EE5609 Assignment 1

SHANTANU YADAV

The python solution code is available at

> https://github.com/Shantanu2508/
> assignment−1/blob/master/stline.py

and latex codes from

> https://github.com/Shantanu2508/assignment
> −1/blob/master/assignment1.tex

1 PROBLEM

Find the equations of the lines which intercepts on the both the axes and whose sum and product are 1 and -6 respectively.

2 SOLUTION

The equation of line in terms of vector notations can be written as

$$\mathbf{n}^T\mathbf{x} = c \qquad (2.0.1)$$

Let the intercepts be $\begin{pmatrix} a \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ b \end{pmatrix}$, respectively.

Given that: $a + b = 1,$ and $ab = -6$

The quadratic equation whose roots are the x and y intercepts can be written as :

$$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$$
$$(2.0.2)$$

$$\implies \qquad x^2 - x - 6 = 0 \qquad (2.0.3)$$

$$\implies \qquad x = (3, -2) \qquad (2.0.4)$$

and corresponding y intercepts are $(-2, 3)$.

The line L1 passes through $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$.

Let direction vector of this line be $\mathbf{m}$.

$$\mathbf{m} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix} \qquad (2.0.5)$$

The normal vector, $\mathbf{n}$:

$$\mathbf{n} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{m} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \qquad (2.0.6)$$

The equation of line in terms of normal vector and passing through a point A is

$$\mathbf{n^T}(\mathbf{x} - A) = 0 \quad \implies \quad \mathbf{n^T}\mathbf{x} = \mathbf{n^T}A \quad (2.0.7)$$

$$\implies \mathbf{n^T}\mathbf{x} = \begin{pmatrix} 2 & -3 \end{pmatrix}\begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (2.0.8)$$

$$\implies \begin{pmatrix} 2 & -3 \end{pmatrix}\mathbf{x} = 6 \quad (2.0.9)$$

Similarly, the equation of second line L2, with x and y intercepts $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ and normal vector $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ is

$$\begin{pmatrix} -3 & 2 \end{pmatrix}\mathbf{x} = 6 \qquad (2.0.10)$$

The equations of lines (2.0.9) and (2.0.10) can be represented collectively as

$$\begin{pmatrix} 2 & -3 \\ -3 & 2 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 6 \\ 6 \end{pmatrix} \qquad (2.0.11)$$

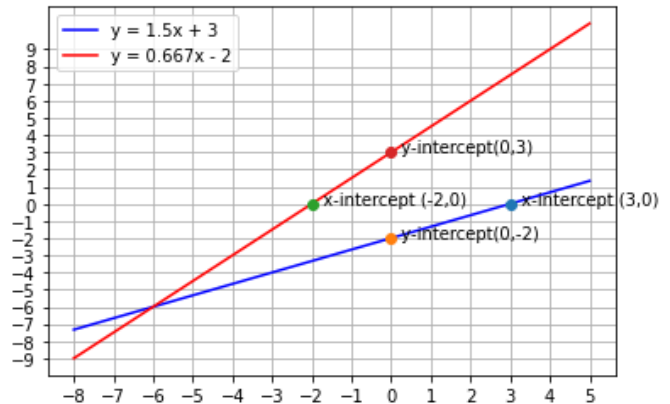| $x-$intercept	$y-$intercept	$\mathbf{n}$
$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -2 \end{pmatrix}$	$\begin{pmatrix} 2 \\ -3 \end{pmatrix}$
$\begin{pmatrix} -2 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$	$\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

TABLE 0

Fig. 0