EE5609 Assignment 4

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1 Problem

D is a point on side BC of ΔABC such that $AD = AC$. Show that $AB > AD$.

2 Solution

Let $\mathbf{D}$ divide BC internally in ratio $1 : k$ where $0 < k < 1$. Then

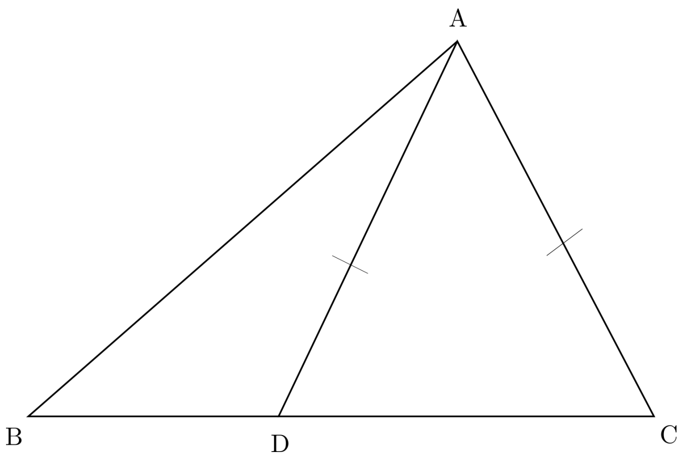

Fig. 0

$$\mathbf{D} = \frac{k\mathbf{B} + \mathbf{C}}{k+1} \tag{2.0.1}$$

The direction vector along AD is $\mathbf{m}_{AD} = \mathbf{D} - \mathbf{A}$.

$$\implies \quad \mathbf{m}_{AD} = \frac{k(\mathbf{B} - \mathbf{A}) + (\mathbf{C} - \mathbf{A})}{k+1} \tag{2.0.2}$$

$$= \frac{k\mathbf{m}_{AB} + \mathbf{m}_{AC}}{k+1} \tag{2.0.3}$$

$$\implies \quad \|\mathbf{m}_{AD}\|^2 = \frac{k^2 \|\mathbf{m}_{AB}\|^2 + \|\mathbf{m}_{AC}\|^2}{(k+1)^2} \tag{2.0.4}$$

Since $\|\mathbf{m}_{AD}\| = \|\mathbf{m}_{AC}\|$, so that

$$\|\mathbf{m}_{AD}\|^2 \{1 - \frac{1}{(k+1)^2}\} = \frac{k^2 \|\mathbf{m}_{AB}\|^2}{(k+1)^2} \tag{2.0.5}$$

$$\implies \quad \|\mathbf{m}_{AB}\|^2 = \{1 + \frac{2}{k}\} \|\mathbf{m}_{AD}\|^2 \tag{2.0.6}$$

$$\implies \quad \|\mathbf{m}_{AB}\|^2 > \|\mathbf{m}_{AD}\|^2 \tag{2.0.7}$$

$$\implies \quad AB > AD \tag{2.0.8}$$