# EE5609 Challenge 1

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## 1 Problem Statement

Computation of Fibonacci sequence using matrix notation.

## 2 Explanation

In Fibonacci series the next element can be represented as the sum of previous two elements. Therfore the $n^{th}$ term can be written as $x_n = x_{n-1} + x_{n-2}$. This can be re-written as

$$\begin{pmatrix} x_1 \\ x_0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.1}$$

$$\begin{pmatrix} x_n \\ x_{n-1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_{n-1} \\ x_{n-2} \end{pmatrix} \tag{2.0.2}$$

The initial terms of a the fibonacci series are $x_0 = 0$ and $x_1 = 1$ respectively. Using 2.0.2 we can write

$$\begin{pmatrix} x_2 \\ x_1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.3}$$

$$\begin{pmatrix} x_3 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_2 \\ x_1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{2.0.4}$$

$$\tag{2.0.5}$$

Therefore it can be re-written as

$$\begin{pmatrix} x_3 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.6}$$

Thus using the results above it can be generalized as

$$\begin{pmatrix} x_n \\ x_{n-1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{n-1} \begin{pmatrix} x_1 \\ x_0 \end{pmatrix} \tag{2.0.7}$$