EE5609 Assignment 4

SHANTANU YADAV, EE20MTECH12001

1 Problem

D is a point on side BC of ΔABC such that $AD = AC$. Show that $AB > AD$.

2 Solution

Let $\mathbf{D}$ divide BC internally in ratio $1 : k$ where $0 < k < 1$. Then

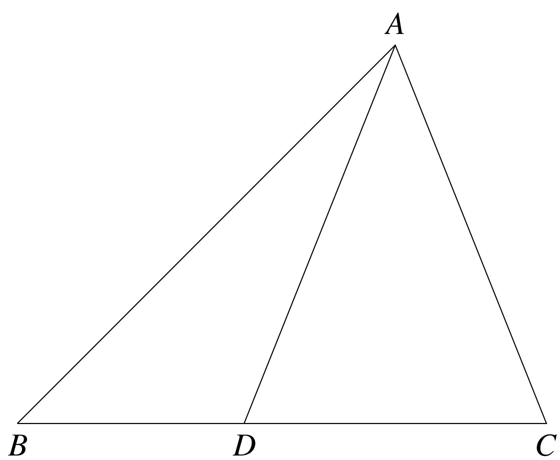

Fig. 0

$$\mathbf{D} = \frac{k\mathbf{B} + \mathbf{C}}{k + 1} \tag{2.0.1}$$

The direction vector along AD is $\mathbf{D} - \mathbf{A}$.

$$\implies \quad \mathbf{D} - \mathbf{A} = \frac{k(\mathbf{B} - \mathbf{A}) + (\mathbf{C} - \mathbf{A})}{k + 1} \tag{2.0.2}$$

$$\implies \quad \|\mathbf{D} - \mathbf{A}\|^2 = \frac{k^2 \|\mathbf{B} - \mathbf{A}\|^2 + \|\mathbf{C} - \mathbf{A}\|^2}{(k + 1)^2} \tag{2.0.3}$$

Since $\|\mathbf{D} - \mathbf{A}\| = \|\mathbf{C} - \mathbf{A}\|$, so that

$$\|\mathbf{D} - \mathbf{A}\|^2 \left\{1 - \frac{1}{(k + 1)^2}\right\} = \frac{k^2 \|\mathbf{B} - \mathbf{A}\|^2}{(k + 1)^2} \tag{2.0.4}$$

$$\implies \quad \|\mathbf{B} - \mathbf{A}\|^2 = \left\{1 + \frac{2}{k}\right\} \|\mathbf{D} - \mathbf{A}\|^2 \tag{2.0.5}$$

$$\implies \quad \|\mathbf{B} - \mathbf{A}\|^2 > \|\mathbf{D} - \mathbf{A}\|^2 \tag{2.0.6}$$

$$\implies \quad AB > AD \tag{2.0.7}$$