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# Metamodel-based parametric study of composite laminates

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**Abstract.** Composite laminates are an integral part of structural engineering. To exploit the full potential of such laminated structures, it is important to understand the effect of various material and geometric parameters on its static and dynamic behaviour. Conventionally high-fidelity numerical procedures like finite element methods have been used to carry out such analysis. Despite their overwhelming accuracy, such methods are computational time intensive making them unsuitable to carry out exhaustive parametric studies. To counter this, a polynomial regression-based metamodeling strategy and its subsequent use as a quick parametric analysis tool is depicted in this study. The metamodels are meticulously built and rigorously validated on the training and testing data using suitable statistical measures like  $R^2$ ,  $[[R]^2]_{adj}$  and  $[[Q^2]_{F3}$ .

## 1 Introduction

The use of composite materials is at an all-time high. Every day more and more of the traditional structural elements are being replaced by the composite materials. This is due to the superior qualities of composite materials like high strength-weight and high stiffness-weight ratios [1]. The use of composite laminates in structural engineering has ushered the need to investigate their behaviour under static and dynamic loading conditions.

Conventionally numerical structural analysis of composite plates has relied on used of high-fidelity but time-intensive approaches like finite element method. Owing to notable advances in various machine learning and allied approaches, a major shift in development and use of local approximate methods has been seen since the last decade. These 'local approximations' or metamodels are special purpose, computationally cheap models often developed to provide a quick but rough estimation of the modelled process or phenomenon. Among these, the artificial neural network [2], radial basis functions [3] and response surface methodology (RSM) [4] [5] [6] are most commonly used.

Response surface methodology starts by choosing a polynomial form to which it tries to fit the training data. In past, RSM has been used by various researchers to approximate the free vibration behaviour of composite plates [7] [8] and shells [9]. Ganguli [10] used a central composite design based RSM methodology to model a helicopter rotor. He fitted a second-order RSM model for 3 design variables to conduct an aeroelastic analysis. Ju et al. [11] used a similar CCD RSM approach to carry out a structural optimization of trusses. Dey et al. [12] used an RSM D-optimal design to model composite shells.

In this paper, we develop a set of RSM metamodels to conduct a parametric study on the free vibration behaviour of composite plates. The built metamodels are validated using independent test datasets.

## 2 Problem Definition

The objective is to develop appropriate RSM metamodels to study the effect of various material and geometric parameters on the fundamental frequency of composite plates. The various material properties considered are— orthotropy ratio  $\left(\frac{E_1}{E_2}\right)$ , major shear modulus to Young's modulus ratio  $\left(\frac{G_{12}}{E_2}\right)$ , minor shear modulus to Young's modulus ratio  $\left(\frac{G_{23}}{E_2}\right)$  and Poisson's ratio  $(\nu_{12})$ . The geometric parameters considered in the study are— composite plate height-to-width ratio  $\left(\frac{b}{a}\right)$ , plate thickness-to-width ratio  $\left(\frac{h}{a}\right)$ , skew angle of plate  $(\alpha)$  and number of plies  $(n)$ .



The range of the various parameters used in the study are-  $\frac{E_1}{E_2} \in \mathbb{R}[20,60]$ ,  $\frac{G_{12}}{E_2} \in \mathbb{R}[0.4,0.8]$ ,  $\frac{G_{23}}{E_2} \in \mathbb{R}[0.3,0.7]$ ,  $\vartheta_{12} \in \mathbb{R}[0.19,0.31]$ ,  $\frac{b}{a} \in \mathbb{R}[1,5]$ ,  $\frac{h}{a} \in \mathbb{R}[0.01,0.1]$ ,  $\alpha \in \mathbb{R}[0,60]$  and  $n \in \mathbb{Z}[2,10]$ . In total four different cases are considered for the parametric study.

**Case 1:** An all side simply-supported, 8-layer symmetric  $([45/-45/45/-45]_s)$ , square composite plate with  $h/a$  of 0.01 is chosen as the structure of interest.

**Case 2:** Same as case 1, except instead of square, a  $30^\circ$  rhombic composite plate is considered.

**Case 3:** For studying the effect of geometric parameters, an all side simply-supported symmetric composite laminate is considered. The material properties are chosen as  $\frac{E_1}{E_2} = 40$ ,  $\frac{G_{12}}{E_2} = 0.6$ ,  $\frac{G_{23}}{E_2} = 0.5$  and  $\vartheta_{12} = 0.25$ .

**Case 4:** Same as case 3, except instead of all side simply-supported, an all side clamped symmetric composite laminate is considered.

### 3 Methodology

A response surface methodology based polynomial regression approach is adopted for metamodeling. A second-order model is selected as the polynomial basis function.

$$y = \beta_0 + \sum_{i=1}^n \beta_i x_i + \sum_{i=1}^n \sum_{j=i+1}^n \beta_{ij} x_i x_j + \sum_{i=1}^n \beta_{ii} x_i^2 + \varepsilon$$

One disadvantage of using a second-order polynomial basis function is that it cannot capture the very high non-linearity and as such would fail miserably on non-linear hard-to-model processes. To counter this, a Box-Cox transformation approach is adopted [13] [14].

$$y' = \begin{cases} (y + C)^\lambda & (\lambda \neq 0), \\ \ln(y + C) & (\lambda = 0); \end{cases}$$

Where  $C$  is a constant and  $\lambda$  is the power of transformation. The commonly used transformations are,  
 $\lambda = 1.0$ , no transformation

$\lambda = 0.5$ , square root

$\lambda = 0$ , natural log

$\lambda = -0.5$ , inverse square root.

$\lambda = -1.0$ , inverse

In the current research, a D-optimal design is used as per the recommendations of Kalita et al. [15] to model the training dataset. The training data needed to fit the above equation for the 4 selected cases is generated by using Kalita et al.'s special purpose finite element program [16]. More details regarding the finite element formulation can be obtained at [17] [18].

The difference between the actual design points (finite element) ( $y_i$ ) and the metamodel predicted design points ( $\hat{y}_i$ ) is called as residue.

$$\varepsilon_i = y_i - \hat{y}_i$$

The  $\beta_i$  estimates in the second-order RSM equations are selected such that the sum of squares of the residuals is minimized. Sum of residuals is given as,

$$SS_R = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

Similarly, the total sum of squares is given as,

$$SS_T = \sum_{i=1}^n (y_i - \bar{y})^2$$

Where,  $\bar{y}$  represents mean of the actual training dataset.

To measure the accuracy of the built RSM metamodels criteria like  $R^2$  and  $R^2_{adj}$  are used.

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$$

$$R^2_{adj} = 1 - \frac{n-1}{n-k-1} (1 - R^2)$$

Since  $R^2$  and  $R^2_{adj}$  provide only a measure of goodness of fit of the training data, it is important to rely on external validation metrics like  $Q^2_{F3}$ .  $Q^2_{F3}$  proposed by Consonni et al. [19] is given as,

$$Q^2_{F3} = 1 - \frac{\sum_{i=1}^{n_{test}} (\hat{y}_i - y_i)^2 / n_{test}}{\sum_{i=1}^{n_{train}} (y_i - \bar{y}_{train})^2 / n_{train}}$$

To calculate the  $Q^2_{F3}$  additional 100 sample points are generated which constitute the testing data.

### 4 Results and Discussion

#### 4.1 RSM metamodels

Using the training data generated by the finite element program, four RSM metamodels are built for the 4 cases discussed in section 2.

##### Case 1:

$$\frac{1}{(\lambda_1)^2} = 0.0005 - 1.23 \times 10^{-5} \left( \frac{E_1}{E_2} \right) - 3.66 \times 10^{-6} \left( \frac{G_{12}}{E_2} \right) - 2.93 \times 10^{-6} \left( \frac{G_{23}}{E_2} \right) - 9.93 \times 10^{-8} (v_{12}) + 1.16 \times 10^{-8} \left( \frac{E_1}{E_2} \right) \left( \frac{G_{12}}{E_2} \right) + 1.84 \times 10^{-6} \left( \frac{G_{12}}{E_2} \right) \left( \frac{G_{23}}{E_2} \right) + 1.02 \times 10^{-7} \left( \frac{E_1}{E_2} \right)^2 + 9.2 \times 10^{-7} \left( \frac{G_{12}}{E_2} \right)^2 + 8.85 \times 10^{-7} \left( \frac{G_{23}}{E_2} \right)^2$$

##### Case 2:

$$(\lambda_1)^2 = 219.8785 + 213.5809 \left( \frac{E_1}{E_2} \right) + 439.0622 \left( \frac{G_{12}}{E_2} \right) + 279.9152 \left( \frac{G_{23}}{E_2} \right) + 250.7809 (v_{12}) + 4.9276 \left( \frac{E_1}{E_2} \right) \left( \frac{G_{12}}{E_2} \right) + 4.5119 \left( \frac{E_1}{E_2} \right) \left( \frac{G_{23}}{E_2} \right) - 246.4965 \left( \frac{G_{12}}{E_2} \right) \left( \frac{G_{23}}{E_2} \right) - 252.7662 \left( \frac{G_{23}}{E_2} \right) (v_{12}) - 0.0551 \left( \frac{E_1}{E_2} \right)^2 - 152.6717 \left( \frac{G_{12}}{E_2} \right)^2 - 147.8168 \left( \frac{G_{23}}{E_2} \right)^2$$

##### Case 3:

$$\frac{1}{\sqrt{\lambda_1}} = 0.1079 + 0.0246 \left( \frac{b}{a} \right) - 0.3217 \left( \frac{h}{a} \right) - 0.0005(\alpha) + 0.0021(n) + 0.01171 \left( \frac{b}{a} \right) \left( \frac{h}{a} \right) - 0.0011 \left( \frac{b}{a} \right) (n) + 0.0089 \left( \frac{h}{a} \right) (\alpha) - 0.00004(\alpha)(n) - 0.0017 \left( \frac{b}{a} \right)^2 - 0.000007(\alpha)^2$$

##### Case 4:

$$\ln(\lambda_1) = 4.4282 - 0.3614 \left( \frac{b}{a} \right) - 1.3347 \left( \frac{h}{a} \right) - 1.2572(\alpha) + 0.1514(n) + 8.6191 \left( \frac{b}{a} \right) (\alpha) - 0.0742 \left( \frac{h}{a} \right) (\alpha) - 0.0391 \left( \frac{h}{a} \right) (n) - 0.0002(\alpha)(n) + 0.0428 \left( \frac{b}{a} \right)^2 + 0.0004(\alpha)^2 - 0.0071(n)^2$$

#### 4.2 Metamodel Validation

The metamodels are validated using  $R^2$  and  $R^2_{adj}$  on the training data and  $Q^2_{F3}$  on the testing data. From Figure 1, it is clear that all the built models are highly accurate. In fact, the metamodels built for case 1 and case 2 have ideal  $R^2$ ,  $R^2_{adj}$  and  $Q^2_{F3}$  values.

To further evaluate the predictive power of the metamodels, scatter plots between the actual finite element models and the predictive response surface methodology based metamodels are constructed. The closer the data points are to the diagonal line better is the accuracy of the metamodels [20]. If the data points lie above the line it means the metamodel is over-predicting while any data point below the diagonal signifies an underprediction by the metamodel. The scatter plots of actual model versus the metamodel for the four cases are reported in Figure 2 and Figure 3.

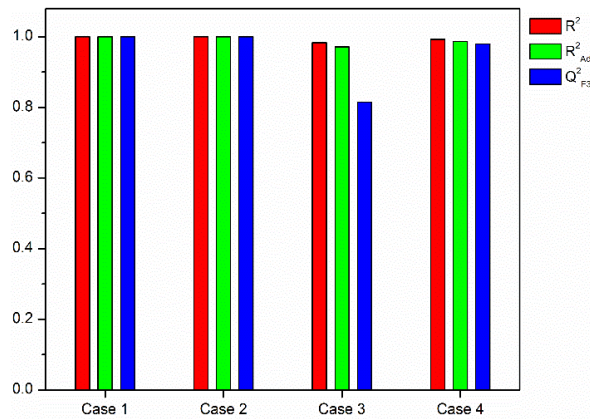


Figure 1.  $R^2$ ,  $R^2_{adj}$  and  $Q^2_{F3}$  of all the four metamodels.

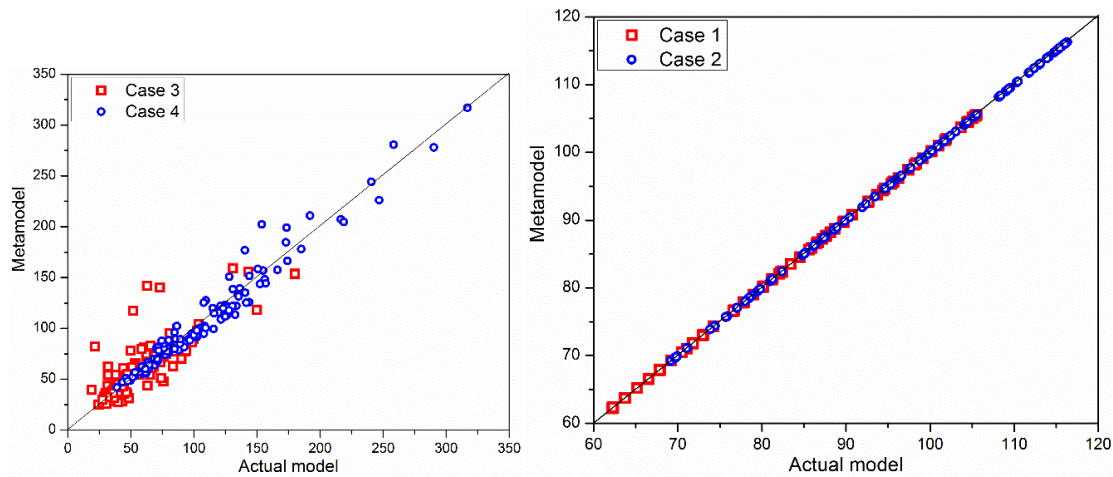


Figure 2. Predictive power of RSM metamodels in Case 1 and Case 2. Figure 3 Predictive power of RSM metamodels in Case 3 and Case 4.

#### 4.3 Parametric study

The metamodels built in section 4.1 are used for conducting a parametric study on free vibration behaviour of composite plates. The effect of orthotropy ratio ( $\frac{E_1}{E_2}$ ) on the fundamental frequency is depicted in Figure 4. It is seen that the fundamental frequency monotonically increases as the orthotropy ratio increases. This is because the increasing orthotropy would increase the stiffness of the composite plates thereby increasing the frequency parameter. Also, it is clear that Case 2 i.e. rhombic composite plate has a higher frequency parameter as compared to Case 1 i.e. Square plate. The effect of major shear modulus to Young's modulus ratio ( $\frac{G_{12}}{E_2}$ ) on fundamental frequency is depicted in Figure 5. Fundamental frequency increases with increase in  $\frac{G_{12}}{E_2}$ . However, the increase is prominent only in case of rhombic plates. Similarly in Figure 6, increase in  $\frac{G_{23}}{E_2}$  nominally increases the fundamental frequency. The effect of Poisson's ratio ( $\nu_{12}$ ) on fundamental frequency is depicted in Figure 7. The Poisson's ratio has a negligible effect on the frequency parameter of the composite plates.

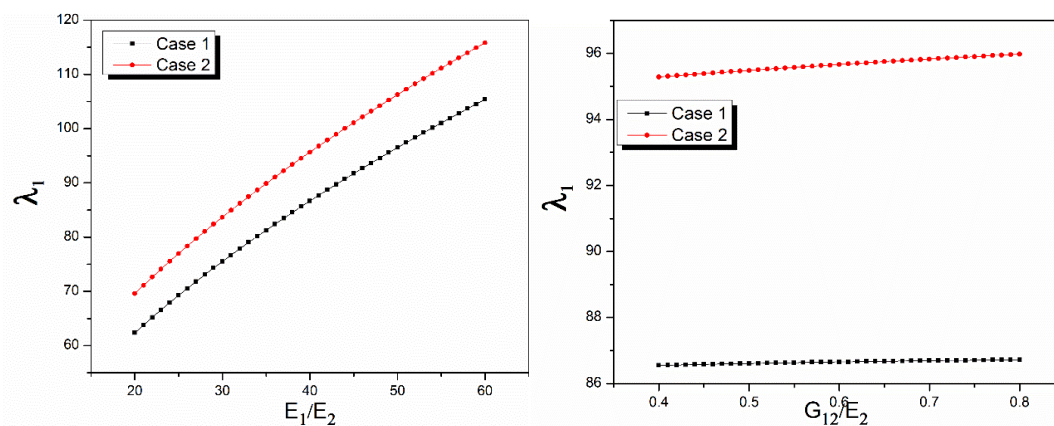


Figure 4. Effect of  $\frac{E_1}{E_2}$  on fundamental frequency. Figure 5. Effect of  $\frac{G_{12}}{E_2}$  on fundamental frequency.

The effect of composite plate height-to-width ratio ( $\frac{b}{a}$ ) on fundamental frequency is illustrated in Figure 8. In general, the increase in  $\frac{b}{a}$  causes decrease in fundamental frequency. However, the boundary condition also plays a dominant role in the behaviour of the composite plate.

The influence of thickness ratio,  $\frac{h}{a}$  on fundamental frequency is illustrated in Figure 9. It is clear that increase in  $\frac{h}{a}$  causes a monotonic decrease in fundamental frequency. This is due to the increase in mass of the composite plate. The drop in fundamental frequency with increase in thickness is more prominent in all sides clamped plate as compared to the all sides simply supported one.



Figure 10 demonstrates the influence of skew angle on the fundamental frequency. Increase in skew angle causes an increase in stiffness of the composite laminates, thereby increasing the fundamental frequency. In all sides simply supported laminates the increase is gradual whereas in clamped plates there is a rapid increase in fundamental frequency between 30° to 60° skew angle.

In general, increase in the number of layers for the same thickness causes a slight increase in fundamental frequency as shown in Figure 11. This is because as more as more plies are added, the rigidity of the composite plate increase which causes the fundamental frequency to increase.

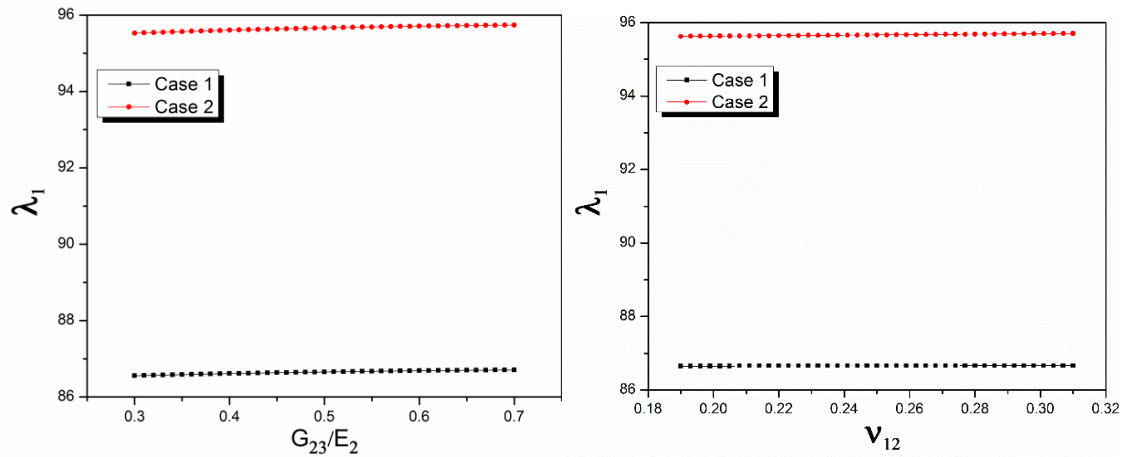


Figure 6. Effect of  $\frac{G_{23}}{E_2}$  on fundamental frequency. Figure 7. Effect of  $\nu_{12}$  on fundamental frequency.

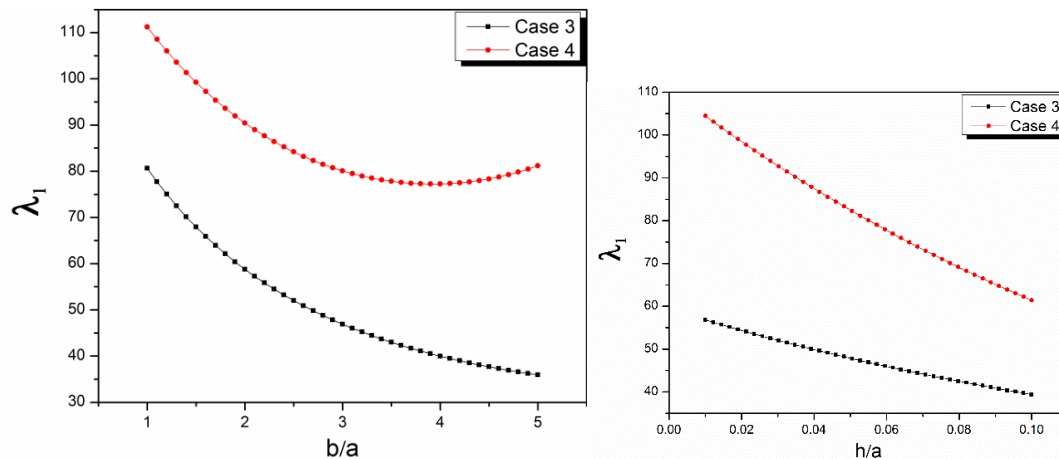


Figure 8. Effect of  $\frac{b}{a}$  on fundamental frequency. Figure 9. Effect of  $\frac{h}{a}$  on fundamental frequency.

It is also worth mentioning here that for all the parametric studies reported in Figure 4–Figure 11, effect of one factor is studied while keeping the other parameters in the metamodel at their mean level.

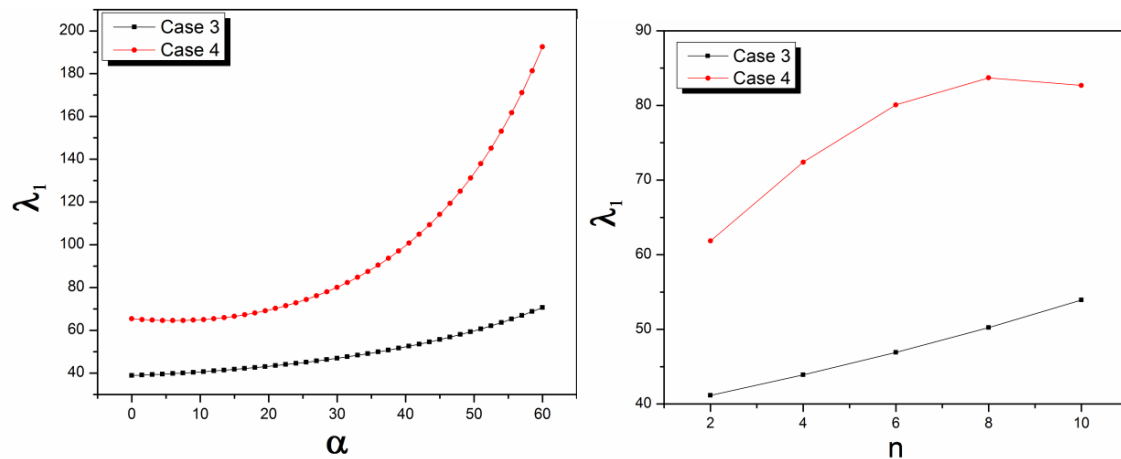


Figure 10. Effect of skew angle,  $\alpha$  on fundamental frequency. Figure 11. Effect of number of layers on fundamental frequency.

## 5 Conclusions

In this work, a numerical study is conducted on the dynamic behaviour of composite laminates. Instead of using time-intensive finite element models, computationally cheap and easy to implement local approximate models (metamodels) are used. The metamodels require only a few sample points to train itself and replace the finite element model. These sample points or the training data may be obtained by conducting finite element simulations or experiments. Similarly, it is well advised to check the performance of the metamodels on independent testing data sets thereby making sure that the metamodels are not ‘over-fitted’ on the training data. Use of statistical metrics like  $R^2$ ,  $R^2_{adj}$  and  $Q^2_{F3}$  provide a quick and reliable knowledge on the usability of the metamodels. Alternately, scatter plots between the actual model and the metamodel provides a quick visual assessment on the predictive power of the metamodel. Any under-prediction or over-prediction by the metamodel can be quickly identified. In the final part of the manuscript, the efficacy of the metamodels in carrying out a quick but approximate parametric analysis of composite laminates is illustrated. Thus, appropriately built robust response surface methodology based metamodels can be a useful alternative to finite element models.

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