

EE5609 Assignment 2

SHANTANU YADAV, EE20MTECH12001

The python solution code is available at

https://github.com/Shantanu2508/Matrix_Theory/blob/master/Assignment%202/assignment2.py

1 PROBLEM

Examine the consistency of the system of the given equations.

$$\begin{aligned} 5x - y + 4z &= 5 \\ 2x + 3y + 5z &= 2 \\ 5x - 2y + 6z &= -1 \end{aligned} \quad (1.0.1)$$

2 SOLUTION

The given equations can be expressed as

$$\mathbf{Ax} = \mathbf{b} \quad (2.0.1)$$

where

$$\mathbf{A} = \begin{pmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix} \quad (2.0.2)$$

By row reducing the augmented matrix :

$$\left(\begin{array}{ccc|c} 5 & -1 & 4 & 5 \\ 2 & 3 & 5 & 2 \\ 5 & -2 & 6 & -1 \end{array} \right) \quad (2.0.3)$$

$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{5}} \left(\begin{array}{ccc|c} 1 & -\frac{1}{5} & \frac{4}{5} & 1 \\ 2 & 3 & 5 & 2 \\ 5 & -2 & 6 & -1 \end{array} \right) \quad (2.0.4)$$

$$\xleftrightarrow{\begin{array}{l} R_2 \leftarrow R_2 - 2R_1 \\ R_3 \leftarrow R_3 - 5R_1 \end{array}} \left(\begin{array}{ccc|c} 1 & -\frac{1}{5} & \frac{4}{5} & 1 \\ 0 & \frac{17}{5} & \frac{17}{5} & 0 \\ 0 & -1 & 2 & -6 \end{array} \right) \quad (2.0.5)$$

$$\xleftrightarrow{R_2 \leftarrow \frac{5}{17}R_2} \left(\begin{array}{ccc|c} 1 & -\frac{1}{5} & \frac{4}{5} & 1 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 2 & -6 \end{array} \right) \quad (2.0.6)$$

$$\xleftrightarrow{R_3 \leftarrow R_3 + R_2} \left(\begin{array}{ccc|c} 1 & -\frac{1}{5} & \frac{4}{5} & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 3 & -6 \end{array} \right) \quad (2.0.7)$$

$$\xleftrightarrow{R_3 \leftarrow \frac{R_3}{3}} \left(\begin{array}{ccc|c} 1 & -\frac{1}{5} & \frac{4}{5} & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -2 \end{array} \right) \quad (2.0.8)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_3} \left(\begin{array}{ccc|c} 1 & -\frac{1}{5} & \frac{4}{5} & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array} \right) \quad (2.0.9)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + \frac{1}{5}R_2 - \frac{4}{5}R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array} \right) \quad (2.0.10)$$

$$\begin{aligned} \Rightarrow \text{rank} \begin{pmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{pmatrix} &= \text{rank} \begin{pmatrix} 5 & -1 & 4 & 5 \\ 2 & 3 & 5 & 2 \\ 5 & -2 & 6 & -1 \end{pmatrix} \\ &= 3 = \dim \begin{pmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{pmatrix} \end{aligned} \quad (2.0.11)$$

i.e., the $\text{rank}(\mathbf{A}) = \text{rank}(\mathbf{A} : \mathbf{b}) = 3$. Hence the system of linear equations is consistent, with a unique solution.

The unique solution is

$$\mathbf{x} = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \quad (2.0.12)$$