QUIZ 2: PROOFS

1. Consider the following "proof" of the statement "$1/4 > 1/2$".

$$2 > 1 \tag{1}$$
$$2\log_{10}(\frac{1}{2}) > 1\log_{10}(\frac{1}{2}) \tag{2}$$
$$\log_{10}((\frac{1}{2})^2) > \log_{10}(\frac{1}{2}) \tag{3}$$
$$\frac{1}{4} = \frac{1}{2}^2 > \frac{1}{2} \tag{4}$$

Which of the below options correctly identifies the mistake in the above proof? Provide justification.

(A) $2 > 1$ is not correct.

(B) $2 > 1$ does not imply $2\log_{10}(\frac{1}{2}) > 1\log_{10}(\frac{1}{2})$.

(C) $2\log_{10}(\frac{1}{2}) > 1\log_{10}(\frac{1}{2})$ does not imply $\log_{10}((\frac{1}{2})^2) > \log_{10}(\frac{1}{2})$.

(D) $\log_{10}((\frac{1}{2})^2) > \log_{10}(\frac{1}{2})$ does not imply $\frac{1}{2}^2 > \frac{1}{2}$.

2. Consider the following "proof" of the statement "If a, b are real numbers such that $a = b$ then $a = 0$".

$$a = b \tag{5}$$
$$a^2 = ab \tag{6}$$
$$a^2 - b^2 = ab - b^2 \tag{7}$$
$$(a-b)(a+b) = (a-b)b \tag{8}$$
$$a + b = b \tag{9}$$
$$a = 0 \tag{10}$$

Which of the below options correctly identifies the mistake in the above proof? Provide justification.

(A) $a = b$ does not imply $a^2 = ab$.

(B) $a^2 = ab$ does not imply $a^2 - b^2 = ab - b^2$.

(C) $a^2 - b^2 = ab - b^2$ does not imply $(a-b)(a+b) = (a-b)b$.

(D) $(a-b)(a+b) = (a-b)b$ does not imply $a + b = b$.

3. Consider the sequence defined inductively as follows: $a_0 = 0$, and $a_n = a_{\lceil n/2 \rceil} + a_{\lfloor n/2 \rfloor}$. We claim that $a_n = n$ for all n. We prove this by induction. For the base case observe that $a_0 = 0$ by definition. Assume that for all $n < k$, we have $a_n = n$. Now $a_k = a_{\lceil k/2 \rceil} + a_{\lfloor k/2 \rfloor}$ by definition. From the induction hypothesis, we have $a_{\lceil k/2 \rceil} = \lceil k/2 \rceil$ and $a_{\lfloor k/2 \rfloor} = \lfloor k/2 \rfloor$. Thus, $a_k = a_{\lceil k/2 \rceil} + a_{\lfloor k/2 \rfloor} = \lceil k/2 \rceil + \lfloor k/2 \rfloor = k$. Thus, the claim is established by induction.

(A) The proof is correct.

(B) For some values of k, the induction hypothesis does not allow us to conclude $a_{\lceil k/2 \rceil} = \lceil k/2 \rceil$

(C) For some values of k, the induction hypothesis does not allow us to conclude $a_{\lfloor k/2 \rfloor} = \lfloor k/2 \rfloor$.

(D) For some values of k, $\lceil k/2 \rceil + \lfloor k/2 \rfloor \neq k$.

Provide justification.