Mid-I

UNIT-I

1. Find the Arithmetic mean of following frequency distribution

x	0-10	10-20	20-30	30-40	40-50	50-60
y	12	18	27	20	17	6

x	$y_{(f_i)}$	x_i	$\Sigma x_i f_i$
0-10	12	5	60
10-20	18	15	270
20-30	27	25	675
30-40	20	35	700
40-50	17	45	765
50-60	6	55	330
	100		2800

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{2800}{100} = \boxed{28}$$

2. In a factory employing 3000 persons in a day, 5% works less than 3hrs, 580 work from 3.01-4.50hrs, 30% works from 4.51-6.00hrs, 500 works from 6.01-7.50hrs, 20% works from 7.51-9.00hrs and the rest work 9.01 or more hrs. Identify the median hours of work.

No of hours of work	no of People	CF	class Interval ll. -0.005, ul. +0.005
<3	5% of 3000 $\boxed{150}$	150	<3.005
3.01-4.50	$\boxed{580}$	730	3.005 - 4.505
4.51-6.00	30% of 3000 $\boxed{900}$	1630	4.505 - 6.005
6.01-7.50	$\boxed{500}$	2130	6.005 - 7.505
7.51-9.00	20% of 3000 $\boxed{600}$	2730	7.505 - 9.005
>9.01	$\boxed{270}$	3000	>9.005
	$\Sigma f_i = 3000$		

$N = \Sigma f_i = 3000$

$\frac{N}{2} = \frac{3000}{2} = 1500$

CF just greater than 1500 is 1630

Median class is 4.505 - 6.005

$l = 4.505 \quad f = 900 \quad cf = 730 \quad h = 1.5$

$$\text{Median} = l + \frac{\left[\frac{N}{2} - cf\right]}{f} h$$

$$= 4.505 + \frac{[1500 - 730]}{900}(1.5)$$

$$\boxed{= 5.788}$$

3. Estimate the Median of the following data

x	1	2	3	4	5	6	7	8
y	4	9	16	25	22	15	7	3

x	y	cf
1	4	4
2	9	13
3	16	29
4	25	54
5	22	76
6	15	91
7	7	98
8	3	101

$\Sigma y = 101$

$N = \Sigma f_i = 101$

$\frac{N}{2} = \frac{101}{2} = 50.5$

CF just greater than 50.5 is 54 so the corresponding 'x' value is '4'

Hence, Median is $\boxed{4}$

4. Find the standard deviation of the following data

x	0-10	10-20	20-30	30-40	40-50	50-60	60-70
y	6	5	8	15	7	6	3

We know that

$$\sigma^2 = h^2 \left[\frac{1}{N} \Sigma f_i d_i^2 - \left(\frac{\Sigma f_i d_i}{N} \right)^2 \right] \quad --(1)$$

$A = 35 \quad h = 10$

x	y	x_i	$d_i = \frac{x_i - A}{h}$	d_i^2	$f_i d_i^2$	$f_i d_i$
0-10	6	5	-3	9	54	-18
10-20	5	15	-2	4	20	-10
20-30	8	25	-1	1	8	-8
30-40	15	35	0	0	0	0
40-50	7	45	1	1	7	7
50-60	6	55	2	4	24	12
60-70	3	65	3	9	27	9
	$\Sigma y = 50$				$\Sigma f_i d_i^2 = 140$	$\Sigma f_i d_i = -8$

from (1)

$$\sigma^2 = 10^2 \left[\frac{140}{50} - \left(\frac{-8}{50} \right)^2 \right]$$

$$\sigma^2 = 277.44$$

$$\sigma = \sqrt{277.44}$$

$$\boxed{\sigma = 16.66}$$

6. Find the quartile deviation of the following data

x	0-10	10-20	20-30	30-40	40-50	50-60	60-70
y	6	5	8	15	7	6	3

x	y	cf
0-10	6	6
10-20	5	11
20-30	8	19
30-40	15	34
40-50	7	41
50-60	6	47
60-70	3	50
	$\sum y = 50$	

Quartile deviation

$$Q = \frac{Q_3 - Q_1}{2}$$

$$Q_1 = l + \frac{\left(\frac{N}{4} - cf\right)}{f} \times h$$

$$\frac{N}{4} = \frac{50}{4} = 12.5$$

cf greater than 12.5 is 19

$l = 20$ $cf = 11$ $h = 10$ $f = 8$

$$Q_1 = 20 + \frac{(12.5 - 11)}{8} \times 10$$

$$\boxed{Q_1 = 21.875}$$

$$Q_3 = l + \frac{\left(\frac{3N}{4} - cf\right)}{f} \times h$$

$$\frac{3N}{4} = 37.5$$

cf just greater than 37.5 is 41

$l = 40$ $cf = 34$ $h = 10$ $f = 7$

$$Q_3 = 40 + \frac{(37.5 - 34)}{7} \times 10$$

$$\boxed{Q_3 = 45}$$

$$\therefore Q = \frac{Q_3 - Q_1}{2}$$

$$= \frac{45 - 21.875}{2}$$

$$\boxed{Q = 11.5625}$$

1) Identify a straight line to the following data and estimate the mean CPU time at $x=3.5$ to the following data by the method of least squares.

No of jobs	1	2	3	4	5
CPU time	2	5	4	9	10

Let the Req st line is $y=a+bx$ --①

By principle of least squares

The normal equations are

$$\Sigma y = na + b\Sigma x \quad --②$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \quad --③$$

x	y	x^2	xy
1	2	1	2
2	5	4	10
3	4	9	12
4	9	16	36
5	10	25	50
15	30	55	110

$\Sigma x = 15$ $\quad \Sigma x^2 = 55$

$\Sigma y = 30$ $\quad \Sigma xy = 110$

$n=5$

$(2) \Rightarrow 5a + 15b = 30$

a+0a

$(3) \Rightarrow 15a + 55b = 110$

By solving $a=0$

$b=2$

$(1) \Rightarrow y = 0 + (2)x$

Given that, mean CPU time at $x=3.5$

$y = 2x$ is the straight line

At $x=3.5 \Rightarrow y = 2(3.5)$

$y = 7.0$ is the mean CPU time.

2. Identify a second degree parabola to the following data

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

Let the Required second degree parabola is $y = a + bx + cx^2$ --(1)

By principle of least squares

The normal equations are

$$\Sigma y = na + b\Sigma x + c\Sigma x^2 \quad \text{---(2)}$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad \text{--(3)}$$

$$\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \quad \text{--(4)}$$

x	y	x^2	x^3	x^4	xy	x^2y
0	1	0	0	0	0	0
1	1.8	1	1	1	1.8	1.8
2	1.3	4	8	16	2.6	5.2
3	2.5	9	27	81	7.5	22.5
4	6.3	16	64	256	25.2	100.8
10	12.9	30	100	354	37.1	130.3

$\Sigma x = 10$ $\quad \Sigma x^2 = 30$ $\quad \Sigma x^3 = 100$ $\quad \Sigma x^4 = 354$

$\Sigma y = 12.9$ $\quad \Sigma xy = 37.1$ $\quad \Sigma x^2y = 130.3$ $\quad n = 5$

From (2) => $5a + 10b + 30c = 12.9$

(3) => $10a + 30b + 100c = 37.1$

(4) => $30a + 100b + 354c = 130.3$

By solving $a = 1.42$

$b = -1.07$ $\quad c = 0.55$

$(1) \Rightarrow y = 1.42 + (-1.07)x + (0.55)x^2$ is the best fit.

3. Identify an exponential curve of the form $y = ae^{bx}$ to the following data

x	1	2	3	4	5	6
y	1.6	4.5	13.8	40.2	125	300

Let the required curve be $y = ae^{bx}$ --①

$\ln y = \ln(ae^{bx})$

$\ln y = \ln a + \ln e^{bx}$

$\ln y = \ln a + bx \ln e$

$\ln y = \ln a + bx$

$Y = A + bx \qquad Y = \ln y$

$A = \ln a$

It is in the form of straight line

It's normal equations are

$\Sigma Y = nA + b\Sigma x$ - ③

$\Sigma xY = A\Sigma x + b\Sigma x^2$ --④

By solving we get

A & b

$\ln a = A$

$\Rightarrow a = e^A$

x	y	$Y = \ln y$	xY	x^2
1	1.6	0.47000	0.47000	1
2	4.5	1.50408	3.00816	4
3	13.8	2.62467	7.87401	9
4	40.2	3.69387	14.77548	16
5	125	4.82831	24.14155	25
6	300	5.70378	34.22268	36
21	485.1	18.82471	84.49188	91

$\Sigma x = 21$ $\quad \Sigma Y = 18.82471$ $\quad \Sigma x^2 = 91$

$\Sigma y = 485.1$ $\quad \Sigma xY = 84.49188$ $\quad n = 6$

$(3) \Rightarrow \; 6A + 21b = 18.82471$

$(4) \Rightarrow \; 21A + 91b = 84.49188$

By solving $A = 0.58362$

$b = 1.06317$

$A = \ln a$

$\Rightarrow a = e^{A} = e^{0.58362} = 1.79252$

$(1) \Rightarrow y = (1.79252)\, e^{(1.06317)x}$

is the best fit

4. Identify an exponential curve of the form $y = ab^x$ to the following data

x	10	12	13	16	17	20	25
y	10	22	24	27	29	33	37

Let the required equation be $y = ab^x$ --①

$\ln y = \ln(ab^x)$

$\ln y = \ln a + \ln b^x$

$\ln y = \ln a + x \ln b$

$Y = A + Bx$ -② $\quad \ln y = Y$

$\ln a = A$

$\ln b = B$

It is in the form of straight line

It's normal equations are

$\Sigma Y = nA + B\Sigma x$ --③

$\Sigma xY = A\Sigma x + B\Sigma x^2$ --④

By solving ③ & ④ we get

A & B

$\ln a = A$	$\ln b = B$
$a = e^A$	$b = e^B$

Substitute a & b in ①

x	y	$Y = \ln y$	xY	x^2
10	10	2.3026	23.026	100
12	22	3.0910	37.092	144
13	24	3.1781	41.3153	169
16	27	3.2958	52.7328	256
17	29	3.3673	57.2441	289
20	33	3.4965	69.93	400
25	37	3.6109	90.2725	625
113		22.3422	371.6127	1983

$\sum x = 113$ $\quad \sum Y = 22.3422$ $\quad \sum xY = 371.6127$

$\sum x^2 = 1983$ $\quad n = 7$

$(3) \Rightarrow \quad 7A + 113B = 22.3422$

$(4) \Rightarrow \quad 113A + 1983B = 371.6127$

$\frac{924939}{444800} = 2.0794$

$\frac{766,203}{11,120,000} = 0.0689$

By solving $\quad A = 2.0794$

$B = 0.0689$

$a = e^A \qquad b = e^B$

$a = 7.9997 \qquad b = 1.0713$

$(1) \Rightarrow y = ab^x$

$y = (7.9997)(1.0713)^x$ is the best fit

5. Identify a power curve of the form $y = ax^b$ to the following data:

x	1	5	7	9	12
y	10	15	12	15	21

Let the required curve is $y = ax^b$ --- ①

$\ln y = \ln(ax^b)$

$\Rightarrow \ln y = \ln a + \ln x^b$

$\Rightarrow \ln y = \ln a + b \ln x$

$\Rightarrow \ln y = \ln a + b \ln x$

$Y = A + bX$ --- ②

It is in the form of straight line

It's normal equations are

$\Sigma Y = nA + b\Sigma X$ --- ③

$\Sigma XY = A\Sigma X + b\Sigma X^2$ --- ④

By solving ③ & ④ we get A and b

$\therefore \ln a = A$

$a = e^A$

x	y	$X = \ln x$	$Y = \ln y$	XY	X^2
1	10	0	2.3026	0	0
5	15	1.6094	2.7081	4.3584	2.5902
7	12	1.9459	2.4849	4.8354	3.7865
9	15	2.1972	2.7081	5.9502	4.8277
12	21	2.4849	3.0445	7.5653	6.1747
34		8.2374	13.2482	22.7093	17.3791

$\Sigma x = 34$ $\quad \Sigma Y = 13.2482$ $\quad \Sigma X^2 = 17.3791$

$\Sigma X = 8.2374$ $\quad \Sigma XY = 22.7093$ $\quad n = 5$

$(3) \Rightarrow \quad 5A + 8.2374b = 13.2482$

$(4) \Rightarrow \quad 8.2374A + 17.3791b = 22.7093$

By solving $\quad A = 2.2676$

$b = 0.2319$

$a = e^A$

$a = e^{2.2676} = 9.6562$

$(1) \Rightarrow \quad y = ax^b$

$= 9.6562\, x^{0.2319}$

6. 10 competitors in a music test were ranked by 3 judges in the following Order

	1	2	3	4	5	6	7	8	9	10
A	1	6	5	10	3	2	4	9	7	8
B	3	5	8	4	7	10	2	1	6	9
C	6	4	9	8	1	2	3	10	5	7

Use the Rank correlation coefficient to determine which pair of judges has the nearest approach to common liking in music

A R_1	B R_2	C R_3	$d = R_1 - R_2 - R_3$	d^2
1	3	6	-8	64
6	5	4	-3	9
5	8	9	-12	144
10	4	8	-2	4
3	7	1	-5	25
2	10	2	-10	100
4	2	3	-1	1
9	1	10	-2	4
7	6	5	-4	16
8	9	7	-8	64
				431

$$\rho = 1 - \frac{6 \Sigma d^2}{n(n^2-1)}$$

$$= 1 - \frac{431}{10(99)}$$

$$= -1.6121$$

7. Estimate the correlation coefficient to the following data

x	65	66	67	67	68	69	71	73
y	67	68	64	68	72	70	69	70

x	y	x^2	y^2	xy
65	67	4225	4489	4355
66	68	4356	4624	4488
67	64	4489	4096	4288
67	68	4489	4624	4556
68	72	4624	5184	4896
69	70	4761	4900	4830
71	69	5041	4761	4899
73	70	5329	4900	5110
546	548	37314	37578	37422

$$r = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sqrt{\sum x^2 - \frac{(\sum x)^2}{n}}\sqrt{\sum y^2 - \frac{(\sum y)^2}{n}}}$$

$$= \frac{37422 - \frac{(546)(548)}{8}}{\sqrt{37314 - \frac{298116}{8}}\sqrt{37578 - \frac{300304}{8}}}$$

$$= \frac{37422 - 37401}{(7.0356)(6.3246)}$$

$$= 0.4719$$

8. Estimate the rank correlation to the following data

x	68	64	75	50	64	80	75	40	55	64
y	62	58	68	45	81	60	68	48	50	70

x	y	R_x	R_y	$d = R_x - R_y$	d^2
68	62	4	5	-1	1
64	58	6	7	-1	1
75	68	2.5	3.5	-1	1
50	45	9	10	-1	1
64	81	6	1	5	25
80	60	1	6	-5	25
75	68	2.5	3.5	-1	1
40	48	10	9	1	1
55	50	8	8	0	0
64	70	6	2	4	16
					72

As '75' is repeated two times: $m_1 = 2$

Avg of 2^{nd} and 3^{rd} ranks $= \frac{2+3}{2} = \frac{5}{2} = 2.5$

As '64' is repeated three times: $m_2 = 3$

Avg of 5^{th}, 6^{th} and 7^{th} ranks $= \frac{5+6+7}{3} = \frac{18}{3} = 6$

As '68' is repeated two times: $m_3 = 2$

Avg of 3^{rd} and 4^{th} ranks $= \frac{3+4}{2} = \frac{7}{2} = 3.5$

7

$$\rho = 1 - \frac{6\left[72 + \frac{1}{12}2(4-1) + \frac{1}{12}3(9-1) + \frac{1}{12}2(4-1)\right]}{10(100-1)}$$

$$\rho = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}m_1(m_1^2-1) + \frac{1}{12}m_2(m_2^2-1) \ldots\right]}{n(n^2-1)}$$

$$= 1 - \frac{6[72 + 0.5 + 2 + 0.5]}{990}$$

$$= 1 - \frac{450}{990}$$

$$= 0.5455$$

9. Identify the regression line Y on X and find Y when X = 6.2

X	1	2	3	4	5	6	7	8	9
Y	9	8	10	12	11	13	14	16	15

x	y	$X = x - \bar{x}$	$Y = y - \bar{y}$	X^2	Y^2	XY
1	9	-4	-3	16	9	12
2	8	-3	-4	9	16	12
3	10	-2	-2	4	4	4
4	12	-1	0	1	0	0
5	11	0	-1	0	1	0
6	13	1	1	1	1	1
7	14	2	2	4	4	4
8	16	3	4	9	16	12
9	15	4	3	16	9	12
45	108	0	0	60	60	57

$\bar{x} = \frac{\Sigma x}{n} = \frac{45}{9}$

$= 5$

$\Rightarrow b_{yx} = \frac{\Sigma xy}{\Sigma x^2}$

$\bar{y} = \frac{\Sigma y}{n} = \frac{108}{9}$

$\Rightarrow y - \bar{y} = b_{yx}(x - \bar{x})$

$= 12$

$b_{yx} = \frac{57}{60} = 0.95$

Regression line Y on X

$\Rightarrow y - 12 = 0.95(x - 5)$

$\Rightarrow y - 12 = 0.95x - 4.75$

$\Rightarrow 0.95x - y - 4.75 + 12 = 0$

$\Rightarrow 0.95x - y - 7.25 = 0$

When $x = 6.2$

$\Rightarrow y = 0.95(6.2) - 7.25$

$= -1.36$

10. Identify the two regression lines to the following data

x	12	10	14	11	12	9
y	18	17	23	19	20	15

x	y	$X = x - \bar{x}$	$Y = y - \bar{y}$	X^2	Y^2	XY
12	18	0.67	-0.67	0.45	0.45	-0.45
10	17	-1.33	-1.67	1.77	2.79	2.22
14	23	2.67	4.33	7.13	18.75	11.56
11	19	-0.33	0.33	0.11	0.11	-0.11
12	20	0.67	1.33	0.45	1.77	0.89
9	15	-2.33	-3.67	5.43	13.47	8.55
68	112			15.34	37.34	22.66

$$\bar{x} = \frac{\Sigma x}{n} = 11.33$$

$$\bar{y} = \frac{\Sigma y}{n} = 18.67$$

$$b_{yx} = \frac{\Sigma XY}{\Sigma X^2} = \frac{22.66}{15.34} = 1.48$$

$$b_{xy} = \frac{\Sigma XY}{\Sigma Y^2} = 0.61 = \frac{22.66}{37.34}$$

Regression equation of y on x

$\Rightarrow y-\bar{y} = b_{yx}(x-\bar{x})$

$\Rightarrow y-18.67 = 1.48(x-11.33)$

$\Rightarrow y-18.67 = 1.48x-16.77$

$\Rightarrow 1.48x-y-16.77+18.67=0$

$\Rightarrow \boxed{1.48x-y+1.90=0}$

Regression equation of x on y

$\Rightarrow x-\bar{x} = b_{xy}(y-\bar{y})$

$\Rightarrow x-11.33 = 0.61(y-18.67)$

$\Rightarrow x-11.33 = 0.61y-11.39$

$\Rightarrow x-0.61y-11.33+11.39=0$

$\Rightarrow \boxed{x-0.61y+0.06=0}$

11. A computer while calculating correlation coefficient between two variables x, y from 25 pairs of observations obtained the following results.

$n = 25$, $\sum x = 125$, $\sum x^2 = 650$, $\sum y = 100$, $\sum y^2 = 460$

$\sum xy = 508$

It was however discovered at the time of checking that he had copied down two pairs as

X	Y
6	14
8	6

while the orginal values are

X	Y
8	12
6	8

Estimate the correct correlation coefficient

Given

$\sum x = 125 - 6 - 8 + 8 + 6 = 125$

$\sum y = 100 - 14 - 6 + 12 + 8 = 100$

[subtract Total values - wrong values + Right values]

$\sum x^2 = 650 - 6^2 - 8^2 + 8^2 + 6^2 = 650$

$\sum y^2 = 460 - 14^2 - 6^2 + 12^2 + 8^2 = 436$

$\sum xy = 508 - 6(14) - 8(6) + 8(12) + 6(8)$

$= 520$

$$r = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sqrt{\sum x^2 - \frac{(\sum x)^2}{n}}\sqrt{\sum y^2 - \frac{(\sum y)^2}{n}}}$$

$$= \frac{520 - \frac{(125)(100)}{25}}{\sqrt{650 - \frac{(125)^2}{25}}\sqrt{436 - \frac{(100)^2}{25}}}$$

$$= \frac{520-500}{\sqrt{25}\sqrt{36}} = \frac{20}{5 \times 6}$$

$$= 0.6667$$

12. Identify a straight line to the following data by the method of least squares.

x	1	2	3	4	5
y	5	7	9	10	11

x	y	x^2	xy
1	5	1	5
2	7	4	14
3	9	9	27
4	10	16	40
5	11	25	55
15	42	55	141

Let the req st line is $y = a + bx$ -- ①

By the principle of least squares

The normal equation's are

$\sum y = na + b\sum x$ - ②

$\sum xy = a\sum x + b\sum x^2$ - ③

$\Sigma x = 15 \qquad \Sigma x^2 = 55$

$\Sigma y = 42 \qquad \Sigma xy = 141$

$n = 5$

$(2) \Rightarrow \quad 5a + 15b = 42$

$(3) \Rightarrow \quad 15a + 55b = 141$

By solving $a = 3.90$

$b = 1.50$

$(1) \Rightarrow y = a + bx$

$y = 3.90 + (1.50)x$ is the required Straight line

UNIT-III

1. If two dice are thrown, what is the probability that the sum is (a) greater than 8, (b) neither 7 nor 11

When two dice are thrown $n(S)=36$

i, let E is the event of getting sum more than 8

$E_1=\{(3,6)(4,5)(5,4)(5,5)(5,6)(6,3)(6,4)(6,5)(6,6)\}$

$n(E_1)=10$

$$\Rightarrow P(E_1)=\frac{n(E_1)}{n(S)}=\frac{10}{36}=\frac{5}{18}$$

ii, E_2 is the event of getting sum 7

$$E_2=\{(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)\}=\frac{6}{36}$$

E_3 is the event of getting sum 11

$$E_3=\{(5,6)(6,5)\} \Rightarrow n(E_3)=2$$

$$P(E_3)=\frac{2}{36}$$

$$\begin{aligned}
P(\text{neither 7 nor 11}) &= P(E_2^c \cap E_3^c) \\
&= P(E_2 \cup E_3)^c \\
&= 1-P(E_2 \cup E_3) \\
&= 1-[P(E_2)+P(E_3)-P(E_2 \cap E_3)] \\
&= 1-[\tfrac{6}{36}+\tfrac{2}{36}-0] \\
&= 1-\frac{8}{36} \\
&= \frac{28}{36}=\frac{14}{18}=\frac{7}{9}
\end{aligned}$$

2. An urn contains 4 tickets numbered 1, 2, 3, 4 and another contains 6 tickets numbered 2, 4, 6, 7, 8, 9. If one of the urn is choosen at random and a ticket is drawn at random from the chosen urn, find the probability that (i) 2 or 4 (ii) 3 (iii) 1 or 9

I	II
1, 2, 3, 4	2, 4, 6, 7, 8, 9

We can select urn I with probability $\frac{1}{2}$ and urn II with probability $\frac{1}{2}$

i, 2 or 4

case-I

If first urn is selected then the probability of selecting 2 (or) 4 is

$$= \frac{1}{2}[P(2 \cup 4)]$$

$$= \frac{1}{2}[P(2) + P(4) - P(2 \cap 4)]$$

$$= \frac{1}{2}\left[\frac{1}{4} + \frac{1}{4} - 0\right] = \frac{1}{2}\left[\frac{2}{4}\right] = \frac{1}{4}$$

case-II

If urn II is selected then the probability of selecting 2 (or) 4 is

$$= \frac{1}{2}[P(2 \cup 4)]$$

$$= \frac{1}{2}[P(2) + P(4) - P(2 \cap 4)]$$

$$= \frac{1}{2}\left[\frac{1}{6} + \frac{1}{6} - 0\right]$$

$$= \frac{1}{2}\left[\frac{2}{6}\right] = \frac{1}{6}$$

$P(2 \cup 4) = \frac{1}{4} + \frac{1}{6} = \frac{3+2}{12} = \frac{5}{12}$

ii, 3

case-I

If urn I is selected then the probability of selecting 3 is

$= \frac{1}{2}[P(3)]$

$= \frac{1}{2}\left[\frac{1}{4}\right]$

$= \frac{1}{8}$

case-II

If urn II is selected then the probability of selecting 3 is

$= \frac{1}{2}[P(3)]$

$= \frac{1}{2}(0)$

$= 0$

$P(3) = \frac{1}{8}$

iii, 1 or 9

case-1

If urn I is selected then the probability of selecting 1 or 9 is

$= \frac{1}{2}[P(1 \cup 9)]$

$= \frac{1}{2}[P(1) + P(9) - P(1 \cap 9)]$

$= \frac{1}{2}\left[\frac{1}{4}\right]$

$= \frac{1}{8}$

case-2

If urn II is selected then the probability of selecting 1 or 9 is

$= \frac{1}{2}[P(1 \cup 9)]$

$= \frac{1}{2}[P(1) + P(9) - P(1 \cap 9)]$

$= \frac{1}{2}\left[\frac{1}{6}\right]$

$= \frac{1}{12}$

$P(1 \cup 9) = \frac{1}{8} + \frac{1}{12} = \frac{3+2}{24} = \frac{5}{24}$

3. A problem in statistics is given to three students A, B, C whose chances of solving it are 1/2, 3/4, 1/4 respectively. What is the probability that the problem will be solved if all of them tries to solve the problem independently.

Let P(A) is the probability that A can solve the problem $= \frac{1}{2}$

Let P(B) is the probability that B can solve the problem

$P(B) = \frac{3}{4}$

Let P(C) is the probability that C can solve the problem

$P(C) = \frac{1}{4}$

$P(A \cup B \cup C)$ is the probability that the problem is solved.

We know that

$P(A \cup B \cup C) + P(\overline{A \cup B \cup C}) = 1$

$P(A \cup B \cup C) = 1 - P(\overline{A \cup B \cup C})$

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= 1 - P(\bar{A})P(\bar{B})P(\bar{C})$

$= 1 - [1 - P(A)][1 - P(B)][1 - P(C)]$

$= 1 - [1 - \frac{1}{2}][1 - \frac{3}{4}][1 - \frac{1}{4}]$

$= 1 - (\frac{1}{2})(\frac{1}{4})(\frac{3}{4})$

$= \frac{29}{32}$

4. A bag contains 10 gold and 8 silver coins. Two sucessive drawings of 4 coins are made such that.

(i) Coins are replaced before the second trial.

(ii) the coins are not placed before the second trial

Find the probability that the first drawing will give 4 gold and the second 4 silver coins.

Let A is the event of getting 4 gold coins in the first trial 4 silver coins in the second trial then

we have to find $P(A \cap B)$

i, With Replacement

Total coins $= 10 + 8 = 18$

We can draw four coins in $^{18}C_4$ ways

$P(A)$ is the probability that the 4 coins are gold

then $P(A) = \frac{^{10}C_4}{^{18}C_4}$

$P(B/A)$ is the probability that the 4 coins are silver in the second trial

$$P(B/A) = P(B)$$

$$= \frac{^8C_4}{^{18}C_4}$$

$$P(A \cap B) = P(A)\,P(B/A)$$

$$= P(A)\,P(B)$$

$$= \frac{^{10}C_4}{^{18}C_4}\,\frac{^8C_4}{^{18}C_4}$$

ii, Without Replacement

$$P(A) = \frac{^{10}C_4}{^{18}C_4}$$

$$P(B/A) = \frac{^8C_4}{^{14}C_4}$$

$$P(A \cap B) = P(A)\,P(B/A)$$

$$= \frac{^{10}C_4}{^{18}C_4}\,\frac{^8C_4}{^{14}C_4}$$

$$= 0.0048$$

5. A box contains 2 white and 4 black balls. Another box contains 5 white and 7 black balls. A black ball is transferred to the box A to the box B. Then a ball is drawn from the box B. Find the Probability that is white.

	W	B
A	2	4
B	5	7

Suppose a black ball is selected from box A and transferred to B. This can be done in $\frac{4C_1}{6C_1}$ ways

Now, Box B contains 5 white and 8 black balls.

Then the probability that white ball is selected is

$$\frac{4C_1}{6C_1}\frac{5C_1}{13C_1} = \frac{4\times 5}{6\times 13}$$

Suppose a white ball is selected from box A and transferred to B. This can be done in $\frac{2C_1}{6C_1}$ ways

Now, Box B contains 6 white and 7 black balls.

Then the probability that white ball is selected is

$$\frac{2C_1}{6C_1}\frac{6C_1}{13C_1} = \frac{2\times 6}{6\times 13}$$

∴ probability of selecting a white

$$= \frac{4\times 5}{6\times 13} + \frac{2\times 6}{6\times 13}$$

$$= \frac{20+12}{6\times 13}$$

$$= \frac{\cancel{32}^{16}}{\cancel{6}_{3}\times 13} = \frac{16}{39}$$

6. In a bolt factory machines A, B and C manufactures respectively 25%, 35% and 40% of the total of their output 5, 4, 7 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machine A, B and C

Let

P(A) be the probability that a bolt is manufac-tured by machine A

P(B) be the probability that a bolt is manufactured by machine B

P(C) be the probability that a bolt is manufactured by machine C

Given that $P(A) = \frac{25}{100}$ $P(B) = \frac{35}{100}$ $P(C) = \frac{40}{100}$

Let D is the event of drawing a defective

$P[D/A] = \frac{5}{100}$ $P\left[\frac{D}{B}\right] = \frac{4}{100}$ $P\left[\frac{D}{C}\right] = \frac{2}{100}$

1) A

Let P[A/D] is the probability that a deffective bolt is manufactured by machine A from

Baye's theorem

$$P[A/D] = \frac{P(D/A)\,P(A)}{\sum P(D/A)\,P(A)}$$

$$= \frac{P[D/A]\,P[A]}{P[D/A]P(A) + P[D/B]P(B) + P[D/C]P(C)}$$

$$= \frac{\left(\frac{5}{100}\right)\frac{25}{100}}{\frac{5}{100}\cdot\frac{25}{100} + \frac{4}{100}\cdot\frac{35}{100} + \frac{2}{100}\cdot\frac{40}{100}}$$

$$= 0.3623$$

② B

Let $P[B/D]$ is the probability that a defective bolt is manufactured by machine B

$$P[B/D] = \frac{P[D/B]\;P[B]}{\sum P[D/B]P[B]}$$

$$= \frac{P[D/B]\,P[B]}{P[D/A]P(A) + P\left[\frac{D}{B}\right]P(B) + P\left[\frac{D}{C}\right]P(C)}$$

$$= \frac{\frac{4}{100}\,\frac{35}{100}}{\frac{5}{100}\,\frac{25}{100} + \frac{4}{100}\,\frac{35}{100} + \frac{2}{100}\,\frac{40}{100}}$$

$$= 0.4058$$

3.c

$P[C/D]$ is the probability that a defective bolt is manufactured by C

from Baye's theorem

$$P[C/D] = \frac{P[D/C]P(C)}{\sum P[D/C]P(C)}$$

$$= \frac{P\left[\frac{D}{C}\right]P(C)}{P[D/A]P(A) + P[D/B]P(B) + P[D/C]P(C)}$$

$$= \frac{\frac{2}{100}\frac{40}{100}}{\frac{5}{100}\frac{25}{100} + \frac{4}{100}\frac{35}{100} + \frac{2}{100}\frac{40}{100}}$$

$$= 0.2319$$