EE5609 Assignment 17

SHANTANU YADAV, EE20MTECH12001

1 Problem

Let m,n,r be natural numbers. Let A be a $m \times n$ matrix with real entries such that $\left(AA^T\right)^r = I$, where I is the $m \times m$ identity marix and A^T is the transpose of the matrix A. We conclude that

1) $m = n$
2) AA^T is invertible
3) A^TA is invertible
4) if $m = n$, then A is invertible.

2 Explanation

Since A is a $m \times n$ matrix, AA^T is $m \times m$ matrix. Also since,

$$\left(AA^T\right)^r = I \tag{2.0.1}$$

$$\implies \left(AA^T\right)\left(AA^T\right)^{r-1} = I \tag{2.0.2}$$

$$\implies \left(AA^T\right)^{-1} = \left(AA^T\right)^{r-1} \tag{2.0.3}$$

Hence AA^T is invertible with inverse equal to $\left(AA^T\right)^{r-1}$. Therefore option (2) is correct.