EE5609 Assignment 4

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1 PROBLEM

D is a point on side BC of ΔABC such that $AD = AC$. Show that $AB > AD$.

2 SOLUTION

Let $\mathbf{D}$ divide BC internally in ratio $1 : k$ where $0 < k < 1$. Then

A

B   D   C
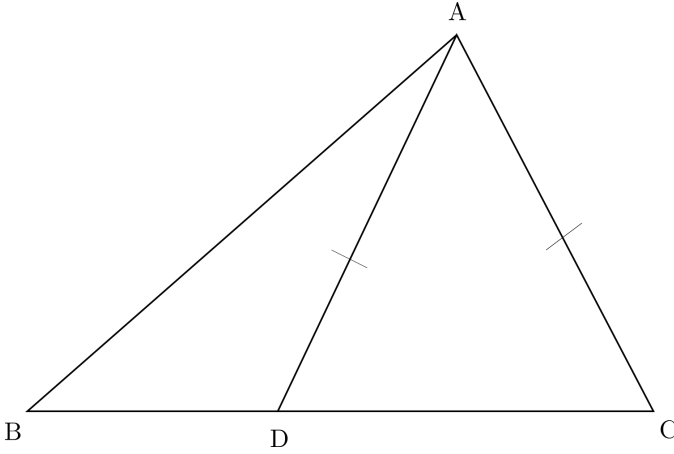
Fig. 0

$$\mathbf{D} = \frac{k\mathbf{B} + \mathbf{C}}{k + 1} \qquad (2.0.1)$$

The direction vector along AD is $\mathbf{D} - \mathbf{A}$.

$$\implies \quad \mathbf{D} - \mathbf{A} = \frac{k(\mathbf{B} - \mathbf{A}) + (\mathbf{C} - \mathbf{A})}{k + 1} \qquad (2.0.2)$$

$$\implies \quad \|\mathbf{D} - \mathbf{A}\|^2 = \frac{k^2 \|\mathbf{B} - \mathbf{A}\|^2 + \|\mathbf{C} - \mathbf{A}\|^2}{(k + 1)^2} \qquad (2.0.3)$$

Since $\|\mathbf{D} - \mathbf{A}\| = \|\mathbf{C} - \mathbf{A}\|$, so that

$$\|\mathbf{D} - \mathbf{A}\|^2 \{1 - \frac{1}{(k + 1)^2}\} = \frac{k^2 \|\mathbf{B} - \mathbf{A}\|^2}{(k + 1)^2} \qquad (2.0.4)$$

$$\implies \quad \|\mathbf{B} - \mathbf{A}\|^2 = \{1 + \frac{2}{k}\} \|\mathbf{D} - \mathbf{A}\|^2 \qquad (2.0.5)$$

$$\implies \quad \|\mathbf{B} - \mathbf{A}\|^2 > \|\mathbf{D} - \mathbf{A}\|^2 \qquad (2.0.6)$$

$$\implies \quad AB > AD \qquad (2.0.7)$$