EE5609 Assignment 2

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The python solution code is available at

https://github.com/Shantanu2508/Matrix_Theory/blob/master/Assignment_2/assignment2.py

1 Problem

Examine the consistency of the system of the given equations.

$$\begin{aligned} 5x - y + 4z &= 5 \\ 2x + 3y + 5z &= 2 \\ 5x - 2y + 6z &= -1 \end{aligned} \tag{1.0.1}$$

2 Solution

The given equations can be written as

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

where

$$\mathbf{A} = \begin{pmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix} \tag{2.0.1}$$

By row reducing the augmented matrix :

$$\begin{pmatrix} 5 & -1 & 4 & 5 \\ 2 & 3 & 5 & 2 \\ 5 & -2 & 6 & -1 \end{pmatrix} \tag{2.0.2}$$

$$\xleftrightarrow[R_3 \leftarrow R_3 - R_1]{R_2 \leftarrow 5R_2 - (R_1 + R_3)} \begin{pmatrix} 5 & -1 & 4 & 5 \\ 0 & 18 & 15 & 6 \\ 0 & -1 & 2 & -6 \end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{R_3 \leftarrow 18R_3 + R_2} \begin{pmatrix} 5 & -1 & 4 & 5 \\ 0 & 18 & 15 & 6 \\ 0 & 0 & 51 & -102 \end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{R_3 \leftarrow \frac{R_3}{51}} \begin{pmatrix} 5 & -1 & 4 & 5 \\ 0 & 18 & 15 & 6 \\ 0 & 0 & 1 & -2 \end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 15R_3} \begin{pmatrix} 5 & -1 & 4 & 5 \\ 0 & 18 & 0 & 36 \\ 0 & 0 & 1 & -2 \end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{18}} \begin{pmatrix} 5 & -1 & 4 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{pmatrix} \tag{2.0.7}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + R_2 - 4R_3} \begin{pmatrix} 5 & 0 & 0 & 15 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{pmatrix} \tag{2.0.8}$$

$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{5}} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{pmatrix} \tag{2.0.9}$$

$$\begin{aligned} \implies rank \begin{pmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{pmatrix} &= rank \begin{pmatrix} 5 & -1 & 4 & 5 \\ 2 & 3 & 5 & 2 \\ 5 & -2 & 6 & -1 \end{pmatrix} \\ &= 3 = dim \begin{pmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{pmatrix} \end{aligned} \tag{2.0.10}$$

i.e., the $rank(\mathbf{A}) = rank(\mathbf{A} : \mathbf{b}) = 3$, which is equal to the row size of $\mathbf{x}$. Hence the system of linear equations is consistent, with a unique solution.

The unique solution is

$$\mathbf{x} = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \tag{2.0.11}$$