CS601-01: HEAP DATA STRUCTURE

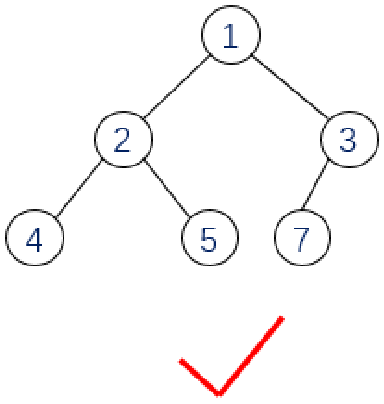
NEHA HATTIHOLI & SWETHA BANAGIRI

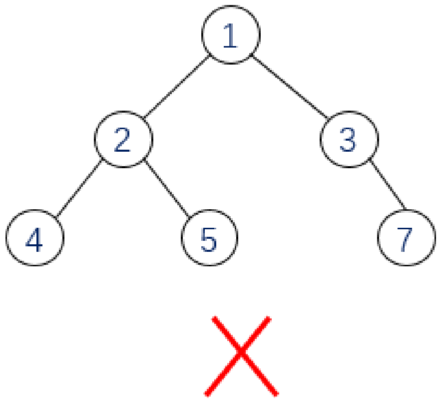
AGENDA

- Overview
- Properties
- Heap Implementation using Array
- Types of Heap – Min Heap/ Max Heap
- Heapify Algorithms
- Insert/Delete Operations
- Time/Space Complexity
- Applications of Heap

OVERVIEW

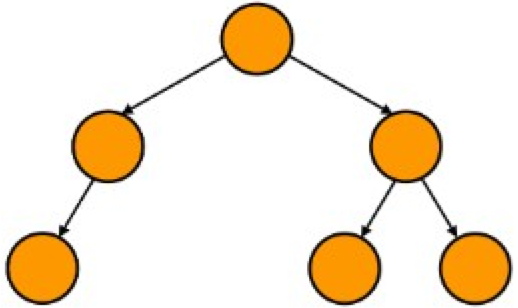
- Binary Tree based Data Structure
- A complete tree that satisfies heap property
- Efficient Implementation of Priority Queue
- Implemented using arrays
- If node A is the parent of node B, then node A is ordered with respect to node B with the same ordering applied to the whole heap





THINK TONK 

From the following images, select the ones that follows the rules of Complete Binary Tree



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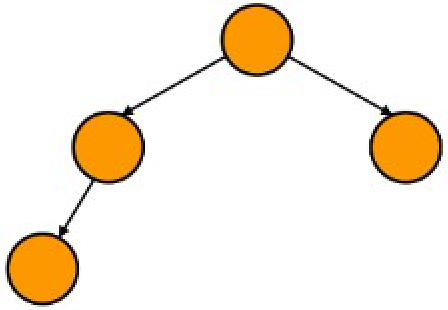

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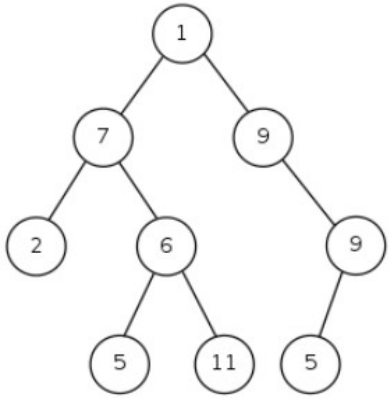

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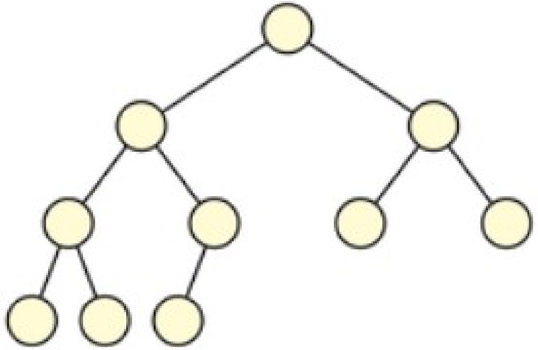

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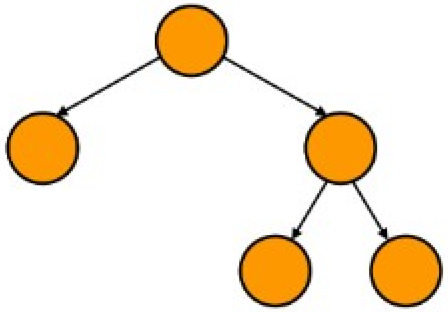

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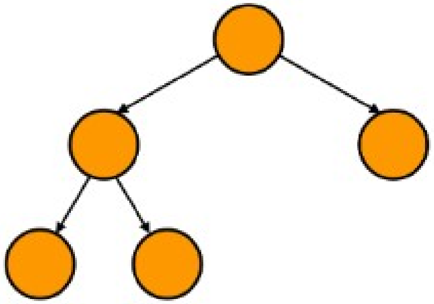

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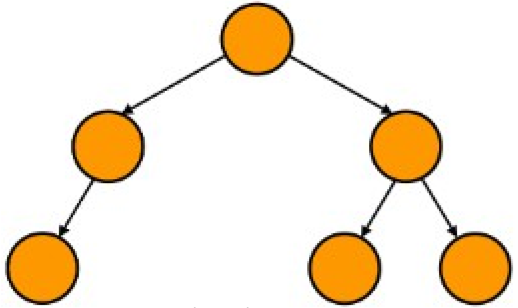
CHECK POINT



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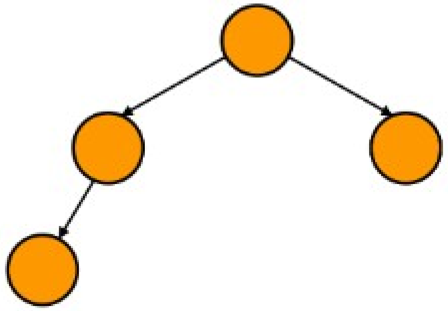
(1)



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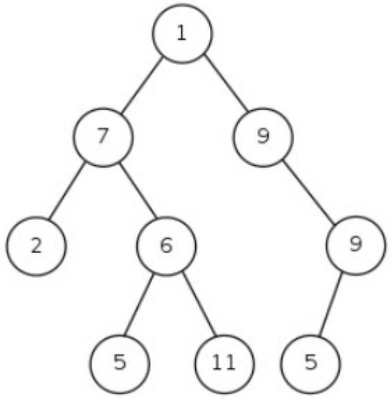

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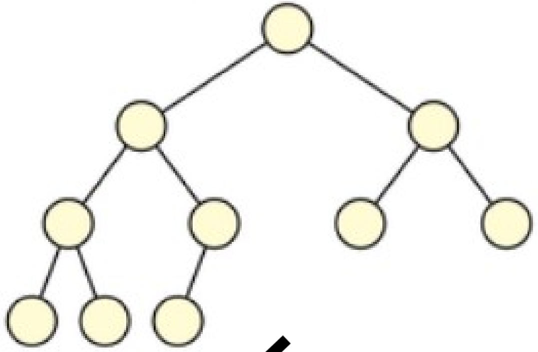

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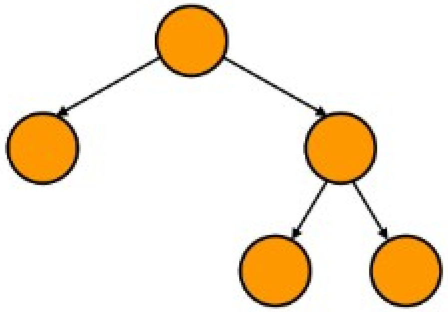

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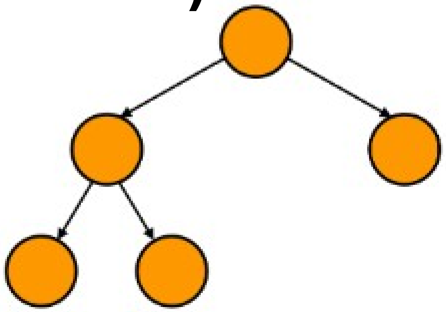

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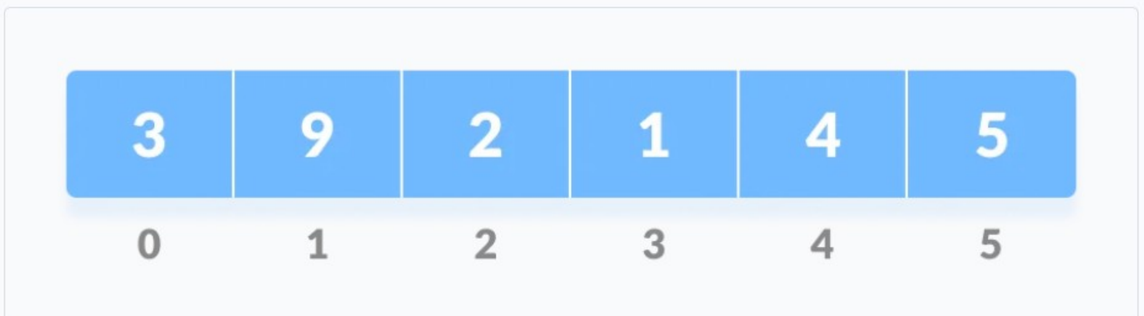
PROPERTIES

Heap Data Structure has the following rules:

- The tree must satisfy the rules of complete binary tree
- Must satisfy Heap Ordering
  - Max Heap : Value of each node is <= Value of its parent
  - Min heap : Value of each node is >= Value of its parent

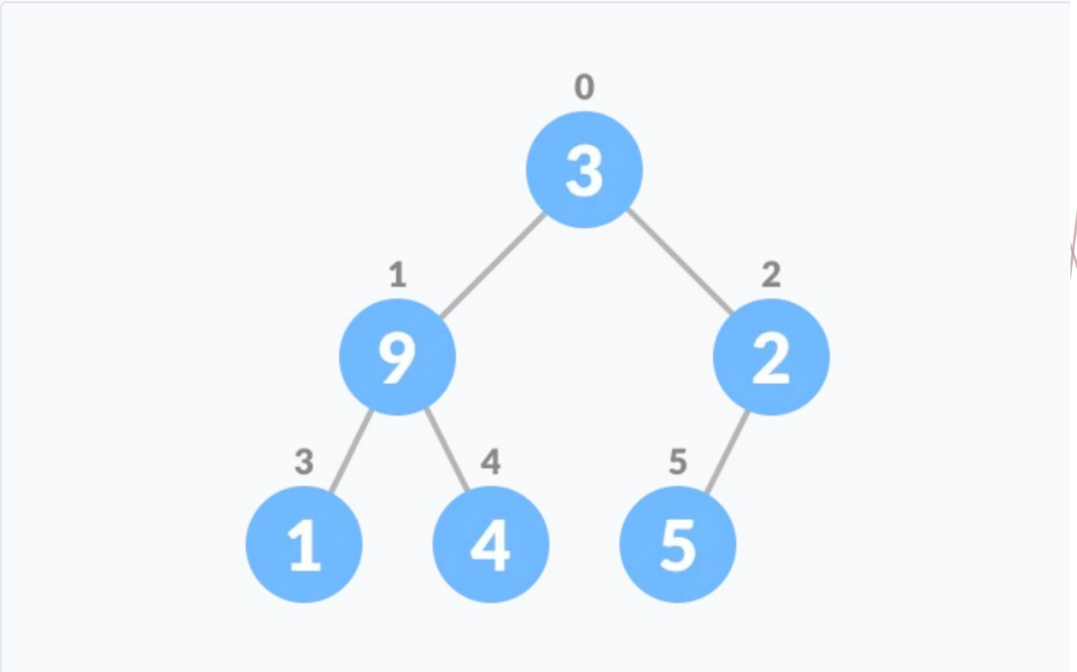
HEAP IMPLEMENTATION USING ARRAY

ARRAY TO HEAP REPRESENTATION



Initial Array

For a root element at index 'i'
Index of LeftChild = 2i + 1
Index of RightChild = 2i+2

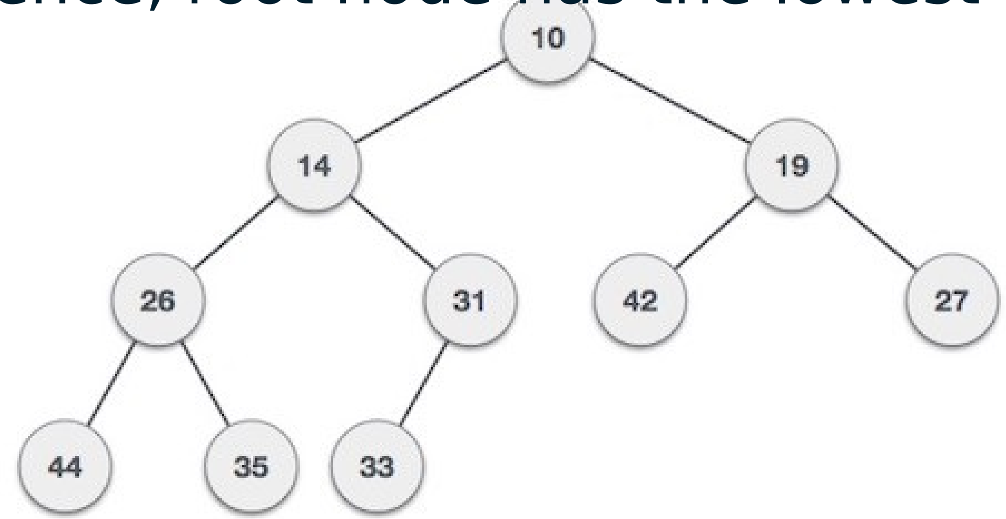


Complete binary tree

TYPES OF HEAP

MIN HEAP

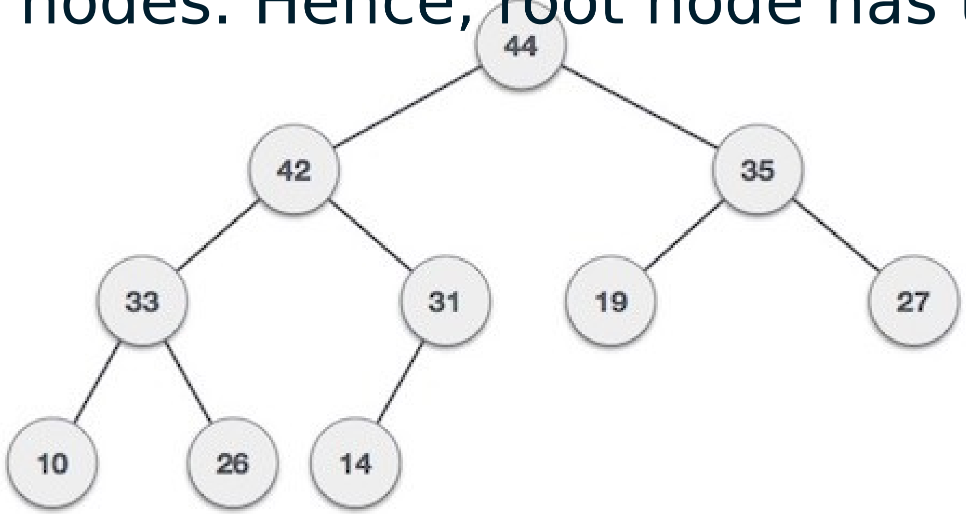
- Value of every node is less than or equal to its decedent nodes. Hence, root node has the lowest value



10	14	19	26	31	42	27	44	35	33
0	1	2	3	4	5	6	7	8	9

MAX HEAP

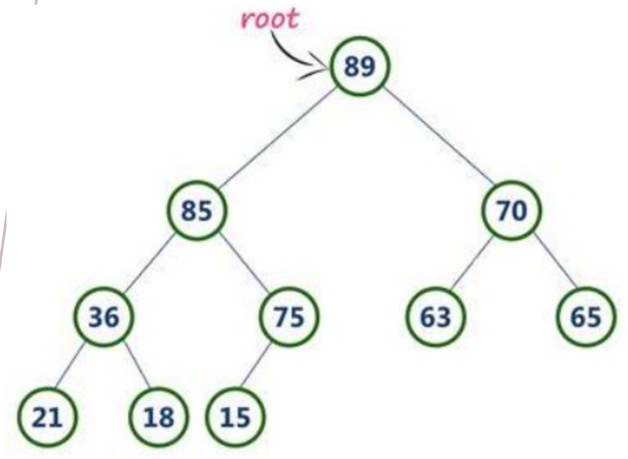
- Value of every node is greater than or equal to its decedent nodes. Hence, root node has the highest value



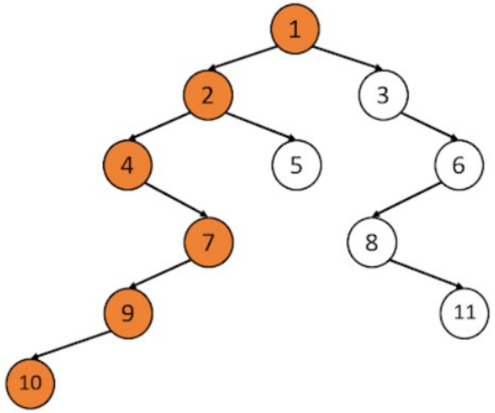
44	**42**	**35**	**33**	**31**	**19**	**27**	**10**	**26**	**14**
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

THINK TONK

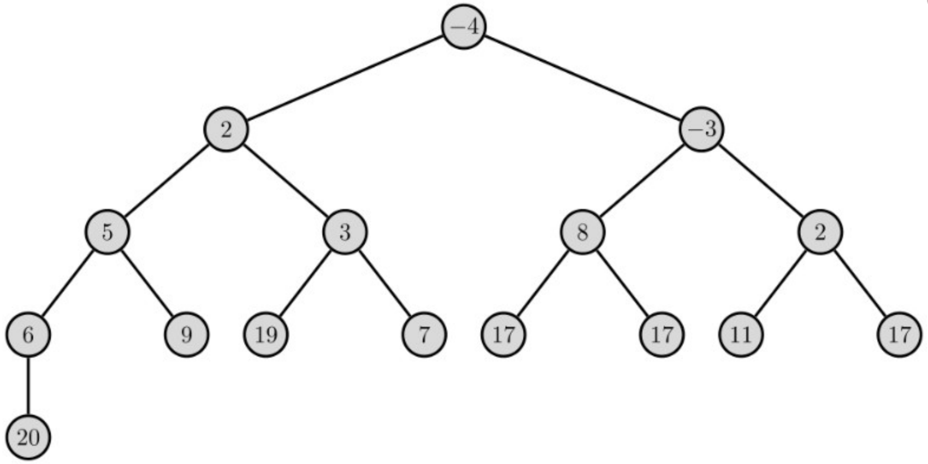
From the following images, validate if it's a heap and if yes, mark them as min/max heaps



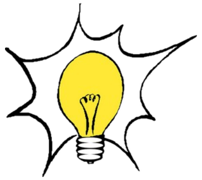
(1)

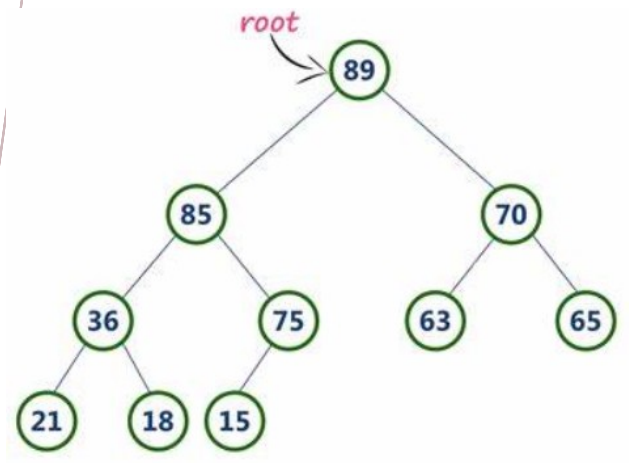


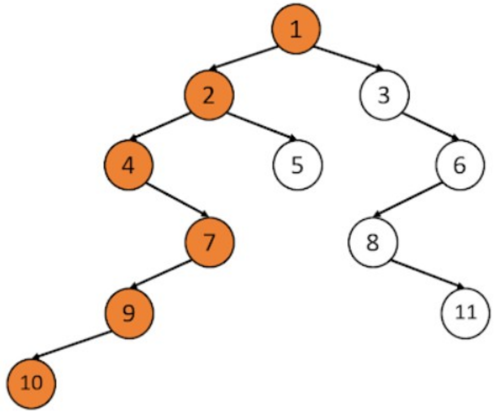
(2)

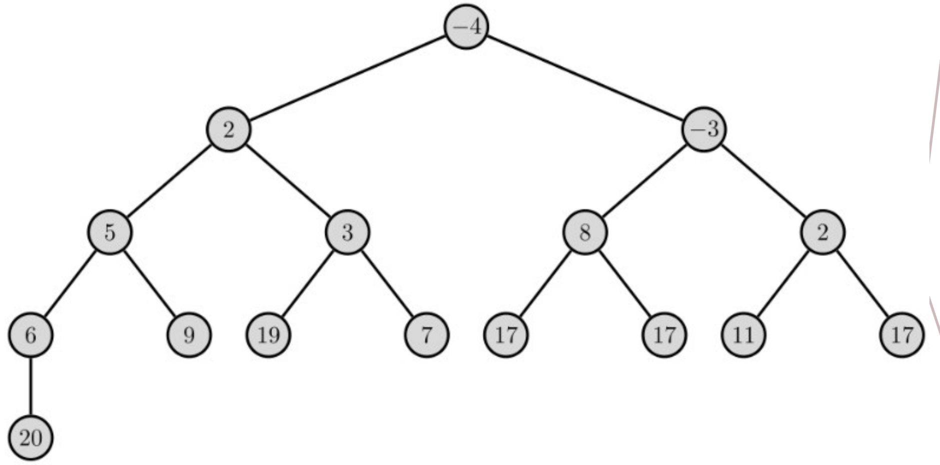


(3)

CHECK POINT 







MAX HEAP (1)

✕ (2)

MIN HEAP (3)

HEAP OPERTIONS

- HEAPIFY
- INSERT ELEMENT INTO HEAP
- DELETE ELEMENT FROM THE HEAP

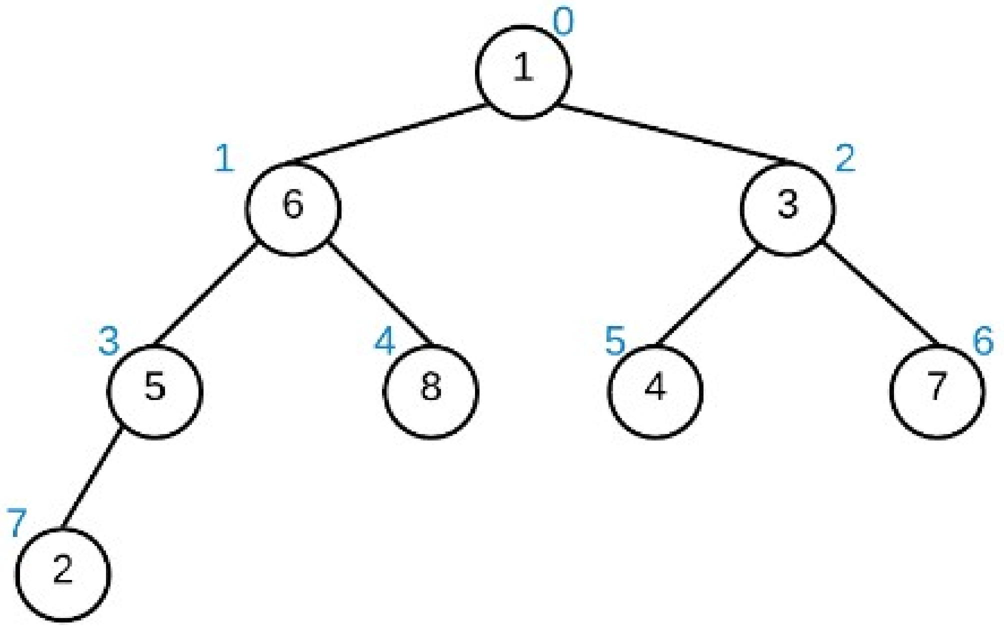
HEAPIFY

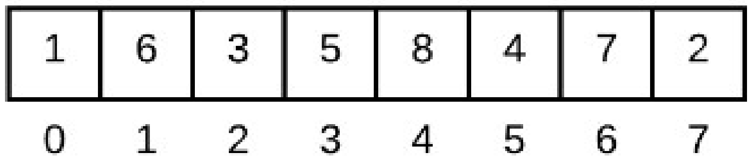
- The process of maintaining Heap ordering given a non-heap structure

- **Algorithm for maintaining Max Heap**
  - if **currentNode** is greater than children, then the heap properties are satisfied
  - Else
    - Swap **currentNode** with **Max** child
    - Heapify() that subtree

HEAPIFY ALGORITHM

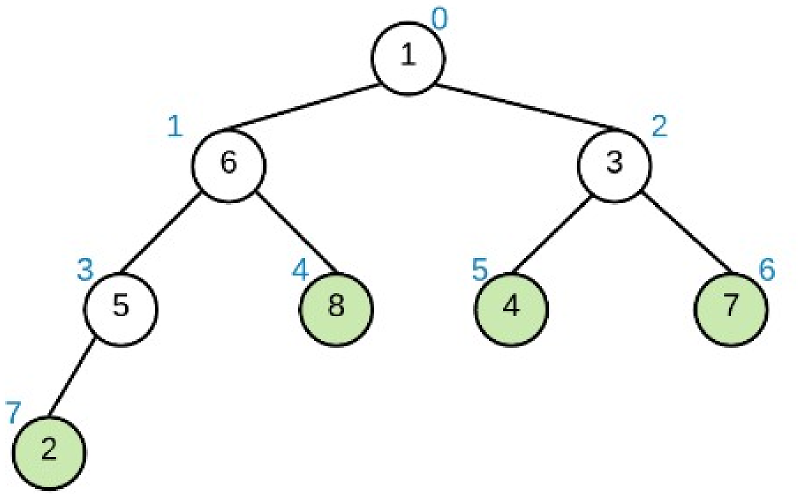
- Following shows the step by step heapify() recursive algorithm on the non-heap array that does not satisfy





HEAPIFY ALGORITHM

- All leaf nodes are valid heaps

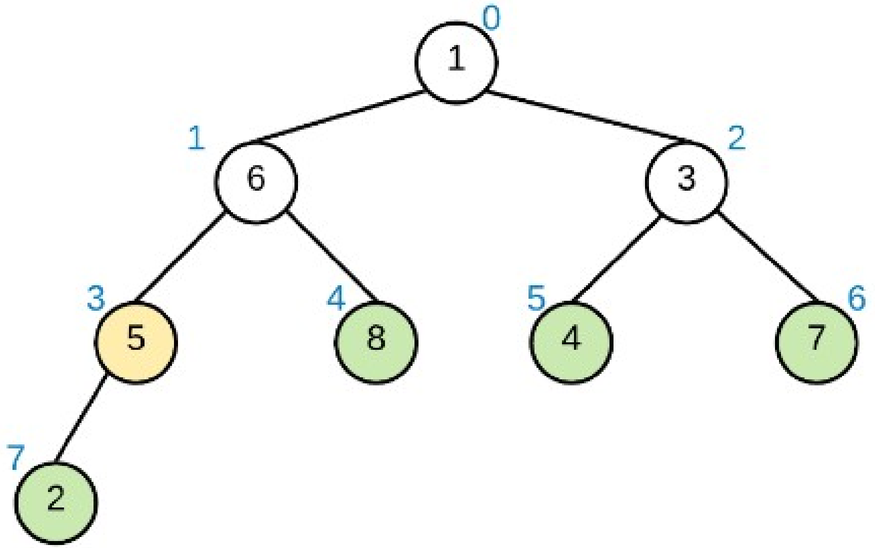


1	6	3	5	8	4	7	2
0	1	2	3	4	5	6	7

- Heapify() loops from first index of non-leaf node to index zero.
  - Index of first non-leaf node : floor[n/2] -1 where n is the total number of elements in the array

HEAPIFY ALGORITHM

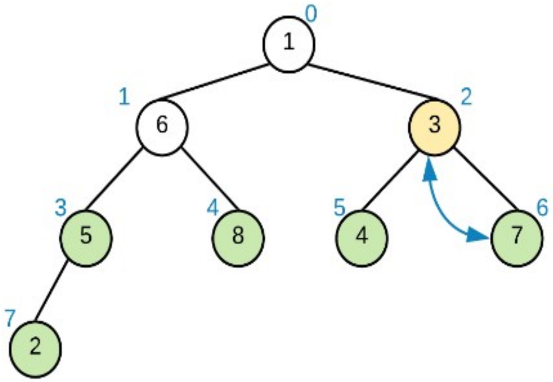
- Heapify(3)
  - First subtree is root element 5 at index 3
  - No swap is needed as the subtree follows heap properties



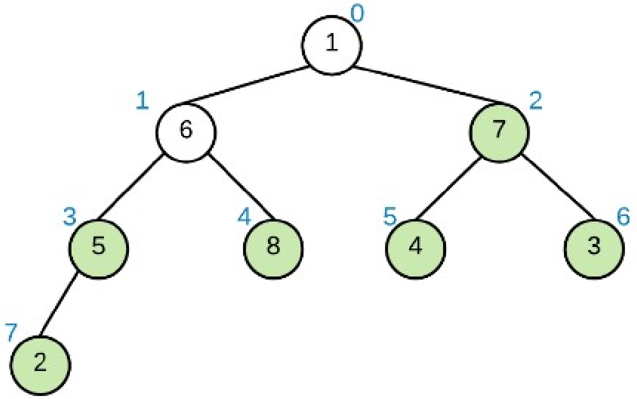
1	6	3	5	8	4	7	2
0	1	2	3	4	5	6	7

HEAPIFY ALGORITHM

- Heapify(2)
  - Second subtree is root element 3 at index 2
  - Children elements 4 and 7 > Parent element 3
  - Swap Parent 3 with element 7



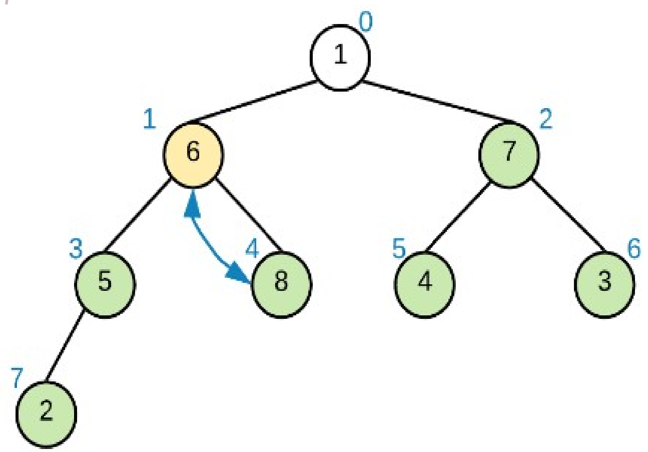
1	6	3	5	8	4	7	2
0	1	2	3	4	5	6	7

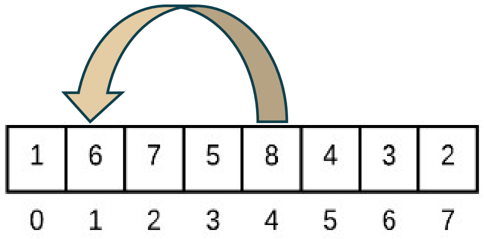


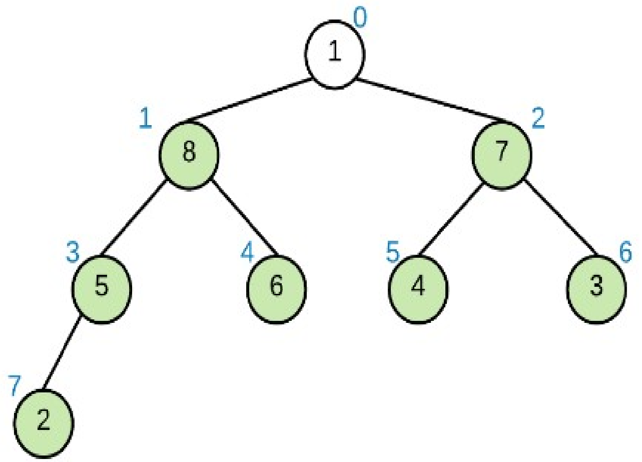
1	6	7	5	8	4	3	2
0	1	2	3	4	5	6	7

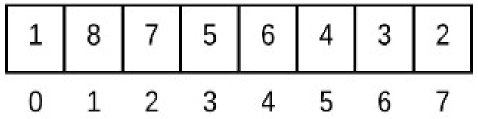
HEAPIFY ALGORITHM

- Heapify(1)
  - Third subtree is root element 6 at index 1
  - Child element 8 > Parent element 6 (Swap)



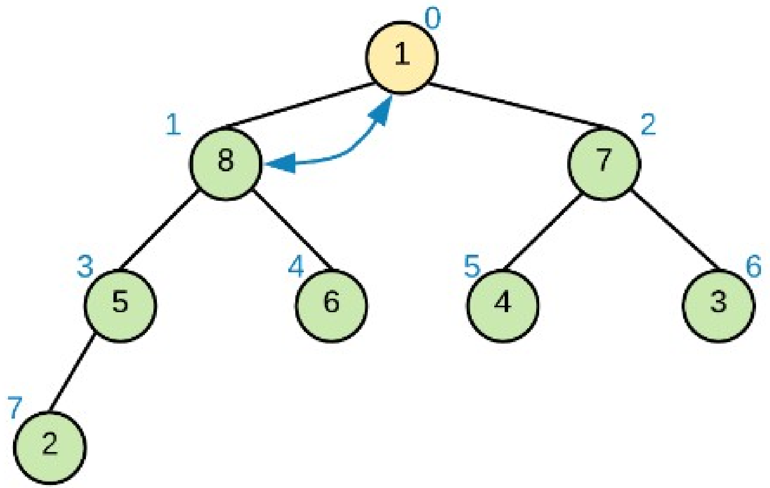


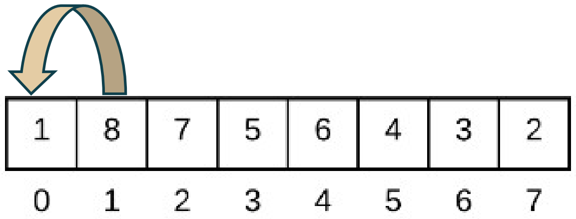




HEAPIFY ALGORITHM

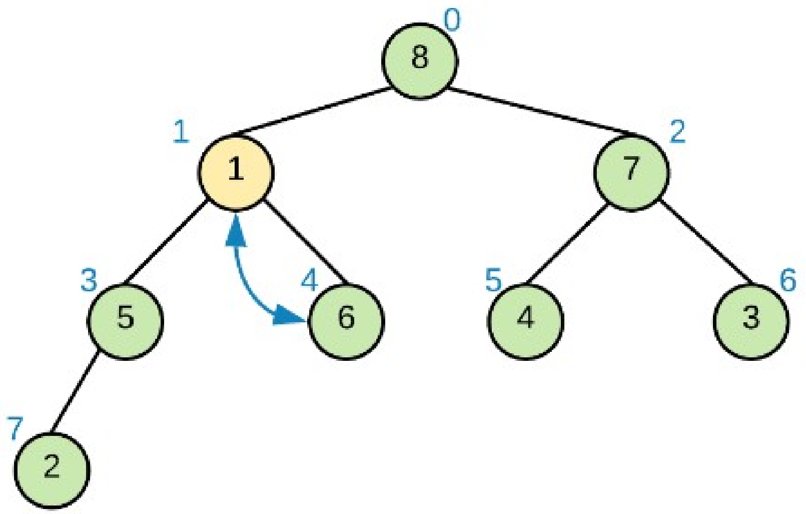
- Heapify(0)
  - Fourth subtree is root element 1 at index 0
  - Child element 8 > Parent element 1 (Swap)

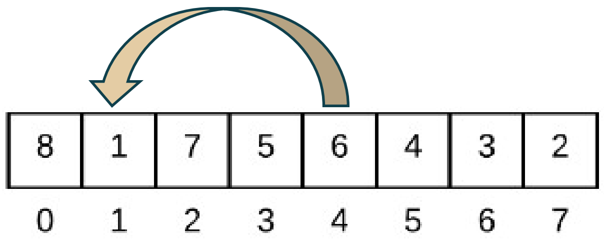




HEAPIFY ALGORITHM

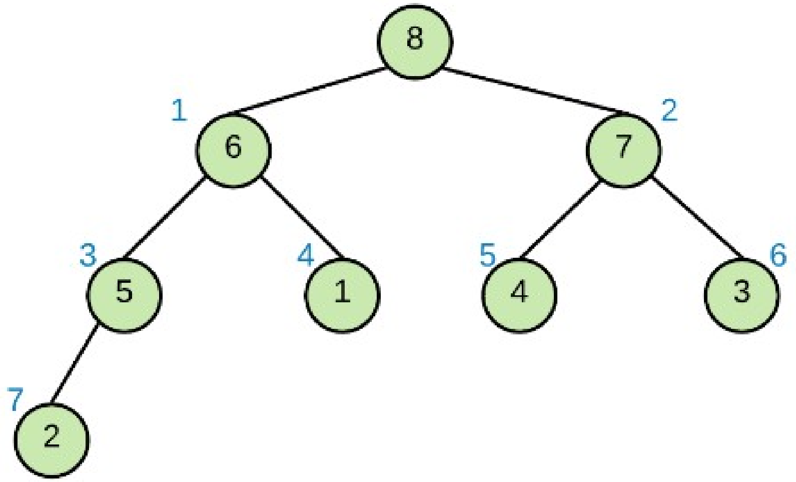
- Heapify(0)
  - Subtree with root element 7 is already heapified
  - Subtree with root element 1 needs heapification
  - Children elements 6 and 5 > Parent element 1 (Swap)





HEAPIFY ALGORITHM

- Final Max Heap after Heapification
  - Subtree with element 7 is already heapified
  - Subtree with element 1 needs heapification
  - Children elements 6 and 5 > Parent element 1 (Swap)



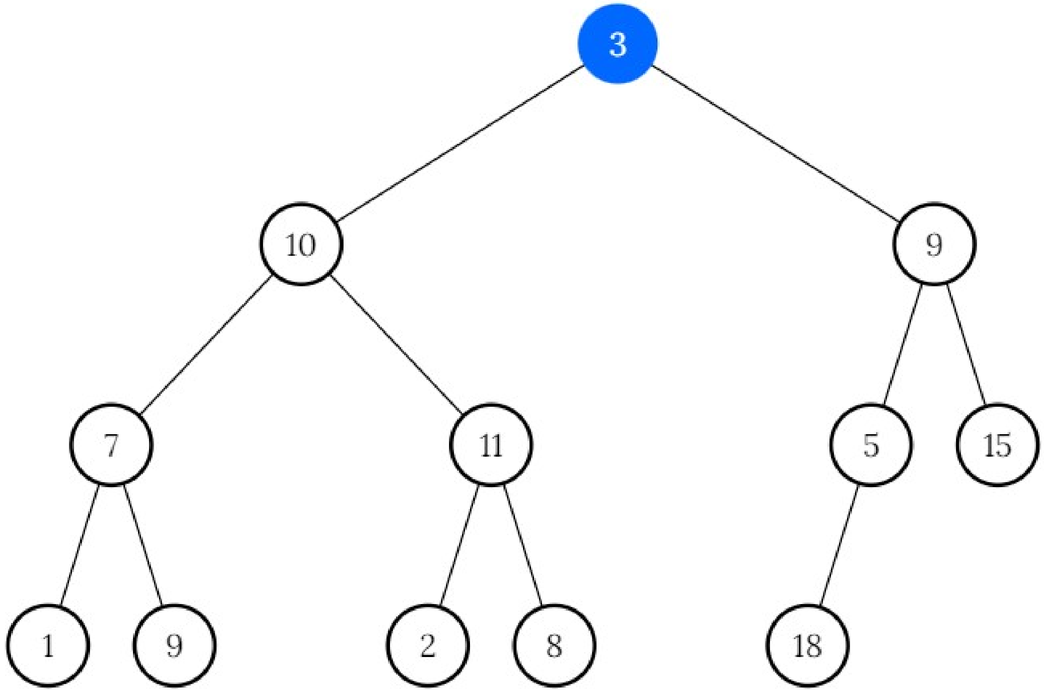
8	6	7	5	1	4	3	2
0	1	2	3	4	5	6	7

HEAPIFY ALGORITHM

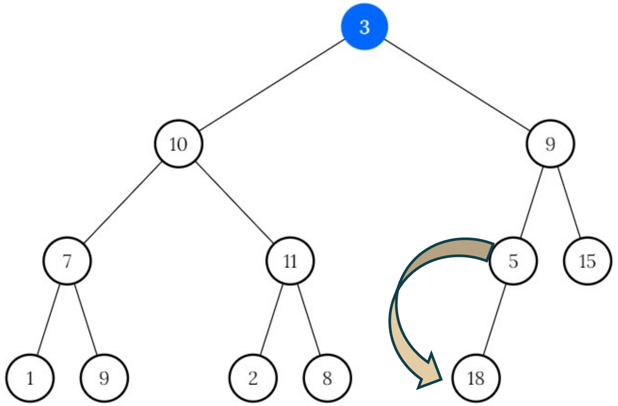
```
Heapify(array, i) {
  largest = i
  leftChild = 2i + 1
  rightChild = 2i + 2

  if array[leftChild] > array[largest] {
    if array[rightChild] > array[leftChild] {
      set rightChild as largest }
    else
      set leftChild as largest }}
  else if array[rightChild] > array[largest] {
    set rightChild as largest  }

  if largest != I {
    swap array[i] and array[largest]
    Heapify(array, largest) } }

MaxHeap(array, size) {
  loop from the i = (n/2)-1 to zero {
    Heapify(array, i) }}
```

LET'S HEAPIFY..

- Heapify the following non-heap structure that follows max heap property



HEAPIFICATION



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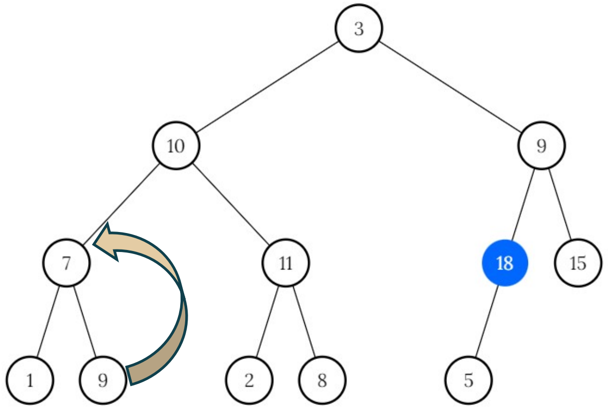

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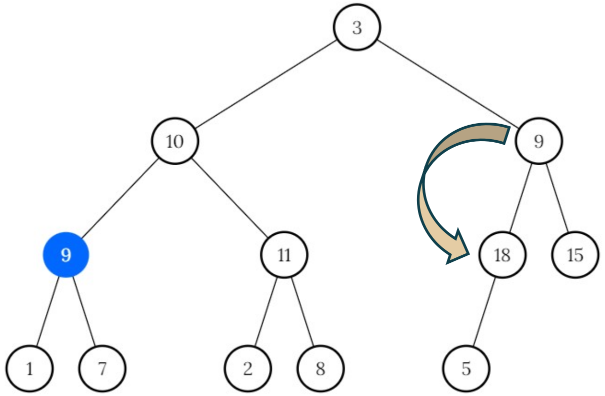

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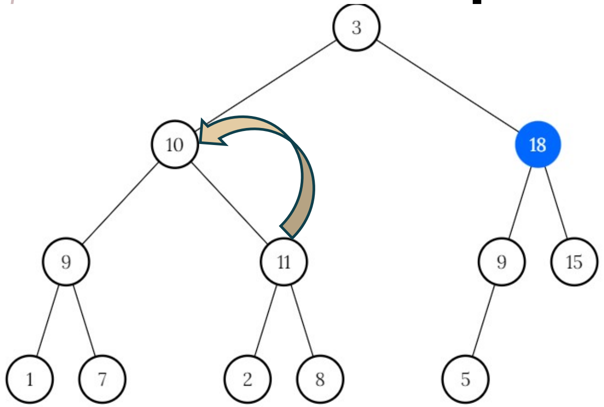

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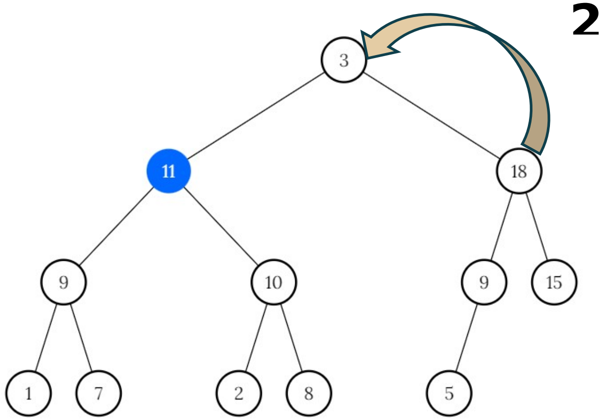

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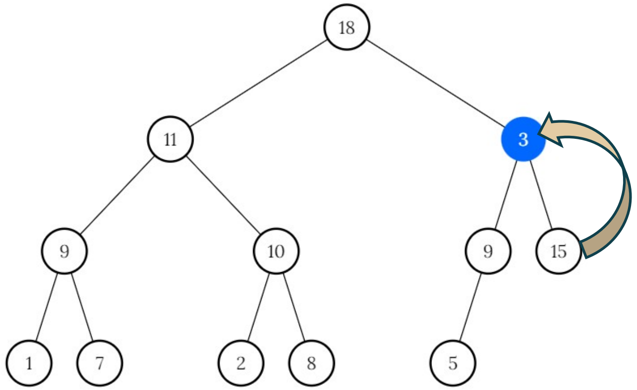

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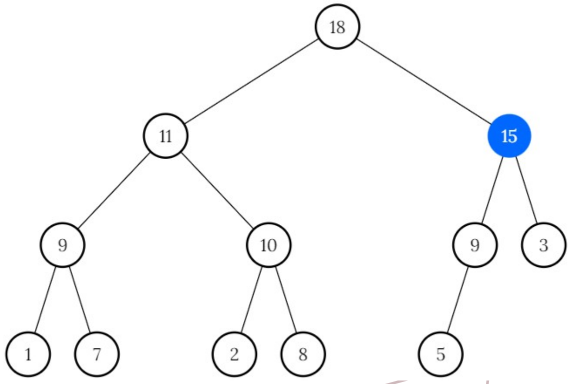
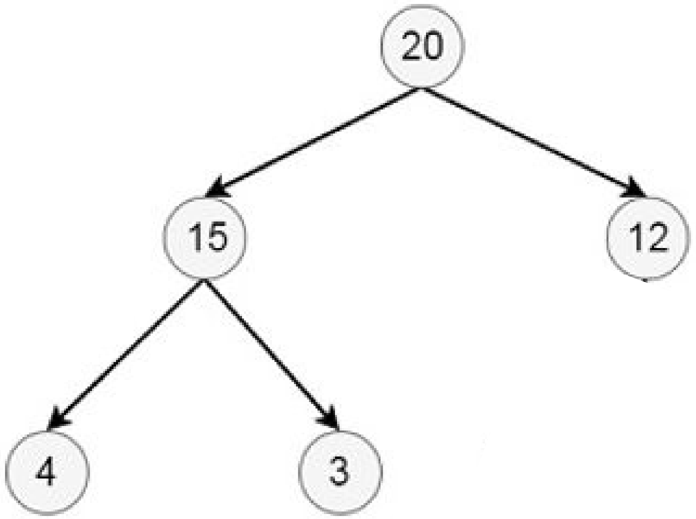


(7

INSERT OPERATION

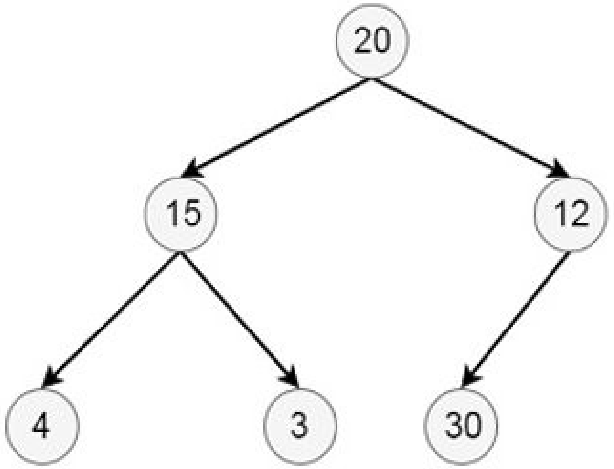
Insert new element at the end of the heap and heapify

- Add element 30 in the following heap



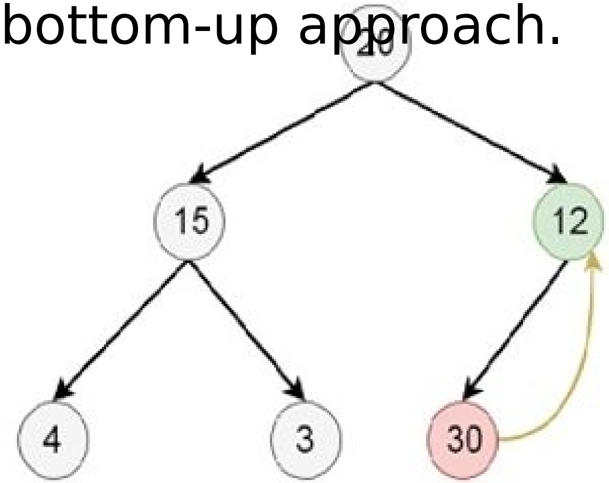
20	**15**	**12**	4	**3**
0	**1**	**2**	3	**4**

- Increase the heap size by 1
- Insert the new element at the end of the Heap.

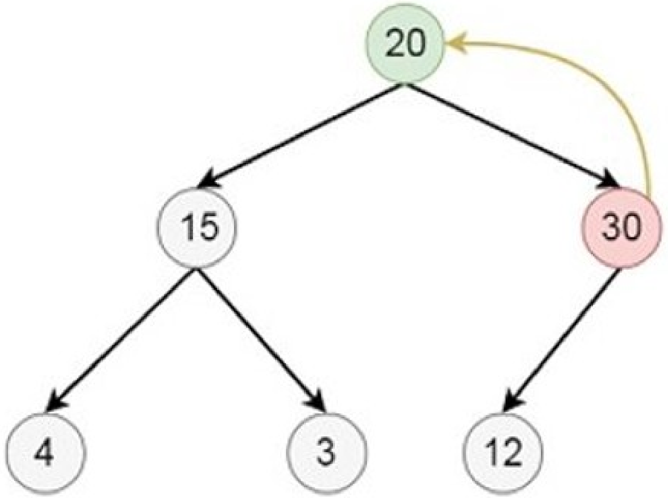


20	15	12	4	3	**30**
0	1	2	3	4	**5**

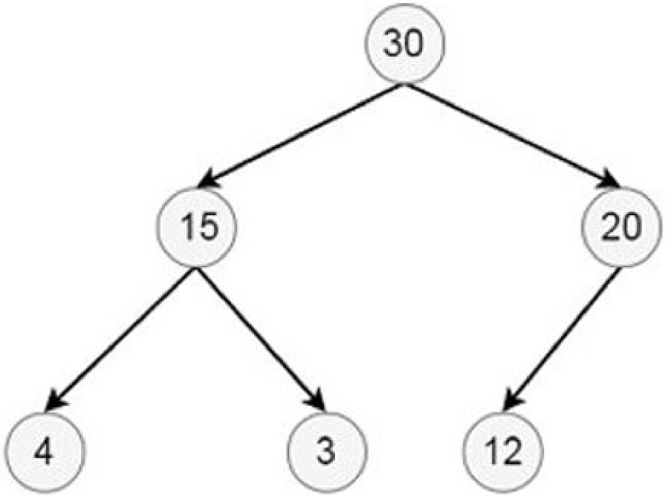
- Heapify this newly inserted element following a bottom-up approach.



20	15	**12**	4	3	**30**
0	1	2	3	4	5



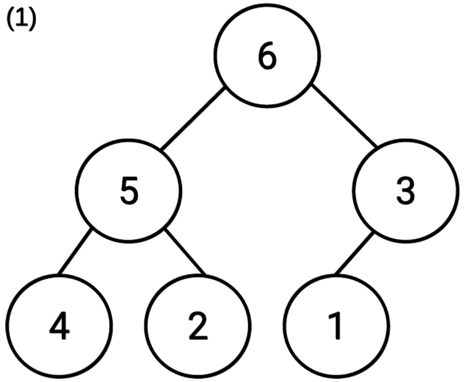
20	15	**30**	4	3	12
0	1	2	3	4	5



30	**15**	**20**	**4**	**3**	**12**
0	**1**	**2**	**3**	**4**	**5**

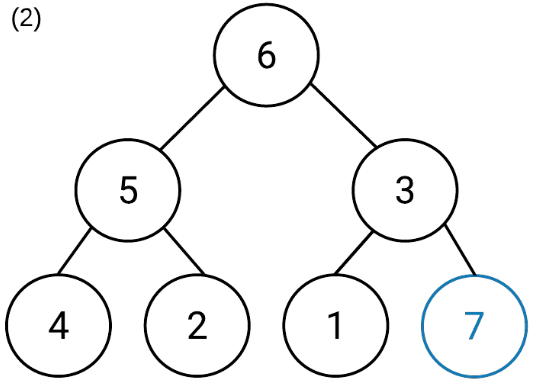
Inserting 7 into this heap

(1)



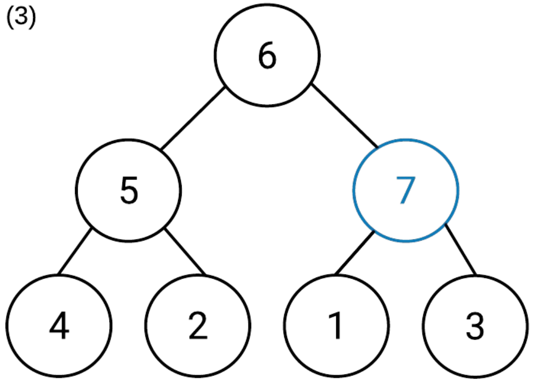
Starting with this max heap

(2)



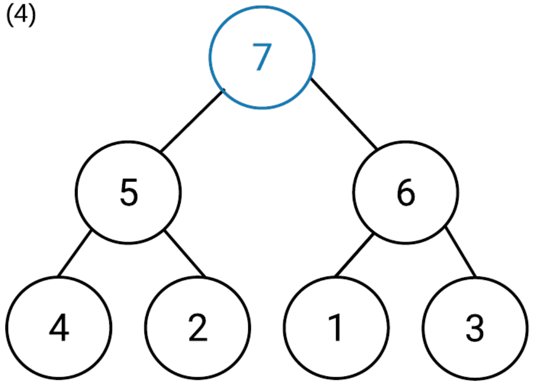
Step 1: 7 is inserted at the bottom most, right most position

(3)



Step 2: Because 7 is bigger than its parent, the 3 node, it gets swapped

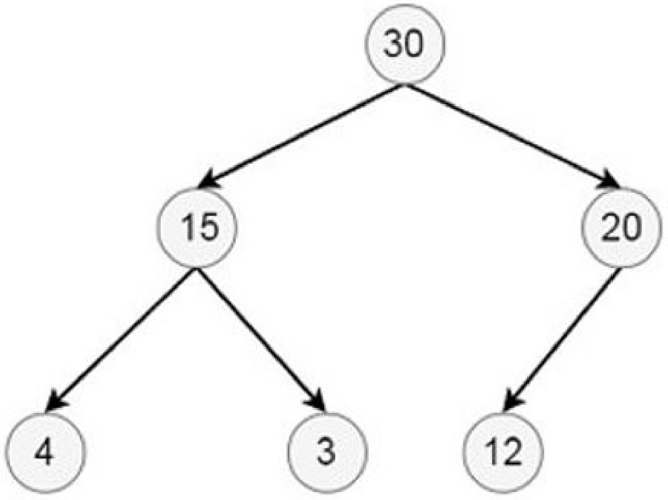
(4)



Step 3: Once again, 7 is bigger than its parent, the 6 node, so it gets swapped

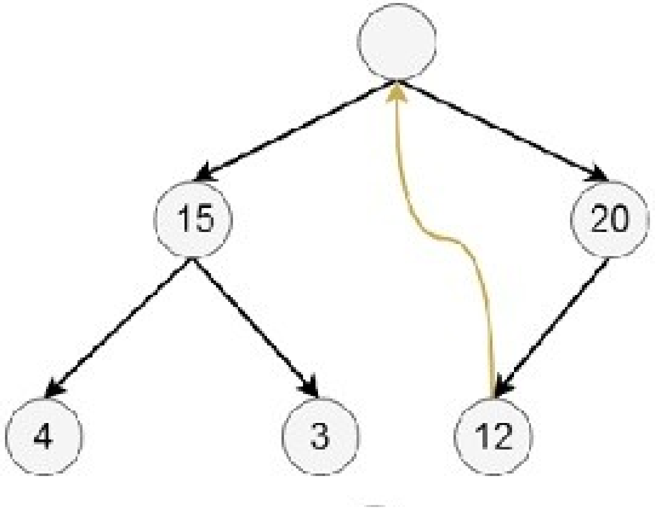
DELETE OPERATION

- Let us delete root element in the below heap



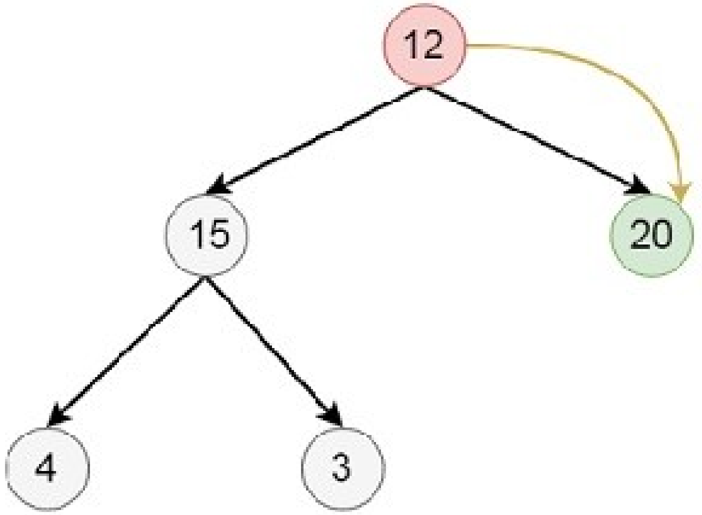
30	15	20	4	3	12
0	1	2	3	4	5

- Replace the root or element to be deleted by the last element.
- Delete the last element from the Heap.

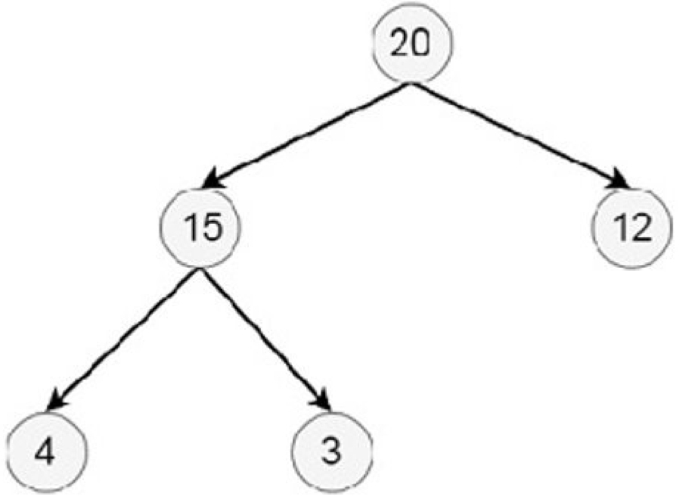


30	15	20	4	3	**12**
0	1	2	3	4	5

- Heapify the last node placed at the new position.



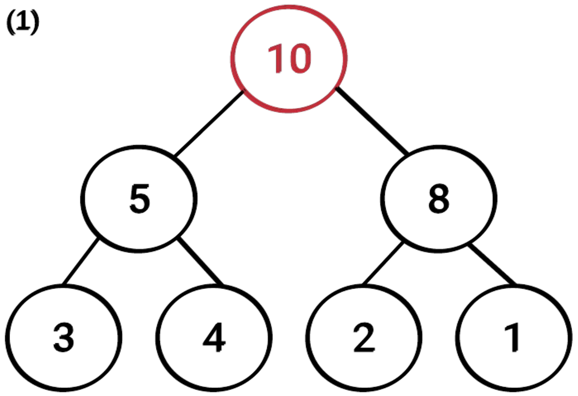
12	15	**20**	4	3
0	1	2	3	4



20	**15**	**12**	**4**	**3**
0	**1**	**2**	**3**	**4**

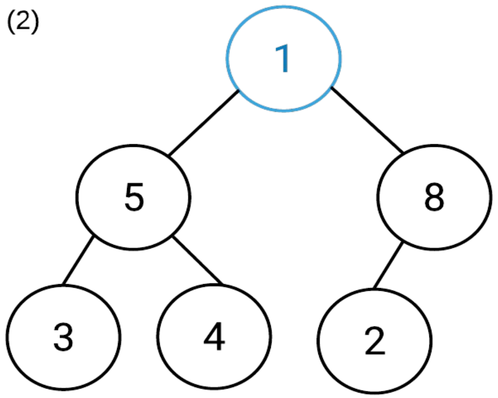
Deleting from this heap

(1)



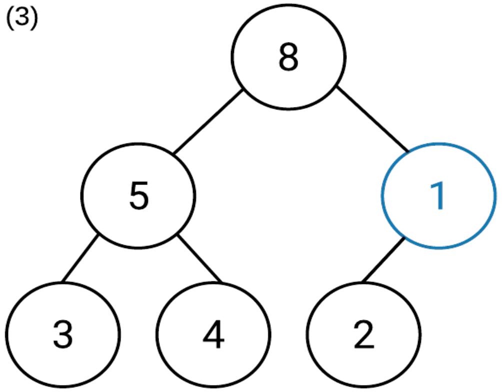
Starting with this max heap

(2)



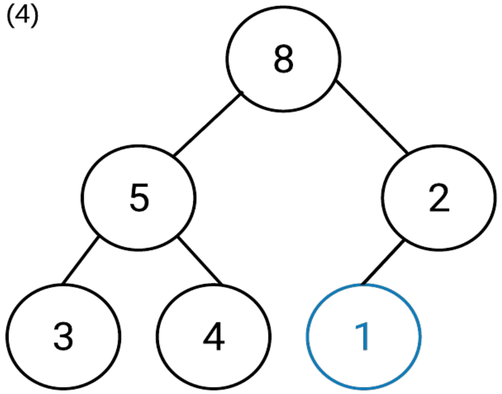
Step 1: the bottom most, left most node, the 1 node, gets placed at the root

(3)



Step 2: Because 1 is less than both of its children, it swaps with the larger element, the 8 node

(4)



Step 3: Once again, 7 is bigger than its parent, the 6 node, so it gets swapped

TIME COMPLEXITY OF HEAP OPERATIONS

- Time Complexity Using Binary Tree

	Best Case	**Worst Case**
Insert	O(1)	$O(\log_2 n)$
Delete	O(1)	$O(\log_2 n)$

APPLICATIONS OF HEAP

- **Heap Sort** - Uses binary heap to sort an array in O(nlogn) time
- **Priority Queue –** Uses binary heap to implement insert/delete/update operations in O(logn) time
- **Graph Algorithms** - Dijkstra's algorithm and Minimum Spanning Tree use heap as internal traversal data structures to reduce time complexity

THANK YOU!