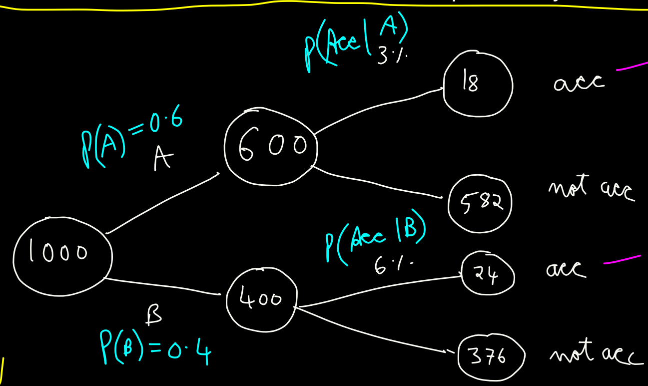
A certain city has two taxi companies, Company A and Company B.
Company A has 60% of the taxis in the city, while Company B has 40%.
Company A's taxis are involved in accidents 3% of the time, while Company B's taxis are involved in 6% accidents 6%.
If a taxi is involved in an accident, what is the probability that it belongs to Company B?



(acc → 18 + 24)

B acc → 24

$\frac{24}{18+24} = 0.57$

$P[B \cap Acc] = P(Acc|B] P[B] = \frac{24}{1000}$

$= (0.4)(0.06)$

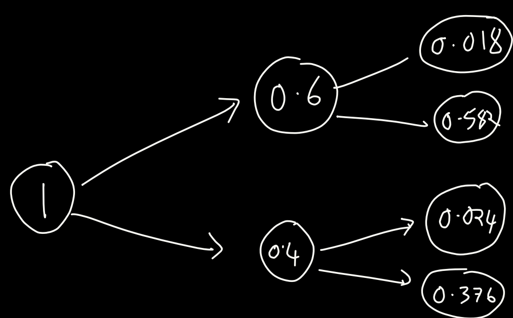
$P[B|Acc] = \frac{P[B \cap Acc]}{P[Acc]} = \frac{0.024}{0.042}$

$P(Acc) = P[Acc \cap A] + P[Acc \cap B]$

$= P(Acc|A) P(A) + P(Acc|B) P(B)$

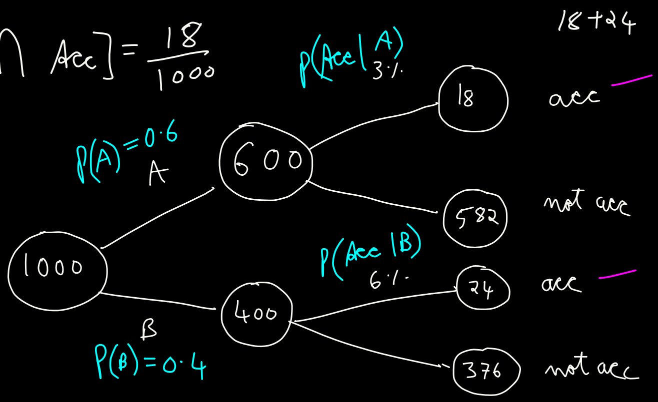
$= (0.03)(0.6) + (0.06)(0.4)$

$P[B|Acc] = \frac{P(Acc|B] P(B)}{P(Acc|A] P(A] + P[Acc|B] P(B)} = \frac{(0.4)(0.06)}{(0.4)(0.06) + (0.03)(0.6)} = \frac{0.024}{0.018 + 0.024}$



$P[B|Acc] = \underline{24}$ $\quad P[A|Acc] = \frac{18}{18+24}$

$P[A \cap \text{Acc}] = \frac{18}{1000}$



$P[A \cap \text{not Acc}] = \frac{582}{1000}$

$\rightarrow P[B \cap \text{acc}] = \frac{24}{1000}$

$\left\{ \begin{array}{l} P[\text{Acc} \mid B] = \frac{24}{400} \\ P[B] = \frac{400}{1000} \end{array} \right.$

Given that a car is not in accident, what is the prob of car A?

$$P[A \mid \text{not acc}] = \frac{582}{582 + 376} = 0.607$$

$P[\text{Pren}] = 0.05 \qquad P[\text{Job} \mid \text{Pren}] = 0.1 \qquad P[\text{Job} \mid \text{non Pren}] = 0.02$

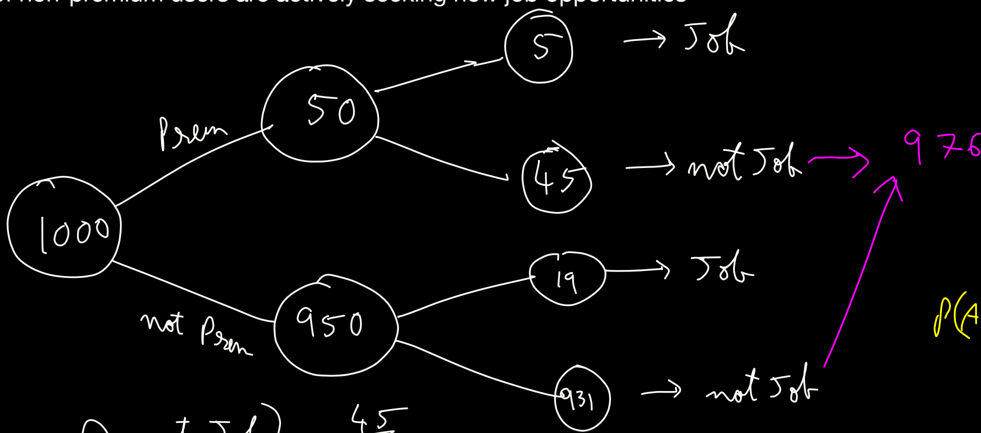
$$P[\text{Pren} \mid \text{not Job}] = \frac{P[\text{Prn} \cap \text{not Job}]}{P[\text{not Job}]} = \frac{P[\text{not Job} \mid \text{Pren}]\, P[\text{Pren}]}{P[\text{not Job} \mid \text{Pren}] P[\text{Prn}] + P[\text{not Job} \mid \text{not Pn}] P[\text{not Pn}]}$$

$= (0.9)(0.05)$

$\overline{(0.9)(0.05)+(0.98)(0.95)}$

not seeking job → prob of prem

It is known that 5% of all LinkedIn users are premium users
10% of premium users are actively seeking new job opportunities.
Only 2% of non-premium users are actively seeking new job opportunities



$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

$P[\text{Prem} \cap \text{not Job}] = \frac{45}{1000}$

$P[\text{not Job} \mid \text{Prem}] = \frac{45}{50}$

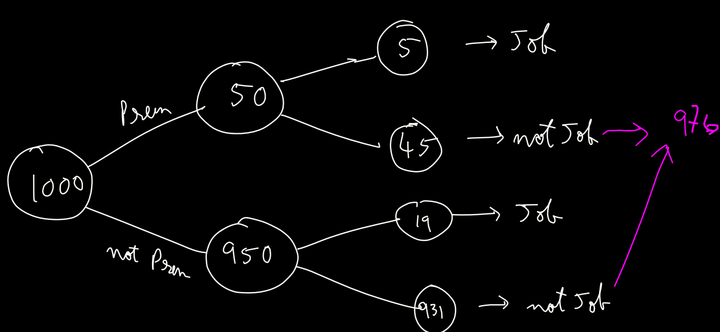
$P[\text{Prem} \mid \text{not Job}] = \frac{45}{976} = 0.046$

$P[\text{not Job}] = \frac{976}{1000}$

$P[\text{Prem}] = \frac{50}{1000}$

$P(\text{Prem} \mid \text{not Job}] = P[\text{Prem} \cap \text{not Job}] / P[\text{not Job}]$

$\frac{45}{976} = \frac{(45/1000)}{976/1000}$



Total Law

$P[C] = P[C \cap A] + P[C \cap A^c]$

$P[C] = P[C \mid A] P[A] + P[C \mid A^c] P[A^c]$

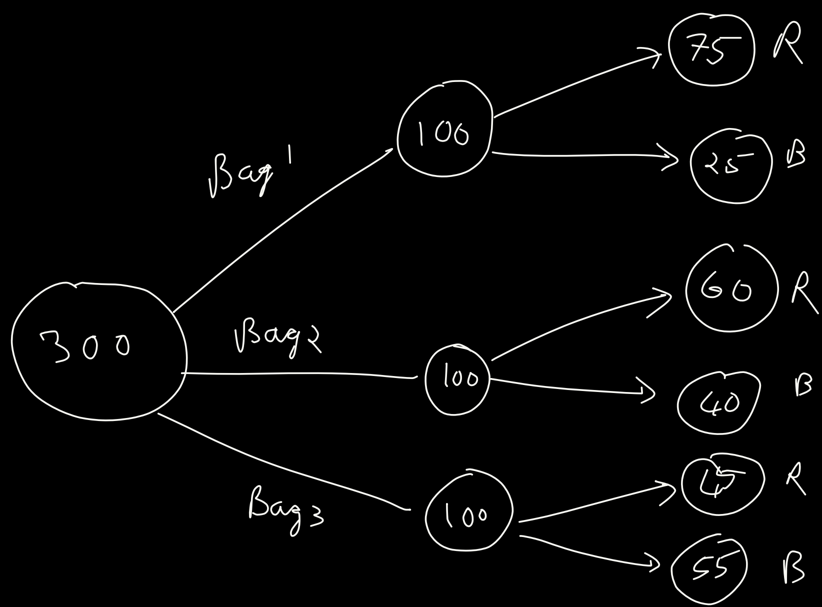
$P[\text{Job}] = P[\text{Job} \cap \text{prem}] + P[\text{Job} \cap \text{not Prem}]$

$= P[\text{Job} \mid \text{prem}] P[\text{prem}] + P[\text{Job} \mid \text{not Prem}] P[\text{not Prem}]$

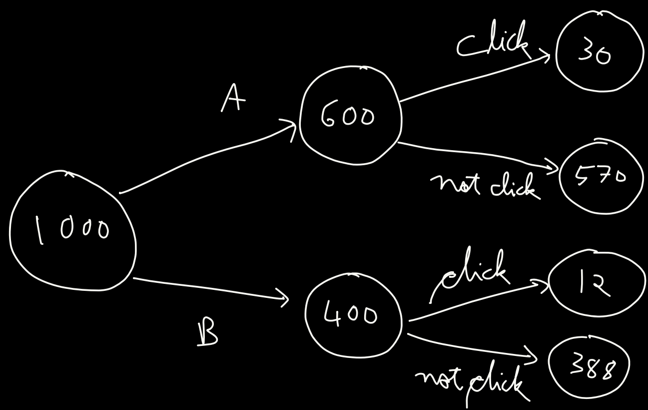
$= \frac{5}{50} \frac{50}{1000} + \frac{19}{950} \frac{950}{1000} = \frac{24}{1000}$

$P[\text{not Job}] = P(\text{not Job} \mid \text{Prem}] P[\text{Prem}] + P[\text{not Job} \mid \text{not Prem}] P(\text{not Prem}]$

$= \frac{45}{50} \frac{50}{1000} + \frac{931}{950} \frac{950}{1000} = \frac{976}{1000}$


300
Bag 1
Bag 2
Bag 3
100
100
100
75
R
25
B
60
R
40
B
45
R
55
B

An e-commerce website shows two types of ads: Type A and Type B.
60% of the visitors see Type A ads, and 40% visitors see Type B ads
The click-through rate for Type A ads is 5%, while the click-through rate for Type B ads is 3%
A visitor to the website does not click the ad. What is the probability that he saw Type A ad?

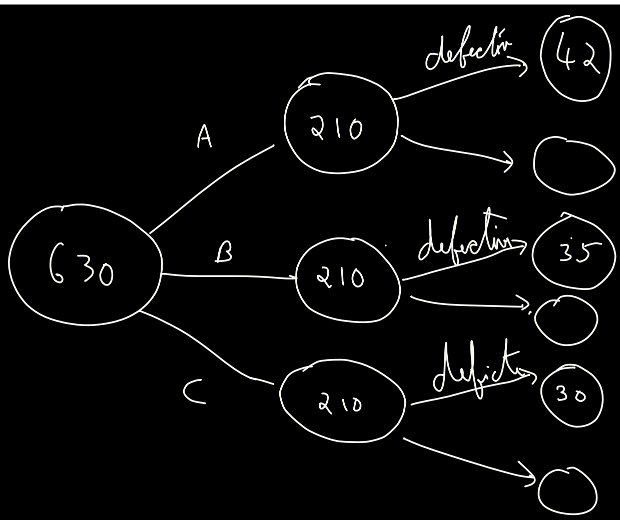


not click → 570 + 388

A ∩ not click → 570

$$P[A \mid \text{not click}] = \frac{570}{570+388} = 0.59$$

The chances of a defective screw in three boxes A, B, C are $\frac{1}{5}$, $\frac{1}{6}$ and $\frac{1}{7}$ respectively. A box is selected at random and a screw drawn from it at random is found to be defective. Find the probability that it came from box A.



$$\frac{42}{42+35+30}$$

A & B are two events

If $P(A|B) = P(A)$, then we say "independent"

Q: coin toss independent of dice?

what is the sample space?

$$S = \{ (H,1), (H,2), (H,3), (H,4), (H,5), (H,6), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6) \}$$

12 outcomes

A: getting heads $A = \{ (H,1), (H,2), (H,3), (H,4), (H,5), (H,6) \}$

B: getting 3 on dice $B = \{ (H,3), (T,3) \}$

$P(A) = \frac{6}{12}$ $\quad P(B) = \frac{2}{12}$ $\quad A \cap B = \{(H,3)\}$

$P(A \cap B) = \frac{1}{12}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/12}{2/12} = \frac{1}{2}$$

Is $P(A|B) = P(A)$? Yes! A & B are independent

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/12}{6/12} = \frac{1}{6} = P(B)$$

A family has 2 children, at least one of them is a girl. What is the probability that both are girls?

$$S = \{BB, \underline{BG, GB, \underline{GG}}\}$$

$A \rightarrow$ Both are girls

$B \rightarrow$ atleast one girl

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P\{GG\}}{P[\{BG, GB, GG\}]} = \frac{1/4}{3/4} = \frac{1}{3}$$