

IIT Hyderabad
SHANTANU YADAV, EE20MTECH12001
Challenge 1

Lines and Planes

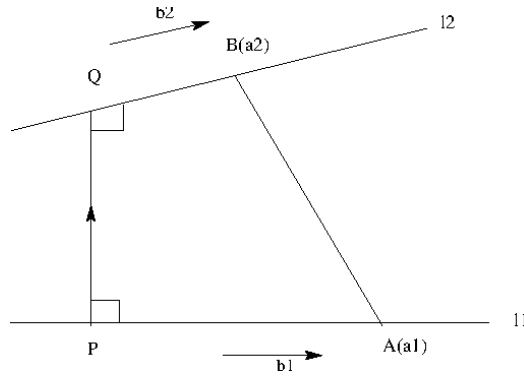
Shortest distance between two skew lines

Let the two lines are L_1 and L_2

$$L1 : \mathbf{x} = \begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \end{pmatrix} + \lambda \begin{pmatrix} b_{11} \\ b_{12} \\ b_{13} \end{pmatrix} \quad (1)$$

and

$$L2 : \mathbf{x} = \begin{pmatrix} a_{21} \\ a_{22} \\ a_{23} \end{pmatrix} + \lambda \begin{pmatrix} b_{21} \\ b_{22} \\ b_{23} \end{pmatrix} \quad (2)$$



Since P lies on L_1 and Q lies on L_2 , the points should satisfy equations (??) and (??), respectively.

$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} a_{11} + \lambda b_{11} \\ a_{12} + \lambda b_{12} \\ a_{13} + \lambda b_{13} \end{pmatrix} \quad (3)$$

and

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} a_{21} + \mu b_{21} \\ a_{22} + \mu b_{22} \\ a_{23} + \mu b_{23} \end{pmatrix} \quad (4)$$

$$\begin{aligned} \mathbf{PQ} &= \mathbf{Q} - \mathbf{P} \\ &= \begin{pmatrix} a_{21} - a_{11} + \mu b_{21} - \lambda b_{11} \\ a_{22} - a_{12} + \mu b_{22} - \lambda b_{12} \\ a_{23} - a_{13} + \mu b_{23} - \lambda b_{13} \end{pmatrix} \end{aligned} \quad (5)$$

The only unknowns are λ and μ .

Since \mathbf{PQ} is perpendicular to \mathbf{b}_1 and \mathbf{b}_2 :

$$\mathbf{PQ} \cdot \mathbf{b}_1 = 0 \quad \text{and} \quad \mathbf{PQ} \cdot \mathbf{b}_2 = 0 \quad (6)$$

these equations can be solved for λ and μ .