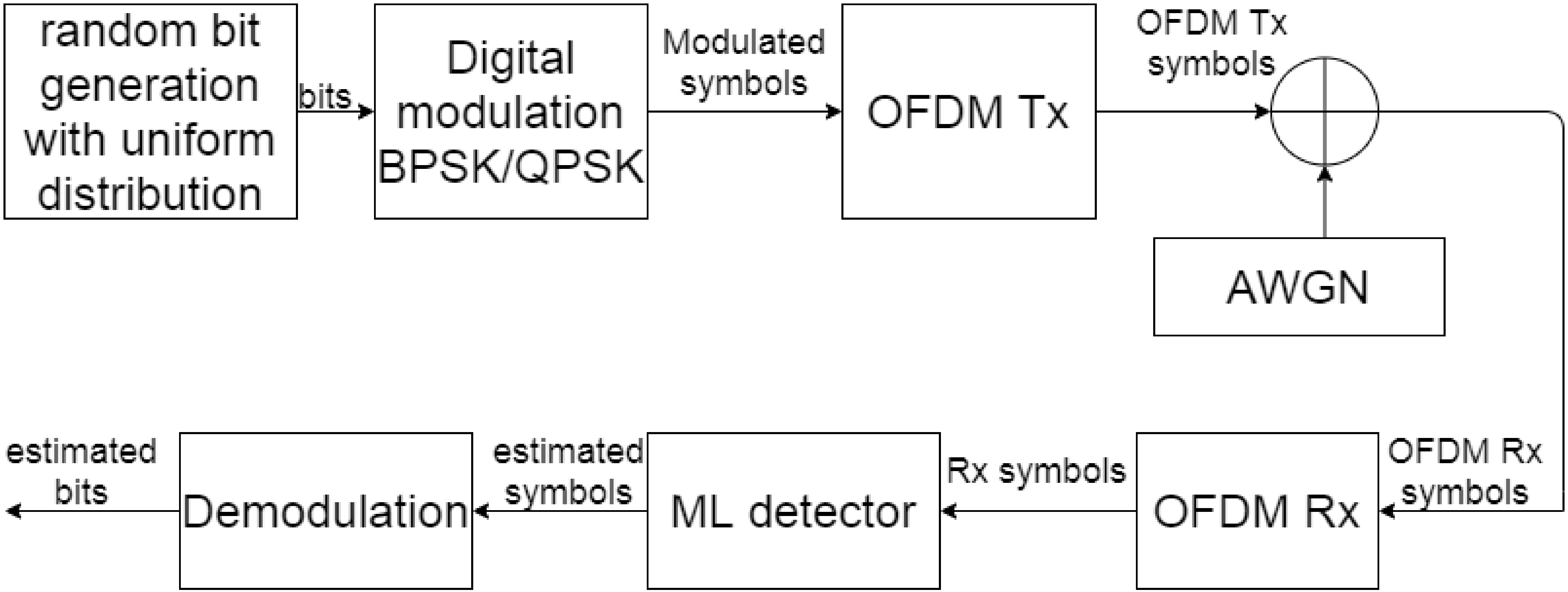
EE5802: DSP Lab
(Jan – May 2021)

Today’s Topic

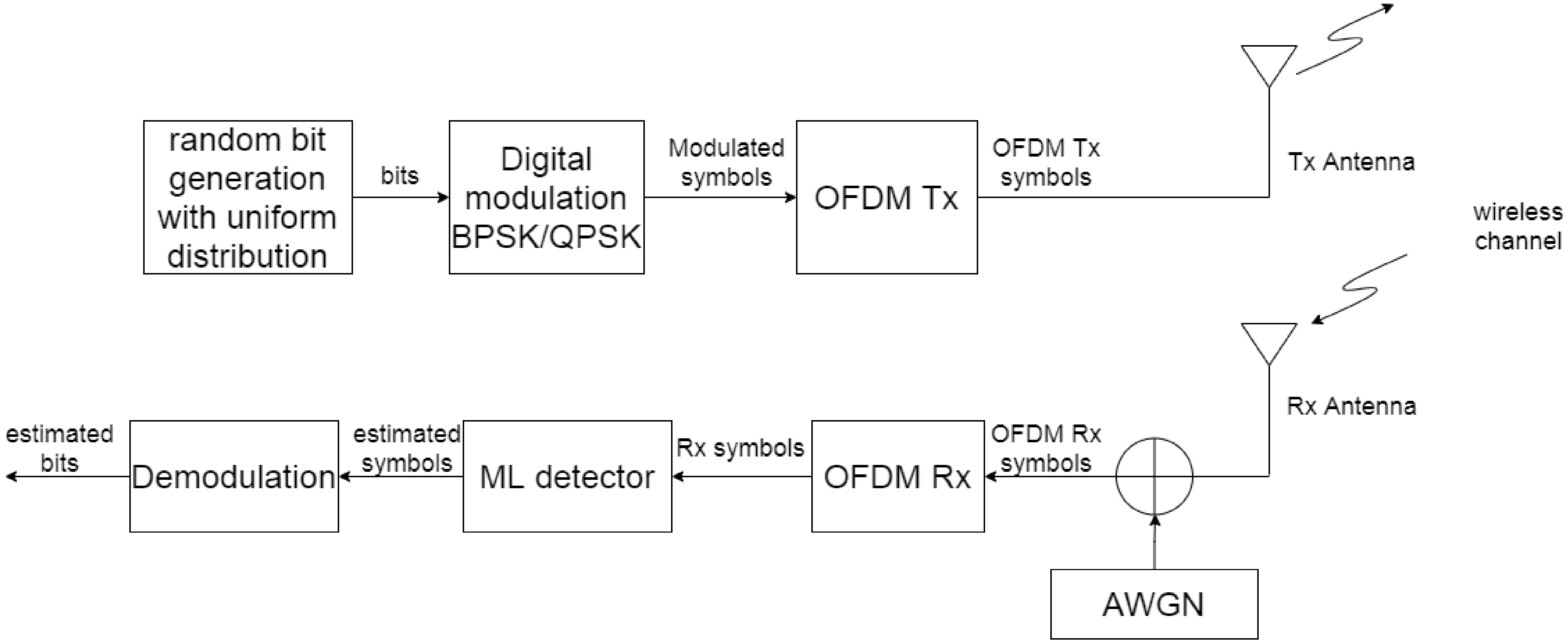
- BER Analysis of OFDM system with wireless channel.

Recap:



random bit generation with uniform distribution
bits
Digital modulation BPSK/QPSK
Modulated symbols
OFDM Tx
OFDM Tx symbols
AWGN
OFDM Rx symbols
OFDM Rx
Rx symbols
ML detector
estimated symbols
Demodulation
estimated bits

Block diagram of OFDM Tx and Rx with wireless channel



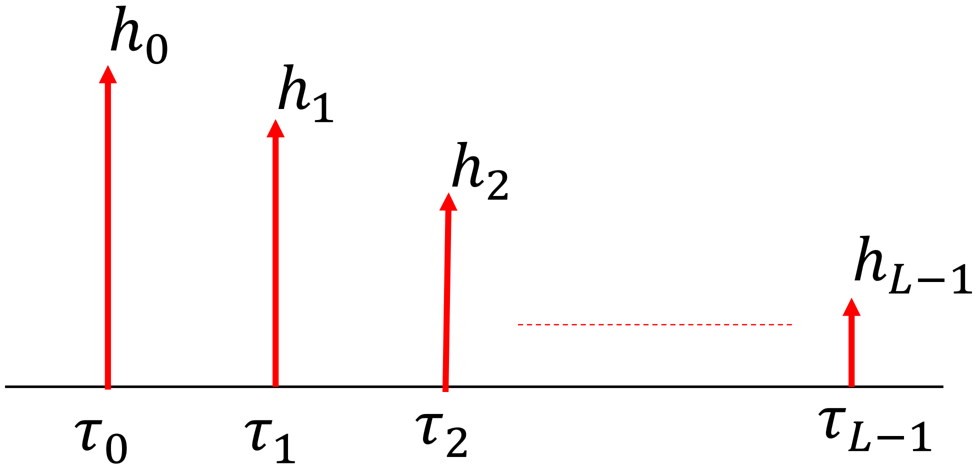
Continuous time system model

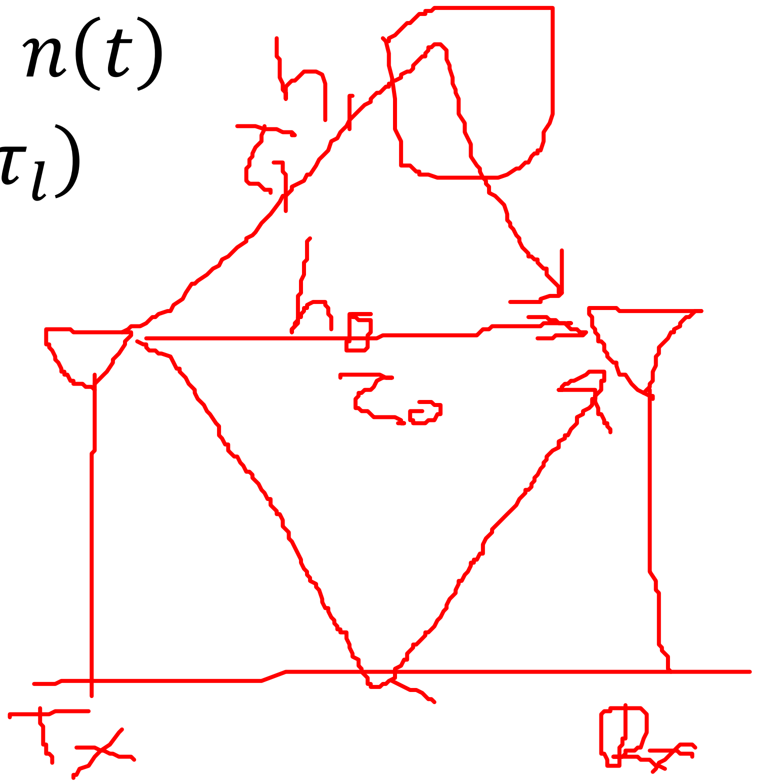
- Wireless SISO channel

$$y(t) = h(t) * x(t) + n(t)$$

$$\text{where, } h(t) = \sum_{l=0}^{L-1} h_l \delta(t - \tau_l)$$

$$h_l \sim CN\left(0, \sigma_l^2\right)$$





Discrete time system model

- Discrete time model

$$\underline{y} = \underline{h} \circledast \underline{x} + \underline{n}$$

where,

$$\underline{x} = Transmited\ symbol\ vector$$

$$\underline{h} = \text{wireless channel gain or channel coefficient vector}$$

$$\underline{y} = Received\ symbol\ vector\ at\ reciever$$

$$\underline{n} = AWGN(Adittive\ White\ Gaussian\ Noise)$$

Linear Convolution:

- $\underline{x} = [x_0,\ x_1, \dots \dots \dots, x_{N-1}]$
- $\underline{h} = [h_0,\ h_1, \dots \dots \dots, h_{L-1}]$
- $y[n] = h_0\, x_n + h_1\, x_{n-1} + \ \dots \dots \dots \dots \dots + h_{L-1}\, x_{n-L+1}$

OFDM:

- $\underline{x} = \left[x_{N-L_{CP}+1}, \dots \dots x_{N-1}, x_0,\ x_1, \dots \dots, x_{N-L_{CP}+1}, \dots \dots x_{N-1}\right]$
- $\underline{h} = [h_0,\ h_1, \dots \dots \dots, h_{L-1}]$
- $y[n] = h_0\, x_n + h_1\, x_{n-1} + \ \dots \dots \dots \dots \dots + h_{L-1}\, x_{n-L+1}$

n=0, $y[0] = h_0\, x_0 + 0 + \ \dots\dots\dots\dots\dots + 0 = h_0\, x_{N-L_{CP}+1}$

n=1, $y[1] = h_0\, x_1 + h_1\, x_0 + 0 + \ \dots\dots\dots\dots + 0$

$= h_0\, x_{N-L_{CP}+2} + h_1\, x_{N-L_{CP}+1}$

n=L-1, $y[L-1] = h_0\, x_{L-1} + h_1\, x_L + \ \dots\dots\dots + h_{L-1}\, x_0 + 0 + \ \dots + 0$

$= h_0\, x_{N-L_{CP}+L} + h_1\, x_{N-L_{CP}+L-1} + \ \dots + h_{L-1}\, x_{N-L_{CP}+1}$

$$\frac{[z_{N-k_p+1}\ \cdots\ z_N][y_{N-k_p+1}\ \cdots\ y_N]}{[h_{L-1}\ \cdots\ h_1\ h_0]\qquad N+L_{cp}}$$

$$h_0 = h$$

Circular convolution example:

$$\underline{x} = [1, 1, 2, 1] \qquad \underline{h} = [1, 2]$$

$$\begin{bmatrix} 3 \\ 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\underline{y} = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \\ 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

Output of DFT at receiver:

$$Y_k = H_k \cdot X_k$$

$$Y_0 = H_0 X_0$$

$$Y_1 = H_1 X_1$$

$$\vdots$$

$$Y_{N-1} = H_{N-1} X_{N-1}$$

Equalization at receiver:

$$\underline{\tilde{Y}} = \frac{\underline{Y}}{\underline{H}}$$

$$\tilde{Y}_k = \frac{Y_k}{H_k}$$

$$\underset{L\times 1}{\underline{h}} = [h_0, h_1, \ldots\ldots, h_{L-1}]$$

$$h_i \sim \mathcal{CN}(0,1)$$

$$L < N \quad i = 0 \rightarrow L-1$$

$$\underline{H} = DFT(\underline{h}, N)$$

Without OFDM:

$$y(1) = h(0)x(1) + h(1)x(0)$$

With OFDM:

$$Y(1) = H(1)X(1)$$

Observation: with OFDM ISI is reduced

- After Equalization, perform ML detection for each $\widetilde{Y_k}$.