Abhishek Tyagi Assignment-2
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DAA matrix chain multiplication

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Q) Consider the four matrix and its dimensions as

Matrix	Dimensions
P	5 x 4
Q	4 x 6
R	6 x 2
T	2 x 7

Find optimal computation using matrix chain method and give the parenthesisation operation.

Sol. Labelling the dimensions,

$d_0 = 5$
$d_1 = 4$
$d_2 = 6$
$d_3 = 2$
$d_4 = 7$

There are 4 matrices, so we construct 2 4x4 tables - 1 for storing intermediate cost (dynamic programming approach), other to store value of k which will be utilised in placing parenthesis.

Cost Table

	1	2	3	4
1	0	120	~~180~~ 88	158 ~~350~~
2		0	~~180~~ 48	~~236~~ 104
3			0	84
4				0

K Table

	1	2	3	4
1		1	~~2~~ 1	3
2			2	3
3				3
4				

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To find $c[i, j]$

$\rightarrow i \le k < j$ min $\begin{cases} c[i, k] + c[k+1, j] \\ + \\ d_{i-1} \times d_j \times d_k \end{cases}$

$d_0 = 5 \quad d_1 = 4 \quad d_2 = 6 \quad d_3 = 2 \quad d_4 = 7$

☆ Put cost at all diagonal cells 0, since cost when ~~matrix~~ is not multiplied with anyone is 0.

Now choose the ones where difference between i and j is 1. So, we need to compute for
$c[1,2]$, $c[2,3]$, $c[3,4]$

- $c[1,2]$

$i=1, j=2 \quad i \le k < j \Rightarrow k = 1$

Therefore, cost =

$c[1,1] + c[2,2] +$
$d_0 \times d_2 \times d_1$
$= 0 + 0 + 5 \times 6 \times 4$
$\Rightarrow 120$

Plugging it into cost table, and note it k as 1 in k-table.

- $c[2,3]$

$i = 2, \quad k = 2, \quad j = 3$

Cost = $c[2,2] + c[3,3] +$
$\cancel{d_0 \times d_2}\ d_3 \times d_2\ d_1 \times d_3 \times d_2$

(4)

$$= \cancel{120}^{\,0} + \cancel{5}^{\,4} \times 2 \times 6$$

$$0\ \cancel{120} + \cancel{60}\ 48 \Rightarrow \cancel{180}\ 48$$

Plugging int table.

- $C[3,4]$

$\Rightarrow i = 3,\ k = 3,\ j = 4$

cost $= C[3,3] + C[4,4] +$
$d_2 \times d_4 \times d_3$
$= 0 + 0 + (6 \times 7 \times 2)$
$\Rightarrow 84$

plug it into tables.

Now, we find those ones whose difference between i and j is 2.
$C[1,3]$ and $C[2,4]$

- $C[1,3]$

k can have 2 values - 1 and 2. we explore both pathways and choose minimum value.

$C[1,3]$ when $k = 1$
$i = 1,\ k = 1,\ j = 3$

$C[1,1] + C[2,3] +$
$d_0 \times d_3 \times d_1$

$= 0 + \cancel{180}^{\,48} + (5 \times 2 \times 4)$
$\quad 48\ \cancel{180} + 40$
$\Rightarrow \cancel{220}\ 88$

$C[1,3]$ when $k = 2$
$i = 1,\ j = 3,\ k = 2$

cost= $c[1,2] + c[3,3] +$
$d_0 \times d_3 \times d_2$

$\Rightarrow 120 + 0 + (5 \times 2 \times 6)$
$120 + 60$
$\Rightarrow 180$

Since this one has minimum cost,
$c[1,3] \Rightarrow \cancel{180}\ 88$
and $k = \cancel{2}\ 1$
Put these into tables.

- $c[2,4]$
$i=2,\ j=4$ and k can be 2 or 3.

→ when $k = 2$
$i=2,\ k=2 \quad j=4$

cost= $c[2,2] + c[3,4] +$
$d_1 \times d_4 \times d_2$

$= 0 + 84 + (4 \times 7 \times 6)$
$= 84 + 168$
$= 252$

→ when $k=3$
$i=2,\ k=3,\ j=4$
cost= $c[2,3] + c[4,4] +$
$d_1 \times d_4 \times d_3$

$= \cancel{180}^{48} + 0 + (4 \times 7 \times 2)$
$= \cancel{180}^{48} + 56$
$= \cancel{236}\ 104$

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This is minimum value. Therefore, cost of $C[2,4]$ is ~~236~~ 104 at $k=3$. Fill it into table.

* Now, we need to choose the ones where difference between i and j is 3.

$\Rightarrow C[1,4]$

$i=1, j=4$ so k can be 1, 2 or 3.

when $k=1$

$cost = C[1,1] + C[2,4] + (d_0 \times d_4 \times d_1)$

$= 0 + \cancel{236}\ 104 + (5 \times 7 \times 4)$

$\cancel{236}\ 104 + 140$

$= \cancel{376}\ 244$

when $k=2$

$cost = C[1,2] + C[3,4] + (d_0 \times d_4 \times d_2)$

$= 120 + 84 + (5 \times 7 \times 6)$

$204 + (30 \times 7)$

$\Rightarrow 204 + 210 \Rightarrow 414$

when $k=3$

$cost = C[1,3] + C[4,4] + d_0 \times d_4 \times d_3$

$= \cancel{158}\ 88 \ \cancel{180} + 0 + (5 \times 7 \times 2)$

$88 \ \cancel{180} + 70$

$= \cancel{250}\ 158$

Therefore, cost for $C[1,4]$ is minimum when $k=3$. Put the values in tables.

Now, we parenthesis it using k table
For general matrices A_i, $i \in [1, 4]$

$A_1 \times A2 \times A3 \times A4$

m[1, 4], value at [1, 4] in k table is 3. so we split at 3.

$(A1 \times A2 \times A3) \cdot A4$

Now for m[1, 3], value in k table for [1, 3] is 1. So we split at A1.

$(A1 \times (A2 \times A3)) \times A4$

Therefore, the optimum order for matrix multiplication is

$\cancel{((A1 \times (A2 \times}$

$((P \times (Q \times R)) \times T)$