EE5609 ASSIGNMENT 1

SHANTANU YADAV, EE20MTECH12001
IIT Hyderabad

Lines and Planes

The python solution code is available at

https://github.com/Shantanu2508/Matrix_Theory/blob/master/stline.py

Problem Statement

Find the equations of the lines which intercepts on the both the axes and whose sum and product are 1 and -6 respectively.

Solution

The equation of line in terms of vector notations can be written as

$$\mathbf{n}^T\mathbf{x} = b \qquad \text{where} \qquad \mathbf{n} = \begin{pmatrix} n_{11} \\ n_{12} \end{pmatrix}, \tag{1}$$

or

$$\begin{pmatrix} n_{11} & n_{12} \end{pmatrix} \mathbf{x} = b \tag{2}$$

Let the intercepts be $\begin{pmatrix} a \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ b \end{pmatrix}$, respectively.
Given that: $\qquad a + b = 1, \qquad$ and $\qquad ab = -6$
The quadratic equation whose roots are the x and y intercepts can be written as :

$$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$$

$$\implies x^2 - x - 6 = 0$$

$$\implies x = (3, -2) \text{ and corresponding y intercepts are } (-2, 3) \tag{3}$$

When the line passes through $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$, respectively, we get, upon substitution in (2):

$$3n_{11} = b \qquad \implies \qquad n_{11} = \frac{b}{3}$$

$$-2n_{12} = b \qquad \implies \qquad n_{12} = -\frac{b}{2}$$

Therefore, the equation of first line is

$$\begin{pmatrix} \frac{b}{3} & \frac{-b}{2} \end{pmatrix} \mathbf{x} = b$$

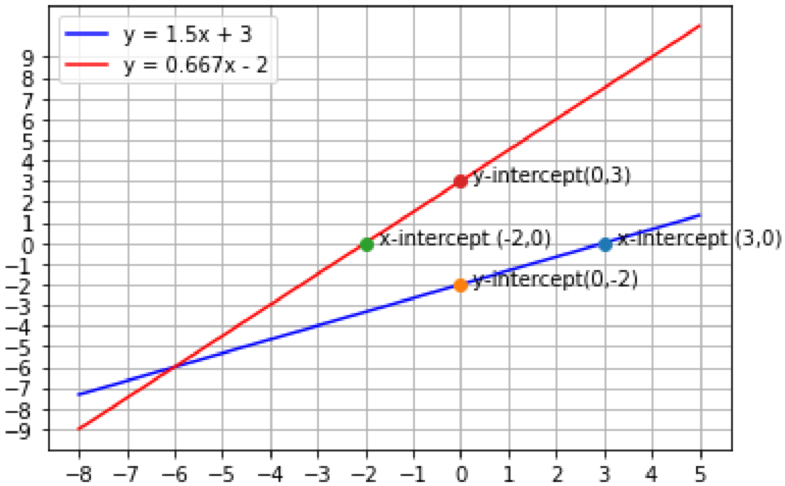

Figure 1:

$\Longrightarrow$

$$\begin{pmatrix}\frac{1}{3} & \frac{-1}{2}\end{pmatrix}\mathbf{x} = 1 \tag{4}$$

Similarly, the equation of second line, which passes through $\begin{pmatrix}-2\\0\end{pmatrix}$ and $\begin{pmatrix}0\\3\end{pmatrix}$ is

$$\begin{pmatrix}\frac{-1}{2} & \frac{1}{3}\end{pmatrix}\mathbf{x} = 1 \tag{5}$$

$x-$intercept	$y-$intercept	n_{11}	n_{12}
3	-2	1/3	-1/2
-2	3	-1/2	1/3

Table 1: