EE5609 Assignment 17

SHANTANU YADAV, EE20MTECH12001

1 Problem

Let $\mathbf{A}$ be a $n \times n$ real matrix with $\mathbf{A}^2 = \mathbf{A}$. Then

1) the eigenvalues of $\mathbf{A}$ are either 0 or 1
2) $\mathbf{A}$ is a diagonal matrix with diagonal entries 0 or 1
3) $rank(\mathbf{A}) = trace(\mathbf{A})$
4) if $rank(\mathbf{I} - \mathbf{A}) = trace(\mathbf{I} - \mathbf{A})$

2 Explanation

Objective	**Explanation**
Eigenvalues of $\mathbf{A}$	Since $$\mathbf{A}^2 = \mathbf{A} \quad (2.0.1)$$ $$\implies \mathbf{A}^2 - \mathbf{A} = \mathbf{O} \quad (2.0.2)$$ From Cayley-Hamilton Theorem we have, $$\lambda^2 - \lambda = 0 \quad (2.0.3)$$ $$\implies \lambda(\lambda - 1) = 0 \quad (2.0.4)$$ $$\implies \lambda = 0, 1 \quad (2.0.5)$$
Relation between rank and trace of $\mathbf{A}$	A matrix $\mathbf{A}$ satisfying $\mathbf{A}^2 = \mathbf{A}$ is an idempotent matrix with eigen values equal to 0 or 1. For such a matrix $$rank(\mathbf{A}) = trace(\mathbf{A}) \quad (2.0.6)$$
Relation between rank and trace of $\mathbf{I} - \mathbf{A}$	Now for the matrix $\mathbf{I} - \mathbf{A}$ we have, $$(\mathbf{I} - \mathbf{A})^2 = (\mathbf{I} - \mathbf{A})(\mathbf{I} - \mathbf{A}) \quad (2.0.7)$$ $$= \mathbf{I}^2 - \mathbf{IA} - \mathbf{AI} + \mathbf{A}^2 \quad (2.0.8)$$ $$= \mathbf{I} - \mathbf{A} - \mathbf{A} + \mathbf{A} \quad (2.0.9)$$ $$= \mathbf{I} - \mathbf{A} \quad (2.0.10)$$ Hence $\mathbf{I} - \mathbf{A}$ is an idempotent matrix. Therefore we conclude, $$rank(\mathbf{I} - \mathbf{A}) = trace(\mathbf{I} - \mathbf{A}) \quad (2.0.11)$$

TABLE I

Thus, options (1),(3) and (4) are correct.