EE5609 ASSIGNMENT 1

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Lines and Planes

The python solution code is available at

https://github.com/Shantanu2508/Matrix_Theory/blob/master/stline.py

Problem Statement

Find the equations of the lines which intercepts on the both the axes and whose sum and product are 1 and -6 respectively.

Solution

The equation of line in terms of vector notations can be written as

$$\mathbf{n}^T\mathbf{x} = c \tag{1}$$

Let the intercepts be $\begin{pmatrix} a \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ b \end{pmatrix}$, respectively.
Given that: $a + b = 1$, and $ab = -6$
The quadratic equation whose roots are the x and y intercepts can be written as :

$$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$$

$$\implies \quad x^2 - x - 6 = 0 \tag{2}$$

$$\implies \quad x = (3, -2) \quad \text{and corresponding } y \text{ intercepts are } (-2, 3)$$

The line L1 passes through $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$.

Let direction vector of this line be $\mathbf{m}$.

$$\mathbf{m} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$$

The normal vector, $\mathbf{n}$:

$$\mathbf{n} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{m} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

The equation of line in terms of normal vector and passing through a point A is

$$\mathbf{n^T}(\mathbf{x} - A) = 0 \qquad \Longrightarrow \qquad \mathbf{n^T}\mathbf{x} = \mathbf{n^T}A$$

$$\Longrightarrow \mathbf{n^T}\mathbf{x} = \begin{pmatrix} 2 & -3 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

$$\Longrightarrow \begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{x} = 6 \tag{3}$$

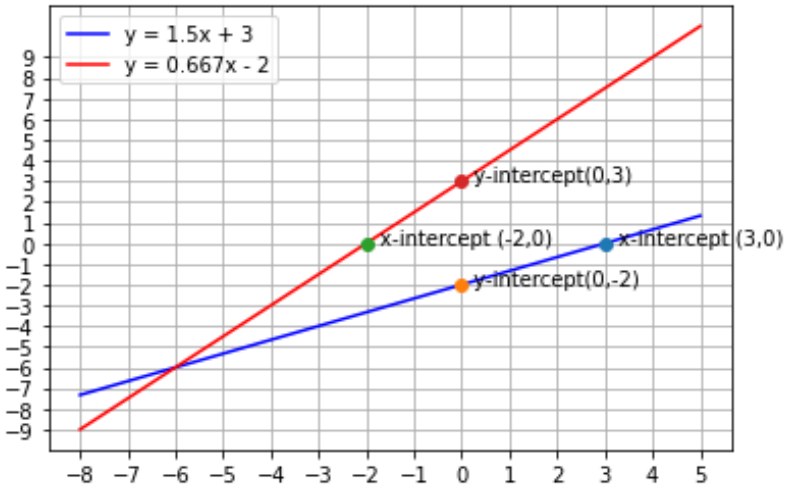

Figure 1:

Similarly, the equation of second line L2, with the x and y intercepts $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ and normal vector $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ is

$$\begin{pmatrix} -3 & 2 \end{pmatrix} \mathbf{x} = 6 \tag{4}$$

The equations of lines (3) and (4) can be represented collectively as

$$\begin{pmatrix} 2 & -3 \\ -3 & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 6 \\ 6 \end{pmatrix} \tag{5}$$

$x-$intercept	$y-$intercept	$\mathbf{n}$
$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -2 \end{pmatrix}$	$\begin{pmatrix} 2 \\ -3 \end{pmatrix}$
$\begin{pmatrix} -2 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$	$\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

Table 1: