Priors and Payoffs in Confidence Judgments

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1 Abstract

Priors and payoffs are known to change perceptual decision-making, but little is understood about how they influence confidence judgments. Human observers performed an orientation-discrimination task with varied priors and payoffs. We investigated the subsequent placement of discrimination and confidence criteria by comparing behavior to several plausible Signal Detection Theory models. A normative account of behavior uses optimal discrimination criteria. Optimal confidence criteria are yoked to the accuracy-maximizing criterion (i.e., are not affected by payoffs). Additionally, in a normative account, the criterion shifts predicted for asymmetric payoffs and priors should sum when both are varied. We found that observers were conservative in discrimination-criterion placement and that criterion shifts due to priors and payoffs did not sum. For confidence judgments, observers exhibited one of two sub-optimal behaviors. One subset of observers used fixed confidence criteria independent of priors and payoffs. The other group of observers always shifted their confidence criteria with the gains-maximizing discrimination criterion. Such metacognitive mistakes about one's perceptual choices could have negative consequences outside the laboratory setting.

Keywords: *decision-making, metacognition, confidence, Signal Detection Theory.*

2 Introduction

In making a perceptual decision, it is wise to consider information beyond the available sensory evidence. To maximize expected gains, one should consider both the baseline probability of each possible world state, i.e. *priors*, as well as the associated risks and rewards for choosing or not choosing each response alternative, i.e., *payoffs*. In the Signal Detection Theory (SDT) framework, priors and payoffs alter the threshold amount of evidence required to choose one alternative versus another, that is, a shift in the *criterion* for reporting option "A" versus option "B" in a binary task. For example, a radiologist may be trying to detect a tumor from an x-ray. The radiologist should be more likely to report a positive result for a suspicious shadow if the patient's file indicates they are a smoker, as this means they have a higher prior probability of cancer. Similarly, the high cost of waiting to treat the cancer should also bias the radiologist towards declaring a positive result. In both real and laboratory environments, observers have been found to factor in priors and payoffs when setting the decision criterion (Maddox and Bohil, 1998, 2000; Maddox and Dodd, 2001; Wolfe et al., 2005; Ackermann and Landy, 2015; Horowitz, 2017), with some caveats we will discuss shortly.

Decisions about the state of the world (cancer or not cancer, cat or dog, clockwise or counter-clockwise of vertical) are based on the stimulus alone and are classified as stimulus-conditioned responses or *Type 1* decisions in the literature. These differ from *Type 2* decisions (or response-conditioned responses), which are judgments about the correctness of Type 1 decisions (Clarke et al., 1959; Mamassian, 2016). In layman's terms, Type 2 responses are the observer's confidence about a decision they've made, which are often operationalized in binary decision-making experiments as a subjective estimate of the probability the Type 1 response was correct (Pouget et al., 2016). Confidence plays a broad role in guiding behavior, subsequent decision-making, and learning in a multitude of scenarios for both humans and animals (Metcalfe and Shimamura, 1996; Smith et al., 2003; Beran et al., 2012).

How does an ideal-observer radiologist modify confidence judgments in response to varying priors or payoffs? Intuitively, a radiologist should be more confident in a positive diagnosis when the patient is a smoker, given the prior scientific literature on the health risks of smoking that the radiologist has read. Additional confirmatory information should boost confidence in that positive diagnosis, and contrary evidence should reduce confidence, because priors (smoker or non-smoker) and sensory evidence (cancerous-looking shadow) are both informative about the likelihood over possible world states. However, this is not the case for payoffs. Incentivizing the different responses with rewards or costs does not change the uncertainty about the world state. The radiologist should not be more or less sure of a cancer diagnosis if the type of cancer would be deadly or benign, even though this should affect their initial diagnosis. In fact, sometimes payoffs will lead the decision-maker to choose the less probable alternative and this should be reflected by low confidence in the decision.

Little is known about how human observers adjust confidence in response to prior-payoff structures. In one perceptual study, the prior probabilities of target present versus absent affected the placement of the criteria for Type 1 and 2 judgments (Sherman et al., 2015), with some evidence that confidence better predicts performance for responses congruent with the more probable outcome than those that are incongruent. In the realm of social judgments, prior probabilities have been shown to modulate the degree of confidence, with higher confidence assigned to more probable outcomes (Manis et al., 1980). However, others have found counter-productive incorporation of priors, with over-confidence for low-probability outcomes and under-confidence for high-probability outcomes (Dunning et al., 1990). In regards to payoffs, early work on monetary incentives in perceptual categorization did collect confidence ratings, however they were not included in any analyses (Lee and Zentall, 1966). Consideration of payoff structures is ubiquitous in animal studies of confidence that employ post-decisional wagering methods (Smith et al., 2003). For example, in the opt-out paradigm, to distinguish between low and high confidence, the animal chooses between a small, certain reward and a risky alternative with either high reward or no reward, for correct and incorrect perceptual responses respectively (Kiani and Shadlen, 2009). However, because animals are motivated by their expected gain and not explicit verbal instructions, it is impossible to isolate decision confidence unconfounded with the subjective value of the reward.

Here, we seek to characterize how human observers adjust perceptual decisions and confidence in response to joint manipulation of priors and payoffs. First, we defined a normative model of confidence judgments that factors in the prior-payoff structure of the environment. Then, we measured how well this model explains human behavior in an orientation-discrimination task, as compared to several sub-optimal decision models. We found that all observers made sub-optimal confidence judgments, but fell into two distinct groups depending on their strategy. These results highlight the importance of considering the effect of priors and payoffs on confidence, particularly in applied or real-world scenarios where they are likely to be non-uniform across the decision alternatives.

3 The Decision Models

In this section we describe the rationale and background for the modeling of Type 1 and Type 2 decision-making. We follow the example of a left-right orientation judgment followed by a binary low-high confidence judgment to match the experimental paradigm used in the present study. First the range of Type 1 models are identified, which assess the placement of the discrimination decision criterion under different prior-payoffs scenarios. Then the Type 2 models are outlined, describing the different potential relationships between the decision criteria for confidence and the criterion for discrimination.

3.1 The Type 1 Decision

To make the Type 1 decision, observers must relate a noisy internal measurement, x, of the stimulus, s, where $s \in \{s_L, s_R\}$, to a binary response, which in the context of our experiment is "tilted left" (say "$s = s_L$") or "tilted right" (say "$s = s_R$"). This is done by a comparison to an internal criterion, k_1, such that if $x < k_1$, the observer will respond with"tilted left", and otherwise "titled right" (Figure 1a). The only component of the Type 1 model where the observer has any control is deciding where to place the criterion. The optimal value of k_1 (k_{opt}) maximizes the expected gain, ensuring the observer makes the most points/money/etc. over the course of the experiment. The value of k_{opt} depends on three things:

(i) The sensitivity of the observer, d'. In the standard model of the decision space, $P(x|s_L) \sim N(\mu_L, \sigma_L)$ and $P(x|s_R) \sim N(\mu_R, \sigma_R)$, with $\mu_L = -\mu_R$ and $\sigma_L = \sigma_R = 1$.

Under this transformation, the sensitivity d' corresponds to the distance between the peaks of the two internal measurement distributions.

(ii) The prior probability of each stimulus alternative, $P(s_L)$ and $P(s_R) = 1 - P(s_L)$.

(iii) The rewards for the four possible stimulus-response pairs, $V_{r,s}$, which are the rewards (positive) or costs (negative) of responding r when the stimulus is s.

An ideal observer that maximizes expected gain (Green and Swets, 1966) uses criterion

$$k_{opt} = \frac{\ln \beta_{opt}}{d'}, \tag{1}$$

where the likelihood ratio β_{opt} at the optimal criterion is a function of priors and payoffs:

$$\beta_{opt} = \frac{P(s_L)}{P(s_R)} \frac{V_{L,L} - V_{L,R}}{V_{R,R} - V_{R,L}}. \tag{2}$$

In our experiment, 0 points are awarded for incorrect answers, allowing us to simplify:

$$\ln \beta_{opt} = \ln \frac{P(s_L) V_{L,L}}{P(s_R) V_{R,R}} = \ln \frac{P(s_L)}{P(s_R)} + \ln \frac{V_{L,L}}{V_{R,R}}. \tag{3}$$

Thus, $k_{opt} = k_p + k_v$, where k_p is the optimal criterion location if only priors were asymmetric and k_v is the optimal criterion if only the payoffs were varied. As can be seen in Eq. 3, the effects of priors and payoffs sum when determining the optimal criterion (illustrated in Figure 1b). When the priors are more similar, or the payoffs are closer to equal, k_{opt} is closer to the neutral criterion $k_{neu} = 0$. Note that in the case of symmetric payoffs, k_{opt} maximizes both expected gain and expected accuracy, whereas when asymmetric payoffs are involved, k_{opt} maximizes expected gain only (i.e., $k_{opt} \neq k_p$). This is because to maximize expected gain, from time to time the observer is incentivized to choose the less probable outcome because it is more rewarded.

3.2 Conservatism

Often, human observers use a sub-optimal value of k_1 when the prior probabilities or payoffs are not identical for each alternative. A common observation is that the criterion is not adjusted far enough from the neutral criterion towards the optimal criterion, $k_{neu} < k_1 < k_{opt}$ or $k_{neu} > k_1 > k_{opt}$, a behavior referred to as conservatism (Green and Swets, 1966; Maddox, 2002). It is useful to express conservatism as a weighted sum of the neutral and optimal criterion:

$$k_1 = (1 - \alpha)k_{neu} + \alpha k_{opt} = \alpha k_{opt}, \tag{4}$$

with $0 < \alpha < 1$ indicating conservative criterion placement. The degree of conservatism is greater the closer α is to 0 (Figure 1c). Several studies have contrasted the conservatism for unequal priors versus unequal payoffs, typically finding greater conservatism for unequal payoffs (Lee and Zentall, 1966; Ulehla, 1966; Healy and Kubovy, 1981; Ackermann and Landy, 2015) with few exceptions (Healy and Kubovy, 1978). This may result from an underlying criterion-adjustment strategy that depends on the shape of the expected gain curve (as a function of criterion placement) and not just on the position of the optimal criterion maximizing expected gain (Busemeyer and Myung, 1992; Ackermann and Landy, 2015) or a strategy that trades off between maximizing expected gain and maximizing expected accuracy (Maddox, 2002; Maddox and Bohil, 2003). Given that the effects of priors and payoffs sum in Eq. 3, we will consider a sub-optimal model of criterion placement that has separate conservatism factors for payoffs and priors:

$$k_1 = \frac{1}{d'}\left[\alpha_p \ln\frac{P(s_L)}{P(s_R)} + \alpha_v \ln\frac{V_{L,L}}{V_{R,R}}\right] = \alpha_p k_p + \alpha_v k_v. \tag{5}$$

The conservatism factors, α_p and α_v, scale these individually before they are summed to give the final conservative criterion placement, taking into account both prior and payoff asymmetries. This formulation allows for differing degrees of conservatism for priors and payoffs.

3.3 Type 1 Decision Models

We consider four models of the Type 1 discrimination decision in this paper, including the optimal model (i) and three sub-optimal models that include varying forms of conservatism (ii-iv):

(i) $\Omega_{1,opt} : k_1 = k_{opt} = k_p + k_v$

(ii) $\Omega_{1,1\alpha} : k_1 = \alpha k_{opt} = \alpha\,(k_p + k_v)$

(iii) $\Omega_{1,2\alpha} : k_1 = \alpha_p k_p + \alpha_v k_v$

(iv) $$\Omega_{1,3\alpha} : \begin{cases} k_1 = \alpha_{pv} k_{opt} & \text{if } k_p \neq 0 \text{ and } k_v \neq 0 \ \ \text{(i.e., both asymmetric)} \\ k_1 = \alpha_p k_p & \text{if } k_v = 0 \ \ \text{(i.e., payoffs symmetric)} \\ k_1 = \alpha_v k_v & \text{if } k_p = 0 \ \ \text{(i.e., priors symmetric).} \end{cases}$$

Thus, we consider models with no conservatism ($\Omega_{1,opt}$), with an identical degree of conservatism due to asymmetric priors and payoffs ($\Omega_{1,1\alpha}$), or different amounts of conservatism for prior versus payoff manipulations ($\Omega_{1,2\alpha}$). In the fourth model, we drop the assumption (that was based on the optimal model) that effects of payoffs and priors on criterion sum, i.e., that behavior with asymmetric priors and payoffs can be predicted from behavior with each effect alone ($\Omega_{1,3\alpha}$). We consider this final model because the additivity of criterion shifts (Eq. 3) has not yet been experimentally confirmed with human observers (Stevenson et al., 1990).

In all models, we also consider an additive bias term, γ, corresponding to a perceptual bias in perceived vertical. The bias is also included in the neutral criterion $k_{neu} = \gamma$. For clarity, however, we have omitted it from the mathematical descriptions of the models. Note that any observer best fit by $\Omega_{1,opt}$ but with a γ significantly different from 0 would no longer be considered as having optimal behavior.

3.4 Confidence Criteria

Confidence judgments should reflect the belief that the selected alternative in the discrimination decision correctly matches the true world state. Generally speaking, the further the internal measurement is from a well-placed decision boundary, the more evidence there is for the discrimination judgment. This is instantiated in the extended SDT framework by the addition of two or more confidence criteria, k_2 (Maniscalco and Lau, 2012, 2014). There are two such criteria for a binary confidence task and more confidence criteria when more than

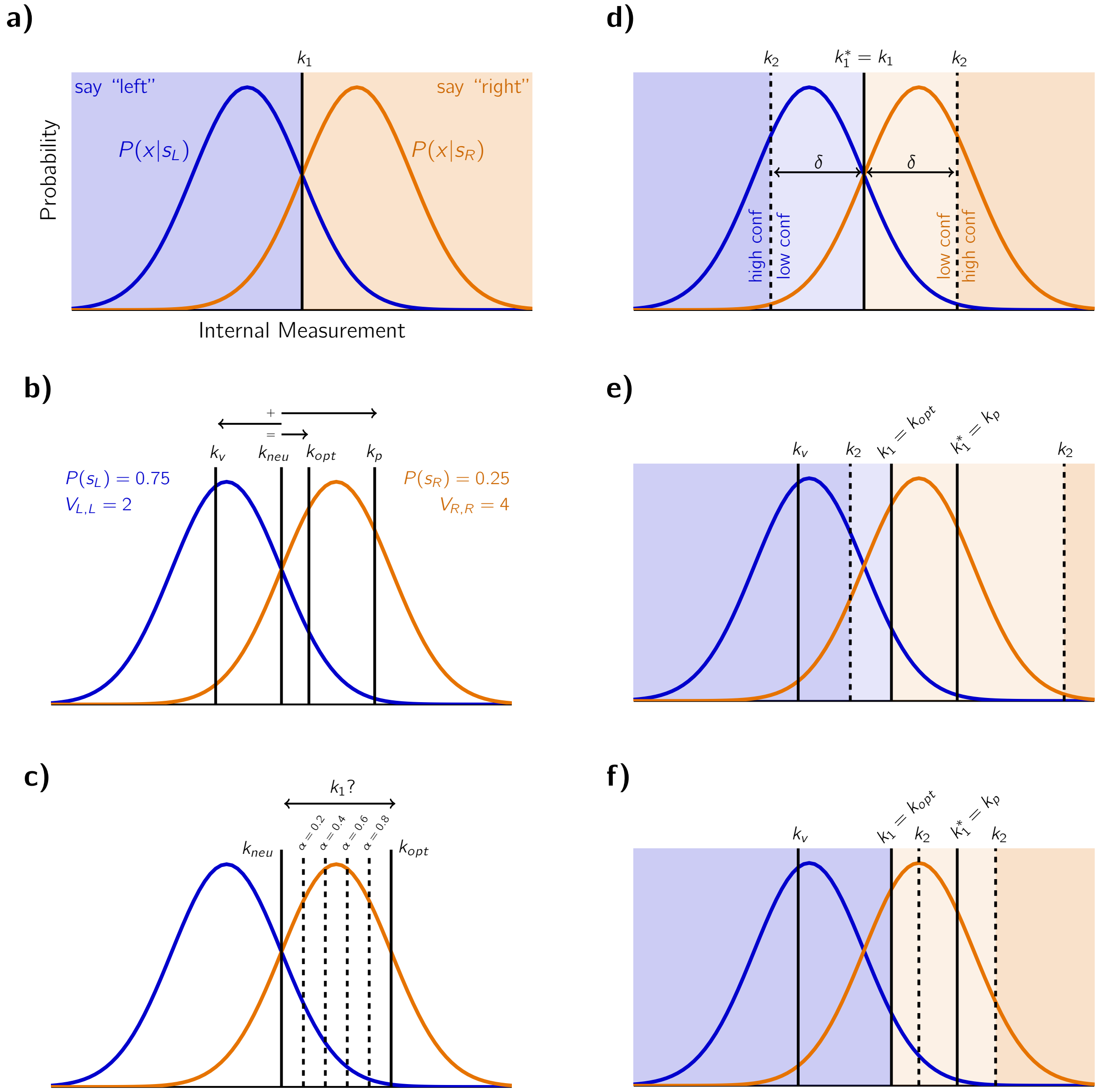


Figure 1: Illustration of the full SDT model. a) On each trial, an internal measurement of stimulus orientation is drawn from a Gaussian probability distribution conditional on the true stimulus value. The Type 1 criterion, k_1, defines a cut-off for reporting "left" or "right". The ideal observer in a symmetrical priors and payoffs scenario is shown. b) The ideal observer's criterion placement with both prior and payoff asymmetry. This prior asymmetry encourages a rightward criterion shift to k_p and the payoff asymmetry a leftward shift to k_v. The optimal criterion placement that maximizes expected gain, k_{opt}, is a sum of these two criterion shifts. For comparison, the neutral criterion, k_{neu} is shown. As the prior asymmetry is greater than the payoff asymmetry, 3:1 vs 1:2, $k_{opt} \neq k_{neu}$. c) A sub-optimal conservative observer will not adjust their Type 1 criterion far enough from k_{neu} to be optimal. The parameter α describes the degree of conservatism, with values closer to 0 being more conservative and closer to 1 less conservative. d) In the case of symmetric payoffs and priors, the Type 2 confidence criteria, k_2, are placed equidistant from the Type 1 decision boundary, carving up the internal measurement space into a low- and high-confidence region for each discrimination response option. e) For the normative Type 2 model, the confidence criteria are placed symmetrically around a hypothetical Type 1 criterion that only maximizes accuracy ($k_1^* = k_p$). This figure shows the division of the measurement space as per the prior-payoff scenario in (b). As a left-tilted stimulus is much more likely, this results in many high-confidence left-tilt judgments and few high-confidence right-tilt judgments. Note that left versus right judgments still depend on k_1. f) The same as in (e) but with small δ value. Note the low-confidence region where confidence should be high (left of the left-hand k_2). This happens because in this region the observer will choose the Type 1 response that conflicts with the accuracy-maximizing criterion, hence they will report low confidence in their decision. Note that the displacements of the criteria from the neutral criterion in this figure are exaggerated for illustrative purposes.

two confidence levels are provided. We restrict our treatment to the binary case, which can be trivially extended to include more gradations of confidence.

As illustrated in Figure 1d for the case of symmetric payoffs and priors, there is a k_2 confidence criterion on each side of the k_1 decision boundary. If the measurement obtained is beyond one of these criteria relative to k_1, then the observer will report high confidence, and otherwise will report low confidence. Stated another way, the addition of the confidence criteria effectively divides the measurement axis into four regions: high-confidence left, low-confidence left, low-confidence right, and high-confidence right. The closer to the discrimination decision boundary that the observer places k_2, the more high-confidence responses they will give. We denote this distance as δ. δ is not always assumed to be identical for both confidence criteria (e.g. Maniscalco and Lau, 2012), but we assumed a single value of δ for model simplicity. Type 2 judgments were not incentivized in our experiment. Thus, there is no explicit cost function to constrain the distance parameter δ, so the precise setting of δ will not factor into the evaluation of how well the normative model fits observer behavior.

3.5 The Counterfactual Type 1 Criterion

The above description of how confidence responses are generated is well suited to cases where the payoffs are symmetric. This is because the optimal decision criterion maximizes both gain and accuracy. For an internal measurement at the discrimination boundary, it is equally probable that the stimulus had a rightward versus leftward orientation. Expressed another way, the log-posterior ratio at k_{opt} is 1. Thus, the distance from the discrimination boundary is a good measure for the probability that the Type 1 response is correct (i.e., confidence as we defined it above). This, however, is not the case when payoffs are asymmetric ($k_1 = k_p + k_v = k_{opt}$ where $k_v \neq 0$), as the ideal observer maximizes gain but not accuracy. The log-posterior ratio is not 1 at k_{opt} but rather it is equal to 1 at k_p.

To extend the SDT model of confidence to asymmetric payoffs, we introduce a new criterion. The counterfactual criterion, k_1^*, is the criterion the ideal observer would have used if they ignored the payoff structure of the environment and exclusively maximized accuracy and not gain (i.e., $k_1^* = k_p$). It is around this criterion that the observer symmetrically places confidence criteria in our normative model (Figure 1e). Whenever payoffs are symmetrical, $k_1 = k_1^*$. Figure 1f illustrates a situation unique to this model that may occur when payoffs are asymmetrical. Here, the value of δ is sufficiently small that both k_2 criteria fall on the same side of k_1. As a result, the region between k_1 and the left-hand k_2 criterion results in a low-confidence response despite being beyond the k_2 boundary (relative to k_1^*). This occurs because this region is to the right of k_1 and thus, due to asymmetric payoffs, the observer will make the *less* probable choice, thus resulting in low confidence in that choice. Effectively, the left-hand confidence criterion is shifted from k_2 to k_1. Here, we rely on the assumption that the confidence system is aware of the Type 1 decision (for further discussion of this issue, see Fleming and Daw, 2017).

The notion of an observer computing additional criteria for counterfactual reasoning is not new. For example, in the model of Type 1 conservatism of Maddox and Bohil (1998), where observers trade off gain versus accuracy, k_1 is a weighted average of the optimal criteria for maximizing expected gain (k_{opt}) and for exclusively maximizing accuracy (k_p). In Zylberberg et al. (2018), observers learned prior probabilities of each stimulus type by an updating decision-making mechanism that computes the confidence the observer would have had if they had used the neutral criterion (k_{neu}) for their Type 1 judgment. We suggest that for determining confidence in the face of asymmetric payoffs, optimal observers compute the confidence they would have reported if they had instead used the k_p criterion for the discrimination judgment.

3.6 Type 2 Decision Models

In addition to the normative model we just described (i), we considered four sub-optimal models (ii-v) for the counterfactual Type 1 criterion about which the Type 2 criteria are symmetrically arranged:

(i) $\Omega_{2,acc} : k_1^* = k_p$

(ii) $\Omega_{2,acc+cons} : k_1^* = \alpha_p k_p$

(iii) $\Omega_{2,gain} : k_1^* = k_{opt}$

(iv) $\Omega_{2,gain+cons} : k_1^* = k_1$

(v) $\Omega_{2,neu} : k_1^* = k_{neu}$

All of these models are characterized by the placement of the counterfactual criterion, k_1^*; the distance δ is the only free parameter for all models. In the normative model ($\Omega_{2,acc}$), the confidence criteria are systematically shifted so that they are centered on the discrimination criterion that maximizes accuracy. We also consider a model in which confidence criteria are centered on the criterion that maximizes gain ($\Omega_{2,gain}$), which is incorrect behavior in the case of asymmetric payoffs. In the neutral model ($\Omega_{2,neu}$), confidence criteria remain fixed around the neutral Type 1 criterion regardless of the prior or payoff manipulation. Finally, for the models that shift in response to priors and payoffs, we consider that conservatism in the discrimination criterion placement also affects k_1^*, either for the accuracy model ($\Omega_{2,acc+cons}$) or the gain model ($\Omega_{2,gain+cons}$). In the latter model, k_1^* is identical to k_1. For the other models, some combinations of priors and payoffs will decouple k_1^* from k_1. For the $\Omega_{2,acc+cons}$ model, the decoupling only occurs for asymmetric payoffs. For the other models, this decoupling occurs whenever priors or payoffs are asymmetric.

Our models assume that δ are placed symmetrically around k_1^*. However, the ability to identify the underlying Type 2 model will not be affected by this assumption. Consider an observer whose low-confidence region to the left of k_1^* was always greater than their low-confidence region to the right of k_1^*, such that $k_1^* - k_{2-} > k_{2+} - k_1^*$. Then, the estimate of δ would be similar because the experiment design tested the mirror prior-payoff condition (i.e., for fixed k_2, one condition would have k_1^* attracted to neutral and the other repelled, which is not the behaviour of k_1^* in any Type 2 model). Thus, the best-fitting model would be unlikely to change when δ is asymmetric, but it would fit less well. Alternatively, an asymmetry in δ could be mirrored about the neutral criterion (e.g., the low confidence region closest to the neutral criterion is always smaller). Then, the δ asymmetry would be indistinguishable from a bias in the conservatism parameters. Ultimately, the confidence criteria are yoked to k_1^*, and it is the patterns of criteria shift from all conditions jointly that are captured by the model comparison.

4 Methods

4.1 Participants

Ten participants (5 female, age range 22-43 years, mean 27.0 years) took part in the experiment. All participants had normal or corrected-to-normal vision, except one amblyopic participant. All participants were naive to the research question, except for three of the authors who participated. On completion of the study, participants received a cash bonus in the range of $0 to $20 based on performance. In accordance with the ethics requirements of the Institutional Review Board at New York University, participants received details of the experimental procedures and gave informed consent prior to the experiment.

4.2 Apparatus

Stimuli were presented on a gamma-corrected CRT monitor (Sony G400, 36 x 27 cm) with a 1280 x 1024 pixel resolution and an 85 Hz refresh rate. The experiment was conducted in a dimly lit room, using custom-written code in MATLAB version R2014b (The MathWorks, Natick, MA), with PsychToolbox version 3.0.11 (Brainard, 1997; Pelli, 1997; Kleiner et al., 2007). A chin-rest was used to stabilize the participant at a viewing distance of 57 cm. Responses were recorded on a standard computer keyboard.

4.3 Stimuli

Stimuli were Gabor patches, either right (clockwise) or left (counterclockwise) of vertical, presented on a mid-gray background at the center of the screen. The sinusoidal grating had a spatial frequency of 2 cycle/deg, a peak contrast of 10%, and a Gaussian envelope (SD: 0.5 deg). The phase of the grating was randomized on each trial to minimize contrast adaptation.

4.4 Experimental Design

Orientation discrimination (Type 1, 2AFC, left/right) and confidence judgments (Type 2, 2AFC, low/high) were collected for seven conditions defined by the prior and payoff struc-

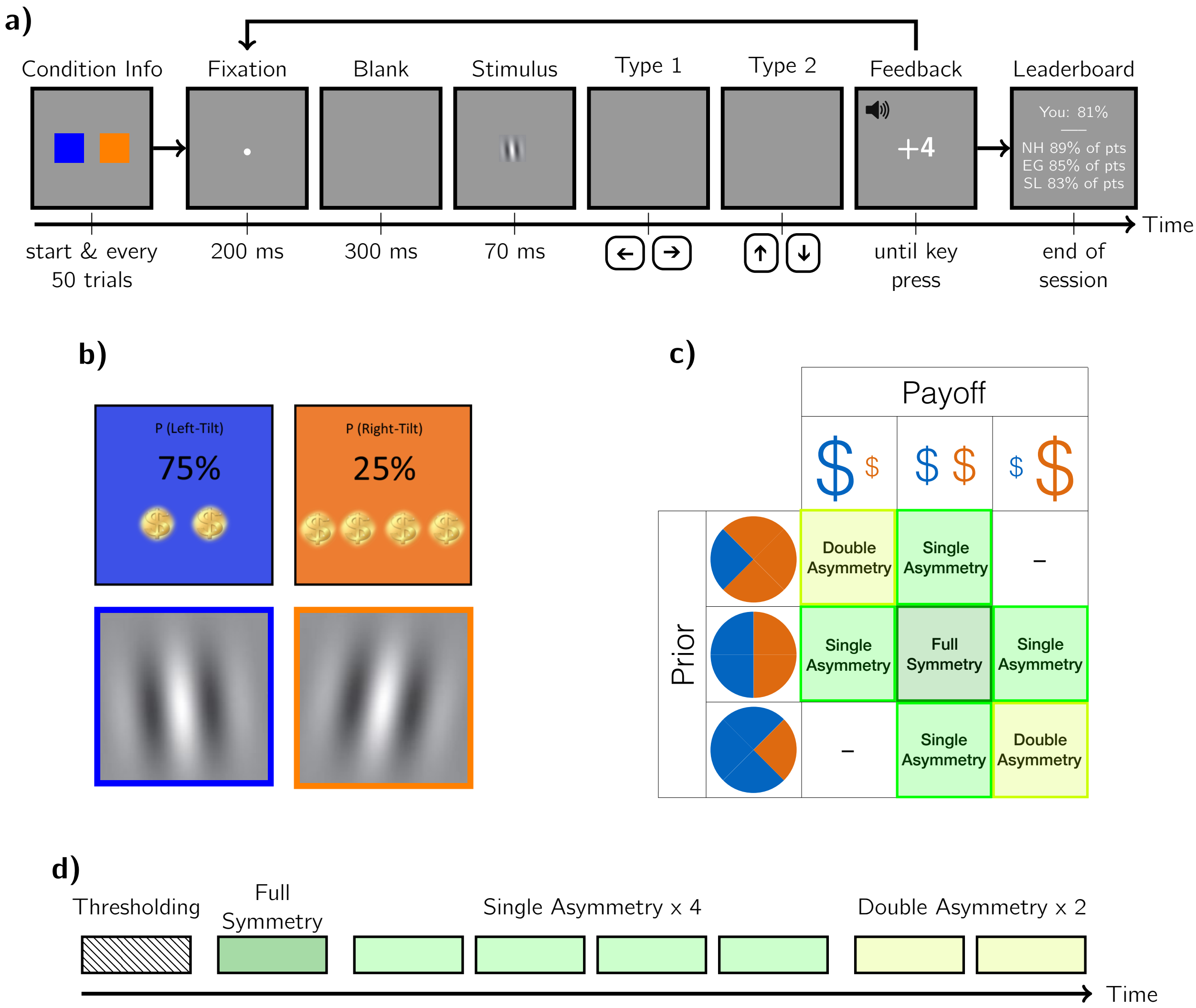


Figure 2: Experimental methods. a) Trial sequence including an outline of the initial condition information screen (see part (b) for details) and final (mock) leaderboard screen. Participants were shown either a right- or left-tilted Gabor and made subsequent Type 1 and Type 2 decisions before being awarded points and given auditory feedback based on the Type 1 discrimination judgment. b) Sample condition information displays from a double-asymmetry condition. Below: Example Gabor stimuli, color-coded blue for left- and orange for right-tilted. The exact stimulus orientations depended on the the participant's sensitivity. c) Condition matrix. Pie charts show the probability of stimulus alternatives (25, 50, or 75%) and dollar symbols represent the payoffs for each alternative (2, 3, or 4 pts). Squares are colored and labeled by the type of symmetry. d) Timeline of the eight sessions. The order of conditions was randomized within the single- and within the double-asymmetry conditions.

ture. The probability of a right-tilted Gabor could be 25, 50, or 75%. The points awarded for correctly identifying a right- versus a left-tilt could be 4:2, 3:3, or 2:4. In the 3:3 payoff scheme, a correct response was awarded 3 points. In the 2:4 and 4:2 schemes, correct responses were awarded 2 or 4 points depending on the stimulus orientation. Incorrect responses were not rewarded (0 points). The prior and payoff structure was explicitly conveyed to the participant before the session began (Fig. 2b) and after every 50 trials. Condition order was randomized within condition type (Fig. 2c): no asymmetry (50%, 3:3), single asymmetry (50%, 4:2; 50%, 2:4; 25%, 3:3; 75%, 3:3), or double asymmetry (25%, 4:2; 75%, 2:4). Note that two of the possible double asymmetry conditions (25%, 2:4; and 75%, 4:2) were not tested because these conditions incentivized one response alternative to such a degree that they would not be informative for model comparison. Participants first completed the full-symmetry condition, followed by the single-asymmetry conditions in random order, and finally the double-asymmetry conditions, also in random order (Fig. 2d). Each condition was tested in a separate session with no more than one session per day.

4.5 Thresholding Procedure

A thresholding procedure was performed prior to the main experiment to equate difficulty across observers to approximately $d' = 1$. Observers performed a similar orientation discrimination judgment as in the main experiment. Absolute tilt magnitude varied in a series of interleaved 1-up-2-down staircases to converge on 71% correct. Each block consisted of three staircases with 60 trials each. Participants performed multiple blocks until it was determined that performance had plateaued (i.e., learning had stopped). Preliminary thresholds were calculated using the last 10 trials of each staircase. At the end of each block, if none of the three preliminary thresholds were better than the best of the previous block's preliminary thresholds, then the stopping rule was met. As a result, participants completed a minimum of two blocks and no participant completed more than five blocks. A cumulative Gaussian psychometric function was fit by maximum likelihood to all trials from the final two blocks (360 trials total). The slope parameter was used to calculate the orientation corresponding to 69% correct for an unbiased observer ($d' = 1$; Macmillan and Creelman, 2005). This orientation was then used for this subject in the main experiment. Thresholds ranged from

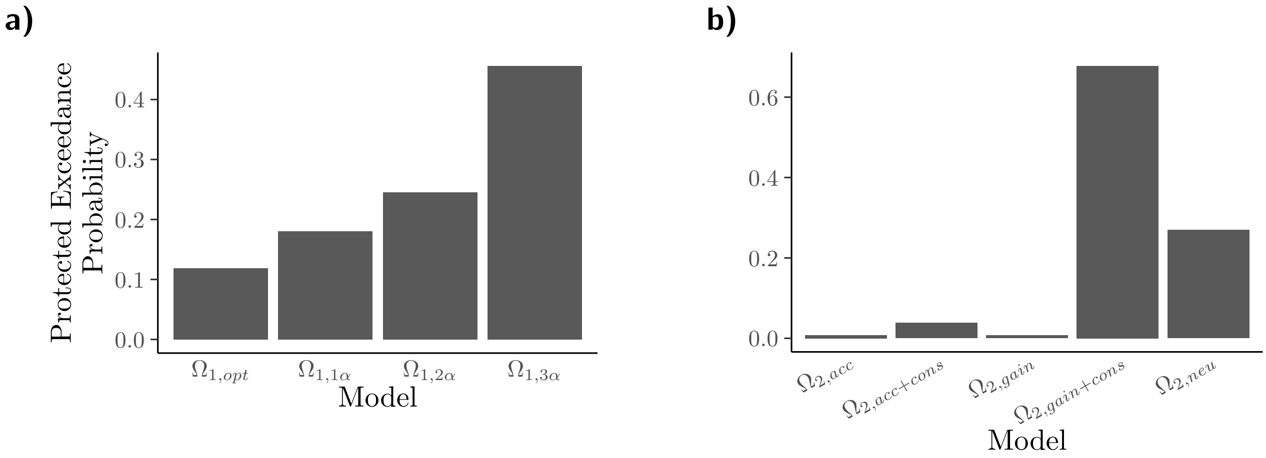
0.36 to 0.78 deg, with a mean of 0.59 deg.

4.6 Main Experiment

Participants completed seven sessions, each of which had 700 trials with the first 100 treated as warm-up and discarded from the analysis. All subjects were instructed to hone their response strategy in the first 50 trials to encourage stable criterion placement. The trial sequence is outlined in Fig. 2a. Each trial began with the presentation of a fixation dot for 200 ms. After a 300 ms inter-stimulus interval, a Gabor stimulus was displayed for 70 ms. Participants judged the orientation (left/right) and then indicated their confidence in that orientation judgment (high/low). Feedback on the orientation judgment was provided at the end of the trial by both an auditory tone and the awarding of points based on the session's payoff structure. Additionally, the running percentage of potential points earned was shown on a leaderboard at the end of each session to foster inter-subject competition. Participants' cash bonus was calculated by selecting one trial selected at random from each session and awarding the winnings from that trial, with a conversion of 1 point to $1, capped at $20 over the sessions. Total testing time per subject was approximately 8 hrs.

4.7 Model Fitting

Detailed description of the model-fitting procedure can be found in Supplementary Information (Sections 1 and 2). Briefly, model fitting was performed in three sequential steps: fitting of d', Type 1 models, then Type 2 decisions. Each session provided one measurement of d', which we used in a hierarchical Bayesian model to estimate each participant's true underlying d' value across the entire experiment. For Type 1 and Type 2 models, we calculated the log likelihood of the data given a dense grid of parameters (e.g., α, γ, and δ) using multinomial distributions defined by the stimulus type, discrimination response, and confidence response. All seven conditions were fit jointly. We then calculated model evidence by marginalizing over all parameter dimensions and then normalizing to account for grid spacing.



c)

Type 1 \ Type 2	$\Omega_{2,acc}$	$\Omega_{2,acc+cons}$	$\Omega_{2,gain}$	$\Omega_{2,gain+cons}$	$\Omega_{2,neu}$	
$\Omega_{1,opt}$	-	-	-	-	-	-
$\Omega_{1,1\alpha}$	-	-	-	2	-	2
$\Omega_{1,2\alpha}$	-	-	-	2	2	4
$\Omega_{1,3\alpha}$	-	1	-	1	2	4
	-	1	-	5	4	

Figure 3: Model comparison for the Type 1 and Type 2 responses. a) The protected exceedance probabilities (PEPs; see text for details) of the four Type 1 models. b) PEPs of the five Type 2 models. Note that model comparisons were performed first for Type 1 and then for Type 2 responses, using the best-fitting Type 1 model and parameters, on a per-subject basis, in the Type 2 model evaluation. c) Best-fitting models for each participant.

5 Results

We sought to understand how observers make perceptual decisions and confidence judgments in the face of asymmetric priors and payoffs. Participants performed an orientation-discrimination task followed by a confidence judgment. To account for the behavior, we defined two sets of models, which were fit in a two-step process. Type 1 models defined the contribution of conservatism to the discrimination responses. Type 2 models defined the role of priors and payoffs in the confidence reports.

5.1 Model Fits

Type 1 models were first fit using the discrimination responses alone. Four models were compared: optimal criterion placement ($\Omega_{1,opt}$), equal conservatism for priors and payoffs ($\Omega_{1,1\alpha}$), different degrees of conservatism for priors and payoffs ($\Omega_{1,2\alpha}$), and a model in which there was a failure of summation of criterion shifts in the double-asymmetry condition ($\Omega_{1,3\alpha}$). Fitting the Type 1 models also provided an estimate of response bias, γ. We performed a Bayesian model selection procedure using the SPM12 Toolbox (Wellcome Trust Centre for

Neuroimaging, London, UK) to calculate the protected exceedance probabilities (PEPs) for each model (Figure 3a). The exceedance probability (EP) is the probability that a particular model is more frequent in the general population than any of the other tested models. The PEP is a conservative measure of model frequency that takes into account the overall ability to reject the null hypothesis that all models are equally likely in the population (Stephan et al., 2009; Rigoux et al., 2014). Overall, an additional parameter in the double-asymmetry conditions was needed to explain Type 1 criterion placement, indicating a failure of summation of criterion shifts (i.e., the best-fitting model was $\Omega_{1,3\alpha}$).

In the second step, the Type 2 models were fit using each participant's best Type 1 model and the associated maximum *a posteriori* (MAP) parameter estimates. The Type 2 models differed in the placement of the Type 2 criteria, which split the internal response axis into "high" and "low" confidence regions, for each "right" and "left" discrimination response. We modeled the two Type 2 criteria as shifting to account for only the prior probability, maximizing accuracy with the confidence response ($\Omega_{2,acc}$; the normative model), shifting the confidence criteria in response to payoff manipulations ($\Omega_{2,gain}$; a sub-optimal model), or failing to move the confidence criteria away from neutral at all ($\Omega_{2,neu}$; a sub-optimal model). We also tested models where the conservatism found in the Type 1 decisions carried over into the confidence decision ($\Omega_{2,acc+cons}$ and $\Omega_{2,gain+cons}$; both sub-optimal). We again compared the models quantitatively with PEPs (Figure 3b). The favored model, $\Omega_{2,gain+cons}$, shifts the confidence criteria in response to both prior and payoff manipulations. Furthermore, the conservatism that participants exhibited in the Type 1 decisions carried over into the placement of the confidence criteria.

Figure 3c shows the best-fitting models for individual participants, according to the amount of relative model evidence (here the marginal log-likelihood). Each of the Type 1 models except the optimal ($\Omega_{1,opt}$) was a best-fitting model for at least one of the ten participants. Similarly, no one was best fit by the normative Type 2 model either ($\Omega_{2,acc}$). Overall, there is no clear pattern between the pairings of Type 1 and Type 2 models.

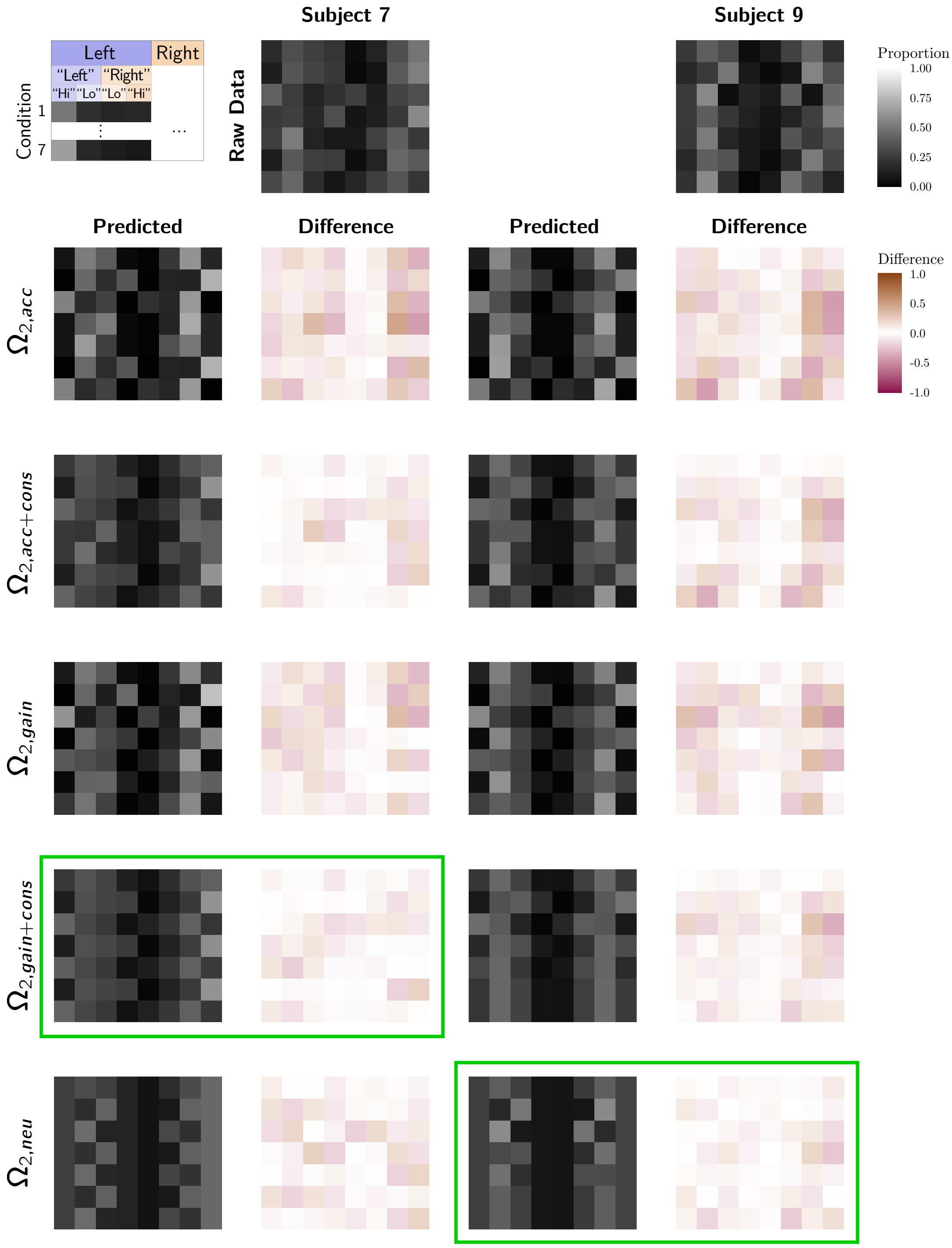


Figure 4: Visualization of the raw and predicted response rates for two example participants. Grids are formed of the seven conditions (rows) and the eight possible stimulus-response-confidence combinations (columns). See Figure S3 in the Supplement for condition order. The fill indicates the proportion of trials for that condition and stimulus that have that combination of response and confidence. Top row: Raw response rates of two example subjects. Subsequent rows, columns 1 and 3: Predicted response rates for each Type 2 model using the best-fitting parameters of the best-fitting Type 1 model for that individual. Columns 2 and 4: Difference between raw and predicted response rates. Green boxes: winning models (Subject 7: $\Omega_{gain+cons}$; subject 9: Ω_{neu}).

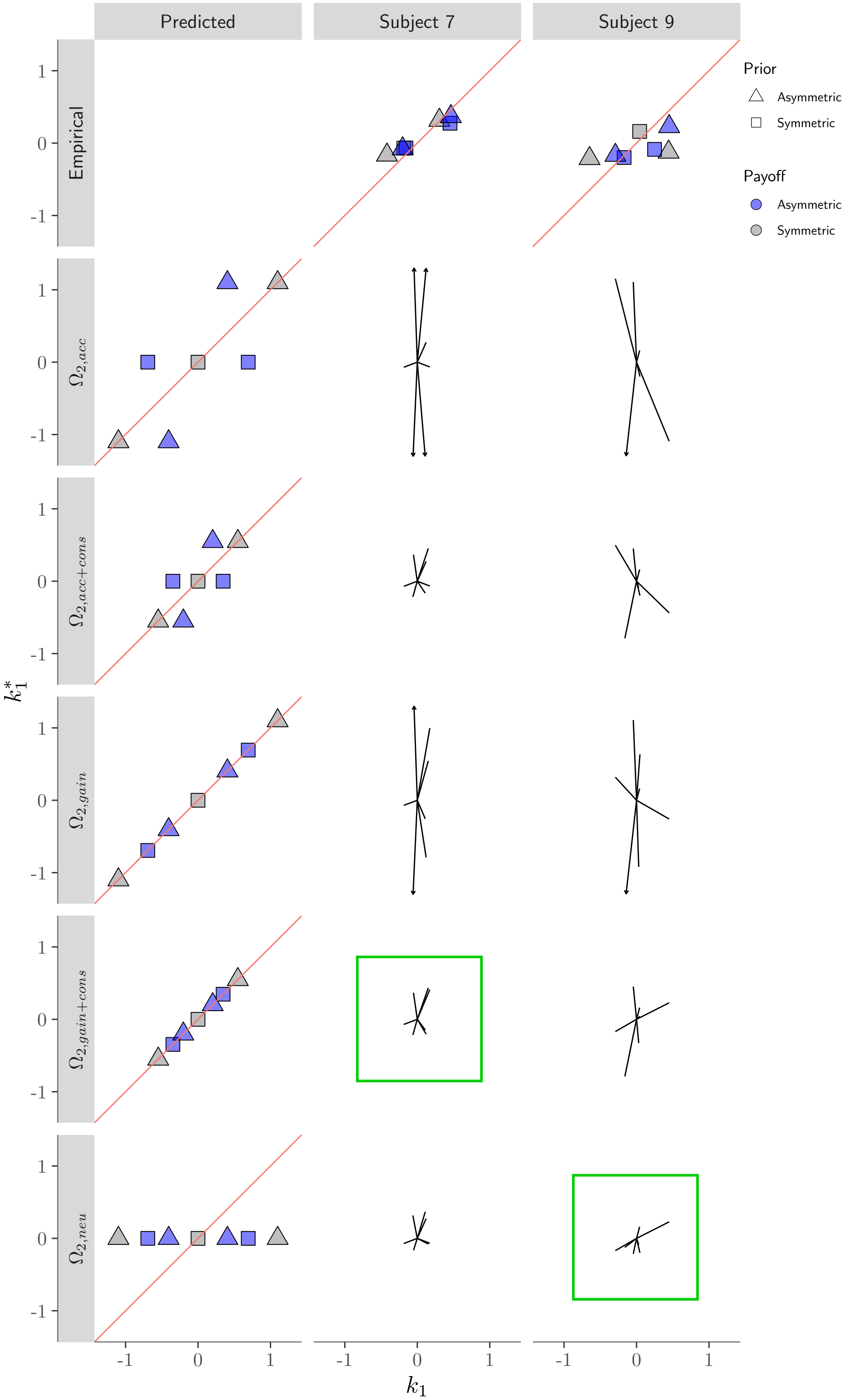


Figure 5: Comparison of the empirical and predicted k_1 and k_1^*. Top row: empirical criteria of two example observers. The k_1^* was calculated as the midpoint between the two empirical k_2 (see Figure S1 for k_2 calculation details). Left column: predicted relationship between the Type 1 and Type 2 criteria ($d' = 1$; either $\Omega_{1,opt}$ or $\Omega_{1,1\alpha}$ with $\alpha = 0.5$). Grey and square symbols: symmetry conditions. Triangles: prior asymmetry. Blue symbols: payoff asymmetry. Polar plots: residuals between empirical data and model prediction based on best-fitting parameters, plotted as vectors. Arrowheads: residuals greater than plot bounds.

5.2 Model Checks

We performed several checks on the fitted data to ensure that parameters were capturing expected behavior and that the models could predict the data well (reported in detail in Section 3 of the Supplementary Information). The quality of a model is not only dependent on how much more likely it is than others, but it is also dependent on its overall predictive ability. To visualize each model's ability to predict the proportion of each response type ("right" vs. "left" x "high" vs. "low"), we calculated the expected proportion of each response type given the MAP parameters for each model and participant. We compared the predicted response proportions to the empirical proportions (Figure 4). Larger residuals are represented by more saturated colors. For the best-fitting models, the residuals are small and unpatterned.

We also compared the Type 1 criteria and the counterfactual confidence criteria (Figure 5). We constrained the empirical counterfactual confidence criterion to be the midpoint between the two Type 2 criteria (i.e., $k_1^* \equiv (k_{2-} + k_{2+})/2$). Using k_1^*, the predictions made by the Type 2 models are highly distinguishable. In the left-most column, predicted k_1 and k_1^* for each session are shown for each model, assuming $d' = 1$ and either $\Omega_{1,opt}$ or $\Omega_{1,1\alpha}$ where $\alpha = 0.5$. In the top row, empirical criteria from the same two example participants as in Figure 4 are shown. Empirical criteria are calculated with the standard SDT method (detailed in Section 1 of the Supplementary Information, see Figure S1).

The visualization in the top row and left-most column of Figure 5 illustrates several behavioral phenomena. The response bias, γ, results in a shift in all criteria in the same direction, translating all data points parallel to the identity line. Conservatism is represented by an attraction of all data toward the origin on the x-axis for Type 1 and the y-axis for Type 2 judgments. The Type 2 models predict qualitatively different arrangements of the data points. If the prior and payoff asymmetries affect the placement of the Type 1 criterion but not the Type 2 criteria ($\Omega_{2,neu}$), the data are clustered along a single value on the y-axis. If the prior and the payoff affect the placement of the Type 1 and Type 2 criteria equally, ($\Omega_{2,gain}$), then the data fall on the identity line. With normative behavior ($\Omega_{2,acc}$), the prior asymmetry conditions (grey triangles) fall on the identity line because confidence tracks the prior, while in the payoff asymmetry conditions (blue squares), the data have the same k_2

midpoint as in the neutral condition (grey squares) because confidence does not track the payoff.

Vectors in all 10 of the bottom right polar plots represent the difference (i.e., the residual) between the empirical and the predicted criteria from the model fits. While the model prediction column is based on fixed parameters, the predicted data used for the 10 polar plots uses parameters that best fit the participant's data using that model. It is immediately clear that the normative model (second row) does a poor job of describing participants' behavior, and that conservatism is a necessary component of the models.

5.3 Conservatism

We first measured the relative magnitude of conservatism due to priors and payoffs. Figure 6a shows fitted α_p and α_v under the most complex conservatism model ($\Omega_{1,3\alpha}$) and Figure 6b shows them under the best-fitting model for each observer. In these figures, eight of the ten participants were conservative in their criterion placement for both prior and payoff manipulations, as indicated by data points in the gray regions. Of the eight participants that displayed conservatism, five were significantly more conservative for payoff asymmetries than prior asymmetries ($\alpha_v < \alpha_p$), whereas only one was significant in the opposite direction ($\alpha_p < \alpha_v$). At the group level, however, we did not find a significant difference between the best fitting α_v and α_p, either for the best-fitting Type 1 model or the winning model (paired t-tests, $p > 0.05$). Note that the negative α values derive from a participant who shifted criteria consistently in the opposite direction expected from a rational observer in response to manipulations of payoffs and priors.

An additional implication of SDT is that an ideal observer's criterion shift due to payoffs and due to priors should sum when both asymmetries are present as in Figure 1b: $k_{pv} = k_p + k_v$ (Stevenson et al., 1990). Figure 6c shows the prediction of this additive rule. Although the difference between the predicted and actual criterion shift is marginally significant ($t = 2.41, p = .039$), this effect is driven by the four observers best fit by $\Omega_{1,3\alpha}$. Each of these four observers had 95% CIs that did not overlap with the identity line. We show the criterion placement in the double-asymmetry cases in Figure 6d. Most observers did not shift their criterion far enough from neutral to the optimal placement, k_{opt}. Three observers, how-

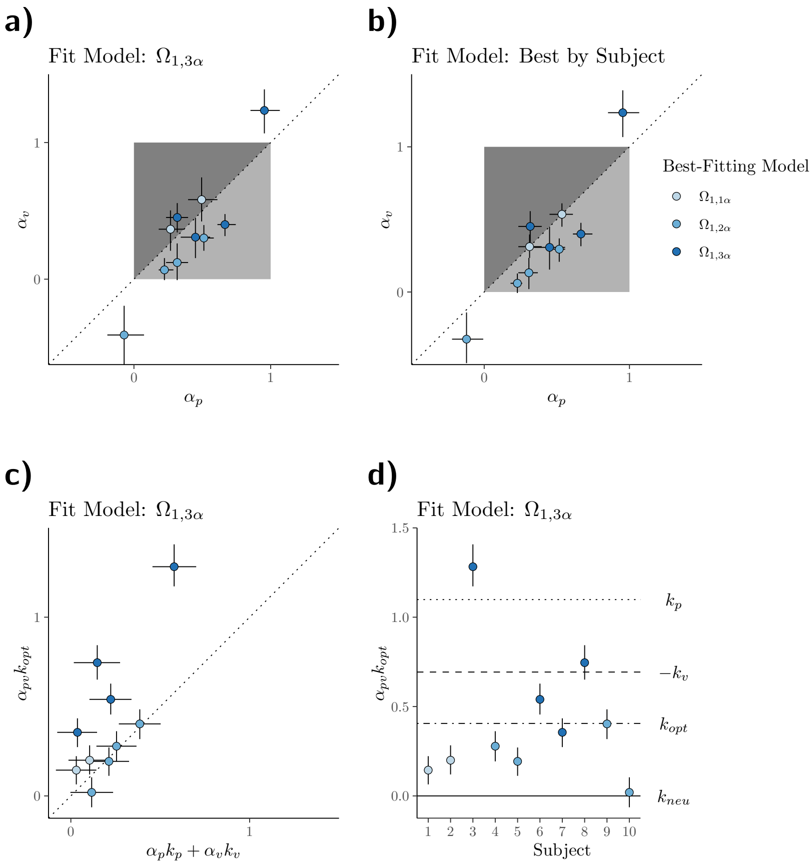


Figure 6: Conservatism for Type 1 decision making. a) A comparison of the extent of conservatism under payoff versus prior asymmetries. Each data point represents the best-fitting conservatism parameters of a single observer when fit by $\Omega_{1,3\alpha}$. These parameters are only contingent on the conservatism in the single-asymmetry conditions. In this model, conservatism in the double-asymmetry conditions is captured by a separate model parameter. Darker marker fill: additional conservatism parameters were required to fit to that observer's data. Dashed line: equality line. Dark grey region: conservatism greater for prior than payoff manipulations (i.e., $\alpha_p < \alpha_v$). Light grey region: conservatism is greater for payoffs (i.e., $\alpha_p > \alpha_v$). Data points outside these regions are not consistent with conservative criterion placement. b) Same as (a) using fit parameters from the best-fitting Type 1 model for each observer. c) Test of summation of criterion shifts using the $\Omega_{1,3\alpha}$ model fits. Observers who required a third α to capture their data (i.e., were best fit by $\Omega_{1,3\alpha}$) had criterion shifts for the double-asymmetry conditions that were not well predicted as the sum of the shifts in the single-asymmetry conditions. d) Criterion placement in the double-asymmetry conditions. These are the same data as in the y-axis of (c), but extended to more easily compare the actual criterion placement with potential other task-relevant criteria. Horizontal criteria lines assume d'=1.

ever, placed their criterion beyond k_{opt}, with two stopping short of the accuracy-maximizing criterion k_p.

In summary, we find that conservatism for priors and conservatism for payoffs do not sum, as traditional SDT predicts. Conservatism applied to priors and payoffs in the discrimination decision was also incorporated into the confidence decision. Participants further deviated from normative behavior by shifting their confidence criteria in response to asymmetric payoffs, which do not inform the probability of a correct discrimination response.

6 Discussion

6.1 Type 1 Judgments

We conducted an orientation-discrimination task in which the prior probability of response alternatives and the payoff matrix varied across sessions. Binary confidence reports were collected after each discrimination judgment to gauge the observer's subjective appraisal of the probability they were correct in their judgment. Observers were found to be conservative in the placement of the discrimination criterion, k_1, as revealed by the Type 1 model comparison. Instead of placing the criterion at the optimal location, as determined by the priors and payoffs, they had the tendency to place k_1 between the optimal criterion and the neutral criterion. While we did find evidence of different degrees of conservatism for payoff versus prior asymmetries at the individual-subject level, we found no evidence at the group level that conservatism was stronger when the payoffs were asymmetrical than when the priors were asymmetrical. Differences in conservatism were more apparent in previous studies (Lee and Zentall, 1966; Ulehla, 1966; Healy and Kubovy, 1981; Maddox, 2002; Ackermann and Landy, 2015), but not all (Healy and Kubovy, 1978). Several factors may be contributing to the observed conservatism of individual observers. Candidate explanations include the hypothesis that observers trade off between maximizing gains and maximizing accuracy (Maddox and Bohil, 1998), as it may be hard for the observer to sacrifice accuracy for expected gain. Alternatively, conservatism could depend on the criterion-adjustment strategy (Busemeyer and Myung, 1992), which may be differentially influenced by subjective factors such as subjective probability and subjective utility (Ackermann and Landy, 2015). This explanation suggests that it is effortful to shift the criterion far from the neutral criterion for an inconsequential gain. Another possibility is that it may be a combination of the two, as suggested by Maddox and Bohil (2003).

In additional analyses we explored the nature of conservatism further, both by fitting Type 1 models with varying levels of complexity as well as testing the predictions of several possible models for the case of both prior and payoff asymmetries. All participants were best described by a model with some form of conservatism, with the majority best fit with two or three separate conservatism parameters. In the extreme case, where three conservatism parameters were needed, we find cases where additivity of criterion shifts was not obtained, as predicted by Healy and Kubovy (1981). By additivity we mean that the criterion shifts induced by priors or payoffs sum when both are present. In our sample population, additivity was not found for 40% of observers, which provides a similarly inconclusive follow-up to previous attempts at testing additivity (Stevenson et al., 1990). Yet, the Bayesian Model Selection procedure indicated that this was the winning model. Taking into account the evidence for each model, as well as penalizing model complexity, we find that the most complex Type 1 model does the best job of describing the behavior of our sample population. Without any sizeable, systematic deviation from additivity (Figure 6c), it is reasonable to suggest this third conservatism parameter is capturing something else, relating to strategy or noise, on the part of the observers.

How consistent is additivity with the various explanations of unequal conservatism for priors and payoffs? In Sect. 1.4 of the Supplementary Information, we demonstrate that the gain-accuracy trade-off strategy is equivalent to our $\Omega_{1,2\alpha}$ model, for which additivity holds. Therefore, observers best fit by this model may be simply trading off between maximizing gain and maximizing accuracy. Turning to the criterion-adjustment strategy explanation of conservatism, behavior might deviate from additivity depending on whether conservatism, which acts as a scale factor on criterion shifts, is applied before or after the individual shifts for priors and the payoffs are combined. If conservatism is applied to these components individually, and then the resulting criteria are summed, this is equivalent to the $\Omega_{1,1\alpha}$ or $\Omega_{1,2\alpha}$ models. If, however, the criterion is adjusted after both the priors or payoffs have been applied, then the rate of change in reward based on the objective or subjective gain functions is in no way constrained to match that of the single-asymmetry cases. Yet, we found that the discrimination criterion in the double-asymmetry cases was placed beyond the optimal criterion for 30% of observers, which is not consistent with a reluctance to shift the criterion sufficiently from neutral. In fact, these criteria are biased in the direction of the accuracy-maximizing criterion, as would be expected under the gain-accuracy trade-off hypothesis. However, we cannot distinguish the trade-off hypothesis from a liberal criterion placement in the double-asymmetry case because, in our task, the prior odds ratio was always more asymmetric than the rewards ratio, always placing the optimal criterion on the same side of the neutral criterion as the accuracy-optimizing criterion.

So far, we have considered explanations of conservatism that are a result of prior and payoff factors. An alternate metacognitive source of conservatism proposed by Kubovy (1977) implicates the d' component of Eq. 5. Observers likely form an estimate of their overall performance from experience with the task. If they happen to overestimate performance (i.e., $\hat{d}' > d'$), then it follows from Eq. 5 that $k_1 < k_{opt}$, and vice versa for underestimation. Note that this is not a form of confidence in the response for a given trial, but a more general metacognitive appraisal of the difficulty of the task. According to this hypothesis, most of the observers would have been overestimating performance, with the one observer with liberal criterion placement underestimating their performance. While it is not uncommon to find overestimation of performance in the metacognitive literature (Mamassian, 2008, 2016), this explanation alone is insufficient as we find differences in the degree of conservatism for priors versus payoffs for some participants. Thus, we conclude that the conservatism observed in this task is likely due to a combination of possible factors, including noisy behavior, strategies to trade off gain versus accuracy, sub-optimal criterion adjustment, and bias in participants' judgments of their own d'.

6.2 Type 2 Judgments

We now turn to the Type 2 results, i.e., how observers form confidence judgments about the discrimination decision. Five Type 2 models were characterized by the placement rule for the counterfactual Type 1 criterion, k_1^*, around which the confidence criteria, k_2, were symmetrically placed. We tested whether this counterfactual criterion coincided with the accuracy-maximizing criterion, the gain-maximizing criterion, either of these options with the Type 1 conservatism applied, or whether it remained fixed at the neutral criterion. We found no observer was best fit by the accuracy-maximizing optimal model ($\Omega_{2,acc}$), with the majority split between the gain-maximizing-with-conservatism model ($\Omega_{2,gain+cons}$) or the fixed neutral model ($\Omega_{2,neu}$). One participant was best fit by the accuracy-maximizing with conservatism model ($\Omega_{2,acc+cons}$). Overall, Bayesian model selection favored the gain-maximizing model with conservatism. When considering the best-fitting Type 1 model, we find no clear pattern between the number of conservatism parameters required to explain behavior and the placement strategy for confidence criteria.

We first turn our focus to the subset of observers who were best fit by the model in which confidence criteria remained fixed around neutral ($\Omega_{2,neu}$). In this model, the perceived magnitude of the tilt was all that was used to compute confidence. These observers correctly did not allow the payoff structure of the environment to affect confidence, unlike the other top-winning model. However, it is sub-optimal not to include the additional information provided via the priors for the response alternatives but the lack of adaptability should not be taken as evidence of an inability to adapt. It is possible that these observers ignored the prior-payoff structure entirely for confidence, and instead opted for a criterion-placement strategy that would work best for all conditions of the experiment. Future experiments could incentivize accurate confidence judgments to test this hypothesis.

In the winning Type 2 model, observers placed k_1^* at the gain-maximizing Type 1 criterion k_p, with an adjustment for conservatism ($\Omega_{2,gain+cons}$). By adjusting the confidence criteria so that the counterfactual Type 1 criterion tracks the actual Type 1 criterion, payoffs are inappropriately incorporated into confidence judgments. As a consequence, higher relative reward or cost will make a person more likely to select that alternative and, on average, more confident about reporting that outcome when they do. In effect, this is a naïve optimism for selecting the highly rewarded outcome and disproportionate pessimism for selecting the costly outcome: "this highly rewarding perceptual alternative that I have selected is certainly the state of the world" or "it is costly to me, so it cannot be true". This bias for higher confidence with greater reward value (or smaller loss value) is consistent with what has been reported previously in the perceptual lottery tasks of Lebreton et al. (2018). Yet, we note that a failure to understand the task instructions could have produced the bias we found. It is possible that observers did not report the probability they were correct, as per the experimenter instructions, but instead reported something about the expected gain of the trial when reflecting on their confidence.

An inability to appropriately dissociate Type 1 and Type 2 responses, in both subsets of observers, is compelling. If this is a true inability for sensory decision-making, then there is a trade-off between maximizing gains for discrimination and accuracy for confidence. That is, if observers cannot selectively decouple their k_1 and k_1^* for asymmetric payoffs, then perhaps they reach a compromise by sacrificing some gains in the Type 1 task by shifting k_1 toward the accuracy-maximizing criterion k_p, thereby shifting k_1^* in a manner that yields confidence reports more consistent with the objective probability of being correct. Consider, for example, judging whether an aircraft is heading for collision with an upcoming mountain peak. The high cost of collision should bias heading judgments toward predicting a collision, so corrective actions can be taken. But you wouldn't want to be confident in that judgment just because it results in high cost, so you reduce the bias and are a bit more confident you'll pass by unscathed. The ideal trade-off between incorporating the payoff structure versus accurately and confidently making a decision will of course depend on the decision at hand. Subsequent laboratory experiments can attempt to shift this trade-off by using more complex reward structures that incorporate both Type 1 and Type 2 judgments.

Finally, we turn to the result that the best-fitting Type 2 model had conservatism applied to the counterfactual criterion k_1^*. It is currently a matter of debate whether the same internal measurement of the sensory event is used by both the perceptual and the metacognitive decision-making systems (e.g., Resulaj et al., 2009; Fleming and Daw, 2017). The SDT framework used here assumes the same internal measurement is used for both judgments. The Type 2 judgment is thought to include additional noise (Maniscalco and Lau, 2012; Fleming and Lau, 2014; Bang et al., 2018), and as such, we incorporated reduced metacognitive sensitivity in our modeling. Additionally, our results suggest several possible scenarios about how the decision boundaries during Type 1 and Type 2 decisions are related. The Type 1 and Type 2 processes may be computed jointly using the same information, but there is considerable evidence that neural processing occurs in distinct regions for perceptual and metacognitive decision-making (Shimamura, 2000; Fleming and Dolan, 2012; Rahnev et al., 2016; Shekhar and Rahnev, 2018). The Type 1 system may convey information to the Type 2 system about its decision boundary, or convey only relative information. Additionally, the processes responsible for conservatism are also applied to the counterfactual criterion in the Type 2 system. Given the complexity of the conservatism we observed, it would appear unlikely for the Type 2 system to recreate the phenomenon of conservatism with information acquired independently from the Type 1 system. Thus, we favor the interpretation that the exact effects of priors and payoffs in perceptual decision-making are also propagated to the metacognitive system. Given that a subset of observers were able to dissociate k_1 and k_1^* by keeping the latter fixed at the neutral criterion, it is less likely that the Type 2 system receives an internal measurement that is coded relative to the discrimination criterion k_1. Yet, if this is the case, why weren't observers able to reduce the influence of the payoff structure at the second processing step? Further work is required to understand why optimal metacognitive behavior was not achieved.

6.3 Conclusion

By manipulating priors and payoffs in a perceptual task, we found sub-optimal decision making at the Type 1 and Type 2 levels. Discrimination judgments were conservative, with no strong tendency for greater conservatism for payoffs than priors. There was also evidence against additivity of criterion shifts for asymmetric priors and payoffs. Confidence judgments were sub-optimal in one of two ways: 1) observers did not consider the role of priors or 2) they incorporated payoffs. Both of these strategies hinder decision-making. For example, a radiologist who ignores prior probabilities when assigning confidence might hesitate recommending further tests for a patient who is a heavy smoker. Similarly, a radiologist who inappropriately incorporates payoffs may be more confident in a positive diagnosis if he receives kickbacks from the imaging center to encourage future scans. The patterns of behavior found in this task point to explanations of why humans may consider trade-offs between maximizing gain and maximizing accuracy, as well as provide new insights about the role of the decision boundary in Type 1 versus Type 2 computations.

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Supplementary Information:

Priors and Payoffs in Confidence Judgments

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Contents

1 Type 1 and Type 2 Sensitivity

To fit the models presented in this paper, we required an estimate of discrimination sensitivity (d') and metacognitive sensitivity (meta-d') for each observer. Each participant completed a threshold procedure to find the Gabor orientation that would yield a d' of 1. We could have used this for all analyses, however we sought to utilize all of the decisions made in the main task to better estimate d', as well as obtain a reasonable estimate of meta-d'. To achieve this, we implemented a hierarchical Bayesian model that leveraged all possible sources of information to yield a single estimate of d' and meta-d' for each participant. We computed the empirical d' for participant i in session j of the main task according to the standard formula

$$d'_{ij} = z(p\mathrm{H}_{ij}) - z(p\mathrm{FA}_{ij}) \tag{S1}$$

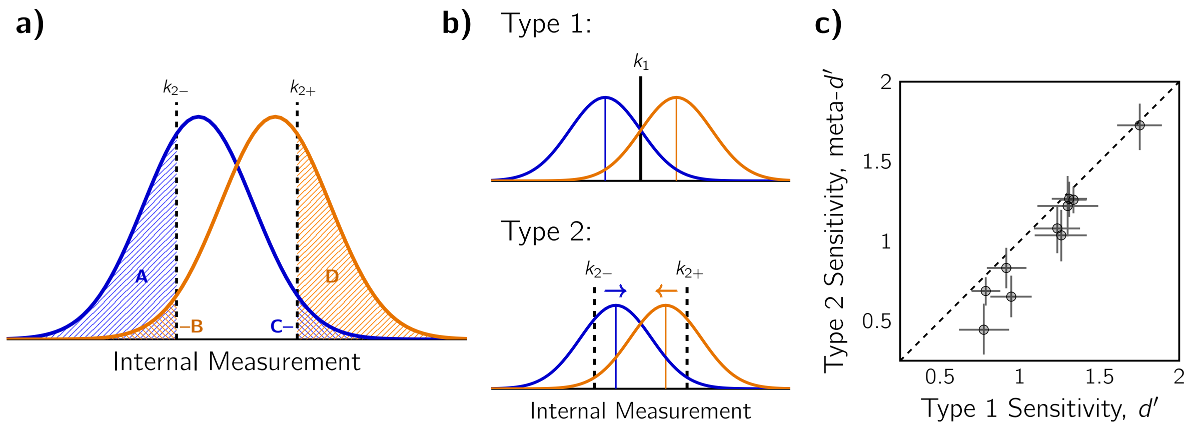

Figure S1: a) Depiction of example regions for the approximate meta-d' calculation. Hatched regions correspond to the probability of a high-confidence judgment for the four possible pairings of stimulus and discrimination response. b) Example of greater sensitivity for perception (Type 1) than confidence (Type 2). In the standard SDT model, this corresponds to an inwards shift of the distributions for confidence. c) Contrast of d' and meta-d' results. Each data point is an observer, with 95% CIs derived from the posterior distribution of parameter estimates. Dashed equality line is also shown for comparison.

where pH was the probability of selecting "right" when the stimulus was truly rightward tilted, pFA was the probability of selecting "right" when the stimulus was leftward tilted, and z refers to the standard z-transform. In a similar fashion, we approximated the meta-d' from the lower and upper confidence criteria, k_{2-} and k_{2+} respectively. These confidence criteria can be empirically calculated as per the standard method for deriving a criterion in Signal Detection Theory (SDT):

$$k_{2-} = \frac{1}{2}\big[z(p\mathrm{A}) + z(p\mathrm{B})\big] \tag{S2}$$

and

$$k_{2+} = \frac{1}{2}\big[z(p\mathrm{C}) + z(p\mathrm{D})\big]. \tag{S3}$$

The corresponding regions A-D are best demonstrated graphically (Figure S1a). To compute meta-d', we used an average of two d'-like measurements, from the empirical upper and lower confidence bounds respectively:

$$\text{meta-}d'_{ij} = \frac{1}{2}\big[z(p\mathrm{A}_{ij}) - z(p\mathrm{B}_{ij}) + z(p\mathrm{D}_{ij}) - z(p\mathrm{C}_{ij})\big]. \tag{S4}$$

The concept behind computing a separate sensitivity parameter for confidence is that additional noise may have been added to the internal measurement between the Type 1 and Type 2 decisions. In the standard SDT framework, the variances of the distributions are fixed, and so the additional noise is modeled as a shift in distributions means (see Figure S1b). As such, we use the confidence bounds to estimate the relative separation of $p(x|S_L)$ and $p(x|S_R)$ with this additional metacognitive noise. These confidence bounds can then be represented in the original Type 1 space by a simple transformation

$$\frac{\text{meta-}d'}{d'} k_{2,\text{space2}} \rightarrow k_{2,\text{space1}}, \tag{S5}$$

as explained by Maniscalco and Lau (2012) and illustrated in Figure S1b.

In the hierarchical Bayesian model, each observation j of d' for participant i was assumed to be drawn from a normally-distributed subject-specific prior,

$$d'_{ij} \sim \mathcal{N}(d'_i,\, \sigma_i^2), \tag{S6}$$

where d'_i is the aggregate estimate of that participant's d' for our next stage in modeling, and σ_i^2 is their sensitivity variance, capturing both noise in the calculation from a limited number of samples and sessional changes in sensitivity (e.g., attention, motivation). Similarly, we modeled the estimates of meta-d' as

$$\text{meta-}d'_{ij} \sim \mathcal{N}(\text{meta-}d'_i,\, \sigma_i^2). \tag{S7}$$

Again, we have a subject-level estimate of sensitivity, meta-d'_i, for our modeling. The same variance parameter was used for both Type 1 and Type 2 estimates, because factors influencing noise in the observations are likely to be similar for both sensitivity measures. We also incorporated hyperpriors for both sensitivity measures, leveraging additional information we had about what to expect for these values. For d', we used a normally-distributed hyperprior with a mean of 1.

$$d'_i \sim \mathcal{N}(1,\, \sigma^2_{\text{Type1}}), \tag{S8}$$

This decision was based on our expectations from the thresholding procedure, where the stimulus was adjusted to find $d' = 1$, and thus, on average, we expected this sensitivity for the observers in the main task. A normal distribution was used because this is the standard model for modeling variability within a population, here referred to as σ^2_{Type1}. We also used the following hyperprior for meta-d':

$$\text{meta-}d'_i \sim \mathcal{N}(0.8d'_i,\, \sigma^2_{\text{Type2}}). \tag{S9}$$

Based on previous results, we expected the meta-d' of a participant to be, on average, about 80% of their d' sensitivity measure (Maniscalco and Lau, 2012). Thus, the mean of the meta-d' hyperprior was adjusted on a per-subject basis. There was a shared variance parameter, σ^2_{Type2}, representing variations in meta-cognition across participants in the same manner as σ^2_{Type1}. To ensure good model behavior, all free parameters had reasonable bounds imposed via a uniform prior either in addition to or in lieu of the other prior distributions described above: $[0, 3]$ for d'_i and meta-d'_i, and $[0.1, 5]$ for σ_i, σ_{Type1}, and σ_{Type2}. The model was fit using custom-written scripts in R and RStan, which implemented an MCMC fitting algorithm with 4000 iterations for each of 4 separate chains, half of which were discarded as warmup. Parameter estimates and confidence intervals were calculated from the marginal posteriors (i.e., from the mean and percentile ranges of the samples).

The results of the model of Type 1 and Type 2 sensitivity are shown in Figure S1c. In general, there was greater sensitivity at the Type 1 level than at the Type 2 level, as expected (Maniscalco and Lau, 2012). The ratio of Type 2 to Type 1 was 0.86 ± 0.04 (mean±SEM). On average, participants' variability in d' over sessions was $\hat{\sigma}_i = 0.19 \pm 0.02$ (mean±SEM). Across participants, we saw a variability in Type 1 sensitivity of $\hat{\sigma}_{\text{Type1}} = 0.37$ (95% CI: [0.23, 0.60] according to the posterior distribution of parameter fits), and at the Type 2 level, $\hat{\sigma}_{\text{Type2}} = 0.12$ (95% CI: [0.1, 0.35]).

2 Multinomial Decision Model

Model fitting was performed in three sequential steps: (1) fitting of d' and meta-d', (2) Type 1 models, and (3) Type 2 models. In each case, the best-fitting parameters (and the best-fitting model in the Type 1 case) from one step were fixed while fitting models in the subsequent step. Fitting d' and meta-d' was explained in the previous section.

For Type 1 fits, we chose a dense grid of parameters, bias (γ) and between zero and three conservatism parameters (α), with which to calculate the likelihood. The likelihood was a binomial across the two possible discrimination responses. We assumed a fixed lapse rate, $\lambda = 0.02$, for all participants, so

$$P(\text{data} \mid \theta) = \prod_{\text{stim} \in \{L,R\}} \prod_{\text{resp} \in \{\text{“}L\text{”},\text{“}R\text{”}\}} \left(\lambda/2 \; + \; (1-\lambda)\, p(\text{resp} \mid \text{stim}, \theta) \right)^{\mathcal{N}_{\text{resp,stim}}}, \qquad \text{(S10)}$$

where $\mathcal{N}_{\text{resp,stim}}$ is the number of trials in which that response was made for the discrimination of that stimulus.

The probability of a response is given by the corresponding area under the normal distribution, as in standard SDT. We fixed the variances of the internal response distributions to be 1, and positioned them based on the participant's sensitivity at locations $\pm d'/2$. Therefore, the probabilities for the correct responses, for example, are:

$$p(\text{“}L\text{”} \mid L) = \Phi\left(\gamma + k_1 + \frac{d'}{2}\right) \qquad \text{(S11)}$$

and

$$p(\text{“}R\text{”} \mid R) = 1 - \Phi\left(\gamma + k_1 - \frac{d'}{2}\right), \qquad \text{(S12)}$$

where Φ is the standard cumulative normal.

The Type 2 fits inherited bias (γ) and various conservatism (α) parameters from the Type 1 model fits. The d' and meta-d' values were inherited from the hierarchical d' model fit. Thus, the counterfactual criterion k_1^* was already fixed, and the Type 2 modeling involved only a single free parameter, δ. Responses were modeled as a multinomial distribution with four possible responses to each stimulus, defined by the combination of the discrimination and confidence responses. We used the same lapse rate, but the probability of a particular random response was now halved because there were twice as many possible outcomes:

$$P(\text{data} \mid \delta) = \prod_{\text{stim} \in \{L,R\}} \prod_{\text{resp} \in \{\text{“}LH\text{”},\text{“}LL\text{”},\text{“}RH\text{”},\text{“}RL\text{”}\}} \Big(\lambda/4 \; + \; (1-\lambda)\, p(\text{resp} \mid \text{stim}, \delta) \Big)^{\mathcal{N}_{\text{resp,stim}}}. \tag{S13}$$

The probabilities of each response depend on the Type 2 criteria, for example:

$$p(\text{“}LH\text{”}|L) = \Phi\left(k_{2-} + \frac{d'}{2}\right) \tag{S14}$$

$$p(\text{“}LL\text{”}|L) = \Phi\left(k_1 + \frac{d'}{2}\right) - \Phi\left(k_{2-} + \frac{d'}{2}\right) \tag{S15}$$

$$p(\text{“}RL\text{”}|R) = \Phi\left(k_{2+} - \frac{d'}{2}\right) - \Phi\left(k_1 - \frac{d'}{2}\right) \tag{S16}$$

$$p(\text{“}RH\text{”}|R) = 1 - \Phi\left(k_{2+} - \frac{d'}{2}\right) \tag{S17}$$

k_{2-} and k_{2+} are the effective left and right confidence criteria respectively, and γ was left out of these equations for readability. In the double-asymmetry conditions, it is possible for an observer's Type 1 criterion to be outside the intended symmetric bounds of the Type 2 criteria with a small enough δ, as in Figure 1f. In this case, the effective k_{2-} is actually equal to k_1. Concretely, this would happen if an observer was highly confident that the stimulus was right-tilted, but the potential rewards are so asymmetric that they respond left-tilted anyway. Because of the potential for these cases, k_{2-} and k_{2+} were not simply $k_1^* \pm \delta$, but rather

$$k_{2+} = \max(k_1, k_1^* + \delta) \tag{S18}$$

$$k_{2-} = \min(k_1, k_1^* - \delta). \tag{S19}$$

We used flat priors on all parameters, so we calculated model evidence by marginalizing across each dimension of the posterior.

$$p(\text{data}|M) = \int p(\text{data}|\theta, M)p(\theta)d\theta \tag{S20}$$

To do this, we numerically integrated the posterior of our parameter grid with a rectangular

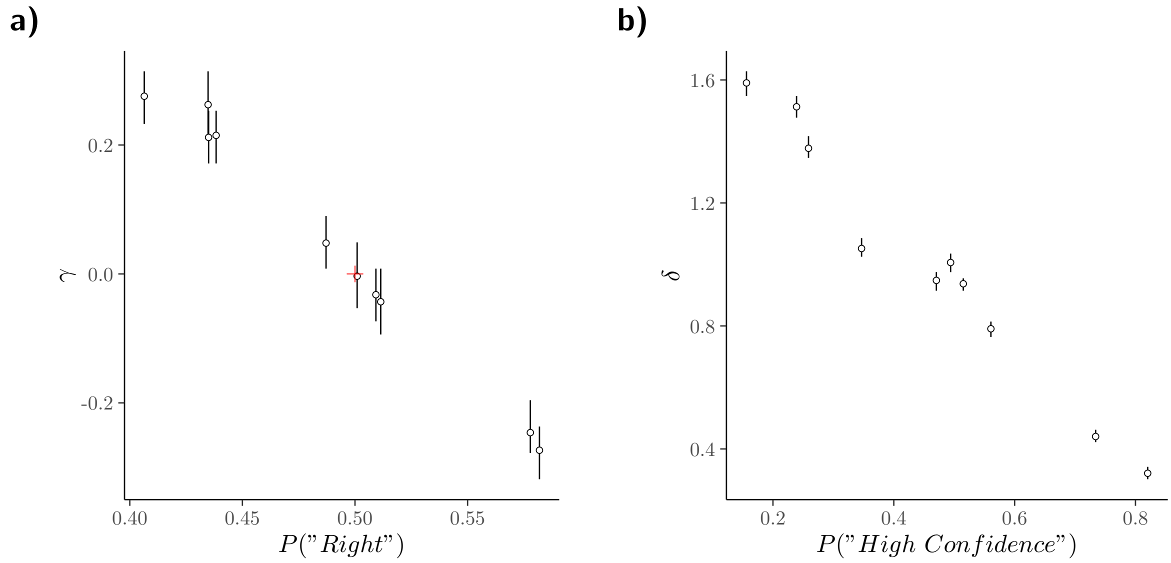

Figure S2: Checks on the fitted model parameters. a) Relationship between the bias in perceived vertical (γ) and the proportion of "right-tilt" judgments. Red cross: results for an unbiased observer. b) Relationship between the confidence criteria width parameter, δ, and the proportion of "high confidence" judgments. Small δ leads to more high confidence reports (over-confidence). This predicted relationship is supported by the data. Error bars: 95% CIs from the posterior.

approximation by summing the volume of each grid element:

$$p(\text{data} \,|\, M) \approx \sum_{\theta} p(\text{data} \,|\, \theta, M)\, \Delta x_{\theta}, \tag{S21}$$

where Δx_{θ} is the product of step sizes for each dimension in the parameter grid. The model evidences for all models and all participants were used to compute the protected exceedance probability with the SPM12 Toolbox (Wellcome Trust Centre for Neuroimaging, London, UK) according to Rigoux et al. (2014).

3 Model Checks and Fits for All Subjects

Two of the model parameters make clear predictions about behavior. The fitted response bias parameter, γ, should be negatively correlated with the total proportion of trials the participants responded "right." Positive γ values indicate a rightward tilted line is perceived as vertical, leading to fewer rightward responses overall. Figure S2a confirms this relationship ($r = -0.995, p < .0001$). The average bias is $\overline{\gamma} = .04 \pm .06$, with 70% of participants significantly biased according to the posterior parameter distribution. Also, δ, half of the distance between the Type 2 criteria, should be inversely correlated with the proportion of "high confidence" reports; larger values of δ expand the low-confidence region (compare Figures 1e and f). This predicted relationship was obtained (Figure S2b; $r = -0.986, p < .0001$; $\overline{\delta} = 1.00 \pm 0.13$). These predictions are not trivial: idiosyncratic biases in one condition may disappear or reverse on a subsequent day in the inverse condition. Nevertheless, we find that the γ and δ parameters are meaningfully capturing patterns of behavior.

The following figures show the results of all subjects in the style of Figures 4 and 5 of the main paper.

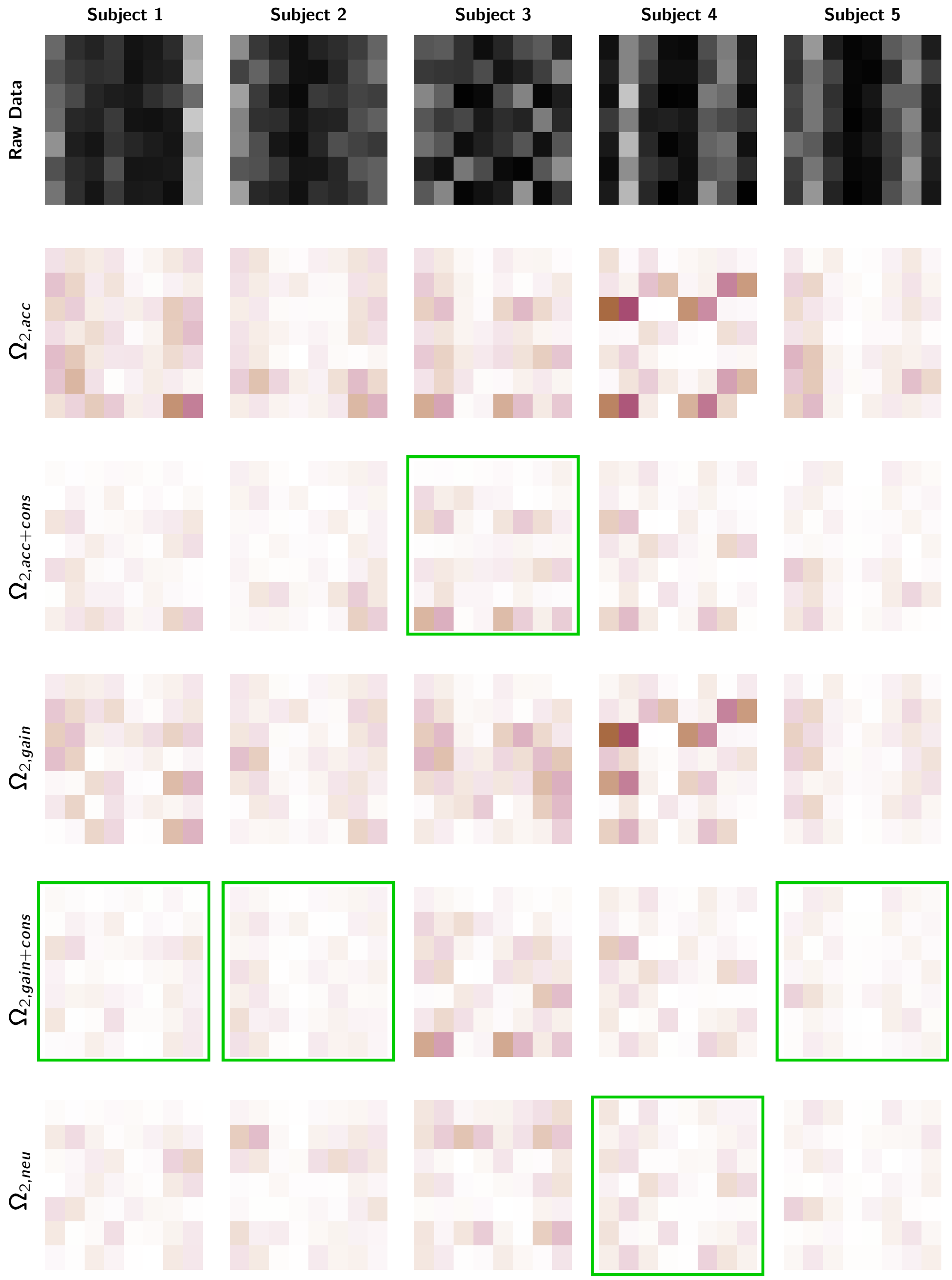

Figure S3: Raw and predicted response rates for participants 1-5. Grids are formed from the seven conditions (rows) and the eight possible stimulus-response-confidence combinations (columns). Condition order: (1) full symmetry, (2) single asymmetry ($p(R) = .75$), (3) single asymmetry ($p(R) = .25$), (4) single asymmetry ($V_R : V_L = 4 : 2$), (5) single asymmetry ($V_R : V_L = 2 : 4$), (6) double asymmetry ($p(R) = .75, V_R : V_L = 2 : 4$), (7) double asymmetry ($p(R) = .25, V_R : V_L = 4 : 2$). Fill: proportion of trials for that condition and stimulus that have that combination of response and confidence. Top row: Raw response rates. Subsequent rows: difference between raw and predicted response rates as per model. Green boxes: winning models.

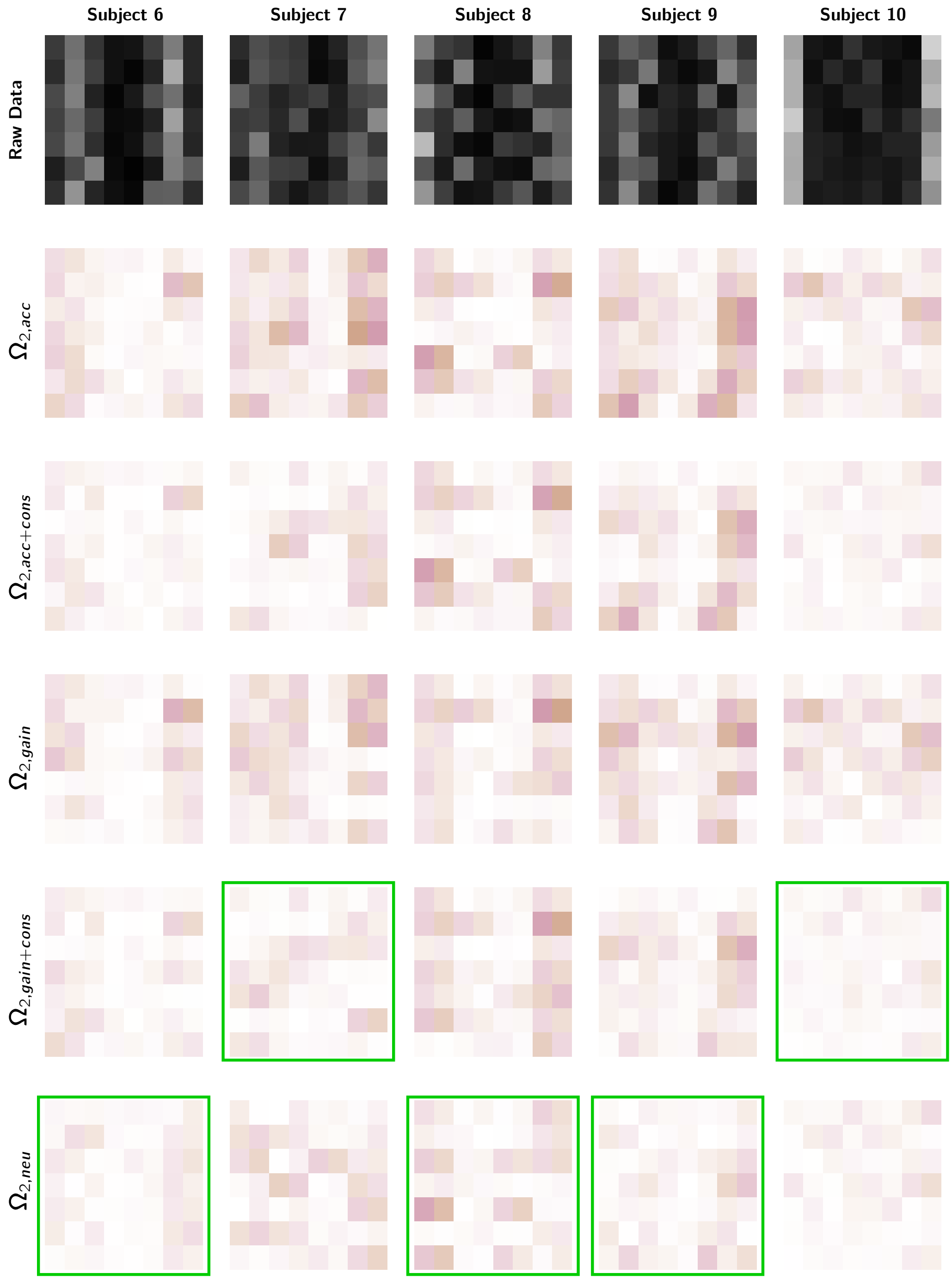

Figure S4: Raw proportions of subjects 6-10 in the style of Figure S3.

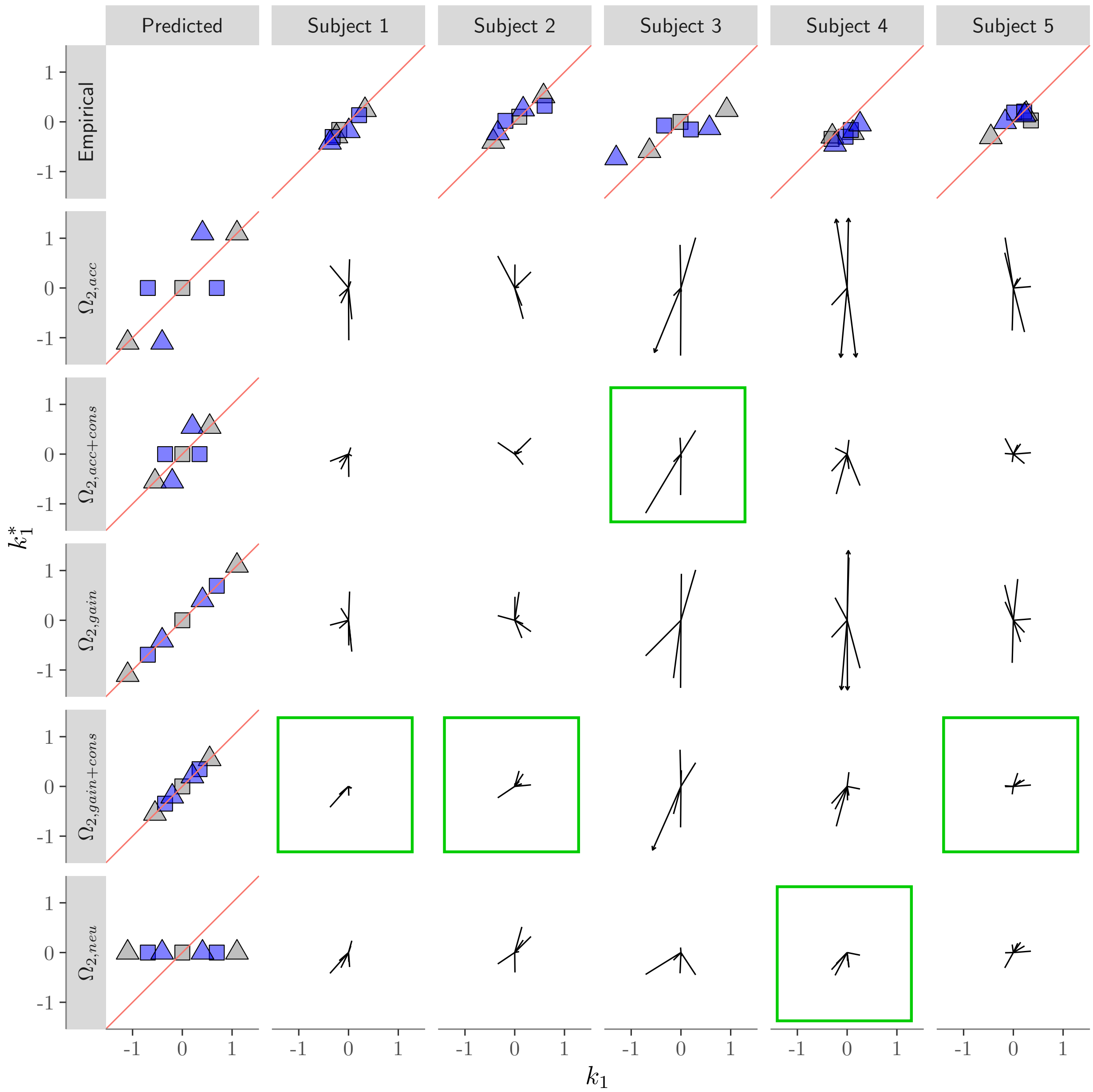

Figure S5: Comparison of the empirical and predicted k_1 and k_1^* for participants 1-5. Top row: empirical criteria. k_1^* was calculated as the midpoint between the two empirical k_2 (see Figure S1 for k_2 calculation details). Left column: predicted relationship between the Type 1 and Type 2 criteria ($d' = 1$; either $\Omega_{1,opt}$ or $\Omega_{1,1\alpha}$ with $\alpha = 0.5$). Grey and square symbols: symmetry conditions. Triangles: prior asymmetry. Blue symbols: payoff asymmetry. Polar plots: residuals between empirical data and model prediction based on best-fitting parameters, plotted as vectors. Arrowheads: residuals greater than plot bounds.

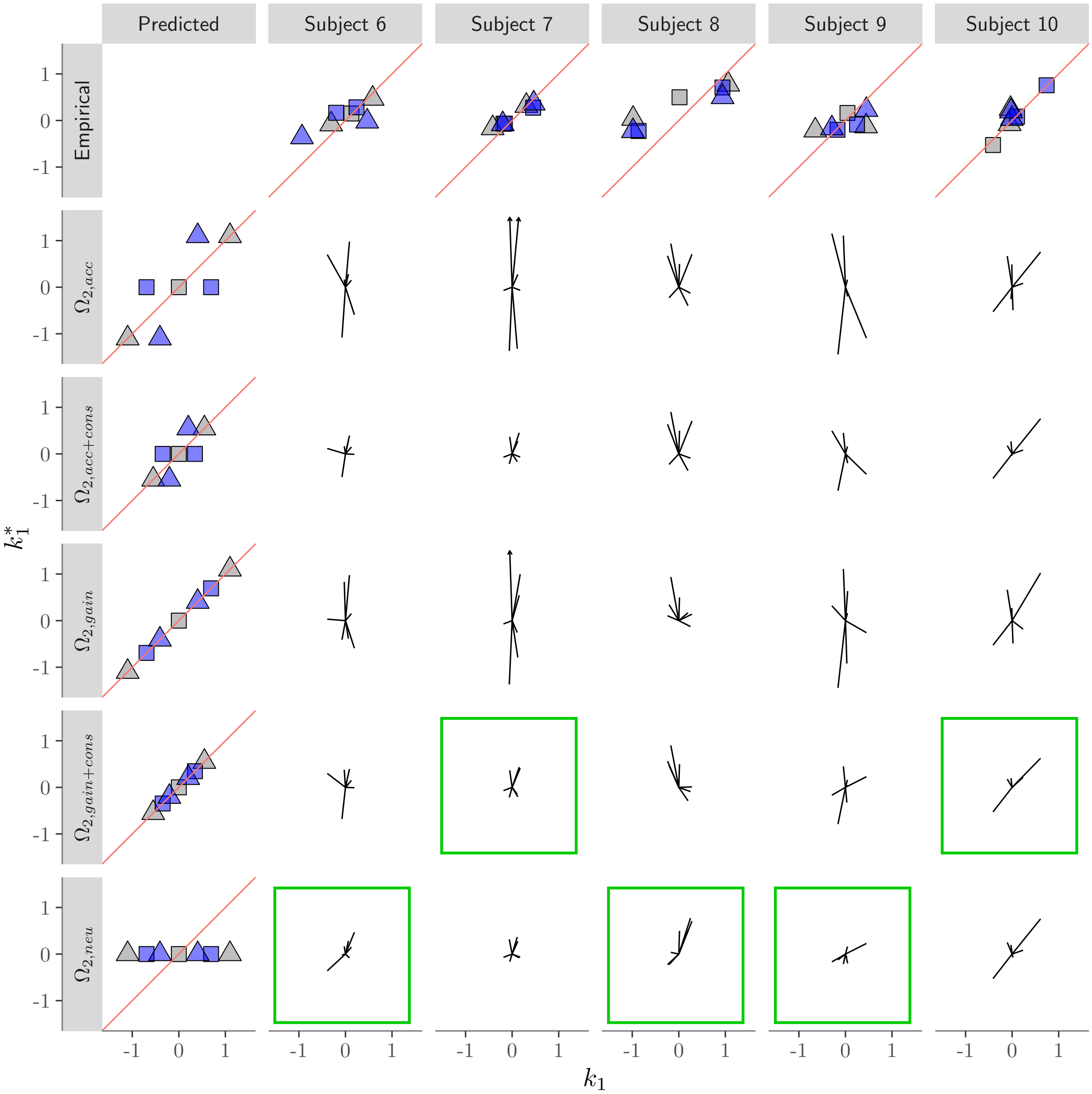

Figure S6: Comparison of k_1 and k_1^* for participants 6-10 in the style of Figure S5.

4 Gains-Accuracy Trade-off Strategy and Conservatism

Here, we show how the gain-accuracy trade-off strategy of Maddox and Bohil (1998) is equivalent to the $\Omega_{1,2\alpha}$ model. The gain-accuracy trade-off strategy can be expressed mathematically as a weighted sum between the gain-maximizing criterion, k_{opt}, and the accuracy maximizing criterion, k_p, with the weight w $(0 \leq w \leq 1)$. We also applied a single general conservatism parameter in this weighting strategy, which can be thought of as acting on each separate component or equivalently to the sum of the components. A simple rearrangement shows how these two models are equivalent:

$$\begin{aligned} \alpha_v k_v + \alpha_p k_p &= w\alpha k_{opt} + (1-w)\alpha k_p \\ &= \alpha(wk_v + wk_p + k_p - wk_p) \\ &= \alpha(wk_v + k_p) \\ &= \alpha w k_v + \alpha k_p \end{aligned} \tag{S22}$$

Therefore, we find that different degrees of conservatism for priors than payoffs can arise as a result of weight values less than 1. Specifically, the weight value contributes to an increase in a general level of conservatism, $\alpha_v = \alpha w$ and $\alpha_p = \alpha$, where the constraint $w \leq 1$ ensures that $\alpha_v \leq \alpha_p$. If $w = 1$, then $\alpha_v = \alpha_p = \alpha$, which is the single conservatism model $\Omega_{1,1\alpha}$.

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