

IIT Hyderabad
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ASSIGNMENT 1

Lines and Planes

Problem Statement

Find the equations of the lines which intercepts on the both the axes and whose sum and product are 1 and -6 respectively.

Solution

The equation of line in terms of vector notations can be written as

$$\mathbf{n}^T \mathbf{x} = b \quad \text{where} \quad \mathbf{n} = \begin{pmatrix} n_{11} \\ n_{12} \end{pmatrix} \quad (1)$$

or

$$\begin{pmatrix} n_{11} & n_{12} \end{pmatrix} \mathbf{x} = b \quad (2)$$

Let the intercepts be $\begin{pmatrix} a \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ b \end{pmatrix}$, respectively.

Given that: $a + b = 1$, and $ab = -6$

$$\begin{aligned} \implies b = \frac{-6}{a} &\implies a^2 - a - 6 = 0 \implies (a - 3)(a + 2) = 0 \\ &\implies (a, b) = (3, -2) \text{ and } (-2, 3) \end{aligned} \quad (3)$$

When the line passes through $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$, respectively, we get, upon substitution in (2):

$$\begin{aligned} 3n_{11} = b &\implies n_{11} = \frac{b}{3} \\ -2n_{12} = b &\implies n_{12} = -\frac{b}{2} \end{aligned}$$

Therefore, the equation of first line is

$$\begin{aligned} &\begin{pmatrix} \frac{b}{3} & -\frac{b}{2} \end{pmatrix} \mathbf{x} = b \\ \implies &\begin{pmatrix} \frac{1}{3} & -\frac{1}{2} \end{pmatrix} \mathbf{x} = 1 \end{aligned} \quad (4)$$

Similarly, the equation of second line, which passes through $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ is

$$\begin{pmatrix} -1 & 1 \\ 2 & 3 \end{pmatrix} \mathbf{x} = 1 \tag{5}$$