Vehicle Rebalancing for Ridesharing Enabled Mobility On Demand Systems

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I. Introduction

Enabling ridesharing for mobility on demand (MOD) systems requires passengers to specify a maximum waiting time, the maximum amount of time they are willing to wait for a vehicle to pick them up. This constraint allows us to develop shareability networks which in turn helps us to decide which passengers can be shared with which rides. The unfortunate consequence of this approach is that it can leave vehicles underutilized and passengers without rides. This fundamental issue can be addressed by intelligently rebalancing vehicles in the road network to ensure that the maximum amount of rides can be serviced. Our approach will have two stages. Firstly, vehicles which have already accepted passengers will be routed through areas that have a high probability of requests that could also be shared with the current passenger demands whilst keeping the path within the time constraints given by the passengers. We will call this *active rebalancing*. Secondly, any vehicles that cannot be assigned to passengers will move to regions that have a high probability of any request without overburdening the congestion limits of the road network. We will call this *passive rebalancing*. These two problems are solved in sequence by first actively rebalancing the currently occupied vehicles, and then using the routes they have chosen to perform passive rebalancing. Both rebalancing schemes are formulated as constrained optimization problems with integer constraints. Particularly, active and passive rebalancing are formulated as a mixed integer linear program (MILP) and a mixed integer quadratically constrained program (MIQCP).

In order to perform active and passive rebalancing, we need to be able to infer the time varying rate at which a particular route is being requested and the rate a certain region is producing requests. We use a particle filter to estimate these rates in real-time and use historical taxi request data to learn a time varying multimodal prior distribution to aid in inferring noisy rates.

II. Problem Formulation

The road network for the MOD system is modelled as a directed graph denoted by $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ where $\mathcal{N} = \{n_0, \dots, n_{N_n}\} \subset \mathbb{R}^2$ is the set of nodes and $\mathcal{E} = \{e_0, \dots, e_{N_e}\}$ is the set of directed edges represented as ordered pairs, $\forall i,\ e_i \in \mathcal{N}^2$. Let the set of incoming requests for rides at time t be denoted as $\mathcal{R}_t \subseteq \mathcal{E}$ and let $\Pi(\mathcal{R}_t) = \{\pi_0, \dots, \pi_{N_v}\}$ be the set of optimal paths given to each vehicle to satisfy $\mathcal{R}_t$ given by the ridsharing algorithm in [1]. Each $\pi_i \in \Pi(\mathcal{R}_t)$ denotes the path vehicle i would be assigned to contribute to satisfying the requests in $\mathcal{R}_t$ with $\pi_i \in 2^{\mathcal{N}}$ and $(\pi_i^k, \pi_i^{k+1}) \in \mathcal{E}$. Note that π_i may be the empty set. Let $||\pi_i||$ represent the total travel time of the path.

A. Estimating Request Rates

The operating area for the MOD system is partitioned into N_k regions denoted as $\mathcal{K} = \{k_0, \dots, k_{N_k}\}$. A *route* is defined as origin-destination pair represented as an ordered pair, $\rho \in \mathcal{K}^2$. Rides are assumed to be requested according to an inhomogeneous Poisson point process with a stochastic time-varying rate, $\hat{\lambda}(t)$. These rates are assumed to drift over time in a short horizon according to a Wiener process. This means $\hat{\lambda}(t') - \hat{\lambda}(t) \sim \mathfrak{N}(0, \sigma^2 \cdot (t' - t))$, where $\mathfrak{N}$ is a normal distribution and σ^2 represents the volatility of the process. We also assume we have access to historical taxi request data in order to build a prior distribution. We seek to determine the rate that different routes are being requested and the rate that regions are producing requests in the presence of noise and uncertainty.

B. Active Rebalancing

As with the work in [1] we assume the there is a maximum acceptable time a potential passenger will spend waiting before being picked up denoted as w. Now for each vehicle, v, we seek to find a Pareto optimal path, π_v, that minimizes the added travel time, $||\pi_v|| - ||\pi_v^*||$, and maximizes the added probability of filling the vehicle to capacity, where $\pi_v^* \in \Pi(\mathcal{R}_t)$ is the original optimal path for the vehicle.

C. Passive Rebalancing

For passive rebalancing, we seek to direct cars to regions where they have a high probability of filling to capacity in the minimum amount of time. This means we are searching for an assignment of vehicles to regions such that the expected time it would take to fill to capacity is minimized. This expected time also includes the time it takes the vehicle to move from its current location to the assigned region. We would also to ensure that the vehicle assignment also takes into account vehicles already assigned to the regions and candidate assignments during the search process since all vehicles in the same region are competing for prospective requests. Lastly, we want a vehicle assignment does not over populate the road network in certain areas (i.e. all vehicles being assigned to a very fruitful region), in order to mitigate possible self inflicted congestion which could in turn reduce the quality of the MOD system.

III. Approach

A. Passive Rebalancing

We seek to find an assignment of vehicles to routes that minimizes the maximum expected time it would take to fill any vehicle to capacity. This includes the time it takes the vehicle to move to the origin of the route and the expected time the vehicle would need to stay in the region to fill to capacity. This assignment should also be valid for vehicles that are already carrying passengers. This means the destination of passengers needs to be taken into consideration. Finally, in order to avoid self inflicted congestion, this assignment should not overburden regions of the road network. We have formulated this problem as a mixed integer quadratically constrained problem (MIQCP) which finds the optimal assignment of vehicles to routes and the expected amount of time a set of vehicles would need to stay in an origin region to all fill to capacity.

Let $V = \{1, \ldots, N_v\}$, $R = \{1, \ldots, N_r\}$, and $O = \{1, \ldots, N_o\}$, be the set of vehicles, routes, and origins respectively. Let $A \in \{0,1\}^{N_v \times N_r}$ be a binary matrix such that if $A_{vr} = 1$, then vehicle v can satisfy route r. This can be computed using the shareability graph. Let $C_{vr} \in \mathbb{R}^{N_v \times N_r}$ be a cost matrix where C_{vr} represents the cost of vehicle v satisfying route r. For our examples we use the distance from v to the origin of r. Let Λ of size N_r be the set of expected request rate for all the routes. More specifically, $\Lambda_r = \mathbb{E}(\hat{\lambda_r})$ where $\hat{\lambda_r}$ is the estimated rate of requests at the current time. Also let Ω of size N_ω be a set such that Ω_ω is the set of requests with origin ω and let K_ω be the remaining number of vehicles we can allow into region ω. The equations below show the MIQCP formulation.

$$
\begin{array}{lll}
\underset{x_{vr},\, T_r}{\arg\min} & \theta & \\
\text{subject to} & x_{vr} \cdot (T_r + C_{vr}) \leq \theta, & \forall v, r \in V \times R \\
& \sum_{v \in V} x_{vr} \cdot \xi_v \leq \Lambda_r \cdot T_r, & \forall r \in R \\
& \sum_{r \in R} A_{vr} \cdot x_{vr} = 1, & \forall v \in V \\
& \sum_{v \in V} \sum_{r \in \Omega_\omega} x_{vr} \leq K_\omega, & \forall \omega \in O \\
& x_{vr} \in \{0,1\} & \forall v, r \in V \times R \\
& T_r \in [1, T_{\max}] & \forall r \in R
\end{array}
$$

B. Active Rebalancing

$$
\begin{array}{ll}
\underset{\mathbf{x},\, \mathbf{y}}{\arg\max} & \sum_{v \in \mathcal{V}} \sum_{\pi \in \Pi_v} \mathbf{x}_{v\pi} \cdot \sum_{r \in \mathcal{R}(\pi)} \dfrac{1 - e^{-\bar{\lambda}_r (T_{\pi r})}}{1 + \sum_{\pi' \in \Pi} \mathbf{y}_{\pi'} \dfrac{\tau(T_{\pi r} \cap T_{\pi' r})}{\tau(T_{\pi r})}} \\
\text{subject to} & \sum_{\pi \in \Pi_v} \mathbf{x}_{v\pi} = 1, \, \forall v \in \mathcal{V} \\
& \sum_{\pi \in \Pi} \mathbf{y}_\pi = N_v \\
& \mathbf{x} \in \{0,1\}^{N_v \times N_\pi} \\
& \mathbf{y} \in \{0,1\}^{N_\pi}
\end{array}
$$

$$
\begin{array}{ll}
\underset{\mathbf{x},\, \mathbf{y}}{\arg\max} & \sum_{v \in \mathcal{V}} \sum_{\pi \in \Pi_v} \mathbf{x}_{v\pi} \cdot \sum_{r \in \mathcal{R}(\pi)} \int_{T_{\pi r}} \dfrac{t \cdot \bar{\lambda}_r(t)}{1 + \sum_{\pi' \in \Pi} \mathbf{y}_{\pi'} \cdot \ell(\pi'(t))} \mathrm{d}t \\
\text{subject to} & \sum_{\pi \in \Pi_v} \mathbf{x}_{v\pi} = 1, \, \forall v \in \mathcal{V} \\
& \sum_{\pi \in \Pi} \mathbf{y}_\pi = N_v \\
& \mathbf{x} \in \{0,1\}^{N_v \times N_\pi} \\
& \mathbf{y} \in \{0,1\}^{N_\pi}
\end{array}
$$

$$
\begin{array}{ll}
\underset{x_\pi}{\arg\min} & \sum_{r=1}^{N_\mathcal{R}} \sum_{i=1}^{N_A^r} (t_{i+1}^r - t_i^r) \mathbb{E}(\lambda_i^r) a_i^r \\
\text{subject to} & \sum_{\pi \in \Pi_i^r} x_\pi = a_i^r, \, \forall r \in \mathcal{R} \\
& \sum_{\pi \in \Pi_v} x_\pi = 1, \, \forall v \in \mathcal{V} \\
& x_\pi \in \{0,1\}, \, \forall \pi \in \Pi
\end{array}
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References

[1] J. Alonso-Mora, S. Samaranayake, A. Wallar, E. Frazzoli, and D. Rus, "Ride-Vehicle Assignment and Analysis of the Benefits of High Capacity Vehicle Pooling," *Proceedings of the National Academy of Sciences*, under review.