

# SHARPE'S SINGLE INDEX MODEL AND ITS APPLICATION TO CONSTRUCT OPTIMAL PORTFOLIO: AN EMPIRICAL STUDY

Niranjan Mandal

B.N. Dutta Smriti Mahavidyalaya, Burdwan

**Abstract.** *An attempt is made here to get an insight into the idea embedded in Sharpe's single index model and to construct an optimal portfolio empirically using this model. Taking BSE SENSEX as market performance index and considering daily indices along with the daily prices of sampled securities for the period of April 2001 to March 2011, the proposed method formulates a unique cut-off rate and selects those securities to construct an optimal portfolio whose excess return to beta ratio is greater than the cut-off rate. Then, proportion of investment in each of the selected securities is computed on the basis of beta value, unsystematic risk, excess return to beta ratio and cut-off rate of each of the securities concerned.*

**Keywords:** Sharpe's Single Index Model, Return and Risk Analysis, Risk Characteristic Line, Portfolio Analysis, Optimal Portfolio Construction

The modern portfolio theory was developed in early 1950s by Nobel Prize Winner Harry Markowitz in which he made a simple premise that almost all investors invest in multiple securities rather than in a single security, to get the benefits from investing in a portfolio consisting of different securities. In this theory, he tried to show that the variance of the rates of return is a meaningful measure of portfolio risk under a reasonable set of assumptions and also derived a formula for computing the variance of a portfolio. His work emphasizes the importance of diversification of investments to reduce the risk of a portfolio and also shows how to diversify such risk effectively. Although Markowitz's model is viewed as a classic attempt to develop a comprehensive technique to incorporate the concept of diversification of investments in a portfolio as a risk-reduction mechanism, it has many limitations that need to be resolved. One of the most significant limitations of Markowitz's model is the increased complexity of computation that the model faces as the number of securities in the portfolio grows. To determine the variance of the portfolio, the covariance between each possible pair of securities must be computed, which is represented in a covariance matrix. Thus, increase in the number of securities results in a large covariance matrix, which in turn, results in a more complex computation. If there are  $n$  securities in a portfolio, the Markowitz's model requires  $n$  average (or expected) returns,  $n$  variance terms and  $\frac{n(n-1)}{2}$  covariance terms (i.e. in total  $\frac{n(n+3)}{2}$  data-inputs). Due to these practical

difficulties, security analysts did not like to perform their tasks using the huge burden of data-inputs required of this model. They searched for a more simplified model to perform their task comfortably. To this direction, in 1963 William F. Sharpe had developed a simplified Single Index Model (SIM) for portfolio analysis taking cue from Markowitz's concept of index for generating covariance terms. This model gave us an estimate of a security's return as well as of the value of index. Markowitz's model was further extended by Sharpe when he introduced the Capital Assets Pricing Model (CAPM) (Sharpe, 1964) to solve the problem behind the determination of correct, arbitrage-free, fair or equilibrium price of an asset (say security). John Lintner in 1965 and Mossin in 1966 also derived similar theories

independently. William F. Sharpe got the Nobel Prize in 1990, shared with Markowitz and Miller, for such a seminal contribution in the field of investment finance in Economics (Brigham and Ehrhardt, 2002). Sharpe's Single Index Model is very useful to construct an optimal portfolio by analyzing how and why securities are included in an optimal portfolio, with their respective weights calculated on the basis of some important variables under consideration.

### ***Objective of the Study***

The main objectives of the study are:

1. To get an insight into the idea embedded in Sharpe's Single Index Model.
2. To construct an optimal portfolio empirically using the Sharpe's Single Index Model.
3. To determine return and risk of the optimal portfolio constructed by using Sharpe's Single Index Model.

### ***Methodology***

Relevant data have been collected from secondary sources of information (i.e. [www.bseindia.com](http://www.bseindia.com) / [www.riskcontrol.com](http://www.riskcontrol.com) ). For this purpose BSE Sensex is taken as the market performance index. Daily indices along with daily prices of 21 sampled securities for the period of April 2001 to March 2011 are taken into consideration for the purpose of computing the daily return of each security as well as determining the daily market return. Taking the computed return of each security and the market, the proposed method formulates a unique Cut off Rate and selects those securities whose 'Excess Return-to-Beta Ratio' is greater than the cut off rate. Then to arrive at the optimal portfolio, the proportion of investment in each of the selected securities in the optimal portfolio is computed on the basis of beta value, unsystematic risk, excess return to beta ratio and the cut off rate of the security concerned. Different Statistical and Financial tools and techniques, charts and diagrams have been used for the purpose of analysis and interpretation of data.

## **SECTION I**

### **SHARPE'S SINGLE INDEX MODEL: THE THEORETICAL INSIGHT**

This simplified model proposes that the relationship between each pair of securities can indirectly be measured by comparing each security to a common factor 'market performance index' that is shared amongst all the securities. As a result, the model can reduce the burden of large input requirements and difficult calculations in Markowitz's mean-variance settings (Sharpe, 1963). This model requires only  $(3n+2)$  data inputs i.e. estimates of Alpha ( $\alpha$ ) and Beta ( $\beta$ ) for each security, estimates of unsystematic risk ( $\sigma_{ei}^2$ ) for each security, estimates for expected return on market index and estimates of variance of return on the market index ( $\sigma_m^2$ ). Due to this simplicity, Sharpe's single index model has gained its popularity to a great extent in the arena of investment finance as compared to Markowitz's model.

### ***Assumptions Made***

The Sharpe's Single Index Model is based on the following assumptions:

1. All investors have homogeneous expectations.

2. A uniform holding period is used in estimating risk and return for each security.
3. The price movements of a security in relation to another do not depend primarily upon the nature of those two securities alone. They could reflect a greater influence that might have cropped up as a result of general business and economic conditions.
4. The relation between securities occurs only through their individual influences along with some indices of business and economic activities.
5. The indices, to which the returns of each security are correlated, are likely to be some securities' market proxy.
6. The random disturbance terms ( $e_i$ ) has an expected value zero (0) and a finite variance. It is not correlated with the return on market portfolio ( $R_m$ ) as well as with the error term ( $e_i$ ) for any other securities.

### ***Symbols and Notations Used***

Following symbols and notations are used to build up this model:

$R_i$  = Return on security i (the dependent variable)

$R_m$  = Return on market index (the independent variable)

$\alpha_i$  = Intercept of the best fitting straight line of  $R_i$  on  $R_m$  drawn on the Ordinary Least Square (OLS) method or 'Alpha Value'. It is that part of security i's return which is independent of market performance.

$\beta_i$  = Slope of the straight line ( $R_i$  on  $R_m$ ) or 'Beta Coefficient'. It measures the expected change in the dependent variable ( $R_i$ ) given a certain change in the independent variable ( $R_m$ ) i.e.  $\frac{dR_i}{dR_m}$ .

$e_i$  = random disturbance term relating to security i

$W_i$  = Proportion (or weights) of investment in securities of a portfolio.

$\sigma_{ei}^2$  = unsystematic risk (in terms of variance) of security i

$R_p$  = Portfolio Return

$\sigma_p^2$  = Portfolio Variance (risk)

$\beta_p$  = Portfolio Beta

$e_p$  = Expected value of all the random disturbance terms relating to portfolio.

$\sigma_{ep}^2$  = Unsystematic risk of the portfolio

$i = 1, 2, 3, \dots, n$

### Mathematical Mechanism Developed

**a) SIM: Return in the context of security.-** According to the assumptions and notations above it is found that the return on security  $R_i$  depends on the market index  $R_m$  and a random disturbance term  $e_i$ . Symbolically,

$$R_i = f(R_m, e_i) \dots\dots\dots (1)$$

Let the econometric model of the above function with  $R_i$  as the explained variable and  $R_m$  as the explanatory variable be:

$$R_i = \alpha_i + \beta_i R_m + e_i \dots\dots\dots (2)$$

where  $\alpha_i$  and  $\beta_i$  are two parameters and  $e_i$  be the random disturbance term which follows all the classical assumptions i.e.  $E(e_i) = 0$ ,  $E(e_i R_m) = 0$ ,  $E(e_i e_j) = 0 \quad \forall \quad i \neq j$  (non-auto correlation) and  $E(e_i e_j) = \sigma_e^2 \quad \forall \quad i = j$  (homoscedasticity).

To determine the value of  $\alpha_i$  and  $\beta_i$  we use the OLS method and get the following two normal equations :

$$\sum R_i = n\alpha_i + \beta_i \sum R_m \dots\dots\dots (3)$$

$$\sum R_i R_m = \alpha_i \sum R_m + \beta_i \sum R_m^2 \dots\dots\dots (4)$$

Solving these two normal equations above by ‘Cramer’s Rule’, we get

$$\alpha_i = \frac{\begin{vmatrix} \sum R_i & \sum R_m \\ \sum R_i R_m & \sum R_m^2 \end{vmatrix}}{\begin{vmatrix} n & \sum R_m \\ \sum R_m & \sum R_m^2 \end{vmatrix}}$$

or  $\alpha_i = \frac{\sum R_i \sum R_m^2 - \sum R_m \sum R_i R_m}{n \sum R_m^2 - (\sum R_m)^2} \dots\dots\dots (5) \text{ and}$

$$\beta_i = \frac{\begin{vmatrix} n & \sum R_i \\ \sum R_m & \sum R_i R_m \end{vmatrix}}{\begin{vmatrix} n & \sum R_m \\ \sum R_m & \sum R_m^2 \end{vmatrix}}$$

or  $\beta_i = \frac{n \sum R_i R_m - \sum R_i \sum R_m}{n \sum R_m^2 - (\sum R_m)^2}$

$$\text{or } \beta_i = \frac{\frac{1}{n} \sum R_i R_m - \left( \frac{\sum R_i}{n} \right) \left( \frac{\sum R_m}{n} \right)}{\frac{1}{n} \sum R_m^2 - \left( \frac{\sum R_m}{n} \right)^2} \dots\dots\dots(6)$$

(Dividing both sides by  $\frac{1}{n^2}$ )

$$\text{or } \beta_i = \frac{Cov(R_i R_m)}{Var(R_m)} \dots\dots\dots(7)$$

$$\text{or } \beta_i = \frac{r \sigma_i \sigma_m}{\sigma_m^2} \dots\dots\dots(8)$$

Putting  $Cov(R_i R_m) = r_{im}$

$$\text{or } \beta_i = r \frac{\sigma_i}{\sigma_m} \dots\dots\dots(9)$$

Following statistical table is prepared to show the necessary calculations for determining the value of  $\alpha_i$  and  $\beta_i$  :

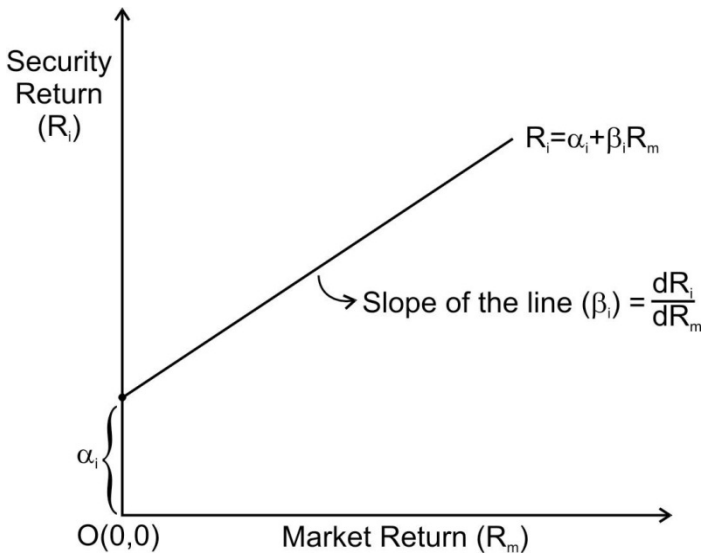
**Table-1 Necessary Calculations for Determining  $\alpha_i$  &  $\beta_i$**

Number of pairs = n	$R_i$	$R_m$	$R_m^2$	$R_i R_m$
	$R_{i_1}$	$R_{m_1}$	$R_{m_1}^2$	$R_{i_1} \times R_{m_1}$
	$R_{i_2}$	$R_{m_2}$	$R_{m_2}^2$	$R_{i_2} \times R_{m_2}$
	.	.	.	.
	.	.	.	.
	.	.	.	.
	$R_{i_n}$	$R_{m_n}$	$R_{m_n}^2$	$R_{i_n} \times R_{m_n}$
	$\sum R_i$	$\sum R_m$	$\sum R_m^2$	$\sum R_i R_m$

Putting these respective values of  $\sum R_i$  ,  $\sum R_m$  ,  $\sum R_m^2$  ,  $\sum R_i R_m$  and n in equation (6) we get the value of  $\alpha_i$  and  $\beta_i$  , then using the value of  $\alpha_i$  and  $\beta_i$  we get required estimated (or regression) line of  $R_i$  on  $R_m$  as under:

$R_i = \alpha_i + \beta_i R_m$  ..... (10), which establishes the linear relationship between security return and market return and is known as the Sharpe's Single Index Model. This model can graphically be represented in Figure-1 as follows:

**Figure-1: Sharpe's Single Index Model (Graphical Exposition)**



Thus, SIM divides the return into two parts:

1. Unique part  $\alpha_i$  and
2. Market related part  $\beta_i R_m$

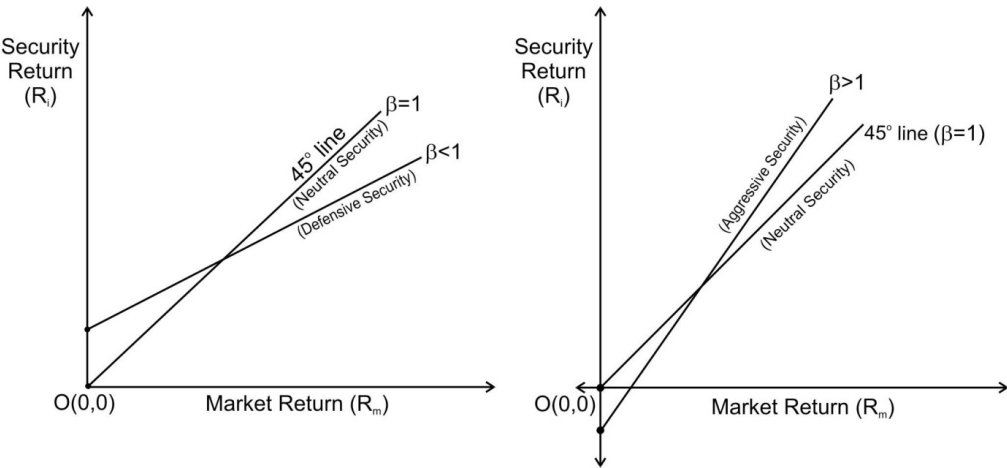
The unique part  $\alpha$ , the intercept term, is called by its Greek name 'Alpha' and is a micro event affecting an individual security but not all securities in general. It is obviously the value of  $R_i$  when  $R_m = 0$ . The market related part  $\beta_i R_m$ , on the other hand, is a macro event that is broad based and affects all or most of the firms. Beta ( $\beta$ ), the slope of the line, is referred to as 'Beta Coefficient'. It is a measure of sensitivity of the security return to the movements in overall market returns. It shows how risky a security is, if the security is held in a well-diversified portfolio.

**b) SIM: The Risk Characteristic Line-** The line representing Sharpe Index Model is also known as the risk characteristic line. The concept of risk characteristic line conveys the message about the nature of security simply by observing its  $\beta$  value as follows:

1. Securities having  $\beta > 1$  are classified as aggressive securities, since they go up faster than the market in a 'bull' (i.e. rising market), go down in a 'bear' (i.e. falling market).
2. Securities having  $\beta < 1$  are categorized as defensive securities, since their returns fluctuate less than the market variability as a whole.
3. Finally, limiting case of securities having  $\beta = 1$ , are neutral securities, since their returns fluctuate at the same rate with the rate of market variability of return.

The risk characteristic lines of the above three cases are represented in Figure-2 below:

**Figure-2: Risk-characteristic Lines of Security having  $\beta < 1$ ,  $\beta > 1$  and  $\beta = 1$**



**c) SIM: Return in the Context of Portfolio.**-For establishing the relation between portfolio return and market return it is required to take the weighted average of the estimated returns of all the securities in the portfolio consisting of n-securities, as under :

$$\sum_{i=1}^n W_i R_i = \sum_{i=1}^n W_i (\alpha_i + \beta_i R_m + e_i) \dots\dots\dots(11)$$

$$\text{or } \sum_{i=1}^n W_i R_i = \sum_{i=1}^n W_i \alpha_i + R_m \sum_{i=1}^n W_i \beta_i + \sum_{i=1}^n W_i e_i$$

$$\text{or } R_p = \alpha_p + \beta_p R_m + e_p \dots\dots\dots (12),$$

$$\text{where } R_p = \sum_{i=1}^n W_i R_i, \quad \alpha_p = \sum_{i=1}^n W_i \alpha_i, \quad \beta_p = \sum_{i=1}^n W_i \beta_i \quad \text{and} \quad e_p = \sum_{i=1}^n W_i e_i$$

According to the classical assumption,  $e_p = \sum_{i=1}^n W_i e_i = 0$  and hence,

$R_p = \alpha_p + \beta_p R_m \dots\dots\dots (13)$ . This is the required estimated (or regression) equation of  $R_p$  on  $R_m$ , which establishes the relationship between portfolio return ( $R_p$ ) and market return ( $R_m$ ).

The following statistical table is prepared to show the necessary calculations for determining  $\alpha_p$  and  $\beta_p$  :

**Table-2: Necessary calculations for determining  $\alpha_p$  &  $\beta_p$**

n = Number of securities in the portfolio	$W_i$	$\alpha_i$	$W_i\alpha_i$	$\beta_i$	$W_i\beta_i$
	$W_1$	$\alpha_1$	$W_1\alpha_1$	$\beta_1$	$W_1\beta_1$
	$W_2$	$\alpha_2$	$W_2\alpha_2$	$\beta_2$	$W_2\beta_2$
	.	.	.	.	.
	.	.	.	.	.
	$W_n$	$\alpha_n$	$W_n\alpha_n$	$\beta_n$	$W_n\beta_n$
	$\sum_{i=1}^n W_i = 1$	--	$\sum_{i=1}^n W_i\alpha_i$	--	$\sum W_i\beta_i$

**d) SIM: Risk in Context of Security.** In Sharpe's index model total risk of a security, as measured by variance, can be deduced by using the relationship between security return and market return (equation 2) as under:

$$Var(R_i) = Var(\alpha_i + \beta_i R_m + e_i)$$

$$\text{or } Var(R_i) = Var(\alpha_i) + Var(\beta_i R_m) + Var(e_i)$$

$$\text{or } Var(R_i) = Var(\beta_i R_m) + Var(e_i) \quad [\because Var(\alpha_i) = 0, \alpha_i \text{ being a parameter}]$$

$$\text{or } \sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{ei}^2 \dots\dots\dots (14)$$

Thus, total variance  $(\sigma_i^2) = \text{Explained variance } (\beta_i^2 \sigma_m^2) + \text{Unexplained variance } (\sigma_{ei}^2)$ .

According to Sharpe, the variance explained by the market index is referred to as systematic risk. The unexplained variance is called residual variance or unsystematic risk. It follows:

$$\text{Total risk of a security } (\sigma_i^2) = \text{Systematic risk } (\beta_i^2 \sigma_m^2) + \text{unsystematic risk } (\sigma_{ei}^2).$$

**e) SIM: Risk in the context of portfolio.** - In SIM, the total portfolio risk (measured by variance) can similarly be deduced by using the relationship between portfolio return and market return [using equation-11] as under:

$$Var\left\{\sum_{i=1}^n W_i R_i\right\} = Var\left\{\sum_{i=1}^n W_i (\alpha_i + \beta_i R_m + e_i)\right\}$$



$$\text{or } Var\left\{\sum_{i=1}^n W_i R_i\right\} = Var\left\{\sum_{i=1}^n W_i \alpha_i\right\} + Var\left\{R_m \sum_{i=1}^n W_i \beta_i\right\} + Var\left\{\sum_{i=1}^n W_i e_i\right\}$$

$$\text{or } \sigma_p^2 = \left(\sum_{i=1}^n W_i \beta_i\right)^2 \sigma_m^2 + \sigma_{ep}^2$$

$$\left[ \because Var \sum_{i=1}^n W_i \alpha_i = 0, \text{ weights being constants and } \alpha_i \text{'s being parameters} \right]$$

$$\text{or } \sigma_p^2 = \beta_p^2 \sigma_m^2 + \sigma_{ep}^2 \dots\dots\dots(15)$$

Sharpe's analysis suggests that total risk of the portfolio also consists of two components: (i) systematic risk and (ii) unsystematic risk of the portfolio. Therefore, we have

$$\text{Total Portfolio Risk } (\sigma_p^2) = \text{Systematic risk of the portfolio } (\beta_p^2 \sigma_m^2) + \text{Unsystematic risk of the portfolio } (\sigma_{ep}^2).$$

### Advantages of SIM

The following are the main advantages of SIM:

a) The apparent advantage to Sharpe's Single Index Model (SIM) is that it considerably simplifies the portfolio problem in comparison to Markowitz's Full-Covariance Model. If we have n securities at our disposal, it requires (3n+2) estimates [i.e.  $\alpha_i$ ,  $\beta_i$ ,  $\sigma_{ei}^2$  for each security] but Markowitz's model requires  $\frac{n(n+3)}{2}$  estimates [i.e.  $R_i$  and  $\sigma_i^2$  for

each security and  $\frac{n(n-1)}{2}$  covariance terms].

b) It simplifies the computational techniques required for solving the problem.

c) It gives us an estimate of security's return as well as of the index value.

d) It greatly helps in obtaining the following inputs required for applying the Markowitz's model:

i) The expected return on each security i.e.  $E(R_i) = \alpha_i + \beta_i E(R_m)$ ,

ii) The variance of return on each security i.e.

$$Var(R_i) = \beta_i^2 Var(R_m) + Var(e_i), \text{ and}$$

iii) The covariance of return between each pair of securities i.e.

$$Cov(R_i R_j) = \beta_i \beta_j Var(R_m)$$

All of the above inputs may be estimated on the basis of historical analysis and/or judgemental evaluation (i.e. ex-post and/or ex-ante analysis).

e) It is very useful in constructing an optimal portfolio for an investor by analyzing the logic behind the inclusion or exclusion of securities in the portfolio with their respective weights.

Sharpe found that there is a considerable similarity between the efficient portfolios generated by SIM and the Markowitz's model. From other studies (viz. Elton et al 1978 and Benari 1988), it is found that SIM performs fairly well. Since SIM simplifies considerably the input requirements and performs well, it represents a greater practical advance in portfolio analysis (Sinacee and Moradi, 2010).

### **Limitations Found**

One of the most important limitations with Sharpe's Single Index model is that it does not consider uncertainty in the market as time progresses; instead the model optimizes for a single point in time (Khan and Jain, 2004). Moreover, this model assumes that security prices move together only because of common co-movement with the market. Many researchers have identified that there are influences beyond the general business and market conditions, like industry-oriented factors that cause securities to move together (Chandra, 2009). However, empirical evidence shows that the more complicated models have not been in a position to outperform the single index model in terms of their ability to predict ex-ante covariances between security returns (Reilly and Brown, 2006).

## **SECTION-II CONSTRUCTION OF OPTIMAL PORTFOLIO BY USING SHARPE'S SINGLE INDEX MODEL**

The construction of optimal portfolio is an easy task if a 'single value' explains the desirability of the inclusion of any security in a portfolio. This single value exists in Sharpe's Single Index Model where it is found that the desirability of any security is directly related to

its 'excess return to beta ratio' (i.e.  $\frac{R_i - r_f}{\beta_i}$ ). This means that the desirability of any security

is solely a function of its excess return-to-beta ratio (Fischer & Jordan, 1995). If securities are ranked on the basis of excess return-to-beta ratio (from highest to lowest), the ranking represents the desirability of including a security in the portfolio. The selection of securities to construct an optimal portfolio depends on a unique cut off rate,  $C^*$  such that all the

securities having  $\frac{R_i - r_f}{\beta_i} \geq C^*$  will be included and at the same time all the securities having

$\frac{R_i - r_f}{\beta_i} < C^*$  will be excluded. Thus, to determine how and in what way judicious inclusion

of securities is made in the construction of the optimal portfolio by using Sharpe's Single Index Model, the following steps are necessary:

1. Calculation of 'Excess Return-to-Beta ratio for each and every security under consideration.
2. Ranking of securities from highest 'Excess Return to Beta'  $\left( i.e. \frac{R_i - r_f}{\beta_i} \right)$  to lowest.

3. Calculation of values of a variable  $C_i$  for each and every security by using following formula:

$$C_i = \frac{\sigma_m^2 \sum_{i=1}^n \frac{(R_i - r_f) \beta_i}{\sigma_{ei}^2}}{1 + \sigma_m^2 \sum_{i=1}^n \frac{\beta_i^2}{\sigma_{ei}^2}}$$

4. Finding out the optimum  $C_i$  (i.e.  $C^*$ ) upto which all the securities under review have excess return to beta above  $C^* \left( i.e. \frac{R_i - r_f}{\beta_i} \geq C^* \right)$  and after that point all the securities have excess return to beta below  $C^* \left( i.e. \frac{R_i - r_f}{\beta_i} < C^* \right)$ .
5. Inclusion of securities having  $\frac{R_i - r_f}{\beta_i} \geq C^*$  in the optimal portfolio.
6. Arriving at the optimal portfolio by computing the proportion of investment in each security included in the portfolio.

The proportion of investment in each of such security is given by :

$$W_i = \frac{Z_i}{\sum_{i=1}^K Z_i}, \text{ where } Z_i = \frac{\beta_i^2}{\sigma_{ei}^2} \left( \frac{R_i - r_f}{\beta_i} - C^* \right) \quad i = 1, 2, \dots, K \text{ out of } n$$

securities under review. It is to be noted that to determine how much to invest in each security the residual variance on each security,  $\sigma_{ei}^2$  has a great role of play here.

### Analysis of Data

To construct the optimal portfolio using Sharpe's single index model various statistical measures have been made on the basis of data relating to daily security prices along with daily market indices under study. Specifically, various statistics such as mean daily return ( $R_i$ ), variance ( $\sigma_i^2$ ) and standard deviation ( $\sigma_i$ ) of daily return, standard deviation of market returns ( $\sigma_m$ ), covariance ( $\sigma_{im}$ ) and correlation ( $r_{im}$ ) between daily security returns and the daily market returns, beta ( $\beta_i$ ), systematic risk ( $\beta_i^2 \sigma_m^2$ ) and unsystematic risk ( $\sigma_{ei}^2$ ) of all the twenty one sampled securities have been calculated on the basis of daily share prices along with the market indices over a period of ten years (i.e. from April 2001 to March 2011). All these statistics as data input have been arranged in Table-1.

From Table-1 it is found that two securities viz. MTNL and DLF have contributed negative returns which could be due to some macroeconomic events in the secondary market. From Table-1 it is also seen that out of twenty one sampled securities only seven securities (viz. BHEL, DLF, Reliance India, HindalCo, ICICI Bank, SBI, Canara Bank) bearing beta

greater than one, have contributed 0.0445% to 0.1725% mean daily return except DLF (-0.0049% return). These seven securities are called the aggressive securities according to their beta values, greater than one. The rest of the securities having beta less than one are called defensive securities. All defensive securities have contributed positive mean daily return, ranging from 0.0402% to 0.1872%, with the exception of MTNL (having return - 0.0197%) and SAIL (having negative beta, -1.3134, with positive return 0.1905%). The systematic ( $\beta_i^2 \sigma_m^2$ ) risk of all the securities ranges from 1.6525% to 4.8112% and the domain of unsystematic risk ( $\sigma_{ei}^2$ ) component of such securities is 2.0813% to 11.7754%. The total risk ( $\sigma_i^2$ ) of all these securities ranges from 4.0188% to 16.8960%. The correlation coefficient of securities with the market ranges from 0.2774 to 0.7360.

As a criterion for the selection of securities, the Sharpe's Single Index Model proposes that the stocks having negative return are to be ignored. Actually, determining which of these securities should be selected in the portfolio depends on the ranking of securities (from highest to lowest) based on excess return to beta ratio  $\left( \frac{R_i - r_f}{\beta_i} \right)$ . The rule of ranking, applied here, states that the securities having highest '*excess return to beta ratio*' would be placed first, then comes the securities having second highest '*excess return to beta ratio*' and so on and so forth.

From Table-2 it is clearly seen that the security of Airtel India, having the highest *excess return to beta ratio* (0.37138904), occupies the first place. The security of Allahabad Bank, having the second highest excess return to beta ratio (0.18050778), occupies the second place. In this way Canara Bank, is third and BPCL fourth and so on. The security of Infosys Tech, having excess return to beta ratio (0.01932743), occupies the last place (18<sup>th</sup>) within the data set.

**Table-1 : Data Need to Construct Optimal Portfolio Using Sharpe's Single Index Model  $\sigma_m = 1.67$**

Sl. No.	Company Security $i$	Mean Daily Return ( $R_i$ )	Variance ( $\sigma_i^2$ )	Standard Deviation ( $\sigma_i$ )	Covariance ( $\sigma_{ijm}$ ) with Market	Correlation with the Market $r_{im}$	Beta ( $\beta_i$ )	$\beta_i \times \sigma_m$	(Systematic risk) <sup>2</sup> $\beta_i^2 \sigma_m^2$	Unsystematic Risk ( $\sigma_{ei}^2$ )
1.	ONGC	0.09354146	8.62416090	2.93669217	0.00023736	0.48387015	0.85067810	1.42063242	2.01819649	6.60596441
2.	BPCL	0.10146048	7.86666643	2.80475782	0.00017244	0.36806358	0.61801100	1.03207837	1.0651857	6.80148073
3.	SAIL	0.19059022	12.72599158	3.56735078	-0.00036954	0.61757295	-1.31344593	-2.1934547	4.81124353	7.91474805
4.	BHEL	0.14064942	7.92329822	2.81483538	0.00028252	0.60086421	1.01252867	1.69092288	2.85922018	5.06407804
5.	NALCO	0.10046810	11.83528449	3.44024483	0.00024778	0.43116904	0.88800397	1.48296663	2.19919002	9.63609447
6.	NTPC	0.07937121	4.59124749	2.14271965	0.00024799	0.6448048	0.76976635	1.28550980	1.65253545	2.93871204
7.	MTNL	-0.01972479	4.27890853	2.06855228	0.00017841	0.60836555	0.88766810	1.48240573	2.19752674	2.08138179
8.	GAIL	0.12277797	7.08111014	2.66103554	0.00024316	0.54702753	0.87142443	1.45527879	2.11783638	4.96327376
9.	COAL INDIA	0.06965161	4.01883828	2.00470404	0.00006830	0.27742122	0.45286009	0.75627635	0.57195392	3.44688436
10.	ENGINEERS INDIA LTD.	0.11720708	13.63777682	3.69293607	0.00022860	0.37010238	0.81716231	1.36466105	1.86229980	11.77547702
11.	ALLAHABAD BANK	0.1872439	8.12317261	2.8501180	0.0002693	0.5510804	0.9158945	1.52954381	2.33950428	5.78366833
12.	AMBUJA CEMENT	0.07048470	8.44529050	2.9060782	0.0002139	0.4405523	0.7664482	1.27996849	1.63831934	6.80697116
13.	DLF	-0.0049767	16.89600801	4.1104754	0.0006214	0.7359169	1.4725510	2.45916017	6.04746874	10.84853927
14.	INFOSYS TECH	0.04019130	9.87370678	3.1422455	0.0002638	0.5026080	0.9454694	1.57893389	2.49303225	7.38067453
15.	RELIANCE INDIA	0.0808451	6.89289948	2.6254332	0.0002844	0.6486778	1.0197465	1.70297665	2.90012948	3.9927700
16.	AIRTEL INDIA	0.1378261	8.35084925	2.8897836	0.0002236	0.2890469	0.3120940	0.52119698	0.27164629	8.07920296
17.	HINDALCO	0.0445422	10.71122376	3.2728006	0.0002805	0.5130665	1.0052432	1.67875614	2.8182222	7.89300156
18.	ICICI BANK	0.1240136	9.13774787	3.0228708	0.0003536	0.7003472	1.2673921	2.11654480	4.47976192	4.65798595
19.	SBI	0.1348221	6.10157828	2.4701373	0.0002960	0.7174836	1.0609895	1.77185246	3.13946115	2.96211713
20.	UCO BANK	0.1411155	9.11336355	3.0188348	0.0003074	0.5754004	0.9815735	1.63922774	2.68706759	6.42629596
21.	CANARA BANK	0.1725621	9.58923626	3.0966492	0.0003314	0.5916341	1.0128097	1.69139220	2.8608075	6.72842876

Compilation is based on:

- i) Data : Daily share prices of sampled securities and daily market index.
- ii) Period : 1<sup>st</sup> April, 2001 – 31<sup>st</sup> March, 2011
- iii) Data Source : [www.bseindia.com](http://www.bseindia.com) and [www.riskcontrol.com](http://www.riskcontrol.com).

**Table-2 : Ranking of Securities on the Basis of Excess Return to Beta Value where  $r_f = 8\%$  p.a. = 0.02192% per day is taken**

Sl. No.	Company Security i having Positive Mean Return and Positive Beta Value	Mean Daily Return ( $R_i$ )	Excess of Mean Daily Return over Risk-free Rate (daily) ( $R_i-r_f$ )	Beta Value ( $\beta_i$ )	Excess Return to Beta $\left(\frac{R_i-r_f}{\beta_i}\right)$	Rank according to Highest to Lowest $\left(\frac{R_i-r_f}{\beta_i}\right)$
1.	ONGC	0.09354146	0.07162365	0.85067810	0.08419595	11
2.	BPCL	0.10146048	0.07954267	0.61801100	0.12870753	4
3.	BHEL	0.14064942	0.11873161	1.01252867	0.11726247	6
4.	NALCO	0.10046810	0.02191781	0.88800397	0.02468211	16
5.	NTPC	0.07937121	0.0574534	0.76976635	0.07463745	13
6.	GAIL	0.12277797	0.10086016	0.87142443	0.11574172	8
7.	COAL INDIA	0.06965161	0.0477338	0.45286009	0.10540518	10
8.	ENGINEERS INDIA LTD.	0.11720708	0.09528927	0.81716231	0.11660997	7
9.	ALLAHABAD BANK	0.1872439	0.16532609	0.9158945	0.18050778	2
10.	AMBUJA CEMENT	0.0704847	0.04856689	0.7664482	0.0633662	14
11.	INFOSYS TECH	0.0401913	0.01827349	0.9454694	0.01932743	18
12.	RELIANCE INDIA	0.0808451	0.05892729	1.0197465	0.05778621	15
13.	AIRTEL INDIA	0.1378261	0.11590829	0.3120940	0.37138904	1
14.	HINDALCO	0.0445422	0.02262439	1.0052432	0.02250638	17
15.	ICICI BANK	0.1240136	0.10209579	1.2673921	0.08055580	12
16.	SBI	0.1348221	0.11290429	1.0609895	0.10641414	9
17.	UCO BANK	0.1411155	0.11919769	0.9815735	0.12143532	5
18.	CANARA BANK	0.1725621	0.15064429	1.0128097	0.14873899	3

Compilation is based on:

- i) Data : Daily share prices of sampled securities and daily market index.
- ii) Period : 1<sup>st</sup> April, 2001 – 31<sup>st</sup> March, 2011
- iii) Data Source : [www.bseindia.com](http://www.bseindia.com) and [www.riskcontrol.com](http://www.riskcontrol.com).

Now it is necessary to determine the securities for which *excess return to beta ratio* is greater than a particular cut off value  $C^*$  of the variable  $C_i$ . In Table-3 securities are arranged according to their rank and finally value of  $C_i$  for each of such securities are

computed using the formula of  $C_i$  given as : 
$$C_i = \frac{\sigma_m^2 \sum_{i=1}^n \frac{(R_i - r_f) \beta_i}{\sigma_{ei}^2}}{1 + \sigma_m^2 \sum_{i=1}^n \frac{\beta_i^2}{\sigma_{ei}^2}}, \text{ where}$$

$\sigma_m^2$  indicates the market index and  $\sigma_{ei}^2$  refers to the variance of a security’s movement that is not associated with the fluctuation of the market index.

**Table-3 : Calculations for Determining the Cut off Rate, C where  $\sigma_m = 1.67$ ,  $r_f = 8\%$  p.a. = 0.02192% per day.**

Rank on the Basis of Excess Return to Beta	Company Security i according to Rank	$\left(\frac{R_i - r_f}{\beta_i}\right)$	$\frac{(R_i - r_f)\beta_i}{\sigma_{ei}^2}$	$\frac{\beta_i^2}{\sigma_{ei}^2}$	$\sum_{i=1}^n \frac{(R_i - r_f)\beta_i}{\sigma_{ei}^2}$	$\sum_{i=1}^n \frac{\beta_i^2}{\sigma_{ei}^2}$	$C_i = \frac{\sigma_m^2 \sum_{i=1}^n \frac{(R_i - r_f)\beta_i}{\sigma_{ei}^2}}{1 + \sigma_m^2 \sum_{i=1}^n \frac{\beta_i^2}{\sigma_{ei}^2}}$
1.	AIRTEL INDIA	0.37138904	0.00447745	0.01205597	0.00447745	0.01205597	0.01208096
2.	ALLAHABAD BANK	0.18050778	0.02618083	0.14503991	0.03065828	0.15709588	0.05945442
3.	CANARA BANK	0.148738988	0.02267602	0.15245513	0.0533343	0.30955101	0.07982798
4.	BPCL	0.12870753	0.00722758	0.05615506	0.06056188	0.36570607	0.08361778
5.	UCO BANK	0.12143532	0.01820664	0.14992875	0.07876852	0.051563482	0.09010363
6.	BHEL	0.117262467	0.02373959	0.20244836	0.10250811	0.71808318	0.09521047
7.	ENGINEERS INDIA LTD.	0.116609967	0.00661262	0.05670719	0.10912073	0.77479037	0.09628118
8.	GAIL	0.11574172	0.01770847	0.15299993	0.1268292	0.9277903	0.09859583
9.	SBI	0.10641414	0.04044076	0.38003180	0.16726996	1.3078221	0.10037885
10.	COAL INDIA	0.10540518	0.00627138	0.05949786	0.17354134	1.36731996	0.10055213
11.	ONGC	0.08419595	0.00922328	0.10954543	0.18276462	1.47686539	0.09957593
12.	ICICI BANK	0.0805558	0.02777926	0.34484491	0.21054388	1.8217103	0.09656759
13.	NTPC	0.07463745	0.01504934	0.20163263	0.22559322	2.02334293	0.09471117
14.	AMBUJA CEMENT	0.0633662	0.00546851	0.08630018	0.23106173	2.10964311	0.09361519
15.	RELIANCE INDIA	0.05778621	0.01504993	0.26044147	0.24611166	2.37008458	0.09019543
16.	NALCO	0.02468211	0.00201981	0.08183305	0.24813147	2.45191763	0.08828787
17.	HINDALCO	0.02250638	0.00288141	0.12802656	0.25101288	2.57994419	0.08542187
18.	INFOSYS	0.019327426	0.00234084	0.12111526	0.253335372	2.70105945	0.08280551

Compilation is based on:

- i) Data : Daily share prices of sampled securities and daily market index.
- ii) Period : 1<sup>st</sup> April, 2001 – 31<sup>st</sup> March, 2011
- iii) Data Source : [www.bseindia.com](http://www.bseindia.com) and [www.riskcontrol.com](http://www.riskcontrol.com).

From Table-3 it is seen that  $\frac{R_i - r_f}{\beta_i}$  values of the first ten securities exceed the  $C_i$  values of the respective securities. The  $C_i$  value of the 10<sup>th</sup> security (COAL INDIA) would be the cut off value  $C^* = C_{10} (= 0.10055213)$  below which ‘excess return to beta ratio’ is less than the respective  $C_i$  value of the security  $\left(\frac{R_i - r_f}{\beta_i} < C_i\right)$  and the collection of these top ten securities, having  $\frac{R_i - r_f}{\beta_i} \geq C^*$ , make it to the optimal portfolio.

After identifying the composition of the optimal portfolio, the next step is to determine the proportion of investment (i.e. weights) in each of the securities in the optimal portfolio as shown in Table-4, using the formula of  $W_i$  cited earlier.

**Table-4 : Calculation of Zi and Wi for the Selected Securities in the Optimal Portfolio**

Selected Securities in the Optimal Portfolio	$\frac{R_i - r_f}{\beta_i}$	$\frac{\beta_i^2}{\sigma_{ei}^2}$	$Z_i = \frac{\beta_i^2}{\sigma_{ei}^2} \left( \frac{R_i - r_f}{\beta_i} - C^* \right)$	Percentage of Investment of each Selected Security $W_i = \frac{Z_i}{\sum Z_i} \times 100$	Approximate Value of $W_i$ (%)
AIRTEL INDIA	0.37138904	0.01205597	0.00326520	9.05	9
ALLAHABAD BANK	0.18050778	0.14503991	0.01159676	32.16	32
CANARA BANK	0.148738988	0.15245513	0.00734633	20.38	20
BPCL	0.12870753	0.05615506	0.00158107	4.38	4
UCO BANK	0.12143532	0.14992875	0.00313099	8.68	9
BHEL	0.117262467	0.20244836	0.00338298	9.38	9
ENGINEERS INDIA LTD.	0.116609967	0.05670719	0.00091059	2.54	3
GAIL	0.11574172	0.15299993	0.00232401	6.45	7
SBI	0.10641414	0.38003180	0.00222775	6.18	6
COAL INDIA	0.10540518	0.05949786	0.00028875	0.80	1
TOTAL	--	--	$\sum Z_i = 0.03605443$	$\sum W_i = 100$	100

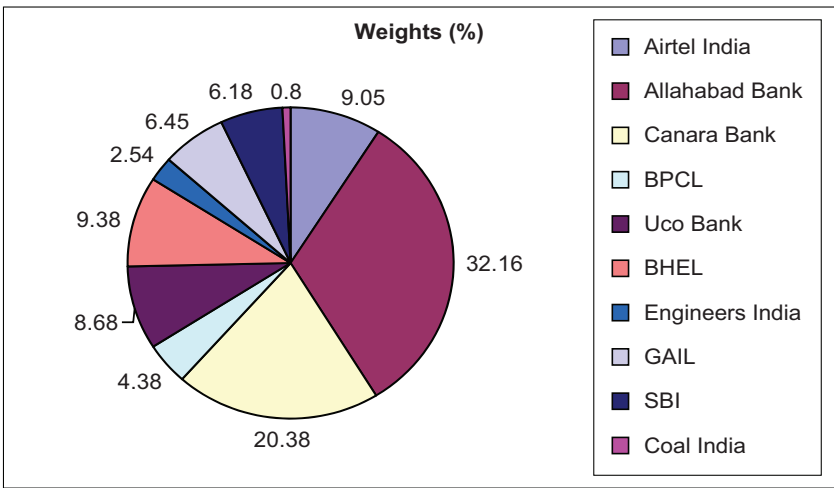
Compilation is based on:

- i) Data : Daily share prices of sampled securities and daily market index.
- ii) Period : 1<sup>st</sup> April, 2001 – 31<sup>st</sup> March, 2011
- iii) Data Source : [www.bseindia.com](http://www.bseindia.com) and [www.riskcontrol.com](http://www.riskcontrol.com).

From Table-4 it is seen that weight  $W_i$  for the selected securities in the optimal portfolio of stocks viz Airtel India, Allahabad Bank, Canara Bank, BPCL, Uco Bank, BHEL, Engineers India, GAIL, SBI, Coal India are 9.05%, 32.16%, 20.38%, 4.38%, 8.68%, 9.38%, 2.54%, 6.45%, 6.18% and 0.80% respectively.

This proportion of stocks in the composition of optimal portfolio can be shown in the following Pie diagram (Figure 3):

**Figure 3 : Stock Composition in the Optimal Portfolio (Constructed)**





Now the portfolio return, portfolio beta and risk components of the optimal portfolio constructed above can be computed on the basis of the compiled data shown in Table-5 below:

**Table-5 : Calculation for Computing Portfolio Return ( $R_p$ ) and Portfolio Risk ( $\beta_p$ )**

Selected securities in the optimal portfolio	Mean Daily Return $R_i$	Standard Deviation of Returns $\sigma_i$	Beta Value $\beta_i$	Proportion of Investment $W_i$	$\sigma_{ei}$	$W_i R_i$	$W_i \beta_i$	$W_i \sigma_{ei}$
Airtel India	0.1378261	2.8897836	0.3120940	0.09	2.8424	0.0124	0.0281	0.2558
Allahabad Bank	0.1872439	2.8501180	0.9158945	0.32	2.4049	0.0599	0.2931	0.7696
Canara Bank	0.1725621	3.0966492	1.0128097	0.20	2.5928	0.0345	0.2026	0.5186
BPCL	0.10146048	2.80475782	0.61801100	0.04	2.6080	0.0041	0.0247	0.1043
UCO Bank	0.1411155	3.0188348	0.9815735	0.09	2.5350	0.0127	0.0883	0.2282
BHEL	0.14064942	2.81483538	1.01252867	0.09	2.2504	0.0126	0.0911	0.2025
Engineers India Ltd.	0.11720708	3.69293607	0.81716231	0.03	3.4315	0.0035	0.0245	0.1029
GAIL	0.12277797	2.66103554	0.87142443	0.07	2.2278	0.0086	0.0610	0.1559
SBI	0.1348221	2.4701373	1.0609895	0.06	1.7211	0.0081	0.0637	0.1033
Coal India	0.06965161	2.00470404	0.45286009	0.01	1.8566	0.0007	0.0045	0.0186
TOTAL	--	--	--	$\sum W_i = 1.00$		$\sum W_i R_i = 0.1571$	$\sum W_i \beta_i = 0.8816$	$\sum W_i \sigma_{ei} = 2.4597$

Compilation is based on:

- i) Data : Daily share prices of sampled securities and daily market index,
- ii) Period : 1<sup>st</sup> April, 2001 – 31<sup>st</sup> March, 2011,
- iii) Data Source : [www.bseindia.com](http://www.bseindia.com) and [www.riskcontrol.com](http://www.riskcontrol.com).

On the basis of information arranged in Table-5 the following results can be extracted:

i) Portfolio Return ( $R_p$ ) =  $\sum W_i R_i = 0.1571\%$  per day  
 $= 4.713\%$  per month  
 $= 0.1571 \times 365$  per annum  
 $= 57.34\%$  per annum

ii) Portfolio beta,  $\beta_p = \sum W_i \beta_i = 0.8816$ , which is less than one indicating the defensive nature of the portfolio.

iii) Systematic risk of the portfolio  $(\beta_p^2 \times \sigma_m^2) = (0.8816)^2 \times (1.67)^2 = 2.1676\%$  (approx), which comes from economy-wide factors.

iv) Unsystematic risk of the portfolio  $(\sigma_{ep}^2) = (\sum W_i \sigma_{ei})^2 = 6.0501\%$ , which comes from firm-specific factors i.e. the internal environmental factors.

v) Total risk of the portfolio  $(\sigma_p^2) = 2.1676 + 6.0501 = 8.2177\%$  (in terms of variance)

Or

Total risk of the portfolio  $(\sigma_p) = \sqrt{8.2177} = 2.87\%$  (in terms of S.D.)

From the above results it is seen that the portfolio return is higher than the average returns of the individual stocks in the optimal portfolio with the exception of Allahabad Bank and Canara Bank. The beta value of the optimal portfolio is less than one which indicates that the returns from the portfolio fluctuates at a slower rate than that of the market index. The unsystematic risk (firm specific) of the optimal portfolio is 6.05% in terms of variance which is much higher than that of the systematic risk (2.1676%), of the portfolio. The total risk of the portfolio (2.87%, in term of SD) is less than that of the securities in the portfolio with the exception of Airtel India, Canara Bank, UCO Bank & Engineers Ltd.

According to Markowitz's Mean-Variance Model, portfolio risk (in terms of variance) is given by:

$$\sigma_p^2 = \sum_{i=1}^n W_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1}^n W_i W_j \sigma_{ij} \dots\dots\dots (16)$$

$i = j \qquad i \neq j$

In terms of S.D. it is as under:

$$\sigma_p = \sqrt{\sum_{i=1}^n W_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1}^n W_i W_j \sigma_{ij}} \dots\dots\dots (16A) \quad [\text{Symbols have their usual meanings.}]$$

$i = j \qquad i \neq j$

Using the inputs from SIM, the covariance variance matrix is shown in Table-6 below. On the basis of information compiled in Table-6 the risk of the 10-security portfolio is calculated to be:

$$\sigma_p^2 = 1.503 + 1.694 = 3.19\%$$

$$\text{or } \sigma_p = \sqrt{3.197\%} = 1.79\%$$

Therefore, it is found that there is a significant difference between the total risk of the optimal portfolio calculated under two different mechanisms found in SIM and Markowitz's model. The total risk of the optimal portfolio is 2.87% (in terms of SD) under SIM and the total risk of the portfolio is found to be 1.79% (in terms of SD) in Markowitz's model taking the necessary input from SIM.

## Findings

i) It is observed that as compared to the Markowitz's Mean-Variance Model, the Sharpe's Single Index model gives an easy mechanism of constructing an optimal portfolio of stocks for a rational investor by analyzing the reason behind the inclusion of securities in the portfolio with their respective weights. Actually, it simplifies the portfolio problems found in the Markowitz's model to a great extent.

ii) So far as the construction of optimal portfolio is concerned, there is a considerable similarity between SIM and the Markowitz's model though, in reality, SIM requires lesser input than the input requirement of Markowitz's model to arrive at the risk and return of the optimal portfolio. From the study, it is observed that only ten securities out of twenty one sampled securities are allowed to be included in the optimal portfolio using the steps behind its construction under SIM. To arrive at the risk and return of this portfolio, the number of inputs required in SIM is 32 (applying  $3n+2$ ) whereas the same is 65 (applying  $\frac{n(n+3)}{2}$ ) in Markowitz's model. Therefore, SIM, obviously, reduces the burden of calculation under Markowitz's model and claims an extra credit in the field of investment finance.

iii) There is a significant difference between the total risk of the optimal portfolio calculated under two different mechanisms found in SIM and Markowitz's model respectively. It is observed that the total risk of the optimal portfolio is 2.87% (in terms of SD) under SIM whereas the same is found to be 1.79% in Markowitz's model taking the necessary input from SIM.

### ***Concluding Remarks***

From the discussion and analysis so far it is clear that the construction of optimal portfolio investment by using Sharpe's Single Index Model is easier and more comfortable than by using Markowitz's Mean-Variance Model. In his seminal contribution Sharpe argued that there is a considerable similarity between efficient portfolios generated by SIM and Markowitz's Model. This model can show how risky a security is, if the security is held in a well-diversified portfolio. This study is made on the basis of small sample ( $n<30$ ) i.e. 21 sampled securities. It can be extended to a large sample to get a more accurate result. Hope this study will contribute a little about a lot in the field of investment finance.

Table-6 Variance – Co-variance Matrix (10×10 order)

	1	2	3	4	5	6	7	8	9	10
1	$W_1^2\sigma_1^2$ 0.068									
2	$W_1W_2$ Cov(1,2) 1.023	$W_2^2\sigma_2^2$ 0.830								
3	$W_1W_3$ Cov(1,3) 0.016	$W_2W_3$ Cov(2,3) 0.166	$W_3^2\sigma_3^2$ 0.384							
4	$W_1W_4$ Cov(1,4) 0.002	$W_2W_4$ Cov(2,4) 0.020	$W_3W_4$ Cov(3,4) 0.014	$W_4^2\sigma_4^2$ 0.013						
5	$W_1W_5$ Cov(1,5) 0.007	$W_2W_5$ Cov(2,5) 0.072	$W_3W_5$ Cov(3,5) 0.050	$W_4W_5$ Cov(4,5) 0.006	$W_5^2\sigma_5^2$ 0.074					
6	$W_1W_6$ Cov(1,6) 0.007	$W_2W_6$ Cov(2,6) 0.053	$W_3W_6$ Cov(3,6) 0.051	$W_4W_6$ Cov(4,6) 0.006	$W_5W_6$ Cov(5,6) 0.02	$W_6^2\sigma_6^2$ 0.064				
7	$W_1W_7$ Cov(1,7) 0.002	$W_2W_7$ Cov(2,7) 0.020	$W_3W_7$ Cov(3,7) 0.014	$W_4W_7$ Cov(4,7) 0.002	$W_5W_7$ Cov(5,7) 0.006	$W_6W_7$ Cov(6,7) 0.006	$W_7^2\sigma_7^2$ 0.012			
8	$W_1W_8$ Cov(1,8) 0.005	$W_2W_8$ Cov(2,8) 0.050	$W_3W_8$ Cov(3,8) 0.034	$W_4W_8$ Cov(4,8) 0.004	$W_5W_8$ Cov(5,8) 0.015	$W_6W_8$ Cov(6,8) 0.016	$W_7W_8$ Cov(7,8) 0.004	$W_8^2\sigma_8^2$ 0.035		
9	$W_1W_9$ Cov(1,9) 0.005	$W_2W_9$ Cov(2,9) 0.052	$W_3W_9$ Cov(3,9) 0.036	$W_4W_9$ Cov(4,9) 0.004	$W_5W_9$ Cov(5,9) 0.016	$W_6W_9$ Cov(6,9) 0.016	$W_7W_9$ Cov(7,9) 0.004	$W_8W_9$ Cov(8,9) 0.011	$W_9^2\sigma_9^2$ 0.022	
10	$W_1W_{10}$ Cov(1,10) 0.0004	$W_2W_{10}$ Cov(2,10) 0.004	$W_3W_{10}$ Cov(3,10) 0.001	$W_4W_{10}$ Cov(4,10) 0.0003	$W_5W_{10}$ Cov(5,10) 0.001	$W_6W_{10}$ Cov(6,10) 0.001	$W_7W_{10}$ Cov(7,10) 0.0003	$W_8W_{10}$ Cov(8,10) 0.001	$W_9W_{10}$ Cov(9,10) 0.001	$W_{10}^2\sigma_{10}^2$ 0.001
Total	0.0674	0.437	0.200	0.0223	0.060	0.039	0.0083	0.012	0.001	0.847

N.B. – 1. Airtel India, 2. Allahabad Bank, 3. Canara Bank, 4. BPCL, 5. UCO Bank, 6. BHEL, 7. Engineers India Ltd., 8. GAIL, 9. SBI, 10. Coal India.

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