EE5609 Assignment 7

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The python solution code is available at

https://github.com/Shantanu2508/Matrix_Theory/blob/master/Assignment_6/qr.py

1 Problem

Perform QR decomposition on matrix A given by

$$A = \begin{pmatrix} 3 & -4 \\ -4 & 3 \end{pmatrix}$$

2 Explanation

Representing matrix A in terms of its column vectors as

$$A = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \tag{2.0.1}$$

Let

$$\mathbf{q_1} = \frac{\mathbf{a}}{\|\mathbf{a}\|} \tag{2.0.2}$$

An orthonormal vector to $\mathbf{q_1}$ can be obtained by subtracting the projection of $\mathbf{b}$ on $\mathbf{q_1}$ from $\mathbf{b}$. Thus

$$\mathbf{q_2} = \frac{\mathbf{b} - k\mathbf{q_1}}{\|\mathbf{b} - k\mathbf{q_1}\|} \tag{2.0.3}$$

where

$$k = \frac{\mathbf{b}^T\mathbf{q_1}}{\|\mathbf{q_1}\|^2} \tag{2.0.4}$$

From (**??**) and (**??**)

$$\mathbf{a} = \|\mathbf{a}\|\,\mathbf{q_1} \tag{2.0.5}$$

$$\mathbf{b} = k\mathbf{q_1} + \|\mathbf{b} - k\mathbf{q_1}\|\,\mathbf{q_2} \tag{2.0.6}$$

$$\implies \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} = \begin{pmatrix} \mathbf{q_1} & \mathbf{q_2} \end{pmatrix} \begin{pmatrix} \|\mathbf{a}\| & k \\ 0 & \|\mathbf{b} - k\mathbf{q_1}\| \end{pmatrix} \tag{2.0.7}$$

$$\implies \mathbf{A} = \mathbf{QR} \tag{2.0.8}$$

QR decomposition of a matrix A is essentially representation of column vectors of matrix A in terms of linear combination of orthonormal basis of column space of A.

3 Solution

For matrix A

$$\mathbf{a} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}, \ \mathbf{b} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \tag{3.0.1}$$

$$\tag{3.0.2}$$

Let

$$\mathbf{q_1} = \frac{1}{\sqrt{5}} \begin{pmatrix} 3 \\ -4 \end{pmatrix} \tag{3.0.3}$$

$$\tag{3.0.4}$$

From (**??**) and (**??**)

$$\mathbf{q_2} = \frac{1}{\sqrt{5}} \begin{pmatrix} -4 \\ 3 \end{pmatrix} \tag{3.0.5}$$

$$\implies \mathbf{Q} = \frac{1}{\sqrt{5}} \begin{pmatrix} 3 & -4 \\ -4 & 3 \end{pmatrix} \tag{3.0.6}$$

$$\mathbf{R} = \begin{pmatrix} \sqrt{5} & 0 \\ 0 & \sqrt{5} \end{pmatrix} \tag{3.0.7}$$

Therefore the matrix A can be decomposed as

$$\mathbf{A} = \begin{pmatrix} \frac{3}{\sqrt{5}} & -\frac{4}{\sqrt{5}} \\ -\frac{4}{\sqrt{5}} & \frac{3}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \sqrt{5} & 0 \\ 0 & \sqrt{5} \end{pmatrix} \tag{3.0.8}$$