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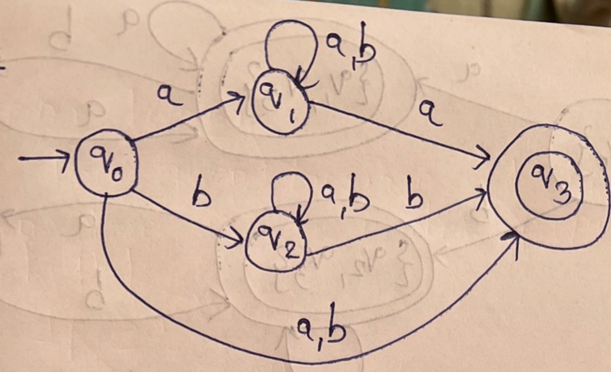
Problem 1.

1) NDFA



NFA to DFA

$Q' = P(Q) = \{\phi, \{q_0\}, \{q_0, q_1\} - - - \}$

$q_0' = \hat{\Delta}(q_0, e) = \{q_0\}$

$\Sigma = \{a, b\}$

$F' = \{A \subseteq Q \mid A \cap F \neq \phi\}$

$= \{\{q_3\}, \{q_0, q_3\} - - \}$

$\delta'(\{q_0\}, a) = \{q_1, q_3\}$

$\delta'(\{q_0\}, b) = \{q_2, q_3\}$

$\delta'(\{q_1, q_3\}, a) = \{q_1, q_3\}$

$\delta'(\{q_1, q_3\}, b) = \{q_1\}$

$\delta'(\{q_2, q_3\}, a) = \{q_2\}$

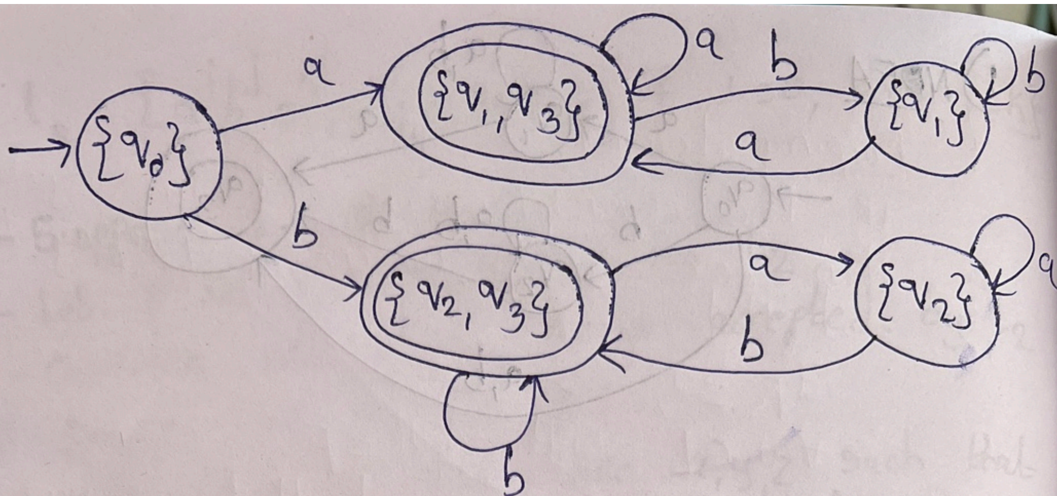
$\delta'(\{q_2, q_3\}, b) = \{q_2, q_3\}$

$\delta'(\{q_1\}, a) = \{q_1, q_3\}$

$\delta'(\{q_1\}, b) = \{q_1\}$

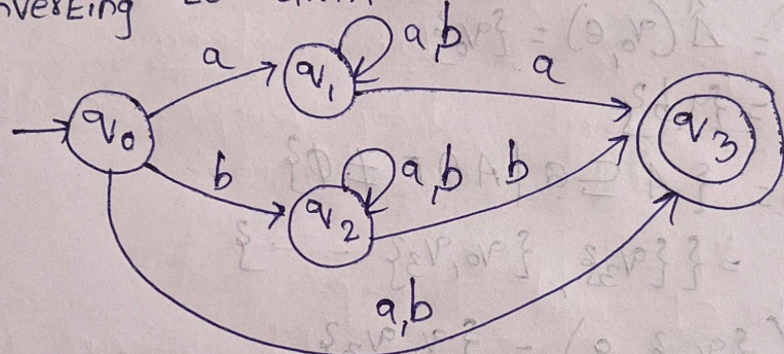
$\delta'(\{q_2\}, a) = \{q_2\}$

$\delta'(\{q_2\}, b) = \{q_2, q_3\}$

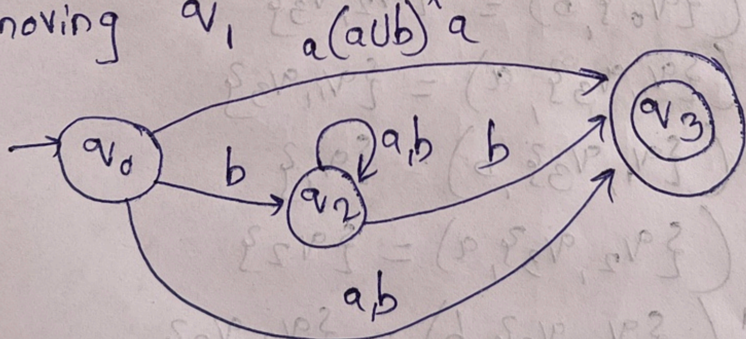


NFA to Regex

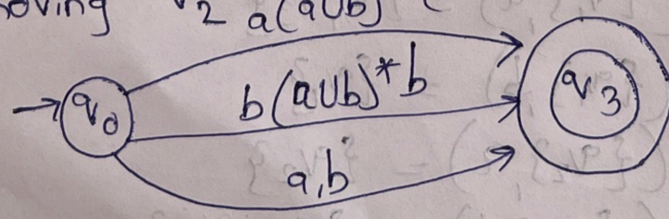
• Converting to GNFA

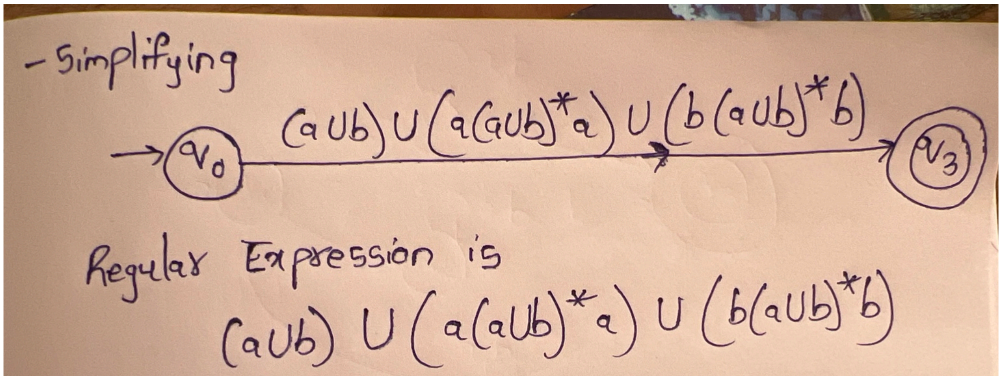


• Removing q_1

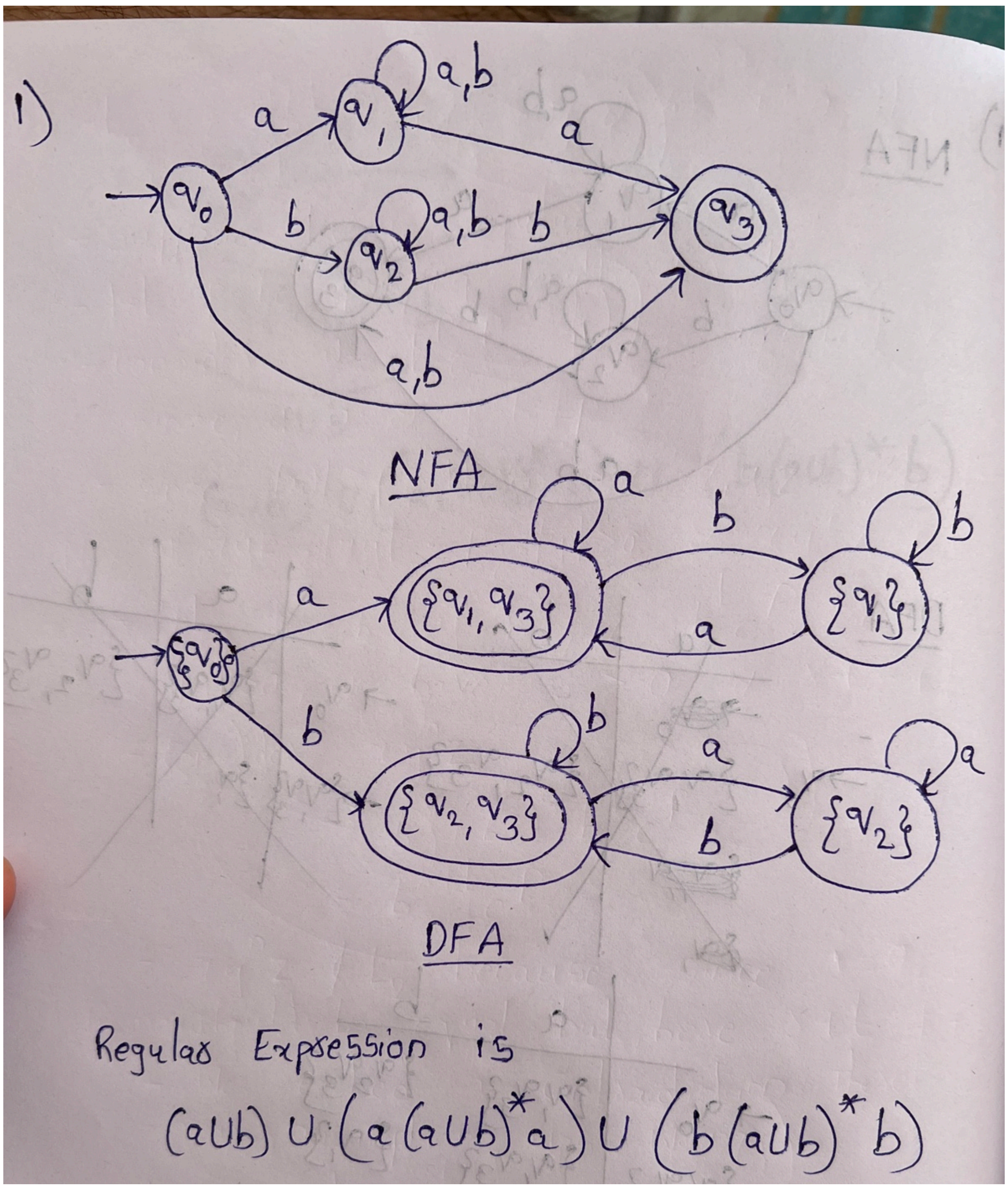


- Removing q_2





The final answers are



Problem 2.

2)

1. Pumping Lemma

if the given Language L is regular then there is a number p (pumping length) such that

$\exists p \geq 0,$

$\forall w \in L$ with $|w| \geq p$

$\exists x, y, z \in \Sigma^*$ such that $w = xyz,$

$|y| > 0$ & $|xy| \leq p,$

$\forall i \geq 0 \quad xy^iz \in L$

Contrapositive of Pumping Lemma

If L doesn't satisfy pumping condition, then L not regular. That is

if $\forall p \geq 0,$

$\exists w \in L$ with $|w| \geq p,$ such that $w = xyz, \ |y| > 0$

$\forall x, y, z \in \Sigma^*$ such that

& $|xy| \leq p,$

$\exists i \geq 0, \quad xy^iz \notin L,$

then L is not regular

2. $L_2 = \{a^i b^j c^k,\ i,j,k \geq 0,$ & if $i=0$, then $j=k\}$

- Suppose L_2 is regular
- Let p be pumping length for L_2
- Consider $w = b^p c^p$. It is accepted by L_2
  since, $i=0$ & $j=k=p$
- Since $|w| > p$, there are x, y, z such that
  $w = xyz$, $|xy| \leq p$, $|y| > 0$ & $xy^i z \in L_2\ \forall i$
- Let $x = b^r$, $y = b^s$, $z = b^t c^p$. Since, as
  $|y| > 0$, we have $s > 0$ & $r+s+t = p$
- Consider, $xy^0 z$

$$\Rightarrow b^r (b^s)^0 b^t c^p$$

$$= b^{r+t} c^p$$

- $xy^0 z \notin L_2$ because since $i=0$, j, k
  should be equal. But, here $r+t \neq p$
- We reached a contradiction. Therefore
  our assumption was wrong

$\therefore L_2$ is not regular

Notes.

<u>Pumping Lemma</u>

- If L is regular then there is a number p (pumping length) such that $\forall w \in L$ with $|w| \geq p$, $\exists x, y, z \in \Sigma^*$ such that $w = xyz$ &
  1) $|y| > 0$, 2) $|xy| \leq p$, 3) $\forall i \geq 0$ $xy^iz \in L$
- L regular implies that L satisfies condition in pumping lemma. If L regular $\Rightarrow$ pumping lemma. Then, Not pumping Lemma $\Rightarrow$ L not regular

<u>Contrapositive Pumping Lemma</u>

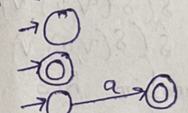
If L doesn't satisfy pumping condition, then L not regular
$\forall p \geq 0$, $\exists w \in L$ with $|w| \geq p$, $\forall x, y, z \in \Sigma^*$ such that $w = xyz$, $|y| > 0$ & $|xy| \leq p$, $\exists i \geq 0$ $xy^iz \notin L$

<u>DFA to Regex</u>

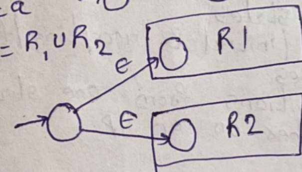
- Given DFA M, to construct Regex R we have 2 steps
  - Construct GNFA ~~G~~ G from M
  - Convert G to Regex R
- GNFA has
  - There is single accept state
  - start state has no incoming transitions & accept state has no outgoing ~~transitions~~ transitions.
  - Transitions are labeled by regex
  -
- A GNFA is $G = (Q, \Sigma, q_0, q_F, P)$ where
  - $q_F \in Q$ a single accepting state
  - $P : (Q \setminus \{q_F\}) \times (Q \setminus \{q_0\}) \to R_\Sigma$ where R_Σ is set of all regex over Σ
- A DFA $M = (Q, \Sigma, \delta, q_0, F)$ can be converted to GNFA $G = (Q', \Sigma, q_0', q_F', P)$: where $Q \cap \{q_0', q_F'\}$
  - $Q' = Q \cup \{q_0', q_F'\}$ $= \emptyset$
  - $P(q_1, q_2) =$ ϵ if $q_1 = q_0'$ & $q_2 = q_0$; ϵ if $q_1 \in F$ & $q_2 = q_F'$; $\bigcup_{\{a \mid \delta(q_1, a) = q_2\}} a$

<u>Kleene closure</u>

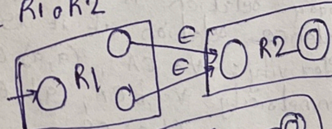
- $L^n = \begin{cases} \{\epsilon\} & \text{if } n = 0 \\ L^{n-1} \circ L & \text{else} \end{cases}$ $\quad L^* = \bigcup_{i \geq 0} L^i$
- $\emptyset^0 = \{\epsilon\}$, $\emptyset^1 = \emptyset$, $\emptyset^* = \{\epsilon\}$
- $\emptyset$ & $\{\epsilon\}$ are the only langs with finite L^*
- A regex is a formula for representing a lang in terms of elementary langs combined using $\cup, \circ, *$
- $L(\emptyset) = \{\}$, $L((R1 \cup R2)) = L(R1) \cup L(R2)$
  $L(\epsilon) = \{\epsilon\}$, $L((R1 \circ R2)) = L(R1) \circ L(R2)$
  $L(a) = \{a\}$, $L((R_1^*)) = L(R1)^*$
- $R \cup S^* \circ T = (R \cup ((S^*) \circ T))$
- L is regular lang iff there is regex R such that $L(R) = L$
  - $R = \emptyset$
  - $R = \epsilon$
  - $R = a$



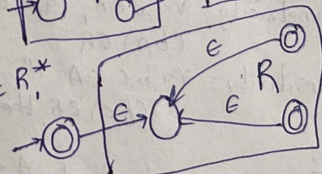
  - $R = R_1 \cup R_2$



  - $R = R1 \circ R2$



  - $R = R_1^*$



<u>~~D~~NFA to ~~N~~DFA</u>

Given NFA, $N = (Q, \Sigma, \delta, q_0, F)$ construct DFA $det(N) = (Q', \Sigma, \delta', q_0', F')$
- $Q' = P(Q)$
- $q_0' = \hat{\Delta}(q_0, \epsilon)$
- $F' = \{A \subseteq Q \mid A \cap F \neq \emptyset\}$
- $\delta'(\{q_1, q_2, \ldots q_K\}, a) = \hat{\Delta}(q_1, a) \cup \hat{\Delta}(q_2, a) \ldots \hat{\Delta}(q_K, a)$

or $\delta'(A, a) = \bigcup_{q \in A} \hat{\Delta}(q, a)$

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NFA

- A NFA is $M = (Q, \Sigma, \delta, q_0, F)$ where
  - $\delta : Q \times (\Sigma \cup \{\epsilon\}) \to P(Q)$ where $P(Q)$ is powerset of Q
- $\hat{\Delta}(q, w) = \{q \in Q \mid q_1 \xrightarrow{w}_M q\}$
- w is accepted iff $\hat{\Delta}(q_0, w) \cap F \neq \phi$
- $\hat{\Delta}(q, uv) = \bigcup_{q' \in \hat{\Delta}(q,u)} \hat{\Delta}(q', v)$

DFA

- A DFA is $M = (Q, \Sigma, \delta, q_0, F)$ where
  - $\delta : Q \times \Sigma \to Q$
- $\hat{\delta}(q, w) = \begin{cases} q & \text{if } w = \epsilon \\ \delta(\hat{\delta}(q,u), a) & \text{if } w = ua \end{cases}$
- $\hat{\delta}(q, uv) = \hat{\delta}(\hat{\delta}(q,u), v)$
- $\hat{\delta}(q, av) = \hat{\delta}(\delta(q,a), v)$

A finite automation has
- finite states
- Start / initial & accepting / final states
- Transitions from one state to another on an i/p

Relations

- A binary relation $R \subseteq A \times A$ is equivalence relation if it is
  - Reflexivity: $\forall a \in A$, $(a,a) \in R$
  - Symmetricity: $\forall a, b \in A$, $(a,b) \in R$ then $(b,a) \in R$
  - Transitivity: $\forall a, b, c \in A$ if $(a,b) \in R$ & $(b,c) \in R$, then $(a,c) \in R$

Functions

- A func F from A to B $F : A \to B$ is a mapping for every $a \in A$, there is a mapping to unique element $b \in B$.
- $F : A \to B$ is one-one, if $\forall a, a' \in A$ $F(a) \neq F(a')$
- F is onto, if $\forall b \in B$ there is $F(a) = b$
- F is bijective if it is both

Proof by Induction

To prove $P(n)$ is true,
- Prove Base case
- For every K if $P(K)$ is true then prove $P(K+1)$ is true

$L = \{0^i \mid i$ is prime$\}$ is not regular

- Suppose L_p is regular
- Let p be pumping len for L
- Consider $w = 0^m$ where $m \geq p+2$ & m is prime
- Since $|w| > p$ there are x, y, z such that $w = xyz$, $|xy| \leq p$, $|y| > 0$ & $xy^iz \in L_p$ $\forall i$
- Thus $x = 0^r$, $y = 0^s$, $z = 0^t$. as $|y| > 0$ we have $s > 0$,
- consider $xy^{r+t}z = 0^{r+s(r+t)+t}$

  Now, $r + s(r+t) + t = (r+t)(s+1)$ & $s > 0$

  Further, $m = r+s+t \geq p+2$ mean $t \geq 2$ & $s+1 \geq 2$

  thus $xy^{r+t}z \notin L_p$. Contradiction