IIT Hyderabad

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Challenge 1

Lines and Planes

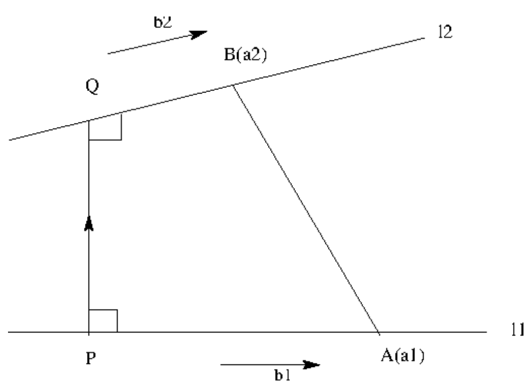
Shortest distance points between two skew lines

Let the two lines are L_1 and L_2

$$L1 : \mathbf{x} = \begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \end{pmatrix} + \lambda \begin{pmatrix} b_{11} \\ b_{12} \\ b_{13} \end{pmatrix} \tag{1}$$

and

$$L2 : \mathbf{x} = \begin{pmatrix} a_{21} \\ a_{22} \\ a_{23} \end{pmatrix} + \lambda \begin{pmatrix} b_{21} \\ b_{22} \\ b_{23} \end{pmatrix} \tag{2}$$



Since P lies on L_1 and Q lies on L_2, the points should satisfy equations (1) and (2), respectively.

$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} a_{11} + \lambda b_{11} \\ a_{12} + \lambda b_{12} \\ a_{13} + \lambda b_{13} \end{pmatrix} \tag{3}$$

and

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} a_{21} + \mu b_{21} \\ a_{22} + \mu b_{22} \\ a_{23} + \mu b_{23} \end{pmatrix} \tag{4}$$

$$\begin{aligned} \mathbf{PQ} &= \mathbf{Q} - \mathbf{P} \\ &= \begin{pmatrix} a_{21} - a_{11} + \mu b_{21} - \lambda b_{11} \\ a_{22} - a_{12} + \mu b_{22} - \lambda b_{12} \\ a_{23} - a_{13} + \mu b_{23} - \lambda b_{13} \end{pmatrix} \end{aligned} \tag{5}$$

The only unknowns are λ and μ.

Since $\mathbf{PQ}$ is perpendicular to $\mathbf{b_1}$ and $\mathbf{b_2}$:

$$\mathbf{PQ} \cdot \mathbf{b}_1 = 0 \qquad \text{and} \qquad \mathbf{PQ} \cdot \mathbf{b}_2 = 0 \tag{6}$$

these equations can be solved for λ and μ.

Problem 3.7.98

Find the shortest distance between the lines

$$\mathbf{x} = \begin{pmatrix} 6 \\ 2 \\ 2 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \quad \text{and}$$

$$\mathbf{x} = \begin{pmatrix} -4 \\ 0 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix}$$

Solution :

From equation (5) we can write $\mathbf{PQ}$ in terms of λ_1 and λ_2

$$\mathbf{PQ} = \begin{pmatrix} -10 + 3\lambda_2 - \lambda_1 \\ -2 - 2\lambda_2 + 2\lambda_1 \\ -1 - 2\lambda_2 - 2\lambda_1 \end{pmatrix} \tag{7}$$

λ_1 and λ_2 can be solved using equation (5)

$$\begin{pmatrix} -9 & 5 \\ -5 & 17 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 8 \\ 24 \end{pmatrix}$$

The value of λ_1 = -0.125 and λ_2 = 1.375.
Substituting λ_1 and λ_2 in equation (7)

$$\mathbf{P} = \begin{pmatrix} 5.875 \\ 2.25 \\ 1.75 \end{pmatrix} \quad \text{and} \quad \mathbf{Q} = \begin{pmatrix} 0.125 \\ -2.75 \\ -3.75 \end{pmatrix}$$

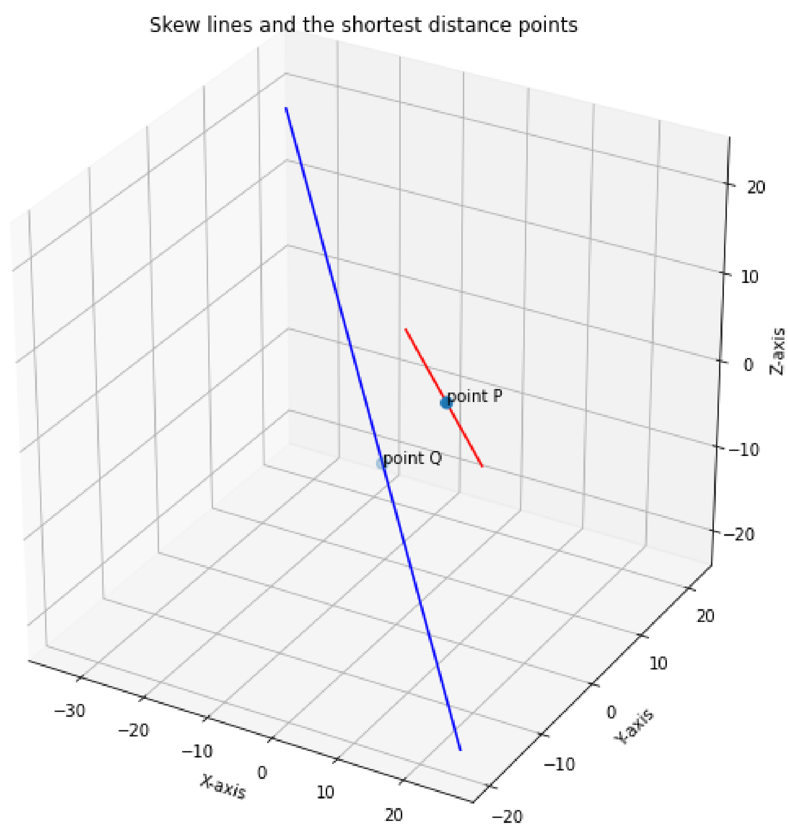


Figure 1: