1. a) <u>Assumption:</u>

i) The proportion of time denoted to 3 items respectively, i.e, x_1, x_2 and x_3 can be fractional values.

ii) Number of produced item of each type is directly proportional to the time denoted to producing the item.

<u>Project Formulation:</u>

Let $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ be the vector of proportion of time devoted to each item each day.

So items produced each day: $\begin{pmatrix} 6000x_1 \\ 5000x_2 \\ 3000x_3 \end{pmatrix}$

Items produced in a week: $\begin{pmatrix} 30000x_1 \\ 25000x_2 \\ 15000x_3 \end{pmatrix}$

$Profit = 120000x_1 + 150000x_2 + 150000x_3$

Therefore the objective is:

$$\max \quad 120000x_1 + 150000x_2 + 150000x_3$$

s.t:

Proportion constraint: $x_1 + x_2 + x_3 \leq 1$

Storage constraint: $1200x_1 + 1125x_2 + 3150x_3 \leq 6000$

Minimum Sale Constraint: $30000x_1 \geq 5000$
$$15000x_3 \geq 4000$$

Demand constraint: $30000x_1 \leq 10000$

$25000x_2 \leq 15000$

$15000x_3 \leq 8000$

Positive constraint: $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

ii) $y_1 =$ number of iPod covers produced over the week

$y_2 =$ number of iPhone covers produced over the week

$y_3 =$ number of iPad covers produced over the week

Profit /week $= 4y_1 + 6y_2 + 10y_3$

So the objective function is

$\max \quad 4y_1 + 6y_2 + 10y_3$

St.

Production constraint /week $= \dfrac{1}{30000y_1} + \dfrac{1}{25000y_2} + \dfrac{1}{15000y_3} \leq 1$

Storage constraint $= 0.04y_1 + 0.045y_2 + 0.21y_3 \leq 6000$

Minimum sale constraint $= y_1 \geq 5000$

$y_2 \geq 4000$

~~$y_3 = 3000$~~

Demand constraint: $y_1 \leq 10000$

$y_2 \leq 15000$

$y_3 \leq 8000$

Positive constraint: $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

c) $z_1 =$ number of hours devoted to production of iPad smart covers in one week

$z_2 =$ number of hours devoted to production of iPhone smart covers in one week

~~$z_3 =$ number of hours devoted to production of iPad smart covers in one week~~

$z_3 =$ total number of production hours employed during the week.

So, number of hours devoted to production of iPad smart covers in one week $= z_3 - z_1 - z_2$

Therefore the objective function is:

$$\max \quad 3000 z_1 + 3750 x_2 + 3750(z_3 - z_1 - z_2)$$
$$st$$

Production constraint : $\dfrac{z_1}{40} \leq 1$

$$\dfrac{z_2}{40} \leq 1$$

Storage constraint : $30 z_1 + 28.125 z_2 + 78.75(z_3 - z_1 - z_2)$
$$\leq 6000$$

Minimum sale constraint: $750 z_1 \geq 5000$
$$375(z_3 - z_1 - z_2) \geq 4000$$

Demand constraint : $750 z_1 \leq 10000$
$$625 z_2 \leq 15000$$
$$375(z_3 - z_1 - z_2) \leq 8000$$

Positive constraint: $\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

d) Since x represents number of hours devoted to each item per day and z represents number of hours devoted to each item per week with working hours $= 8/$ day.

→ working hours / week $= 5 \times 8 \text{ hrs} = 40 \text{ hrs}.$

$\Rightarrow \quad z_1 = 40 x_1$

$\quad\quad z_2 = 40 x_2$

Total time devoted per day in (a) $= x_1 + x_2 + x_3$

$\Rightarrow z_3 = 40 (x_1 + x_2 + x_3).$