EE5609 Assignment 3

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The python solution code is available at

https://github.com/Shantanu2508/Matrix_Theory/blob/master/Assignment_3/assignment3.py

1 Problem

Using properties of determinants, prove that:

$$\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma) \tag{1.0.1}$$

2 Solution

Using transformations and properties of determinants:

$$\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} \tag{2.0.1}$$

$$\xleftrightarrow{C_3 \leftarrow C_1 + C_3} \begin{vmatrix} \alpha & \alpha^2 & \alpha+\beta+\gamma \\ \beta & \beta^2 & \alpha+\beta+\gamma \\ \gamma & \gamma^2 & \alpha+\beta+\gamma \end{vmatrix} \tag{2.0.2}$$

$$\implies \quad (\alpha+\beta+\gamma)\begin{vmatrix} \alpha & \alpha^2 & 1 \\ \beta & \beta^2 & 1 \\ \gamma & \gamma^2 & 1 \end{vmatrix} \tag{2.0.3}$$

$$\xleftrightarrow[R_2 \leftarrow R_2 - R_1]{R_3 \leftarrow R_3 - R_1} (\alpha+\beta+\gamma)\begin{vmatrix} \alpha & \alpha^2 & 1 \\ \beta-\alpha & \beta^2-\alpha^2 & 0 \\ \gamma-\alpha & \gamma^2-\alpha^2 & 0 \end{vmatrix} \tag{2.0.4}$$

$$\implies \quad (\beta-\alpha)(\gamma-\alpha)(\alpha+\beta+\gamma)\begin{vmatrix} \alpha & \alpha^2 & 1 \\ 1 & \beta+\alpha & 0 \\ 1 & \gamma+\alpha & 0 \end{vmatrix} \tag{2.0.5}$$

$$\implies (\beta-\alpha)(\gamma-\alpha)(\alpha+\beta+\gamma)(-1)^{1+3}\begin{vmatrix} 1 & \beta+\alpha \\ 1 & \gamma+\alpha \end{vmatrix} \tag{2.0.6}$$

$$\implies \quad (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma) \tag{2.0.7}$$