

**IIT Hyderabad**  
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**ASSIGNMENT 1**

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**Lines and Planes**

**Problem Statement**

Find the equations of the lines which intercepts on the both the axes and whose sum and product are 1 and -6 respectively.

**Solution**

The equation of line in terms of vector notations can be written as

$$\mathbf{n}^T \mathbf{x} = b \quad \text{where} \quad \mathbf{n} = \begin{pmatrix} n_{11} \\ n_{12} \end{pmatrix}, \quad (1)$$

or

$$\begin{pmatrix} n_{11} & n_{12} \end{pmatrix} \mathbf{x} = b \quad (2)$$

Let the intercepts be  $\begin{pmatrix} a \\ 0 \end{pmatrix}$  and  $\begin{pmatrix} 0 \\ b \end{pmatrix}$ , respectively.

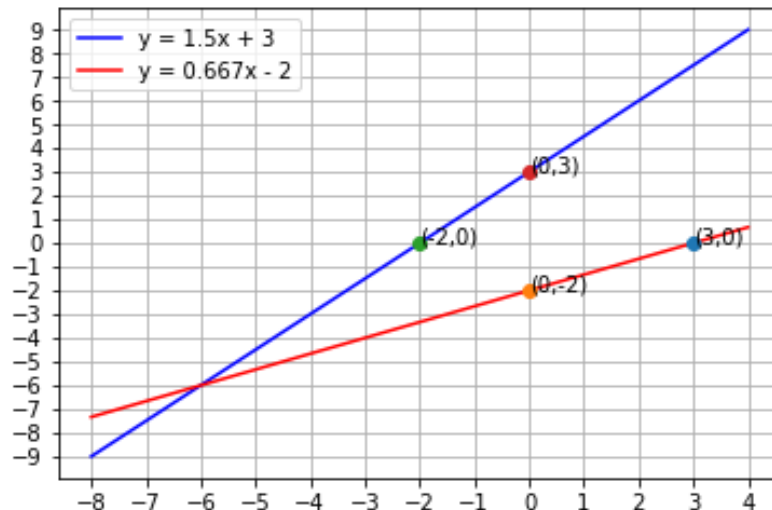


Figure 1:

Given that:  $a + b = 1$ , and  $ab = -6$

$$\begin{aligned}
\Rightarrow b = \frac{-6}{a} &\Rightarrow a^2 - a - 6 = 0 \Rightarrow (a - 3)(a + 2) = 0 \\
&\Rightarrow (a, b) = (3, -2) \text{ and } (-2, 3)
\end{aligned} \tag{3}$$

When the line passes through  $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$  and  $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ , respectively, we get, upon substitution in (2):

$$\begin{aligned}
3n_{11} = b &\Rightarrow n_{11} = \frac{b}{3} \\
-2n_{12} = b &\Rightarrow n_{12} = -\frac{b}{2}
\end{aligned}$$

Therefore, the equation of first line is

$$\left(\frac{b}{3} \quad -\frac{b}{2}\right) \mathbf{x} = b$$

$\Rightarrow$

$$\left(\frac{1}{3} \quad -\frac{1}{2}\right) \mathbf{x} = 1 \tag{4}$$

Similarly, the equation of second line, which passes through  $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$  and  $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$  is

$$\left(\frac{-1}{2} \quad \frac{1}{3}\right) \mathbf{x} = 1 \tag{5}$$