

# EE5609 Assignment 17

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## 1 PROBLEM

Let  $f : [-1, 1] \rightarrow \mathbf{R}$  be a function given by

$$f(x) = \begin{cases} x^2 \cos\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Then

- 1)  $f$  is of bounded variation on  $[-1, 1]$
- 2)  $f'$  is of bounded variation on  $[-1, 1]$
- 3)  $|f'(x)| \leq 1 \quad \forall x \in [-1, 1]$
- 4)  $|f'(x)| \leq 3 \quad \forall x \in [-1, 1]$

## 2 EXPLANATION

We know that

$$-1 \leq \cos\left(\frac{1}{x}\right) \leq 1 \quad (2.0.1)$$

$$\implies -x^2 \leq x^2 \cos\left(\frac{1}{x}\right) \leq x^2 \quad (2.0.2)$$

But since  $x \in [-1, 1]$

$$-1 \leq x^2 \cos\left(\frac{1}{x}\right) \leq 1 \quad (2.0.3)$$

Upon differentiating (2.0.2) we get

$$-2x \leq f' \leq 2x \quad (2.0.4)$$

$$\implies -2 \leq f' \leq 2 \quad (2.0.5)$$

$$\implies |f'| \leq 2 \quad \forall x \in [-1, 1] \quad (2.0.6)$$

Thus  $f$  is of bounded variation on  $[-1, 1]$ .