m(t) = A_m\cos\omega_m t) \\
&= A\left(1+\mu\cos\omega_m t\right)\cos\omega_c t
\end{aligned}$$

where, $\mu = \frac{A_m}{A}$ = modulation index.

Note that the spectrum of AM signal is same as that of DSB-SC signal except two additional impulses at $\pm f_c$ will be present. Mathematically the spectrum of AM signal is given by,

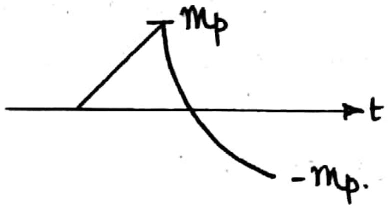
$$\varphi_{AM}(t) \longleftrightarrow \frac{1}{2}\left[M(f+f_c)+M(f-f_c)\right] + \frac{A}{2}\left[\delta(f+f_c)+\delta(f-f_c)\right]$$

If we carefully observe the mathematical representation of the $\varphi_{AM}(t)$ and $\varphi_{DSB\text{-}SC}(t)$, it is clear that the AM signal is identical to the DSB-SC signal with $(A + m(t))$ as the modulating signal instead of $m(t)$. The value of 'A' is always chosen to be positive. It is important to note that the value of A affects the time domain envelope of the modulated signal.

To study the effect of 'A' in the modulation, we consider two cases. In one case, $A + m(t) \geq 0$ since we have considered the value of A large enough. On the other case, it is assumed that $A + m(t) \not\geq 0$ since the value of A is not large enough.

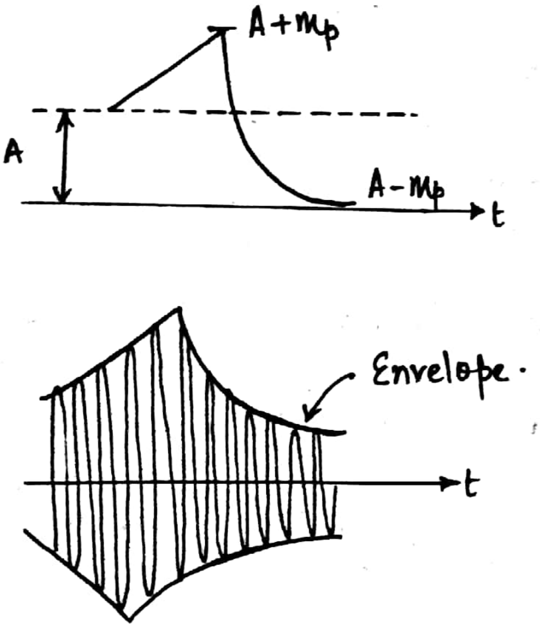
In the first case, the modulated signal envelope has the same as m(t). However in the second case, the envelope shape differs from the m(t), as the negative part of A+m(t) is rectified. Thus we can detect the desired signal m(t) by detecting the envelope of the modulated signal if $A + m(t) \geq 0$. However this scheme fails if $A + m(t) \not\geq 0$. Thus the required condition for the envelope detection is,

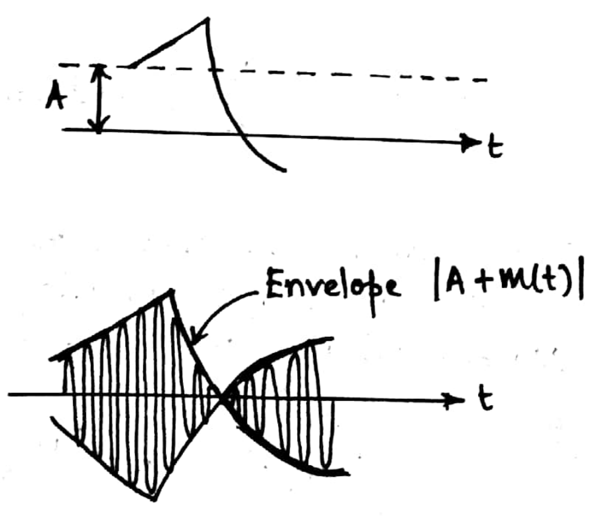
$$A + m(t) \geq 0 \text{ for all values of 't'}.$$



$A + m(t) \geq 0$

$A + m(t) \not\geq 0$





CASE-I

CASE-II

Modulation Index and Percentage Modulation

The modulation index or depth of modulation or modulation factor, or coefficient of modulation (μ, or m_a) is defined as the extent of amplitude variation in AM about an un modulated carrier amplitude. Mathematically we can write,

$$\mu = \frac{\text{max}^m\text{. excursion in AM Ampl. about A}}{\text{Carrier signal Ampl.}}$$

$$= \frac{A_m \text{ (volts)}}{A_c \text{ (volts)}}$$

Similarly, the percentage modulation is given by, $\boxed{M_a = \mu \times 100}$.

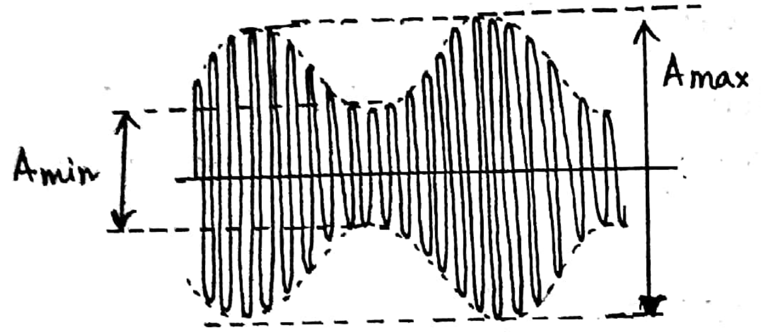
The term 'μ' signifies the amount by which the amplitude of the carrier signal is varied in the process of modulation. Based on the value of 'μ' the AM signal modulation can be classified into three categories. When the AM signal has $\mu < 1$; those AM signal will be referred as undermodulated signal. For the AM signal with $\mu = 1$, it is often referred as 100% modulated AM signal or perfectly modulated AM signal. Similarly, for AM signal with $\mu > 1$, the modulated signal is called overmodulated signal. Generally the required condition for distortion-less envelope detection is,

$$0 \leq \mu \leq 1$$

In certain cases, like undermodulated AM signal, the modulation index is given by,

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

Where Amax and Amin are shown below.



Power Relations in AM

The advantage of envelope detection in AM signal comes at a price of higher power requirement. Since the carrier power is not conveying any information so the power associated with carrier signal is a waste.

Recall that

$$\varphi_{AM}(t) = \underbrace{A\cos\omega_c t}_{\text{Carrier Signal}} + \underbrace{m(t)\cos\omega_c t}_{\text{Sideband}}$$

The power associated with carrier signal and sidebands are estimated as,

$$P_c = \frac{A^2}{2}$$

$$P_s = \frac{\widetilde{m^2(t)}}{2}$$

Hence the total power required at the transmitting end is given by,

$$P_T = P_s + P_c$$

Power efficiency of the AM signal is given by,

$$\eta = \frac{P_s}{P_T} = \frac{P_s}{P_s + P_c} = \left[\frac{\widetilde{m^2(t)}}{A^2 + \widetilde{m^2(t)}} \times 100\right]\%$$

In case of tone modulation,

$$m(t) = \mu A\cos\omega_m t$$

Hence,
$$P_s = \frac{\widetilde{m^2(t)}}{2} = \frac{\mu^2 A^2}{2}$$

Thus,
$$\eta = \frac{P_s}{P_s + P_c} = \left[\frac{\mu^2}{\mu^2 + 2}\right] \times 100\%$$

Where the max^{m}. condition for μ become 1 (since we consider envelope detection). Therefore the maximum power efficiency for AM signal becomes,

$$\eta_{max} = \left(\frac{1}{3} \times 100\right) = \underline{33.33\%}$$