EE5609 Assignment 11

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The python solution code is available at

https://github.com/Shantanu2508/Matrix_Theory/blob/master/Assignment_11/assignment11.py

1 Problem

Is the vector $\begin{pmatrix} 3 \\ -1 \\ 0 \\ -1 \end{pmatrix}$ in the subspace of $\mathbf{R}^3$ spanned by the vectors $\begin{pmatrix} 2 \\ -1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \\ -3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 9 \\ -5 \end{pmatrix}$?

2 Explanation

Let

$$S = \left\{ \begin{pmatrix} 2 \\ -1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \\ -3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 9 \\ -5 \end{pmatrix} \right\} \tag{2.0.1}$$

If $\begin{pmatrix} 3 \\ -1 \\ 0 \\ 1 \end{pmatrix} \in span(S)$ there exists a unique solution $\mathbf{x}$ such that

$$\begin{pmatrix} 2 & -1 & 1 \\ -1 & 1 & 1 \\ 3 & 1 & 9 \\ 2 & -9 & -5 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ -1 \\ 0 \\ 1 \end{pmatrix} \tag{2.0.2}$$

Using row-reduction on augmented matrix

$$\begin{pmatrix} 2 & -1 & 1 & | & 3 \\ -1 & 1 & 1 & | & -1 \\ 3 & 1 & 9 & | & 0 \\ 2 & -3 & -5 & | & 1 \end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{2}} \begin{pmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & | & \frac{3}{2} \\ -1 & 1 & 1 & | & -1 \\ 3 & 1 & 9 & | & 0 \\ 2 & -3 & -5 & | & 1 \end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow[R_3 \leftarrow R_3 - 3R_1]{R_2 \leftarrow R_2 + R_1} \begin{pmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & | & \frac{3}{2} \\ 0 & \frac{1}{2} & \frac{3}{2} & | & \frac{1}{2} \\ 0 & \frac{5}{2} & \frac{15}{2} & | & -\frac{9}{2} \\ 2 & -3 & -5 & | & 1 \end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_4 \leftarrow R_4 - 2R_1} \begin{pmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & | & \frac{3}{2} \\ 0 & \frac{1}{2} & \frac{3}{2} & | & \frac{1}{2} \\ 0 & \frac{5}{2} & \frac{15}{2} & | & -\frac{9}{2} \\ 0 & -2 & -6 & | & -2 \end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow[R_4 \leftarrow -\frac{R_4}{2}]{R_2 \leftarrow 2R_2, R_3 \leftarrow 2R_3} \begin{pmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & | & \frac{3}{2} \\ 0 & 1 & 3 & | & 1 \\ 0 & 5 & 15 & | & -9 \\ 0 & 1 & 3 & | & 1 \end{pmatrix} \tag{2.0.7}$$

$$\xleftrightarrow[R_4 \leftarrow R_4 - R_2]{R_3 \leftarrow R_3 - 5R_2} \begin{pmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & | & \frac{3}{2} \\ 0 & 1 & 3 & | & 1 \\ 0 & 0 & 0 & | & -14 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \tag{2.0.8}$$

$$\xleftrightarrow[R3 \leftarrow -\frac{R_3}{14}]{R_1 \leftarrow R_1 + \frac{R_2}{2}} \begin{pmatrix} 1 & 0 & 2 & | & 2 \\ 0 & 1 & 3 & | & 1 \\ 0 & 0 & 0 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \tag{2.0.9}$$

$$\xleftrightarrow[R_2 \leftarrow R_2 - R_3]{R_1 \leftarrow R_1 - 2R_3} \begin{pmatrix} 1 & 0 & 2 & | & 0 \\ 0 & 1 & 3 & | & 0 \\ 0 & 0 & 0 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \tag{2.0.10}$$

Since $rank(A) = 2$ and $rank(A|B) = 3$ solution to above system of linear equation does not exist. Therefore $\begin{pmatrix} 3 \\ -1 \\ 0 \\ 1 \end{pmatrix} \notin span(S)$.