EE5609 Assignment 1

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The python solution code is available at

https://github.com/Shantanu2508/assignment−1/blob/master/stline.py

and latex codes from

https://github.com/Shantanu2508/assignment−1/blob/master/assignment1.tex

1 Problem

Find the equations of the lines which intercepts on the both the axes and whose sum and product are 1 and -6 respectively.

2 Solution

The equation of line in terms of vector notations can be written as

$$\mathbf{n}^T\mathbf{x} = c \tag{2.0.1}$$

Let the intercepts be $\begin{pmatrix} a \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ b \end{pmatrix}$, respectively.

Given that: $a + b = 1, \quad \text{and} \quad ab = -6$

The quadratic equation whose roots are the x and y intercepts can be written as:

$$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0 \tag{2.0.2}$$

$$\implies \quad x^2 - x - 6 = 0 \tag{2.0.3}$$

$$\implies \quad x = (3, -2) \tag{2.0.4}$$

and corresponding y intercepts are $(-2, 3)$.

The line L_1 passes through $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$.

Let direction vector of this line be $\mathbf{m}$.

$$\mathbf{m} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix} \tag{2.0.5}$$

The normal vector, $\mathbf{n}$:

$$\mathbf{n} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\mathbf{m} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \tag{2.0.6}$$

The equation of line in terms of normal vector and passing through a point A is

$$\mathbf{n}^\mathbf{T}(\mathbf{x} - A) = 0 \quad \implies \quad \mathbf{n}^\mathbf{T}\mathbf{x} = \mathbf{n}^\mathbf{T}A \tag{2.0.7}$$

$$\implies \mathbf{n}^\mathbf{T}\mathbf{x} = \begin{pmatrix} 2 & -3 \end{pmatrix}\begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{2.0.8}$$

$$\implies \begin{pmatrix} 2 & -3 \end{pmatrix}\mathbf{x} = 6 \tag{2.0.9}$$

Similarly, the equation of second line L_2, with x and y intercepts $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ and normal vector $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ is

$$\begin{pmatrix} -3 & 2 \end{pmatrix}\mathbf{x} = 6 \tag{2.0.10}$$

The equations of lines (2.0.9) and (2.0.10) can be represented collectively as

$$\begin{pmatrix} 2 & -3 \\ -3 & 2 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 6 \\ 6 \end{pmatrix} \tag{2.0.11}$$

$x-$intercept	$y-$intercept	$\mathbf{n}$
$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -2 \end{pmatrix}$	$\begin{pmatrix} 2 \\ -3 \end{pmatrix}$
$\begin{pmatrix} -2 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$	$\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

TABLE 0

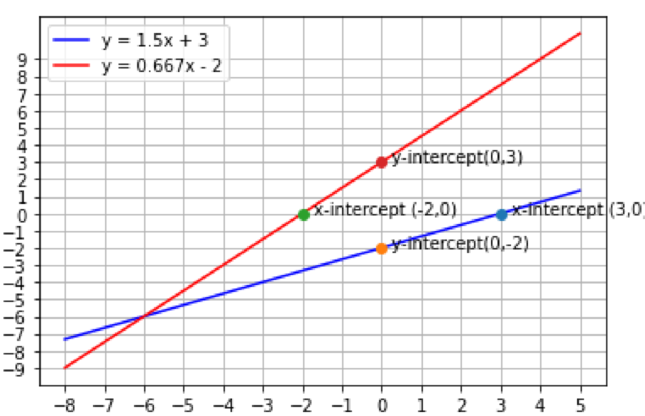

Fig. 0