DSP ASSIGNMENT : Time Offset in OFDM

EE20MTECH12001
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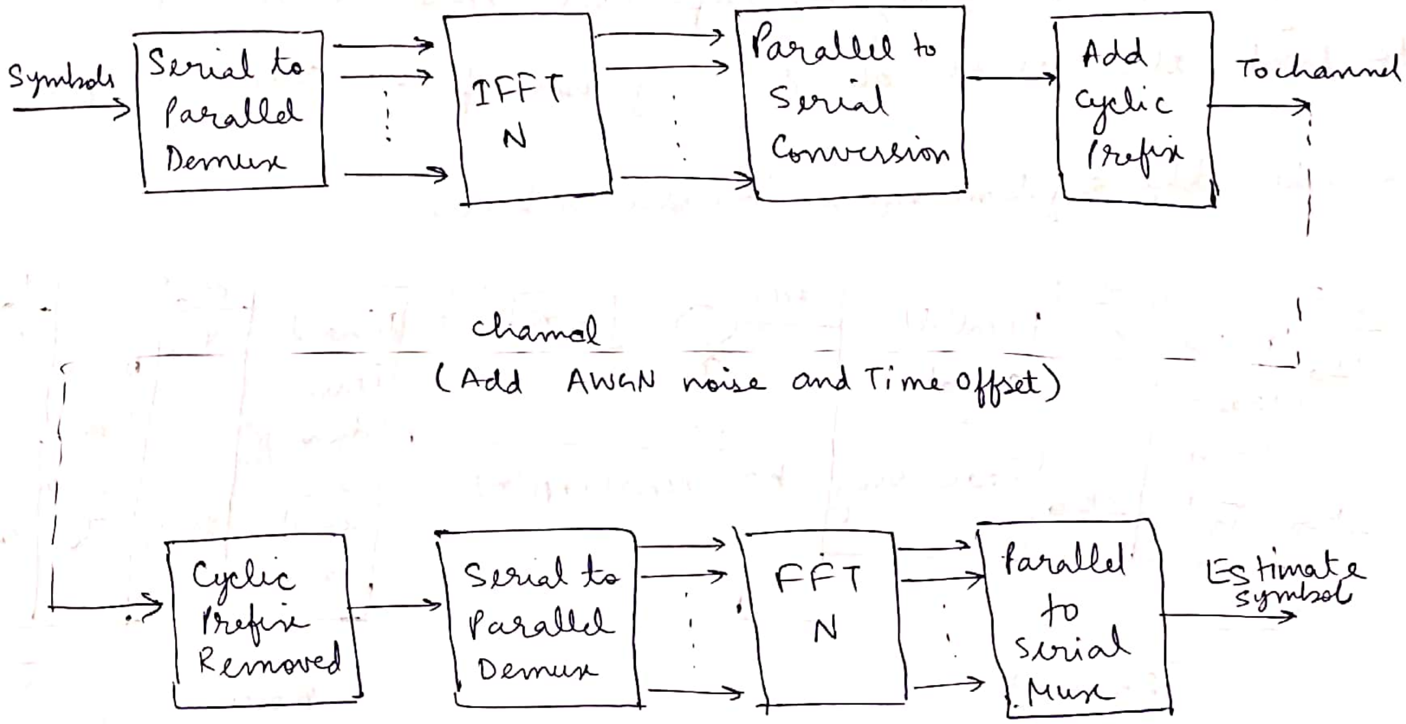
Flow diagram of OFDM



After OFDM modulation, we are unknowingly introducing time-offset error which causes some loss of information.

This error causes:

i) It rotates the data symbols

ii) Since accumulated sampling period offset is not constant during OFDM symbol and it increases from sample to sample which affect orthogonality of Source Coding.

How time offset is generated?

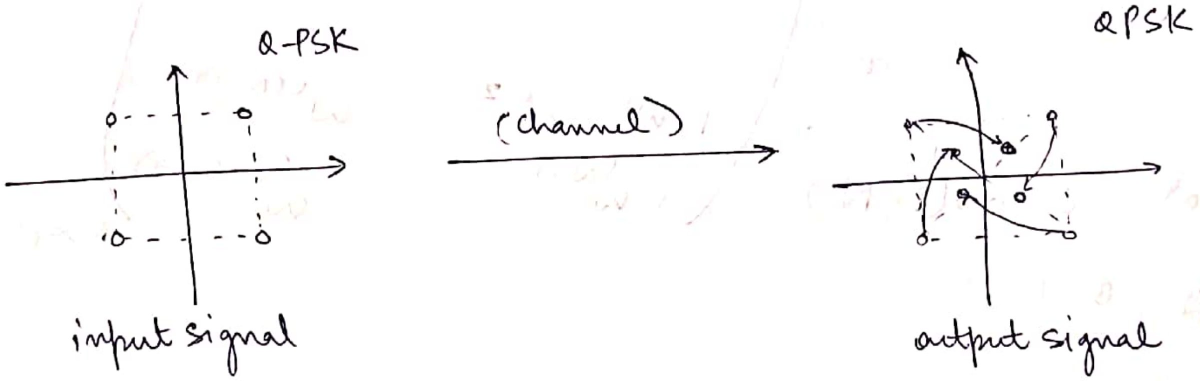
As we know that in order to decode the signal we must have the information regarding the channel. Consider the impulse response of the channel as

$$h(\theta) = |h| \angle \theta \quad \text{— (1)}$$

and let the input signal be x and so the output signal be $y = x|h|\angle\theta$

$$\Rightarrow \frac{y}{x} = |h|\angle\theta \quad \text{— (2)}.$$

<u>Eg</u>: if $h(\theta) = \frac{1}{4} \angle \pi/2$

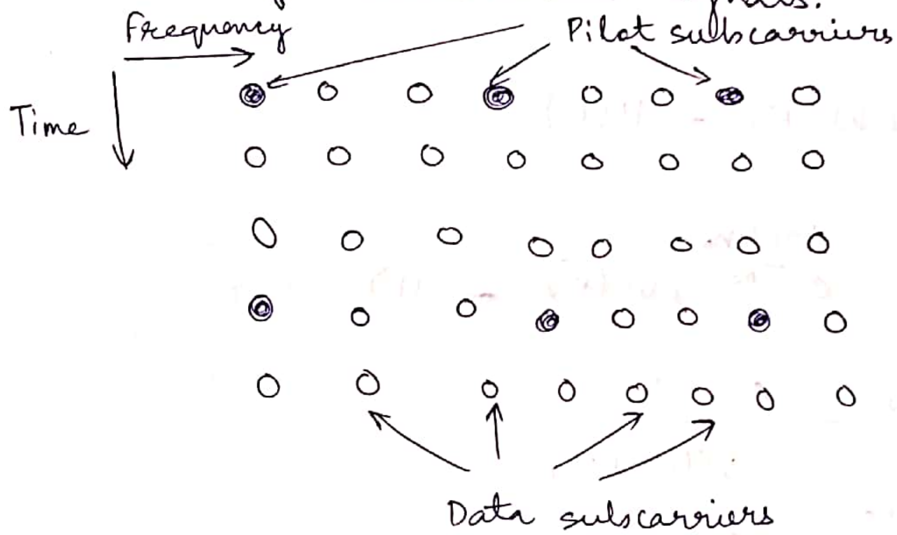


This shows that the output signal is some factor of input signal's magnitude with shifted-phase.

So after modulation, due to the above phenomena the output signal is a shifted version of input signal with AWGN noise. This happens because the receiver may not sample at the appropriate time instant.

Q) How to estimate timing-offset error?

If OFDM system has regular pilots (known signals to reciever) inserted on all subcarriers, the FFT output signals and estimated channel responses will simultaneously carry the same rotated phase which can be ~~shared~~ channel equalization in obtaining the estimates of transmitted signals.



- Pilot subcarriers and data subcarriers in an OFDM signal.

- In order to obtain the channel information we ~~obtain~~ transmit a "known" signal and observe the output—signal. From this we can obtain the channel response. Such signals which are used to determine channel characteristics are known as pilot-signals.

- But transmitting pilot signals does not send any information. To ~~send~~ resolve this issue pilot signal is sent along, with fraction of information signal.

Let the recieved signal be $y(n) = x(n-n_0) + w(n)$

where n_0 is the time-offset and

$w(n)$ is the AWGN noise

$x(n)$ is the input signal.

Using $x(n-n_0) \xrightarrow{DFT} e^{-j\frac{2\pi}{N}kn_0} X(k)$ we have,

$$Y(k) = e^{-j\frac{2\pi}{N}kn_0} X(k) + W(k)$$

$$\frac{Y(k)}{X(k)} = e^{-j\frac{2\pi k n_0}{N}} + W'(k) = H(k)$$

So we get $H(k) = e^{-j\frac{2\pi k n_0}{N}} + W'(k)$ — (1)

$$H(0)\overline{H(1)} = e^{\frac{j2\pi n_0}{N}} + (\quad)$$

W'(k) term

$$H(1)\overline{H(2)} = e^{\frac{j2\pi n_0}{N}} + (\quad)$$

$\vdots$

$$H(k)\overline{H(k)} = e^{\frac{j2\pi n_0}{N}} + (\quad)$$

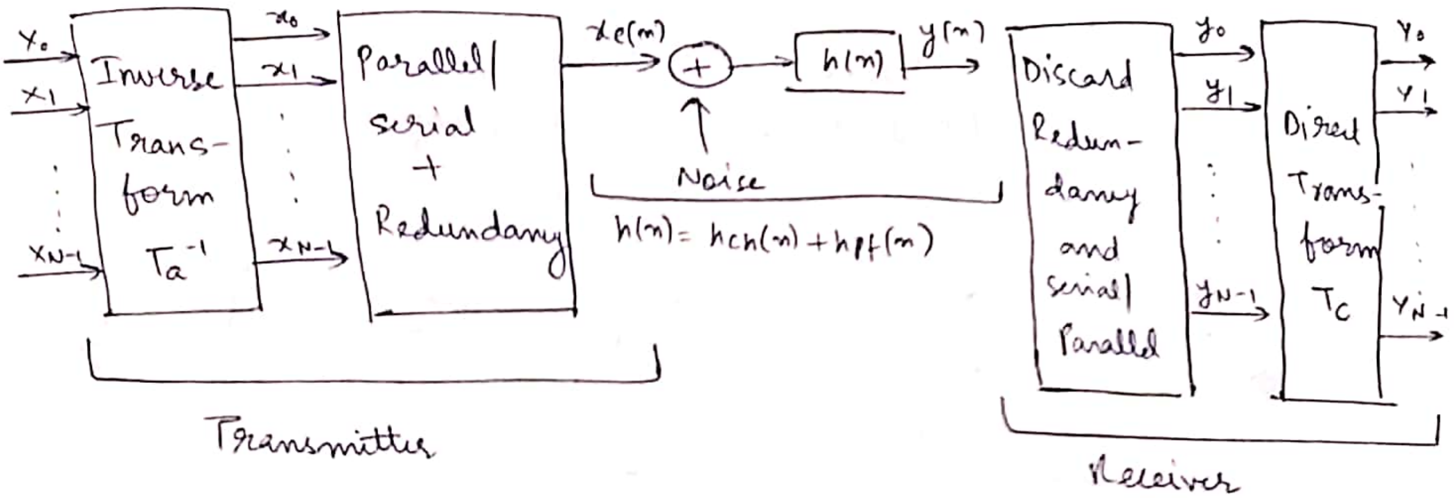
$\Rightarrow H(0)\overline{H(1)} + H(1)\overline{H(2)} + \cdots + H(k-1)\overline{H(k)} = Ne^{\frac{j2\pi n_0}{N}} + E(W(k)\text{term})$

$$\Rightarrow n_0 = \frac{N}{j2\pi} \log\left(\sum_{k=0}^{k-1} H(k)\overline{H(k+1)}\right)$$

Additional :

In any multicarrier modulation (MCM) systems, symbol timing estimators play an important role in the receiver to find the start of the symbol of the received signal.

Consider the system model as shown below:



In DCT-MCM the maximum value or peak is reached (in the absence of noise) when there exists a set of samples that are pairwise correlated. The time position of this maximum value is useful in finding the symbol timing and the phase of the correlation could yield the frequency estimate.

At the start of the symbol, specifically at instants $n_0 - 1$ and n_0 we have,

$$y(n_0 - 1) = y(n_0)$$

Let us consider absence of noise and the fact that for any integer number δ we have,

$$y(n_0-1) = y(n_0)$$

$$= \sum_{k=1}^{y} h(k)\left(x_e(n_0+\delta-k-1) + x_e(n_0+\delta+k-1)\right)$$

$$+ h(0)\, x_e(n_0+\delta-1)$$

In addition, at the end of the symbol, the receiving signal presents identical or opposite values in two consecutive samples n_0+N-1 and n_0+N. That is we have,

$$y(n_0+N-1) = \gamma y(n_0+N) = \sum_{k=1}^{y} h(k).\left(x_e(n_0+N+\delta-k-1) + x_e(n_0+N+\delta+k-1)\right)$$

$$+ h(0)\, x_e(n_0+N+\delta-1)$$

where $\gamma = 1$ for HS extension or $\gamma = -1$ for the HA extension.

So, $y(n_0-l-1) = y_0(n_0+l)$, $\delta \le l \le \delta+N-\gamma-1$