EE5609 Challenge Problem

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1 Problem

If $x(n) * h_1(n) = y(n)$ and $x(n) * h_2(n) = y(n)$. Is $h_1(n) = h_2(n)$?

2 Relation between convolution and polynomial

Let us consider a simple example

$$x[n] = [2, 3] \tag{2.0.1}$$
$$h_1[n] = [4, 5] \tag{2.0.2}$$
$$h_2[n] = [a, b, c] \tag{2.0.3}$$

From convolution

$$x[n] * h_1[n] = [8, 22, 15] \tag{2.0.4}$$

Now let us consider

$$x[n] = 2x + 3 \tag{2.0.5}$$
$$h_1[n] = 4x + 5 \tag{2.0.6}$$
$$h_2[n] = ax^2 + bx + c \tag{2.0.7}$$

Calculating

$$x[n] \times h_1[n] = (2x + 3)(4x + 5) = 8x^2 + 22x + 15 \tag{2.0.8}$$

This is an important result which shows that coefficients of polynomial are the weights of convolution operation.
Now,

$$x[n] \times h_2[n] = 2ax^3 + (2b + 3a)x^2 + (2c + 3b)x + 3c \tag{2.0.9}$$

If there exists $h_2[n]$ such that

$$x[n] * h_2[n] = x[n] * h_1[n] \tag{2.0.10}$$

then, on comparision

$$a = 0;\ b = 4;\ c = 5 \tag{2.0.11}$$
$$h_2[n] = 4x + 5 \tag{2.0.12}$$

3 Generalizing the Result

Let $x[n]$, $h[n]$, $h_2[n]$ be represented as polynomials of degree n, m and p respectively.

$$x(n) = (c_0 + c_1 x)^n \tag{3.0.1}$$
$$h_1(n) = (a_0 + a_1 x)^m \tag{3.0.2}$$
$$h_2(n) = (b_0 + b_1 x)^p \tag{3.0.3}$$

From binomial theorem and relation between convolution operation and polynomial coefficients

$$x(n) \times h_1(n) = \binom{n}{j}\binom{m}{j} c_1^i c_0^{n-i} a_1^j a_0^{m-j} x^{i+j} \tag{3.0.4}$$
$$x(n) \times h_2(n) = \binom{n}{j}\binom{p}{k} c_1^i c_0^{n-i} b_1^k b_0^{p-k} x^{i+k} \tag{3.0.5}$$
$$\tag{3.0.6}$$

Both the equations are equal iff

$$\binom{m}{j} a_1^j a_0^{m-j} = \binom{p}{k} b_1^k b_0^{p-k} \tag{3.0.7}$$
$$\implies \quad m = p;\ a_1 = b_1;\ a_0 = b_0; \tag{3.0.8}$$
$$\implies \quad h_1(n) = h_2(n) \tag{3.0.9}$$