

EE5609 Assignment 16

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1 PROBLEM

Let \mathbf{W} be the space of $n \times n$ matrices over the field \mathbf{F} , and let \mathbf{W}_0 be the subspace spanned by the matrices C of the form $C = AB - BA$. Prove that \mathbf{W}_0 is exactly the subspace of matrices which have trace zero.

2 EXPLANATION

Let there be two subspaces defined as

$$\mathbf{W}_0 = \{A \in \mathbf{W} : \text{trace}(A) = 0\} \quad (2.0.1)$$

and

$$\mathbf{W}_1 = \{C \in \mathbf{W} : C = AB - BA\} \quad (2.0.2)$$

Consider $C \in \mathbf{W}$ such that $C = AB - BA$ where $A, B \in \mathbf{R}^{N \times N}$ and since

$$\text{trace}(AB) = \text{trace}(BA) \quad (2.0.3)$$

$$\implies \text{trace}(AB) - \text{trace}(BA) = 0 \quad (2.0.4)$$

$$\implies \text{trace}(C) = 0 \quad (2.0.5)$$

Thus any linear combination of matrices $C = AB - BA$ represents the subspace of all $N \times N$ matrices with trace equal to 0. Hence, the subspace \mathbf{W}_0 and \mathbf{W}_1 are equivalent.