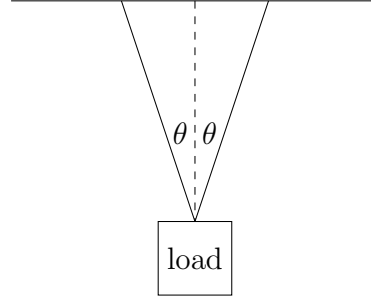


3. *Minimum time maneuver for a crane.* A crane manipulates a load with mass $m > 0$ in two dimensions using two cables attached to the load. The cables maintain angles $\pm\theta$ with respect to vertical, as shown below.



The (scalar) tensions T^{left} and T^{right} in the two cables are independently controllable, from 0 up to a given maximum tension T^{max} . The total force on the load is

$$F = T^{\text{left}} \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix} + T^{\text{right}} \begin{bmatrix} \sin \theta \\ \cos \theta \end{bmatrix} + mg,$$

where $g = (0, -9.8)$ is the acceleration due to gravity. The acceleration of the load is then F/m .

We approximate the motion of the load using

$$p_{i+1} = p_i + hv_i, \quad v_{i+1} = v_i + (h/m)F_i, \quad i = 1, 2, \dots,$$

where $p_i \in \mathbf{R}^2$ is the position of the load, $v_i \in \mathbf{R}^2$ is the velocity of the load, and $F_i \in \mathbf{R}^2$ is the force on the load, at time $t = ih$. Here $h > 0$ is a small (given) time step.

The goal is to move the load, which is initially at rest at position p^{init} to the position p^{des} , also at rest, in minimum time. In other words, we seek the smallest k for which

$$p_1 = p^{\text{init}}, \quad p_k = p^{\text{des}}, \quad v_1 = v_k = (0, 0)$$

is possible, subject to the constraints described above.

- Explain how to solve this problem using convex (or quasiconvex) optimization.
- Carry out the method of part (a) for the problem instance with

$$m = 0.1, \quad \theta = 15^\circ, \quad T^{\text{max}} = 2, \quad p^{\text{init}} = (0, 0), \quad p^{\text{des}} = (10, 2),$$

with time step $h = 0.1$. Report the minimum time k^* . Plot the tensions versus time, and the load trajectory, *i.e.*, the points p_1, \dots, p_k in \mathbf{R}^2 . Does the load move along the line segment between p^{init} and p^{des} (*i.e.*, the shortest path from p^{init} and p^{des})? Comment briefly.