IIT Hyderabad

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Challenge 1
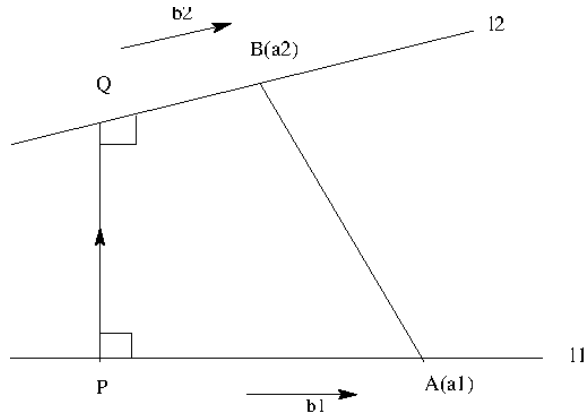
Lines and Planes

Shortest distance between two skew lines

Let the two lines are L_1 and L_2

$$L1 : \mathbf{x} = \begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \end{pmatrix} + \lambda \begin{pmatrix} b_{11} \\ b_{12} \\ b_{13} \end{pmatrix} \tag{1}$$

and

$$L2 : \mathbf{x} = \begin{pmatrix} a_{21} \\ a_{22} \\ a_{23} \end{pmatrix} + \lambda \begin{pmatrix} b_{21} \\ b_{22} \\ b_{23} \end{pmatrix} \tag{2}$$

Since P lies on L_1 and Q lies on L_2, the points should satisfy equations (1) and (2), respectively.

$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} a_{11} + \lambda b_{11} \\ a_{12} + \lambda b_{12} \\ a_{13} + \lambda b_{13} \end{pmatrix} \tag{3}$$

and

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} a_{21} + \mu b_{21} \\ a_{22} + \mu b_{22} \\ a_{23} + \mu b_{23} \end{pmatrix} \tag{4}$$

$$\mathbf{PQ} = \mathbf{Q} - \mathbf{P}$$

$$= \begin{pmatrix} a_{21} - a_{11} + \mu b_{21} - \lambda b_{11} \\ a_{22} - a_{12} + \mu b_{22} - \lambda b_{12} \\ a_{23} - a_{13} + \mu b_{23} - \lambda b_{13} \end{pmatrix} \tag{5}$$

The only unknowns are λ and μ.

Since $\mathbf{PQ}$ is perpendicular to $\mathbf{b_1}$ and $\mathbf{b_2}$:

$$\mathbf{PQ} \cdot \mathbf{b}_1 = 0 \qquad \text{and} \qquad \mathbf{PQ} \cdot \mathbf{b}_2 = 0 \tag{6}$$

these equations can be solved for λ and μ.