
HOMEWORK 5

CS611: THEORY OF COMPUTATION

Instructions: This homework has problems that should be solved individually.

Recommended Reading: Lecture 10, 11.

Problem 1. [Category: Design+Proof] Let L be the language consisting all strings over $\{a, b\}$ that have twice as many a s as b s. For example, $aababa \in L$ and $\epsilon \in L$ but $a \notin L$.

1. Design a context-free grammar for L . [5 points]
2. (OPTIONAL) Prove that your grammar is correct. [15 points]

Problem 2. [Category: Comprehension+Design] Let $G = (V, \Sigma, R, \langle \text{STMT} \rangle)$ be the following grammar

$$\begin{aligned}\langle \text{STMT} \rangle &\rightarrow \langle \text{ASSIGN} \rangle \mid \langle \text{IF-THEN} \rangle \mid \langle \text{IF-THEN-ELSE} \rangle \\ \langle \text{IF-THEN} \rangle &\rightarrow \text{if condition then } \langle \text{STMT} \rangle \\ \langle \text{IF-THEN-ELSE} \rangle &\rightarrow \text{if condition then } \langle \text{STMT} \rangle \text{ else } \langle \text{STMT} \rangle \\ \langle \text{assign} \rangle &\rightarrow \text{a} := 1\end{aligned}$$

where $\Sigma = \{\text{if, then, else, condition, a} := 1\}$ and $V = \{\langle \text{STMT} \rangle, \langle \text{IF-THEN} \rangle, \langle \text{IF-THEN-ELSE} \rangle, \langle \text{ASSIGN} \rangle\}$. G is a natural looking grammar for a fragment of a programming language, but G is ambiguous.

1. Show that G is ambiguous. [5 points]
2. Give a new unambiguous grammar for the same language. You need not prove that your grammar is correct but explain your construction. [5 points]

Problem 3. [Category: Design+proof] Give a context-free grammar that generates the language $A = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$. Is your grammar ambiguous? Why or Why not? [10 points]