

EE5609 Assignment 7

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The python solution code is available at

https://github.com/Shantanu2508/Matrix_Theory/blob/master/Assignment_7/qr.py

3 SOLUTION

For matrix A

$$\mathbf{a} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \quad (3.0.1)$$

$$(3.0.2)$$

1 PROBLEM

Perform QR decomposition on matrix A given by

$$A = \begin{pmatrix} 3 & -4 \\ -4 & 3 \end{pmatrix}$$

Let

$$\mathbf{q}_1 = \frac{1}{\sqrt{5}} \begin{pmatrix} 3 \\ -4 \end{pmatrix} \quad (3.0.3)$$

$$(3.0.4)$$

From (2.0.3) and (2.0.4)

$$\mathbf{q}_2 = \frac{1}{\sqrt{5}} \begin{pmatrix} -4 \\ 3 \end{pmatrix} \quad (3.0.5)$$

$$\Rightarrow \mathbf{Q} = \frac{1}{\sqrt{5}} \begin{pmatrix} 3 & -4 \\ -4 & 3 \end{pmatrix} \quad (3.0.6)$$

$$\mathbf{R} = \begin{pmatrix} \sqrt{5} & 0 \\ 0 & \sqrt{5} \end{pmatrix} \quad (3.0.7)$$

Therefore the matrix A can be decomposed as

$$\mathbf{A} = \begin{pmatrix} \frac{3}{\sqrt{5}} & -\frac{4}{\sqrt{5}} \\ -\frac{4}{\sqrt{5}} & \frac{3}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \sqrt{5} & 0 \\ 0 & \sqrt{5} \end{pmatrix} \quad (3.0.8)$$

$$\mathbf{q}_2 = \frac{\mathbf{b} - k\mathbf{q}_1}{\|\mathbf{b} - k\mathbf{q}_1\|} \quad (2.0.3)$$

where

$$k = \frac{\mathbf{b}^T \mathbf{q}_1}{\|\mathbf{q}_1\|^2} \quad (2.0.4)$$

From (2.0.2) and (2.0.3)

$$\mathbf{a} = \|\mathbf{a}\| \mathbf{q}_1 \quad (2.0.5)$$

$$\mathbf{b} = k\mathbf{q}_1 + \|\mathbf{b} - k\mathbf{q}_1\| \mathbf{q}_2 \quad (2.0.6)$$

$$\Rightarrow (\mathbf{a} \ \mathbf{b}) = (\mathbf{q}_1 \ \mathbf{q}_2) \begin{pmatrix} \|\mathbf{a}\| & k \\ 0 & \|\mathbf{b} - k\mathbf{q}_1\| \end{pmatrix} \quad (2.0.7)$$

$$\Rightarrow \mathbf{A} = \mathbf{QR} \quad (2.0.8)$$

QR decomposition of a matrix A is essentially representation of column vectors of matrix A in terms of linear combination of orthonormal basis of column space of A.