

### Quiz-3

1.

A)  $\delta(q_0, 10) = \text{Undefined}$

Since,  $M$  has only 0, 1 in its alphabet & 10 is not part of it.

B)  $\hat{\delta}(q_1, 101)$

We know that,

$$\hat{\delta}(q, w) = \begin{cases} q & \text{if } w = \epsilon \\ \delta(\hat{\delta}(q, u), a) & \text{if } w = ua \end{cases}$$

$$\begin{aligned} \text{So, } \hat{\delta}(q_1, 101) &= \hat{\delta}(\hat{\delta}(q_1, 10), 1) \\ &= \delta(\hat{\delta}(\hat{\delta}(q_1, 1), 0), 1) \\ &= \delta(\delta(\delta(\hat{\delta}(q_1, \epsilon), 1), 0), 1) \\ &= \delta(\delta(\delta(q_1, 1), 0), 1) \\ &= \delta(\delta(q_2, 0), 1) \\ &= \delta(q_3, 1) \\ &= q_3 \end{aligned}$$

$$\therefore \hat{\delta}(q_1, 101) = q_3$$

C) The language of  $M$ ,  $L(M) = \{01\}$



2.

$$A) \hat{\delta}(a, \epsilon) = a$$

$$B) \hat{\delta}(a, ax) = \hat{\delta}(\delta(a, a), x)$$

Proof:

We know that,

$$\hat{\delta}(a, uv) = \hat{\delta}(\hat{\delta}(a, u), v)$$

Let  $u=a$

$$\hat{\delta}(a, av) = \hat{\delta}(\hat{\delta}(a, a), v)$$

We know that,

$$\begin{aligned}\hat{\delta}(a, a) &= \delta(\hat{\delta}(a, \epsilon), a) \\ &= \delta(a, a)\end{aligned}$$

$$\text{So, } \hat{\delta}(a, av) = \hat{\delta}(\delta(a, a), v)$$