IIT Hyderabad

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ASSIGNMENT 1

Lines and Planes

Problem Statement

Find the equations of the lines which intercepts on the both the axes and whose sum and product are 1 and -6 respectively.

Solution

The equation of line in terms of vector notations can be written as

$$\mathbf{n}^T\mathbf{x} = b \qquad \text{where} \qquad \mathbf{n} = \begin{pmatrix} n_{11} \\ n_{12} \end{pmatrix} \tag{1}$$

or

$$\begin{pmatrix} n_{11} & n_{12} \end{pmatrix} \mathbf{x} = b \tag{2}$$

Let the intercepts be $\begin{pmatrix} a \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ b \end{pmatrix}$, respectively.

Given that: $a + b = 1$, and $ab = -6$

$$\implies b = \frac{-6}{a} \implies a^2 - a - 6 = 0 \implies (a-3)(a+2) = 0$$

$$\implies (a, b) = (3, -2) \text{ and } (-2, 3) \tag{3}$$

When the line passes through $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$, respectively, we get, upon substitution in (2):

$$3n_{11} = b \qquad \implies \qquad n_{11} = \frac{b}{3}$$

$$-2n_{12} = b \qquad \implies \qquad n_{12} = -\frac{b}{2}$$

Therefore, the equation of first line is

$$\begin{pmatrix} \frac{b}{3} & \frac{-b}{2} \end{pmatrix} \mathbf{x} = b$$

$\implies$

$$\begin{pmatrix} \frac{1}{3} & \frac{-1}{2} \end{pmatrix} \mathbf{x} = 1 \tag{4}$$

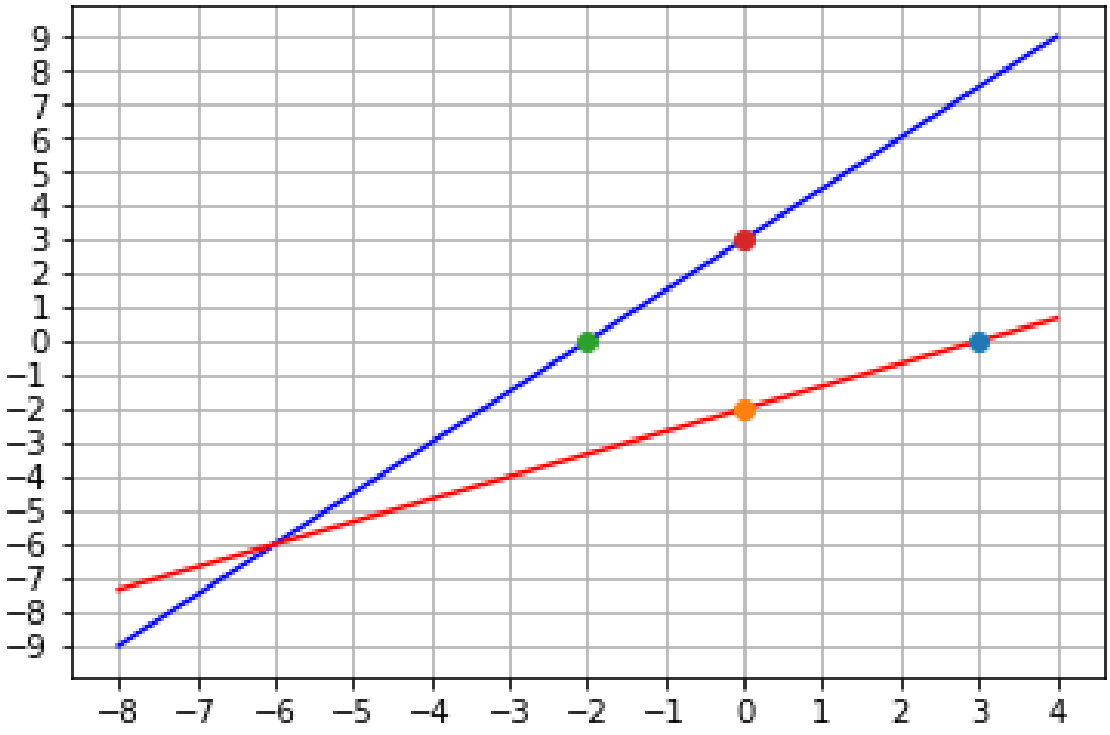

Figure 1:

Similarly, the equation of second line, which passes through $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ is

$$\begin{pmatrix} \frac{-1}{2} & \frac{1}{3} \end{pmatrix} \mathbf{x} = 1 \tag{5}$$