Quantimanina Assigement 1

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1 Question

1.1

We consider to have $\{|K\rangle\}$ set of vectors as an orthonormal basis,

Now we know we can write $|a\rangle$ and $|b\rangle$ as,

$$|a\rangle = \Sigma_k a_k |K\rangle$$

$$|b\rangle = \Sigma_k b_k |K\rangle$$

Now we have from the definition from the book that the inner product of say $|a\rangle$ and $|b\rangle$ as,

$$\langle a|b\rangle = (\Sigma_k a_k |K\rangle, \Sigma_k b_k |K\rangle = \Sigma_k a_k^* b_k \delta_{kk} = \langle a|b\rangle = \Sigma_k \langle a|K\rangle\langle K|b\rangle$$

and as we have i = j that we consider in the formula thus $\delta_{kk} = 1$ and we will also only consider this case as other will automatically go to equate to 0(in the case of orthogonal states).

Now as we have considered $\{|K\rangle\}$ as orthonormal basis we can write, $\langle k|a\rangle = a_k$ therefore with the help of the above equations we can write,

$$(\Sigma_k |k\rangle\langle k||a\rangle\ ) = \Sigma_k |k\rangle\langle k|a\rangle = \Sigma_k a_k |k\rangle = |a\rangle$$

As the above equation is true for all $|a\rangle$ we have,

$$\Sigma_k \langle k|k\rangle = I$$

This gives us the completion theorem and proves that the condition $\langle a|b\rangle = \Sigma_k \langle a|K\rangle\langle K|b\rangle$ is enough to prove that the given set of vectors $\{|K\rangle\}$ are orthonormal basis.

1.2

We have been given a relation such as $e^A = B$ where A and B are any matrix and to prove that the log is a non unique fucntion we can use the preiodicity of complex exponential functions i.e.

$$e^x = e^{x+i2\pi ny}$$

where y could be any other matrix. So if we use this equation here in our given equation we will get,

$$e^A = e^(A + i2\pi nX) = B$$

So here we can take X to be any other matrix and thus we can see that we have more than one matrix A that could satisfy the condition given to us.

1.3

We can use the concept of tensor product for finding the eignenvalues of the matrices,

1.

$$\begin{bmatrix} 0 & 2 & 0 & 3 \\ 2 & 0 & 3 & 0 \\ 0 & 1 & 0 & 4 \\ 1 & 0 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Thus from the above tensor product will be get the eigenvalue of the matrices as 1, 5, -1, -5.

2.

$$\begin{bmatrix} 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 4 \\ 2 & 3 & 0 & 0 \\ 1 & 4 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

This will give us the eigenvalues as 1, -1, 5, -5.

3.

$$\begin{bmatrix} 4 & 6 & 6 & 9 \\ 2 & 8 & 3 & 12 \\ 2 & 3 & 8 & 12 \\ 1 & 4 & 4 & 16 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \otimes \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

This will give us the eigenvalues as 1, 5, 5, 25.

2 Question

2.1

The starting state that we have been given $|\Psi\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$ and now using the postulate 2 we can write that $|\Psi^{'}\rangle = U|\Psi\rangle$ where U is any unitary operator which in this we will take the Hadamard operator given to us as,

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

So we can operate H on $|\Psi\rangle$ to get $|\Psi^{'}\rangle$. Thus we will get,

$$|\Psi^{'}\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

and by using diagonal representation,

$$|\Psi^{'}\rangle = \frac{|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|}{\sqrt{2}} \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$|\Psi^{'}\rangle = |1\rangle$$

So we have our new state after applying the unitary transformation.

2.2

We have been given the initial state $|\Psi\rangle$ and the transformed state $|\Psi^{'}\rangle$ in the computational basis $\{|0\rangle, |1\rangle\}$

We can write the state $|\Psi\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = a|0\rangle - b|1\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|b\rangle$

We the measurement operators being $M_o = |0\rangle\langle 0|$ and $M_1 = |1\rangle\langle 1|$

Using the postulate 3 we can say that the probability distribution for the state $|\Psi\rangle$ will be as,

1. The probability of measuring $|0\rangle$ is $|\langle 0|\Psi\rangle|^2 = |a|^2 = \frac{1}{2}$

2. The probability of measuring $|1\rangle$ is $|\langle 1|\Psi\rangle|^2 = |b|^2 = \frac{1}{2}$

Now the transformed state can be obtained as,

$$\frac{M_o|\Psi\rangle}{|a|} = \frac{a}{|a|}|0\rangle$$

$$\frac{M_1|\Psi\rangle}{|b|} = \frac{b}{|b|}|1\rangle$$

$$|\Psi^{'}\rangle = 1$$

So the probability distribution for the transformed state will be,

1. The probability of measuring $|0\rangle$ is $|\langle 0|\Psi^{'}\rangle|^2 = 0$
2. The probability of measuring $|1\rangle$ is $|\langle 1|\Psi^{'}\rangle|^2 = 1$