(Deep) Neural Networks

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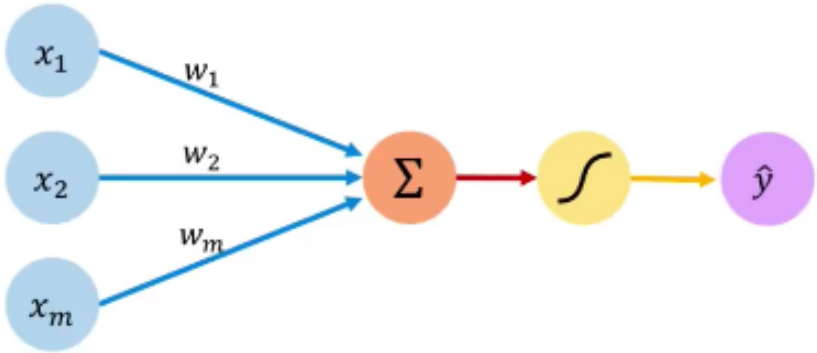
Summary of Linear/Logistic Regression



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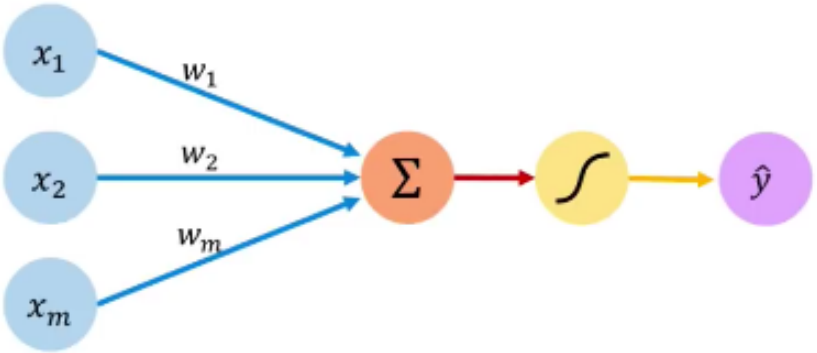

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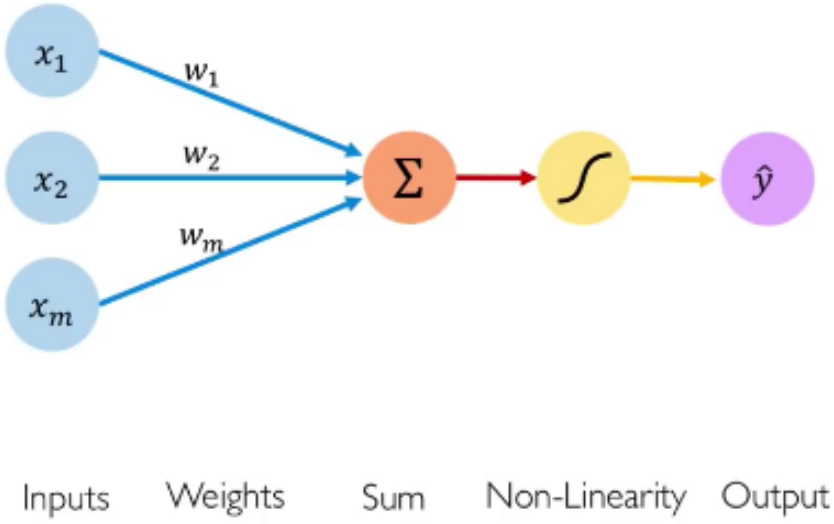

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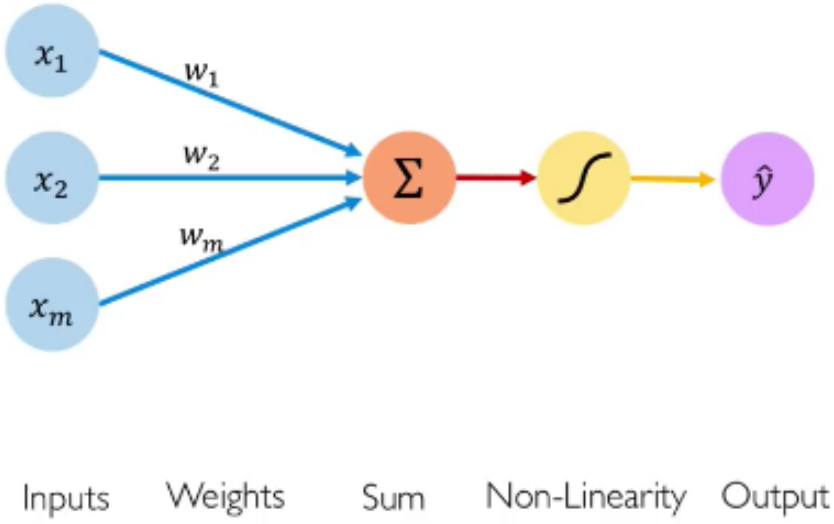

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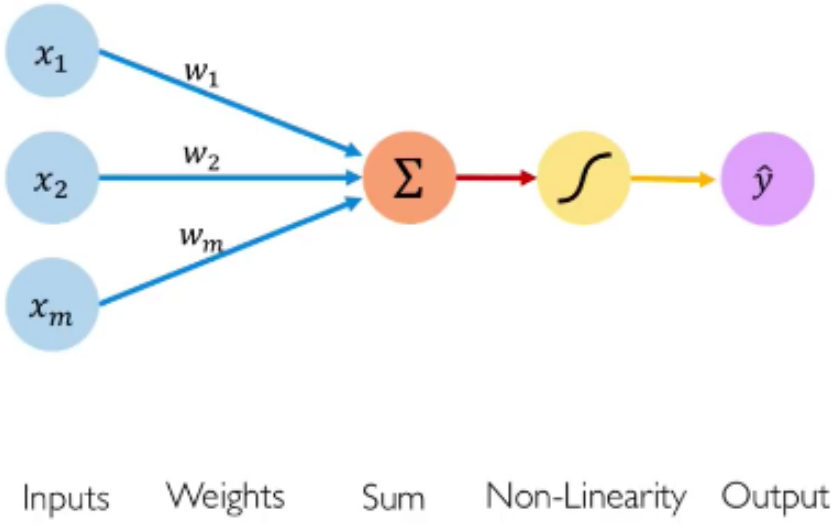

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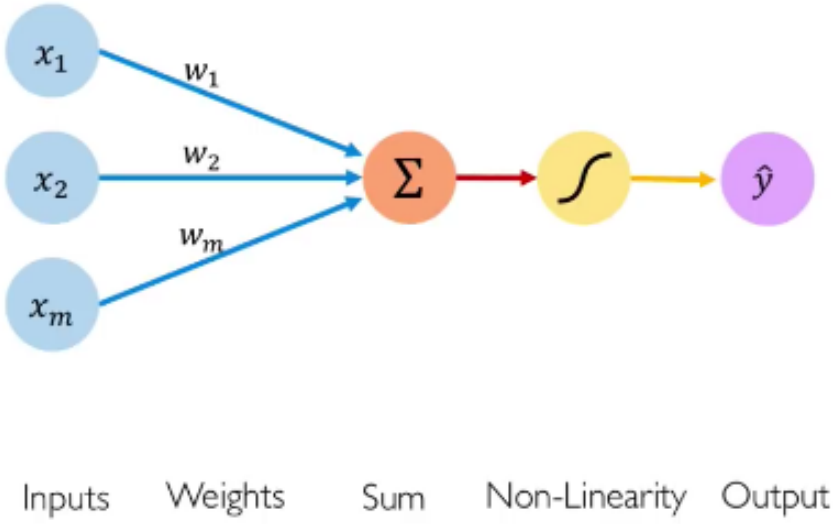

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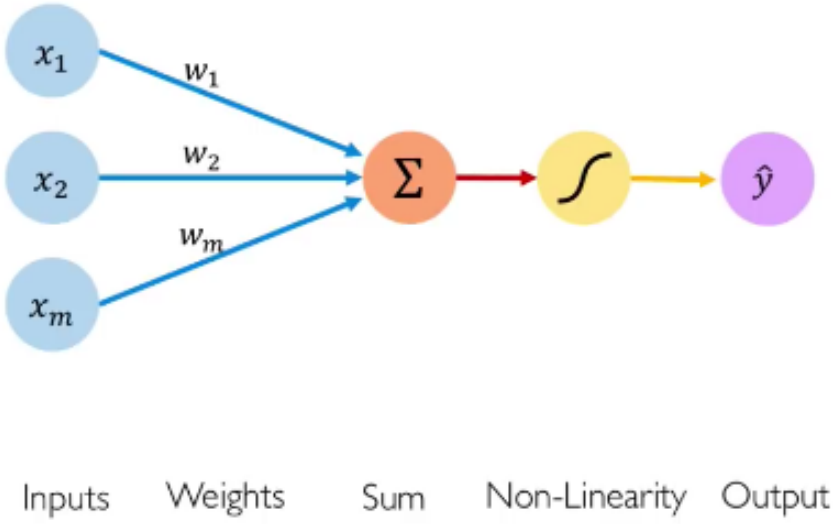

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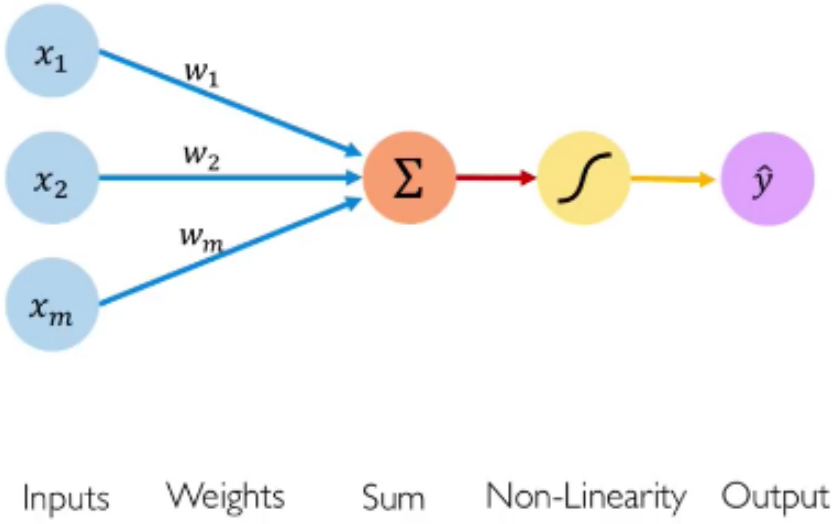

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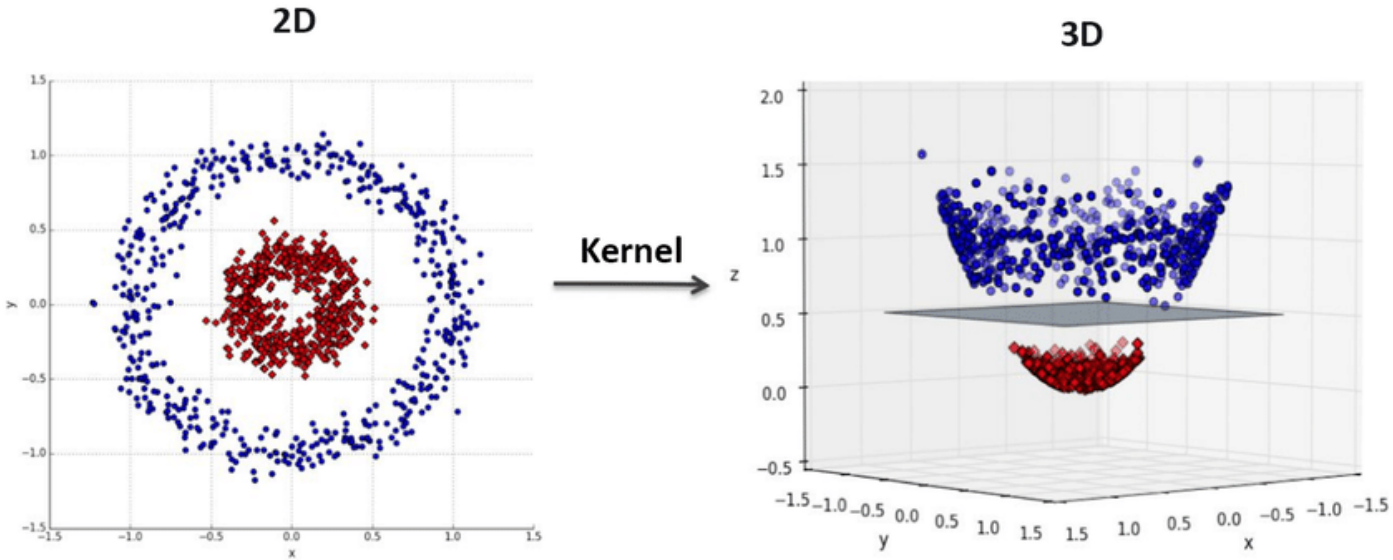
- Models linear ip-op relation or linearly separable boundaries

Cover's Theorem



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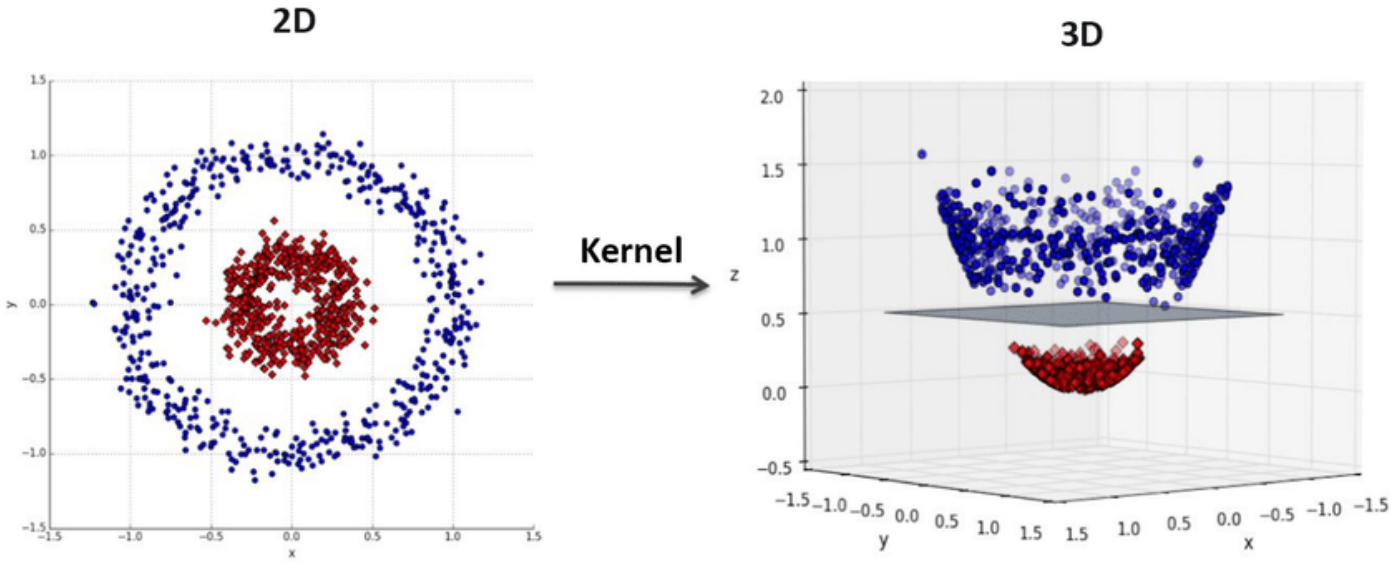

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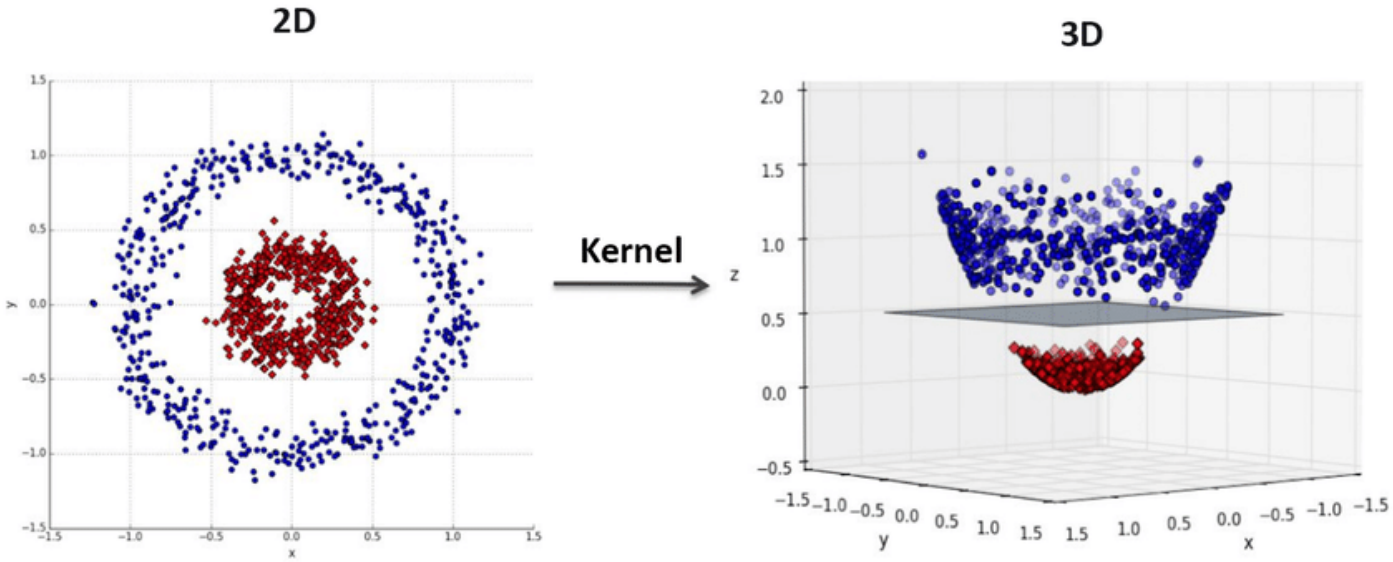

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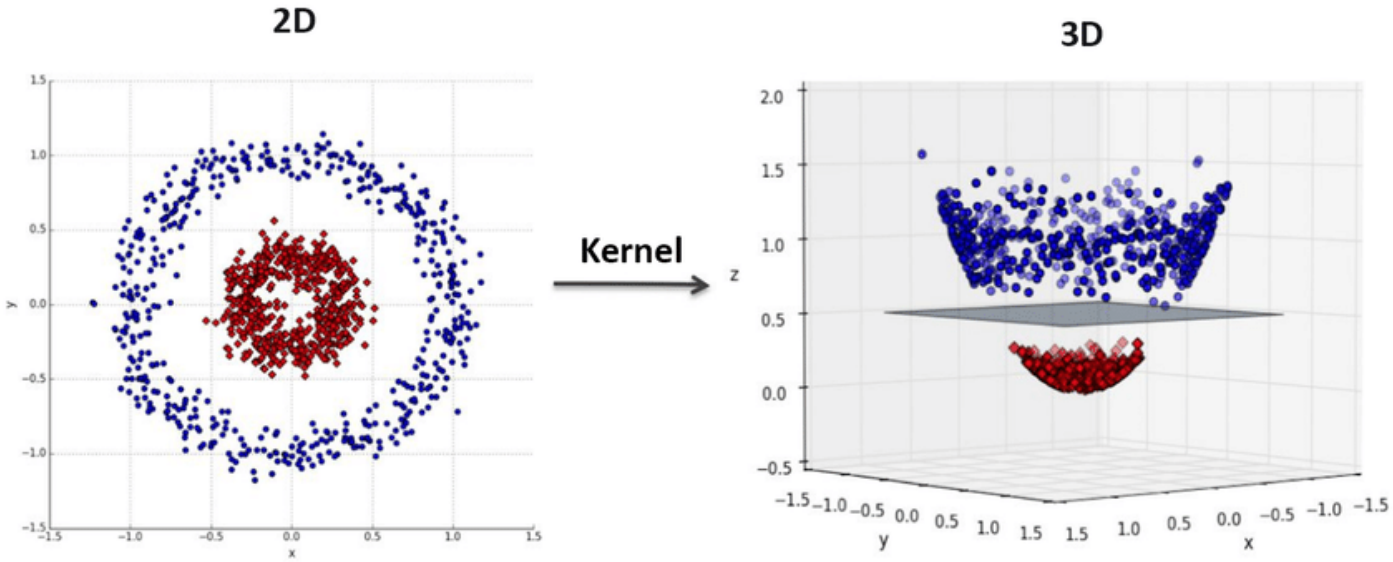

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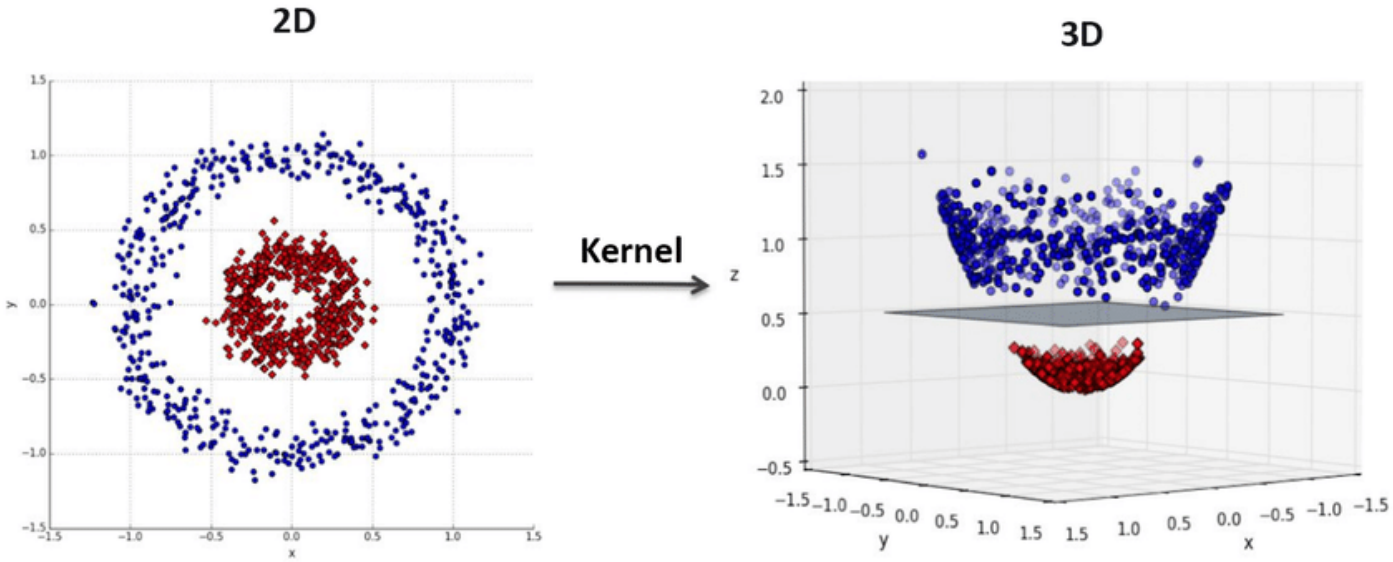

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  - The transformation from input space to feature space is nonlinear
  - The dimensionality of the feature space is high enough
  - $\mathbf{x} = [x_1\ x_2] \rightarrow \phi(\mathbf{x}) = [x_1^2\ x_2^2\ \sqrt{2}x_1x_2]$

Data Independent Transformation

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- Apply linear models in feature space $\phi(\mathbf{x})$:

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$$\hat{t} = y(\phi(\mathbf{x}), \mathbf{w}) = h\left(\mathbf{w}^{\mathsf{T}}\phi(\mathbf{x})\right)$$

- Polynomial kernel of order n: $k(\mathbf{x}, \mathbf{y}) = (\mathbf{x}^{\mathsf{T}}\mathbf{y} + c)^n$
  - For $\mathbf{x} \in \mathbb{R}^2$, $n = 2$ and $c = 1$: $k(\mathbf{x}, \mathbf{y}) = (x_1y_1 + x_2y_2 + 1)^2$
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  - Polynomial kernels are explicit kernels
- Gaussian kernel: $k(\mathbf{x}, \mathbf{y}) = \exp\left(-\frac{\|\mathbf{x}-\mathbf{y}\|^2}{h}\right)$
  - Gaussian kernel is an infinite dimensional kernel
  - Feature representation $\phi(\mathbf{x})$ is not available: *Implicit kernel*
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- There is no reason to believe that the same transformation suits well for all domains - image, speech, medical, forensic, financial etc.,

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- Desired output is estimated as linear combination of nonlinear basis functions of the inputs $\mathbf{x} \in \mathbb{R}^D \rightarrow \boldsymbol{\phi}(\mathbf{x}) \in \mathbb{R}^M$

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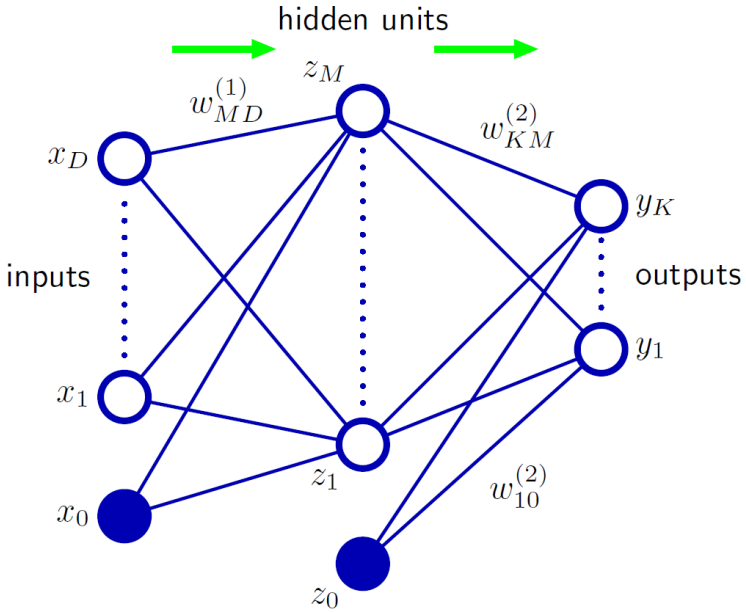
- $h(.)$ can be sigmoid, tanh, relu, softplus etc.,

2-Layer Feed Forward Network



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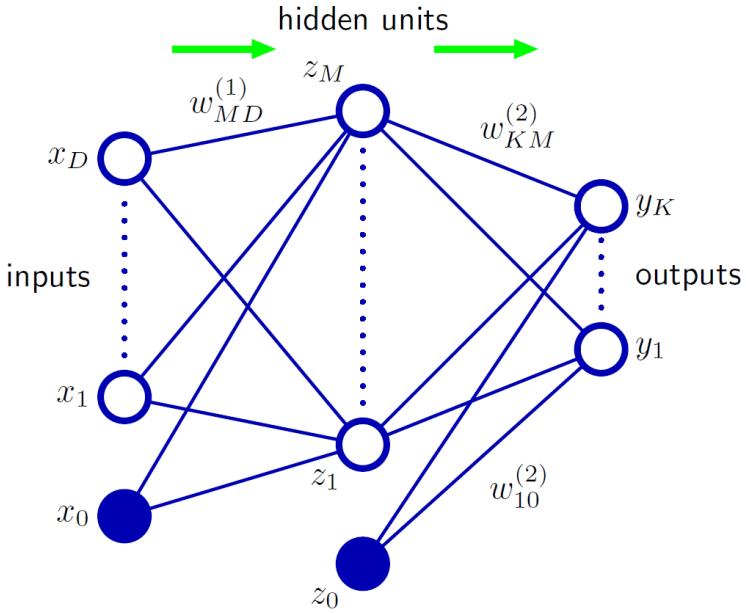

- Input: $\mathbf{x} \in \mathbf{R}^D$
- Hidden: $\mathbf{z} = \phi(\mathbf{x}) \in \mathbf{R}^M$
- Output: $\mathbf{y} \in R^K$
- $w^{(1)}_{ji} : x_i \rightarrow z_j$
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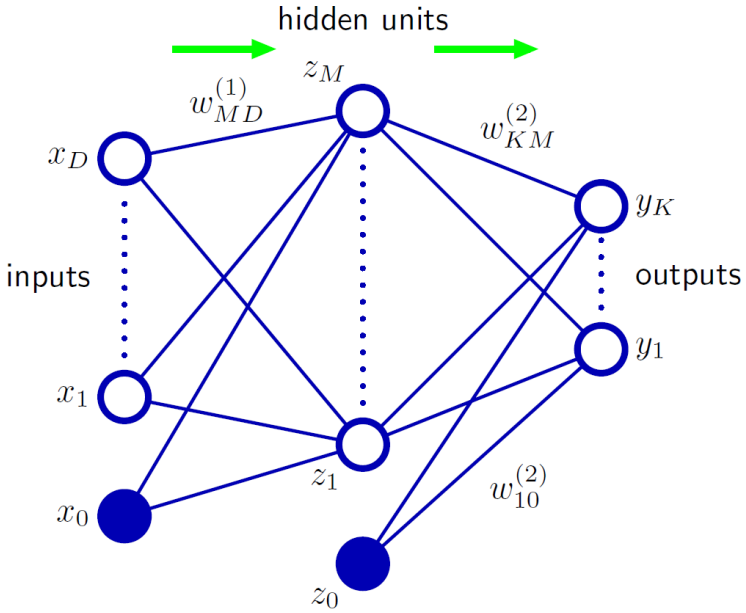

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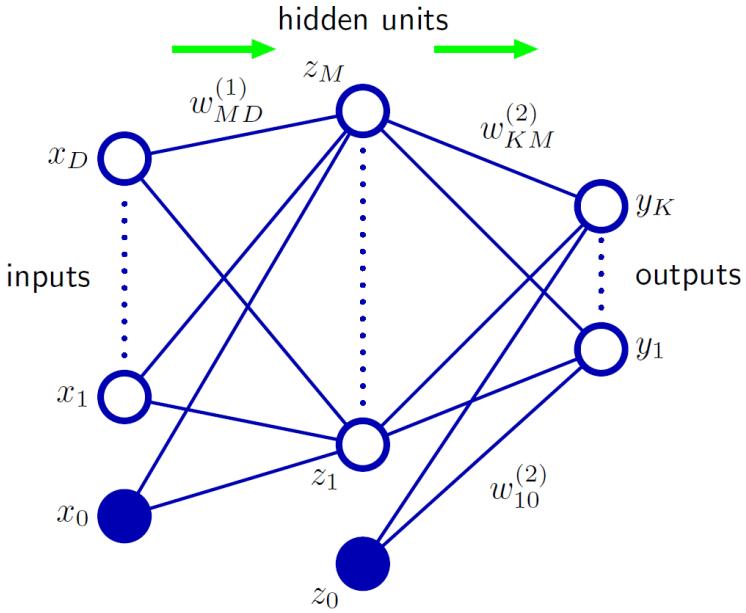

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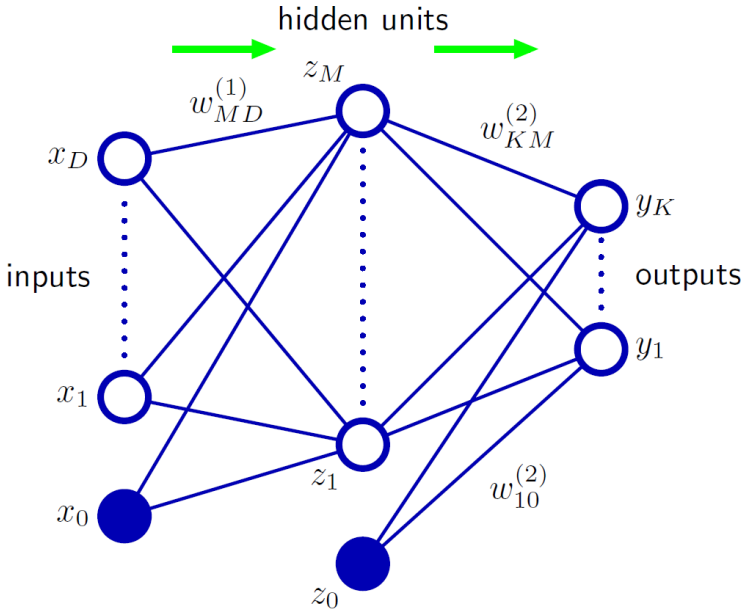

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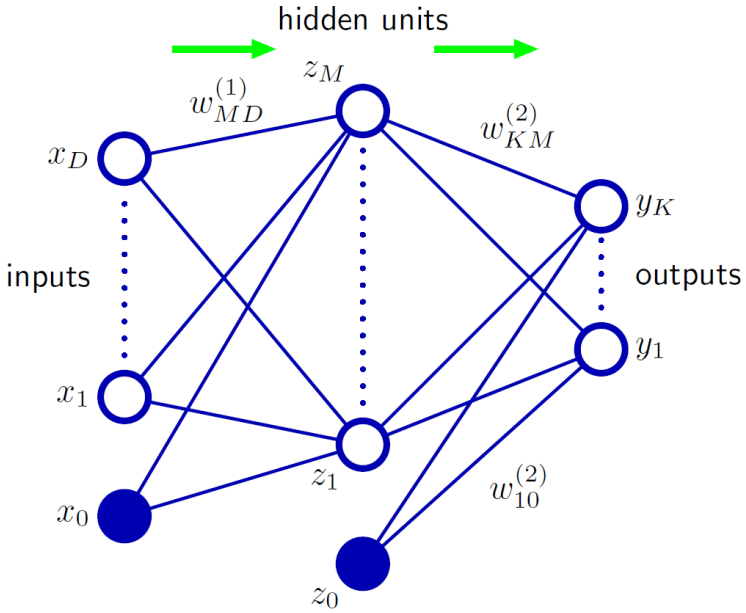

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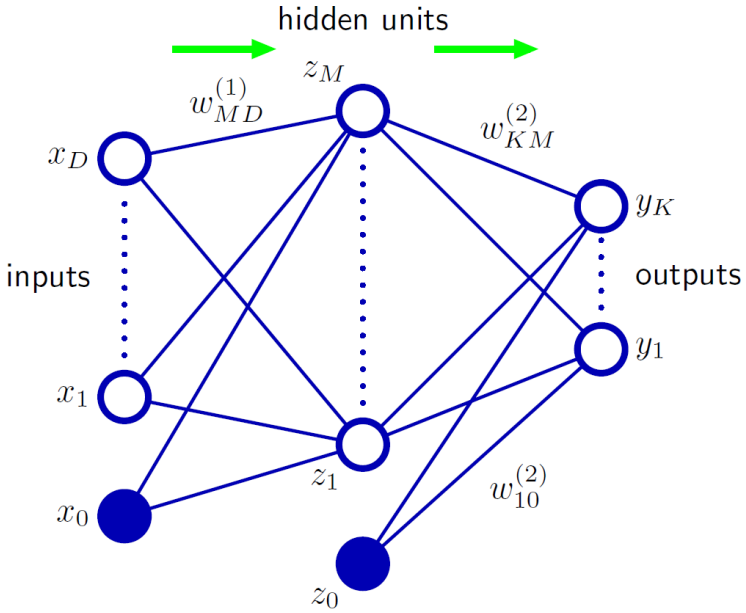

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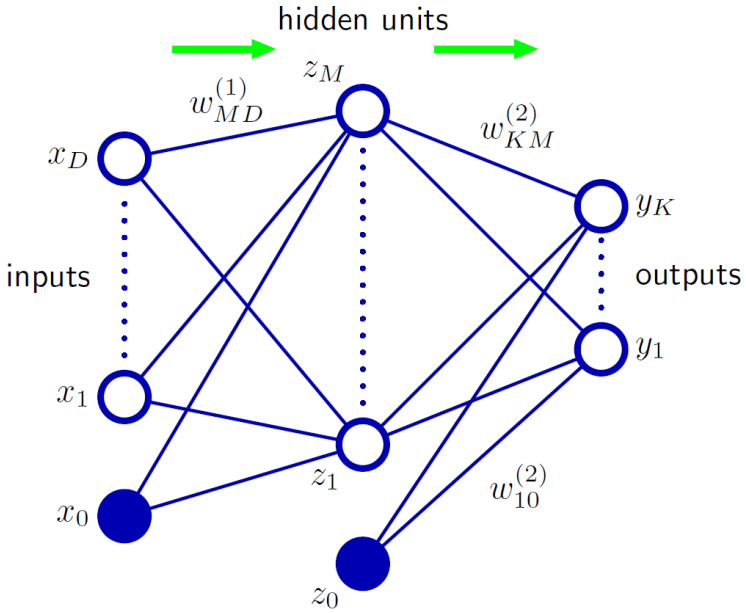

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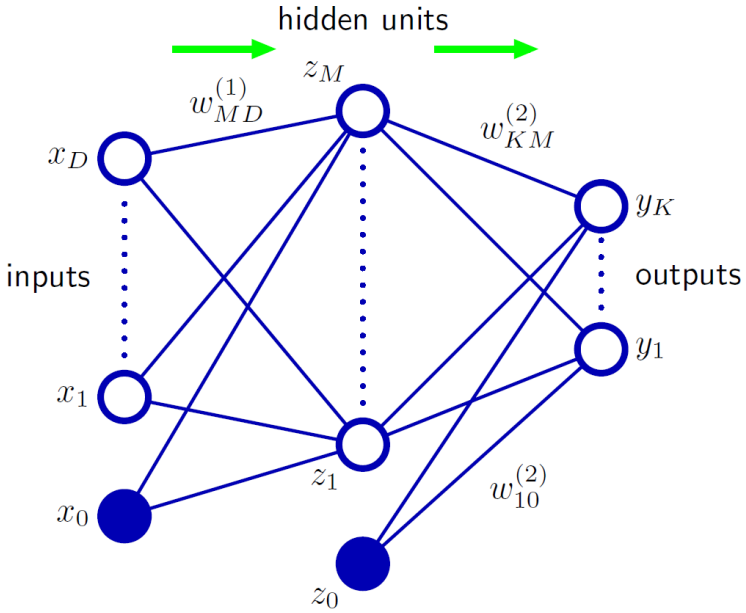

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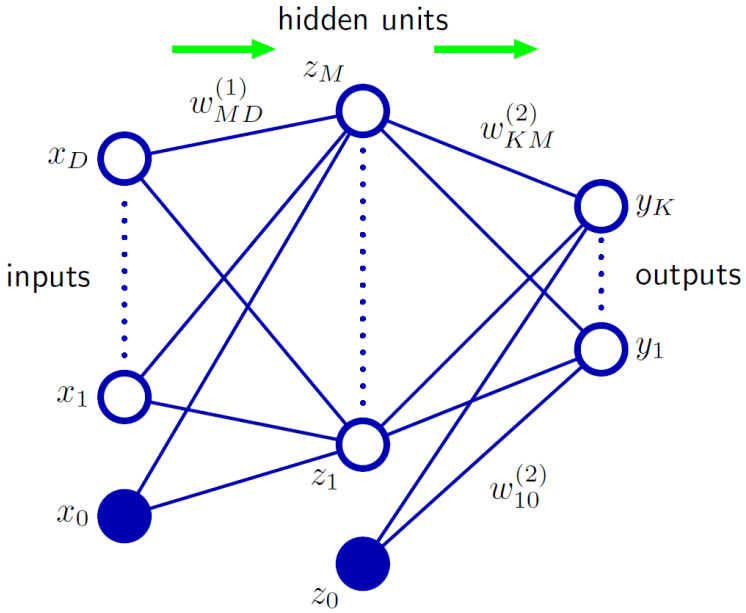
  - Softmax for Multiclass Classification

2-Layer Feed Forward Network



- Input: $\mathbf{x} \in \mathbf{R}^D$
- Hidden: $\mathbf{z} = \phi(\mathbf{x}) \in \mathbf{R}^M$
- Output: $\mathbf{y} \in R^K$
- $w^{(1)}_{ji} : x_i \rightarrow z_j$
- $w^{(2)}_{kj} : z_j \rightarrow y_k$

- At j^{th} hidden unit:

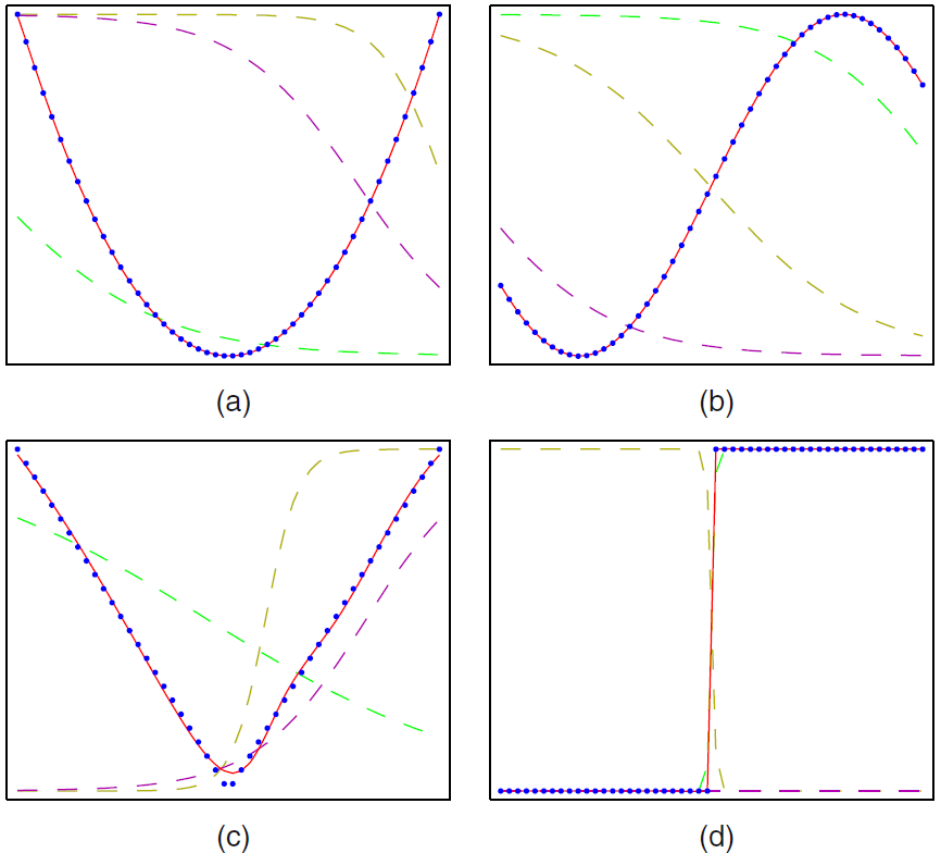
$$a_j = \sum_{i=0}^{D} w^{(1)}_{ji} x_i \qquad z_j = h(a_j)$$

- $h(.)$ is a nonlinear function
- At k^{th} output unit:

$$a_k = \sum_{i=0}^{M} w^{(2)}_{kj} z_j \qquad y_k = f(a_k)$$

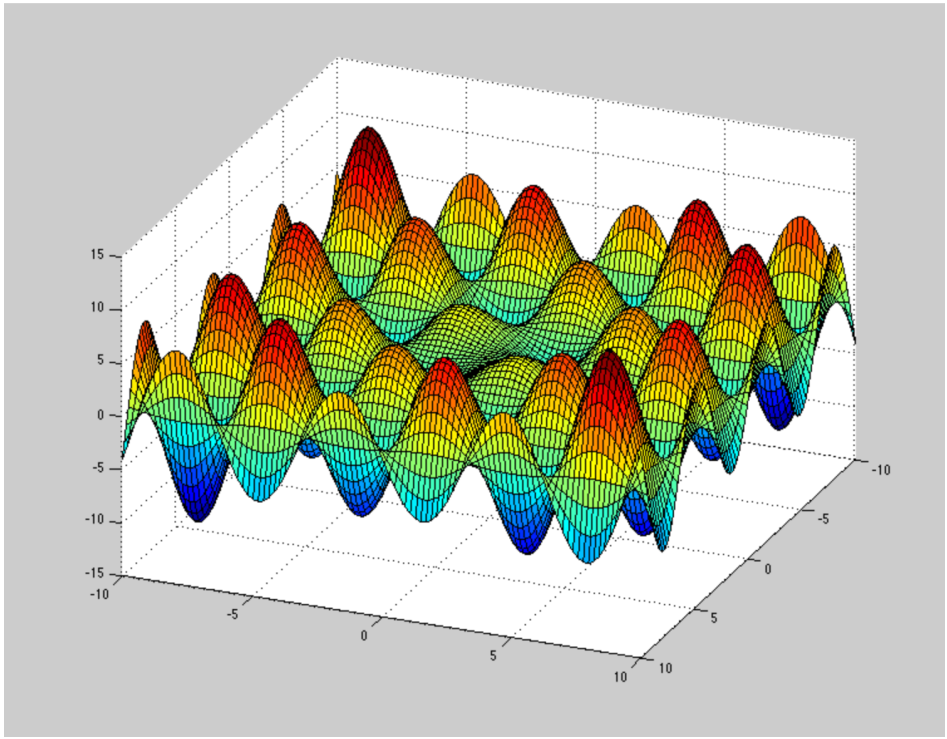
- Choice of $f(.)$ depends on task
  - Linear for Regression
  - Sigmoid for Binary Classification
  - Softmax for Multiclass Classification
- $y_k = f\left(\sum_{i=0}^{M} w^{(2)}_{kj} h\left(\sum_{i=0}^{D} w^{(1)}_{ji} x_i\right)\right)$

Universal Approximator



- (a) $f(x) = x^2$, (b) $f(x) = \sin(x)$ (c) $f(x) = |x|$, (d) $f(x) = sign(x)$
- One hidden layer with 3 tanh(.) units

Weight Space Symmetry



- Error $J(\mathbf{W}) = \mathbb{E}[\|\mathbf{t} - \mathbf{y}\|]$ is nonconvex in $\mathbf{W} = [\mathbf{W}^{(1)}\ \mathbf{W}^{(2)}]$.
- There are $2^M\ M!$ symmetric points with the same error
  - Order of neuronal units in hidden layer does not matter: $M!$
  - Weights leading to and going out of a hidden unit can be negated: 2^M
- Architecture, nonlinearity, loss function and dataset

Parameter Estimation

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- Network weights $\mathbf{W} = [\mathbf{W}^{(1)}\ \mathbf{W}^{(2)}]$ have to be adjusted to minimize

$$J(\mathbf{W}) = \sum_{n=1}^{N} J_n(\mathbf{W})$$

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- Gradients $\frac{\partial J(\mathbf{W})}{\partial w_{ji}^{(1)}}$ and $\frac{\partial J(\mathbf{W})}{\partial w_{kj}^{(2)}}$ are computed using error backpropagation

Backpropagation

Forward pass input $\mathbf{x}$

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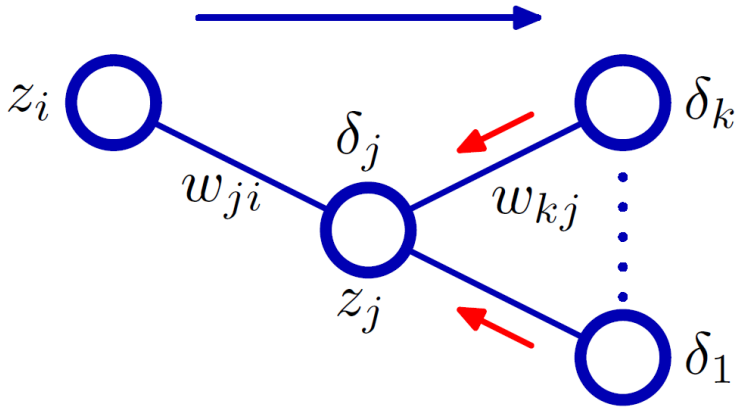

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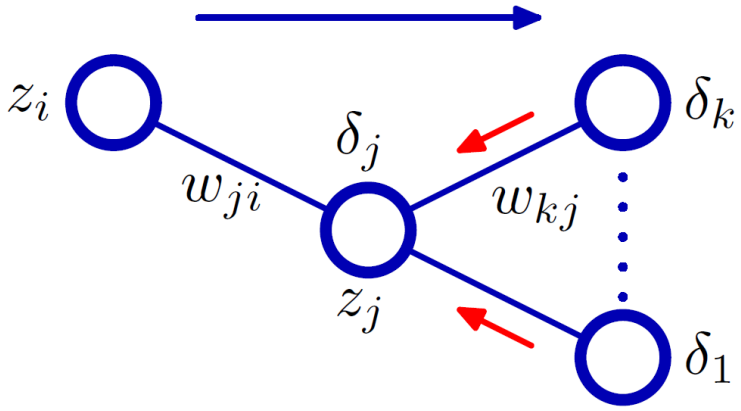

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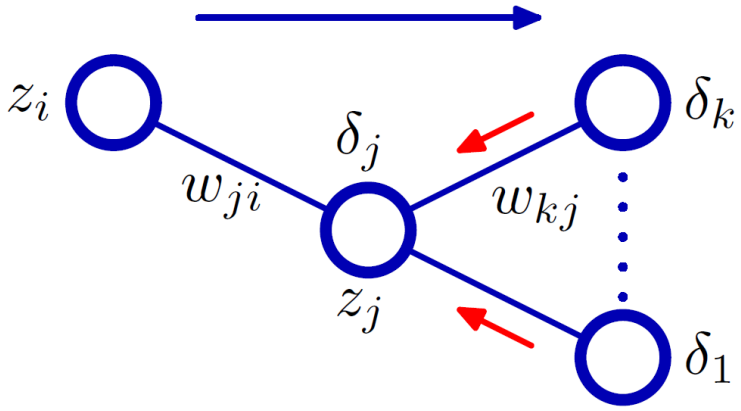

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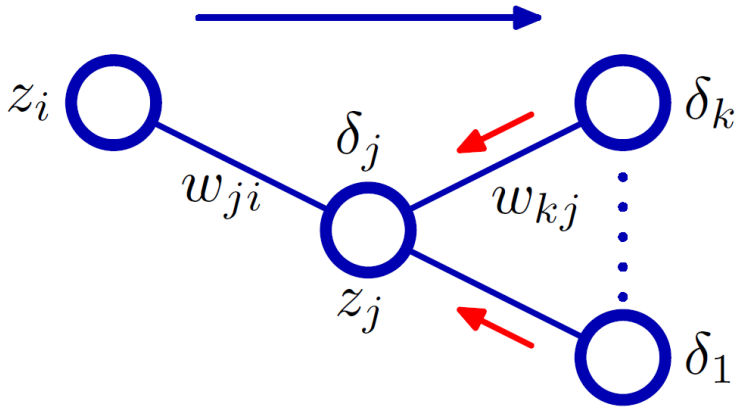

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- $\mathcal{B}$ denotes a random mini-batch of samples drawn from dataset.

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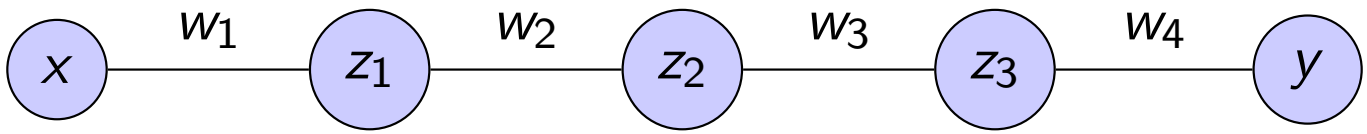
  - Weight space dynamics is influenced by weight initialization, batch size, order of presentation, learning rate schedule.

Vanishing Gradients



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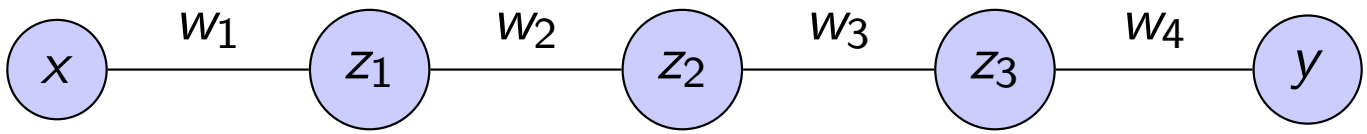

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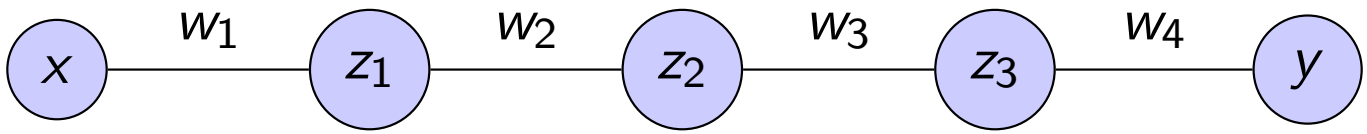

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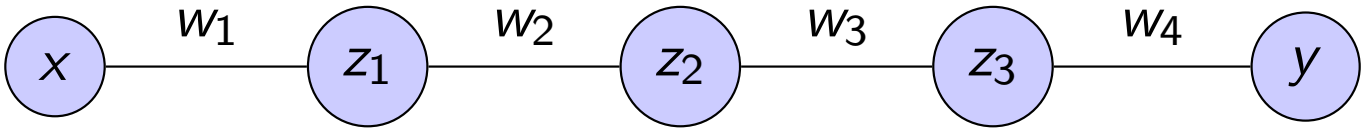

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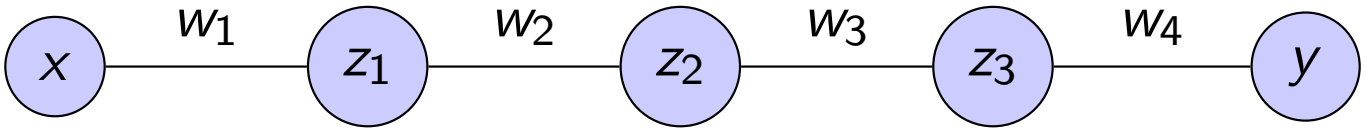

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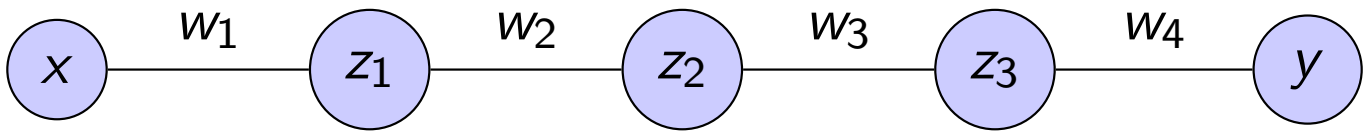

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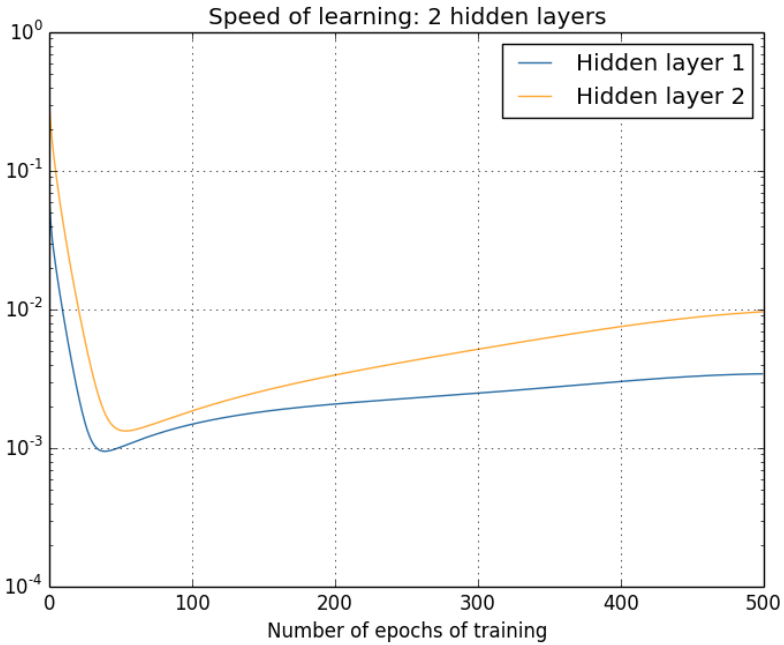
$$= (y-t)\, w_4\, h'(a_3)\, w_3\, h'(a_2)\, w_2\, h'(a_1)\, x$$

- Initial layer gradients are much smaller

Training Speed



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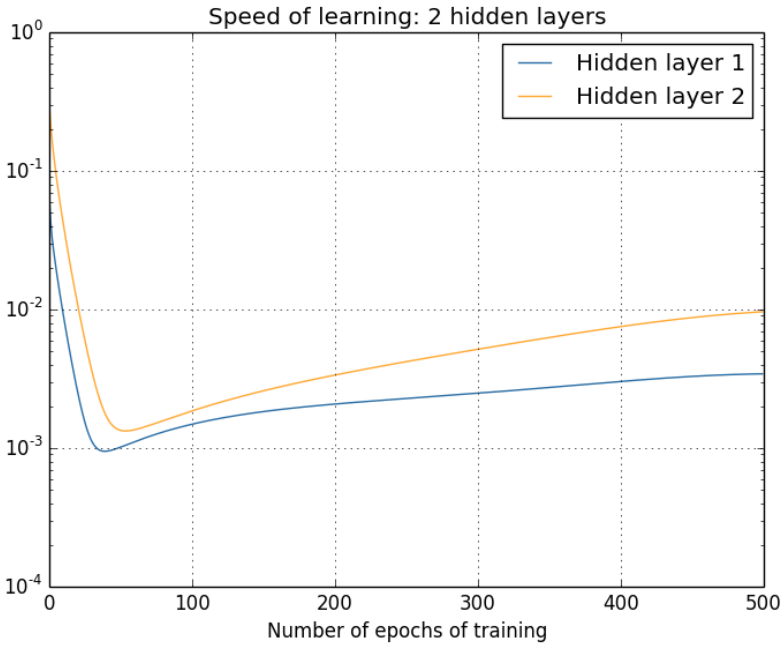

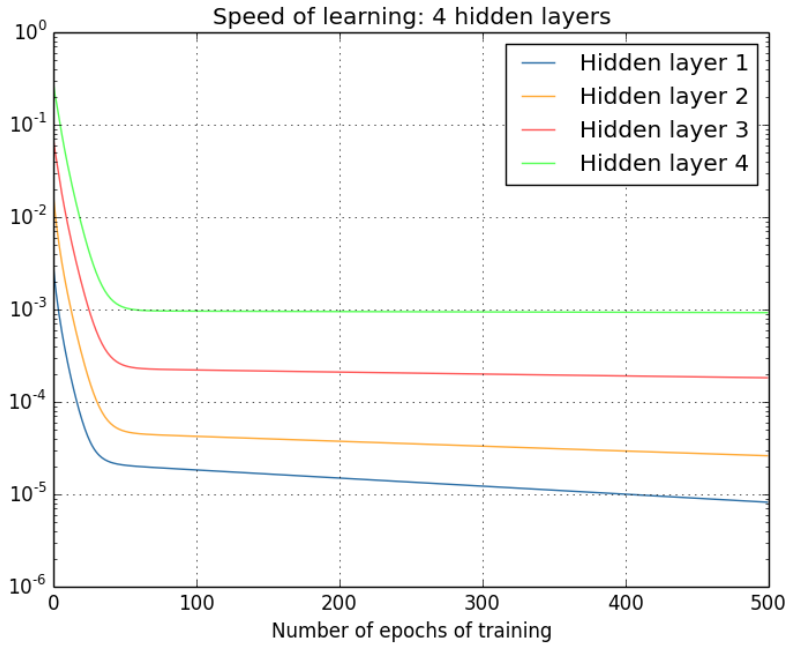

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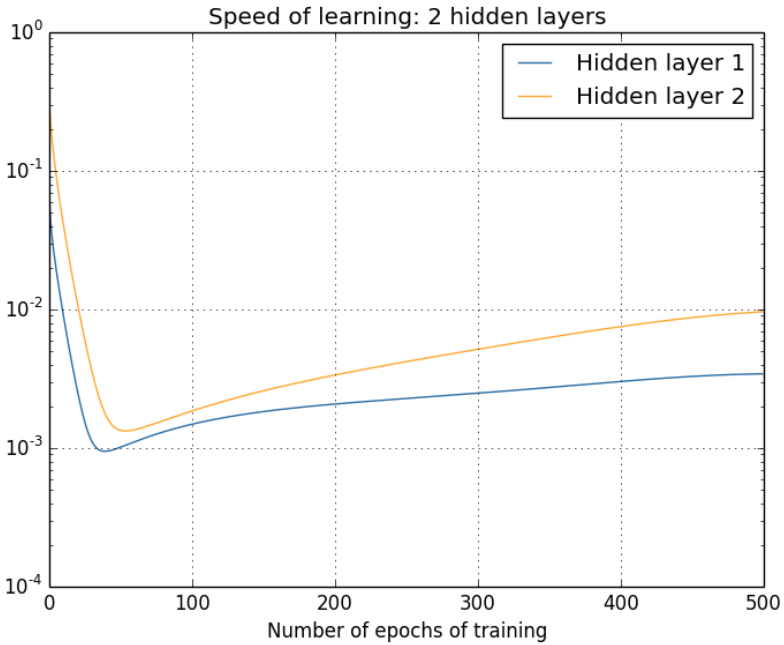
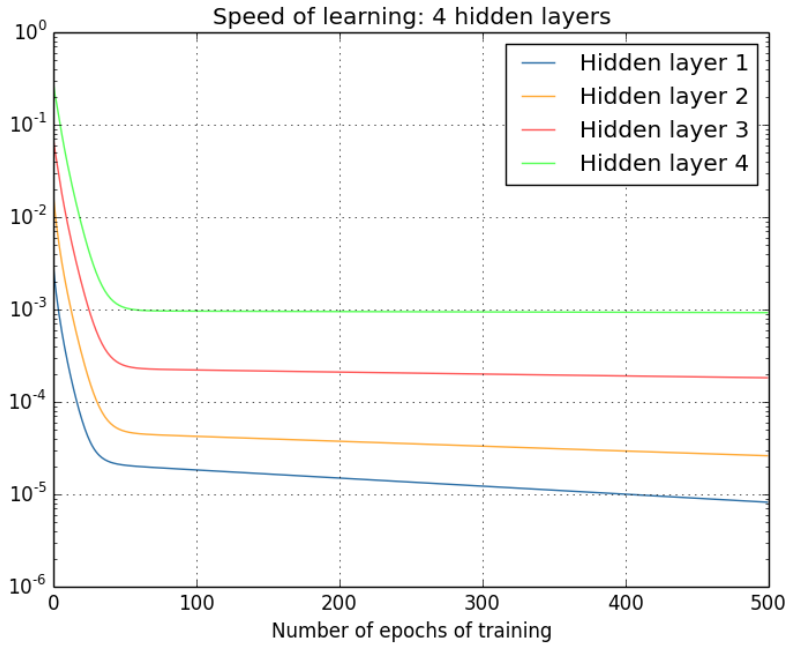
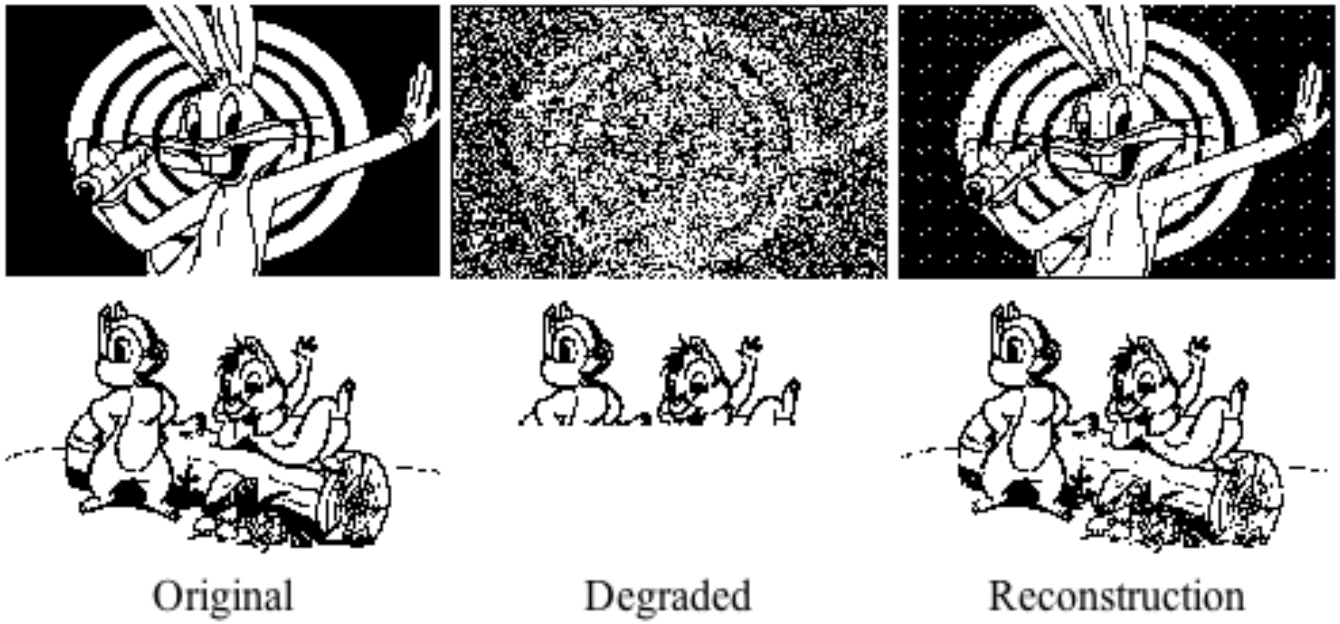




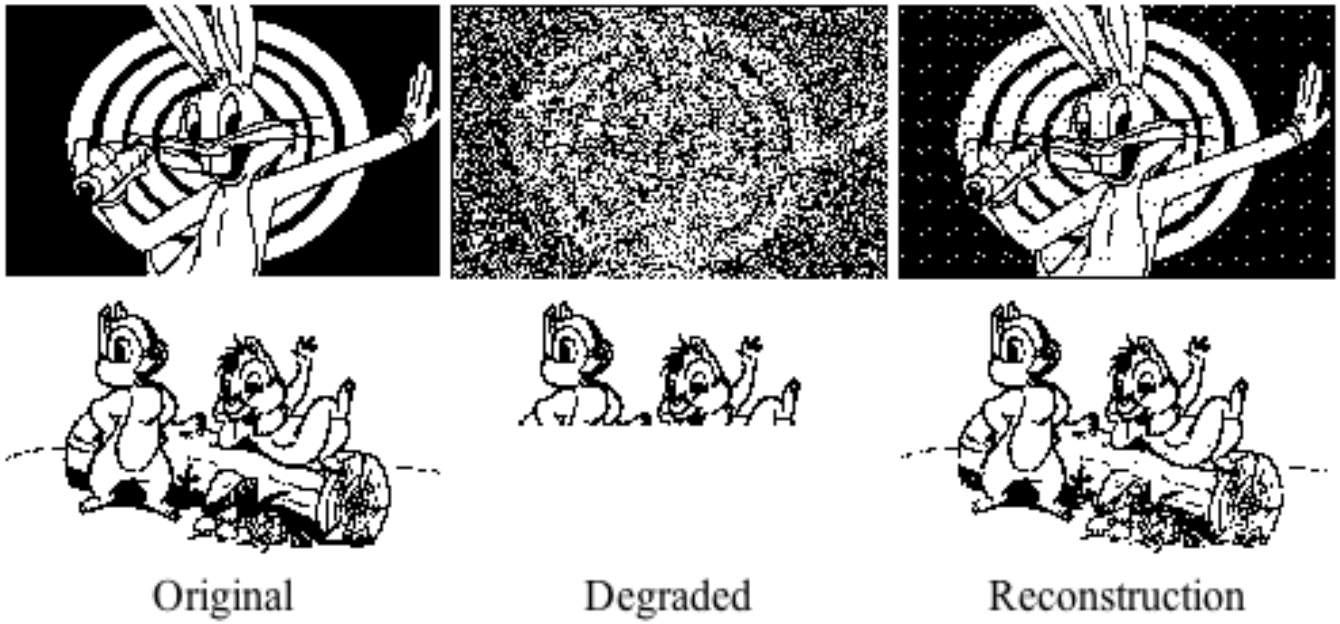
- Training speed is quantified using the norm of the weight update ΔW
- The updates are much smaller for initial layers - hence not trained
- The representations supplied to the deeper layers are not reliable

Pattern Recall - Energy-Based Models



Original Degraded Reconstruction

Pattern Recall - Energy-Based Models



- A good representation should capture correlations in the input

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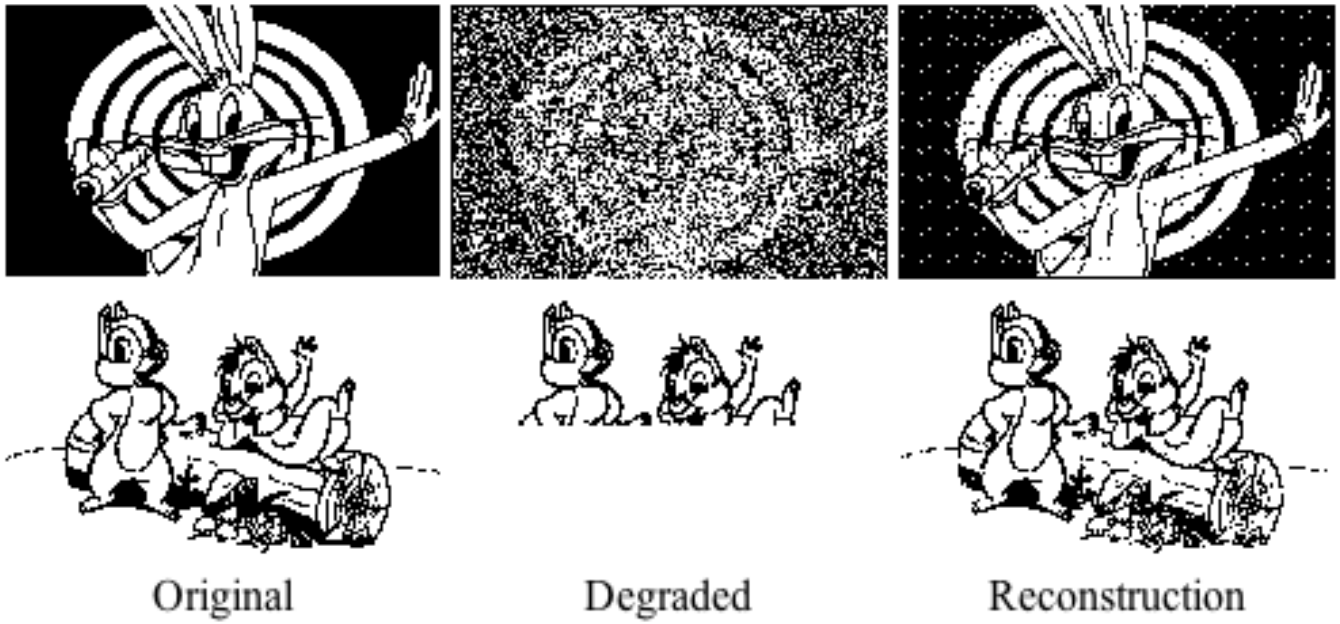

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  - Natural patterns admits much lower dimensional representations

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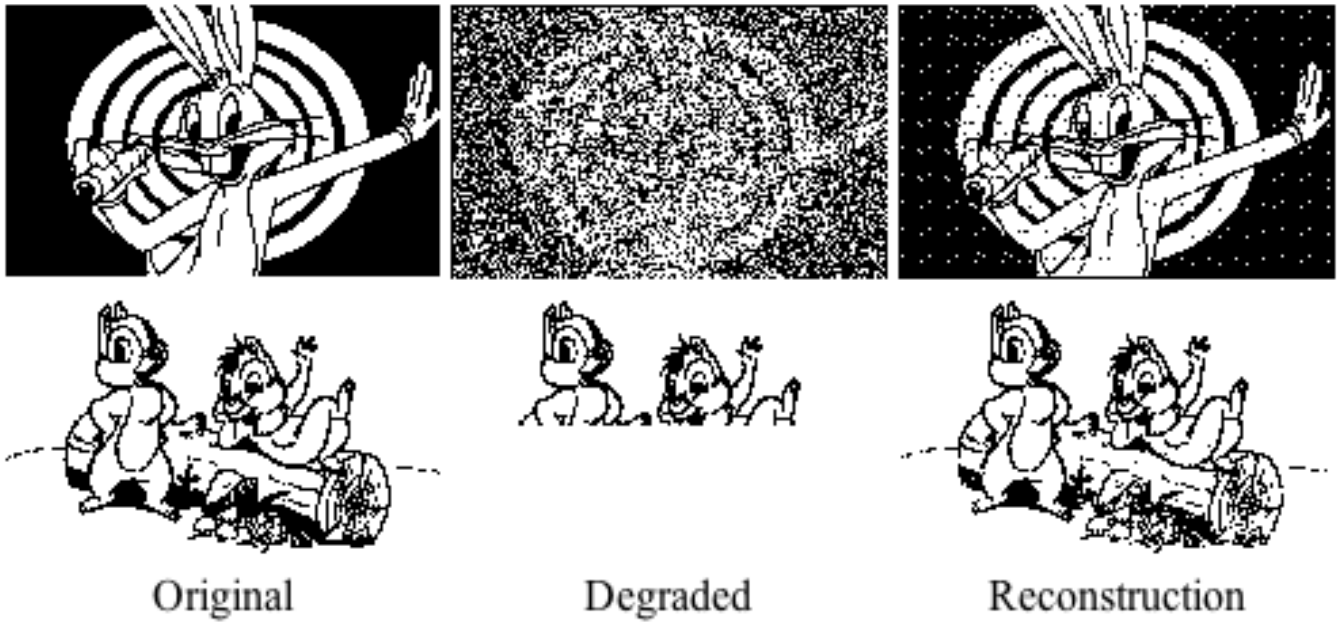

- A good representation should capture correlations in the input
  - Natural patterns admits much lower dimensional representations
  - Should be able to recall the pattern from partial/degraded input

Hopfiled Network

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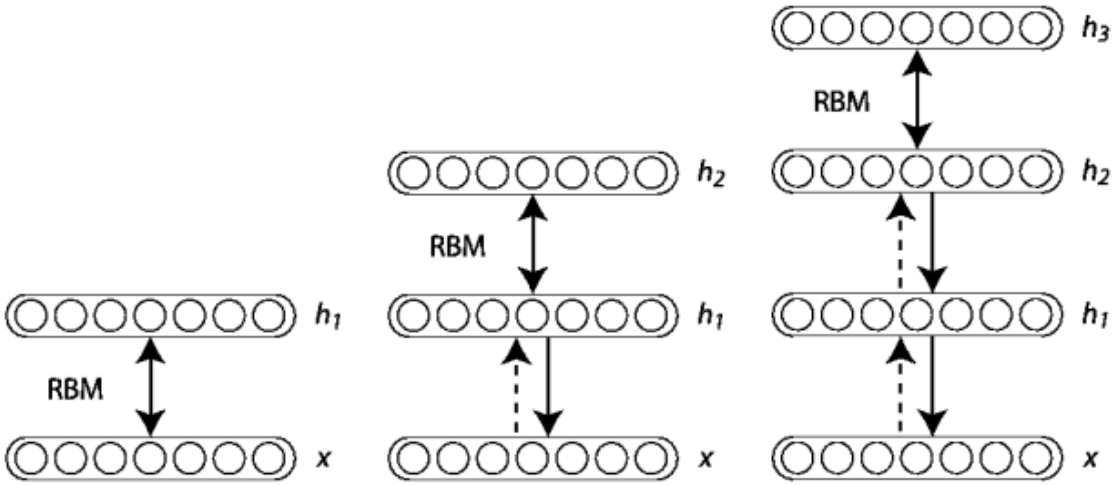
- Hardball can get stuck in the local minima of the energy landscape

Restricted Boltzmann Machine



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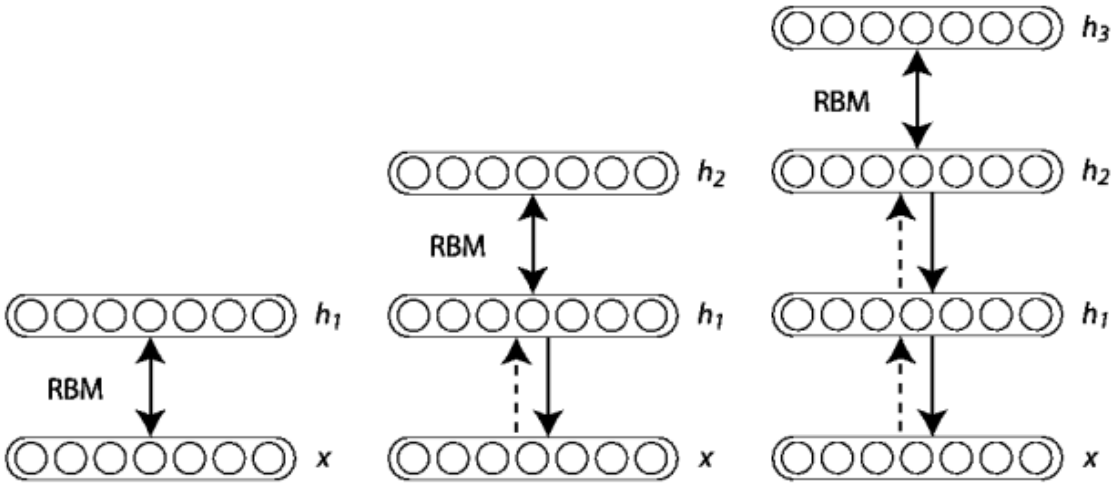

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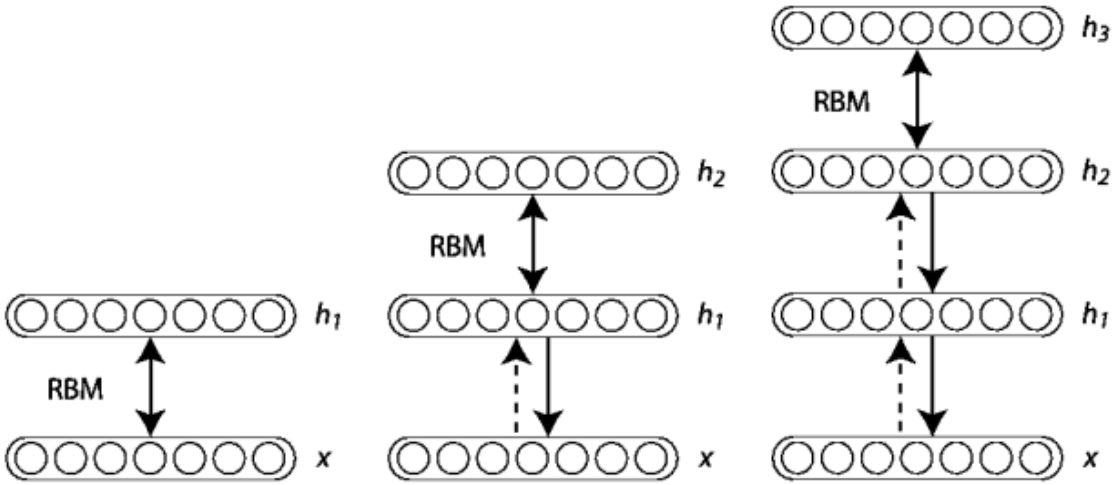

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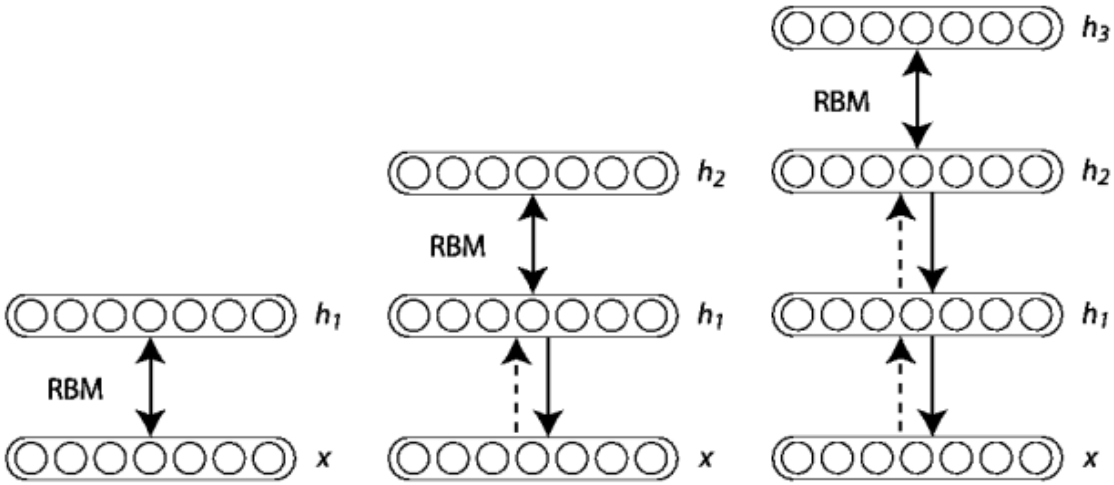

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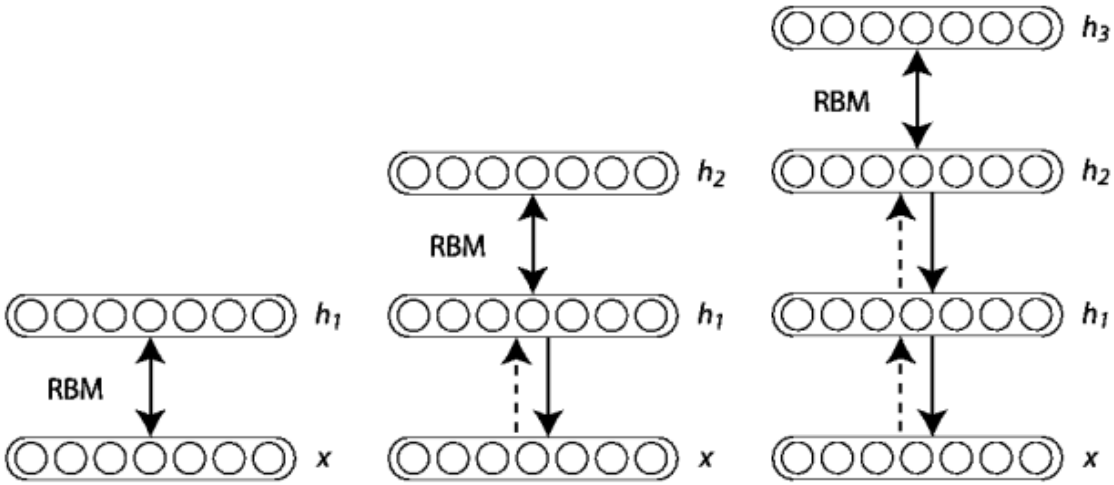

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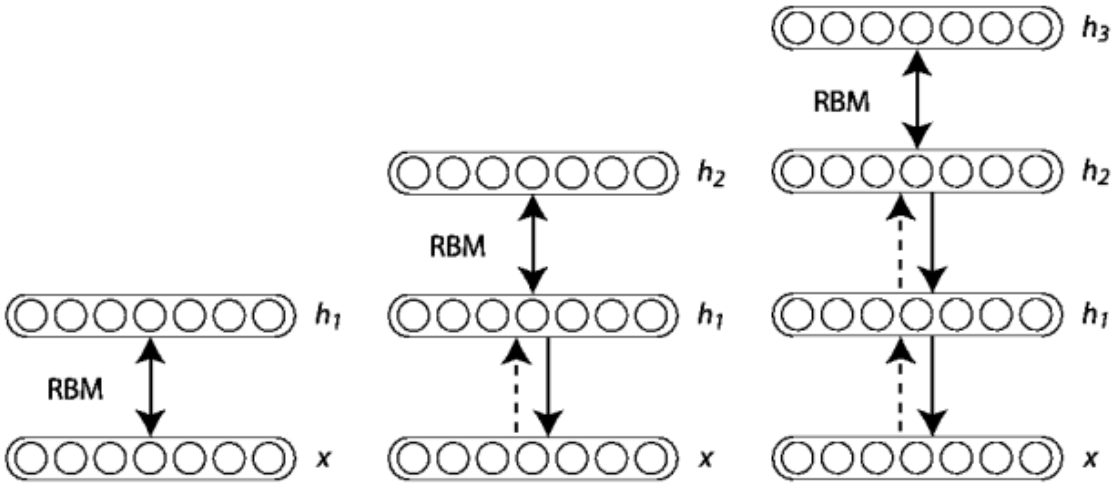

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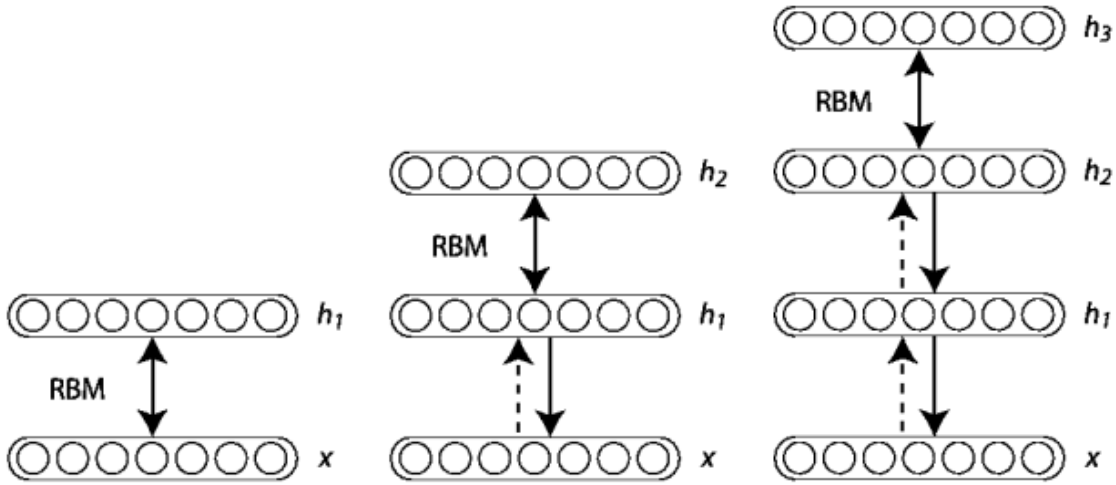

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- Estimate $\mathbf{W}$ to maximize $p(\mathbf{v}) = \sum_{\mathbf{h}} p(\mathbf{v}, \mathbf{h})$
- Contrastive divergence is used to estimate the parameters

Unsupervised Pretraining

- Unsupervised pretraining forces model close to $p(\mathbf{x})$ (Hinton 2006)
- Representations good for $p(\mathbf{x})$ are good for $p(\mathbf{t}/\mathbf{x})$?
- Initializes near better local minima for $p(\mathbf{t}/\mathbf{x})$?

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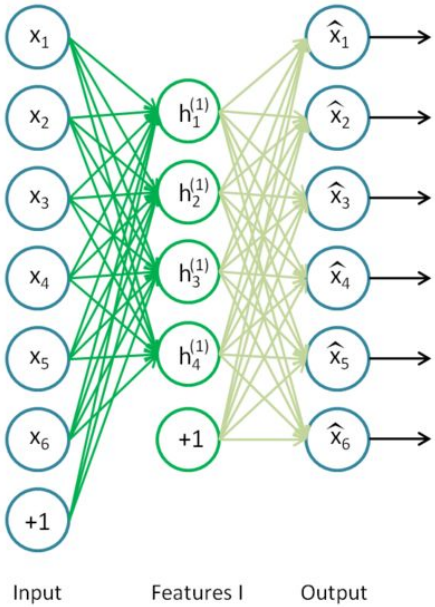
- Initializes near better local minima for $p(\mathbf{t}/\mathbf{x})$?
- Offers easy layer-wise training with local criterion
- Large amounts of unlabeled data can be used in pretraining
- Achieved significant improvements in speech recognition (Hinton 2008)

Pretraining using Autoencoders



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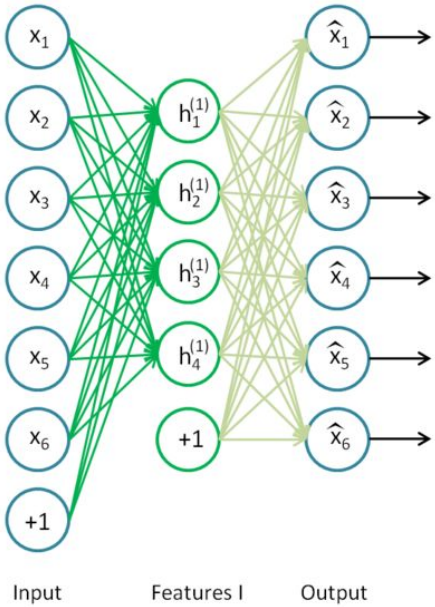

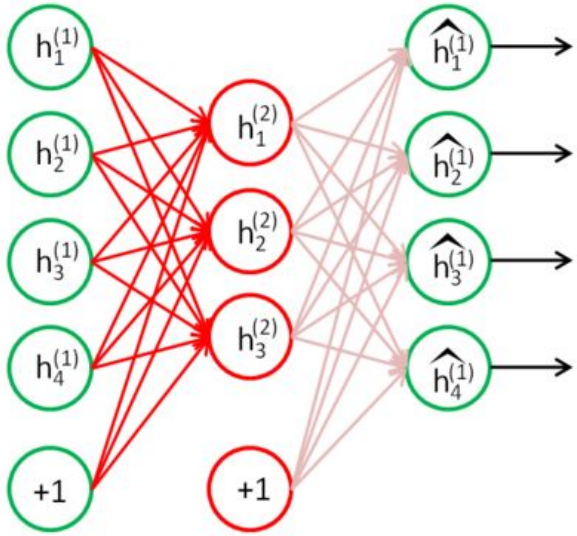

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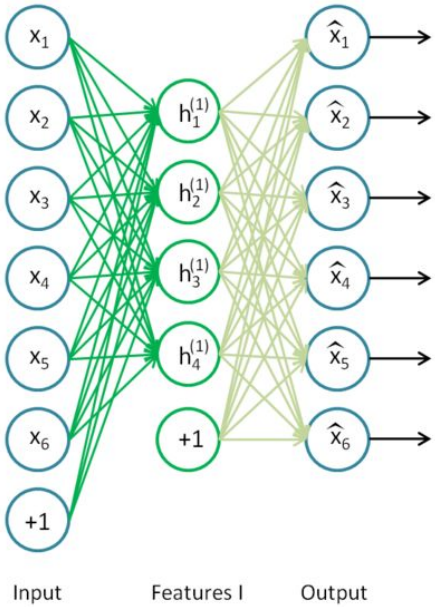

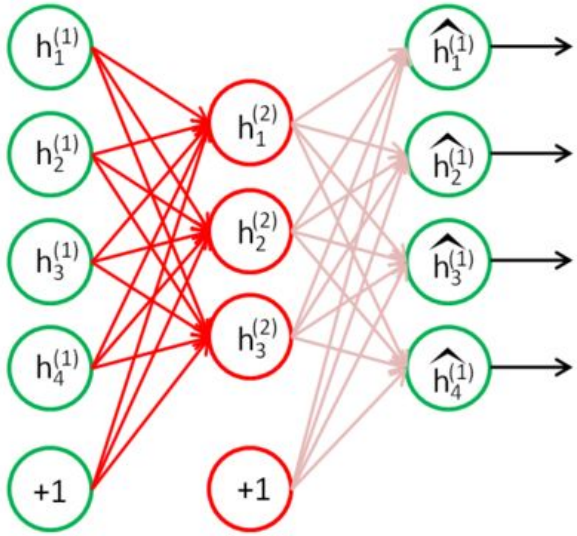

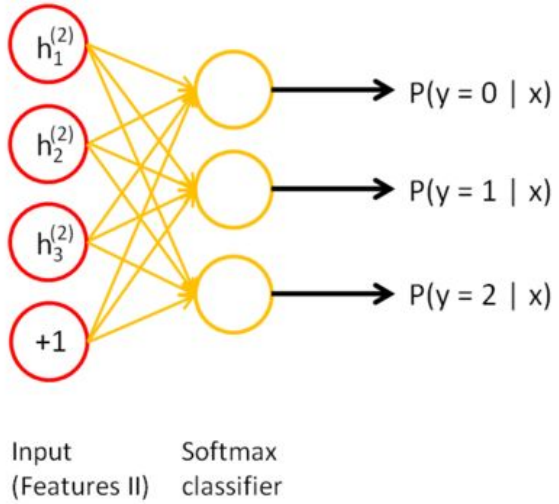

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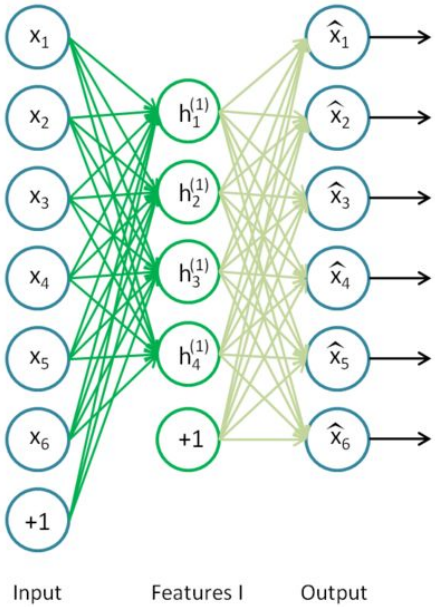
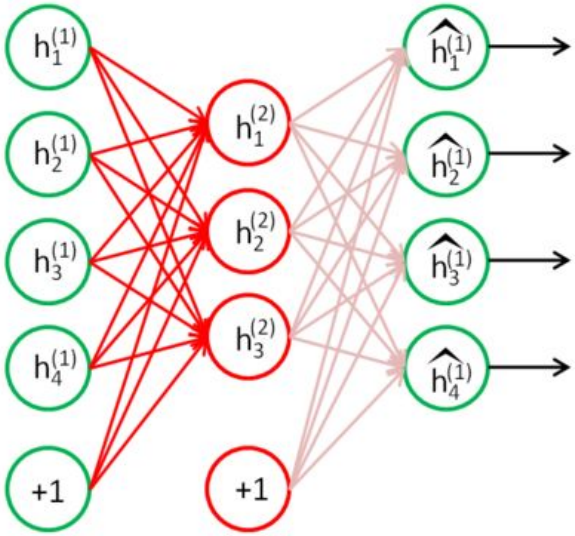
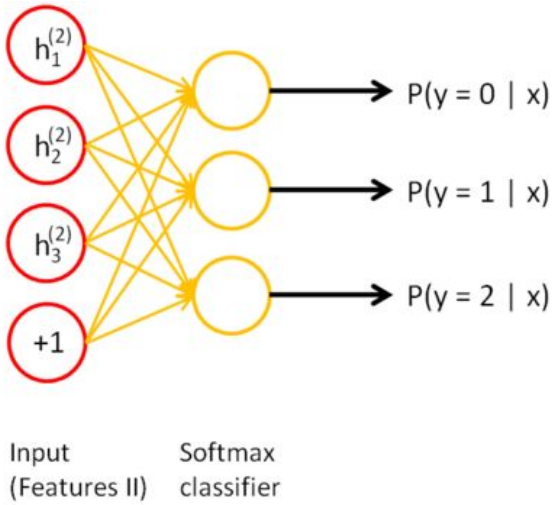
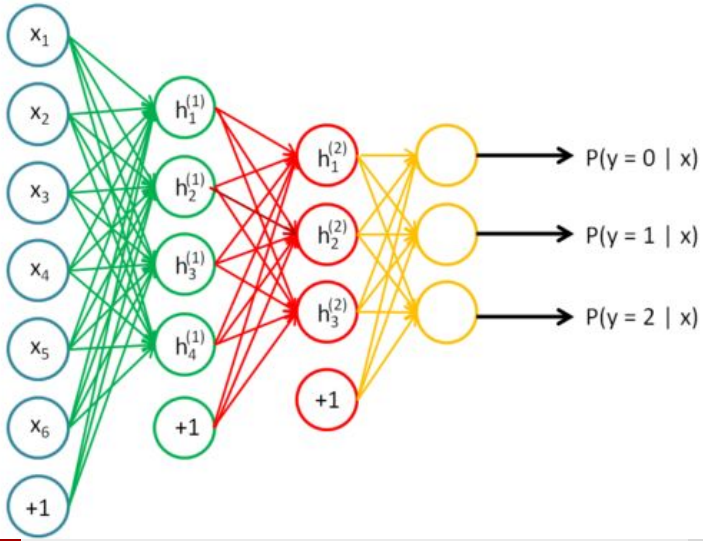
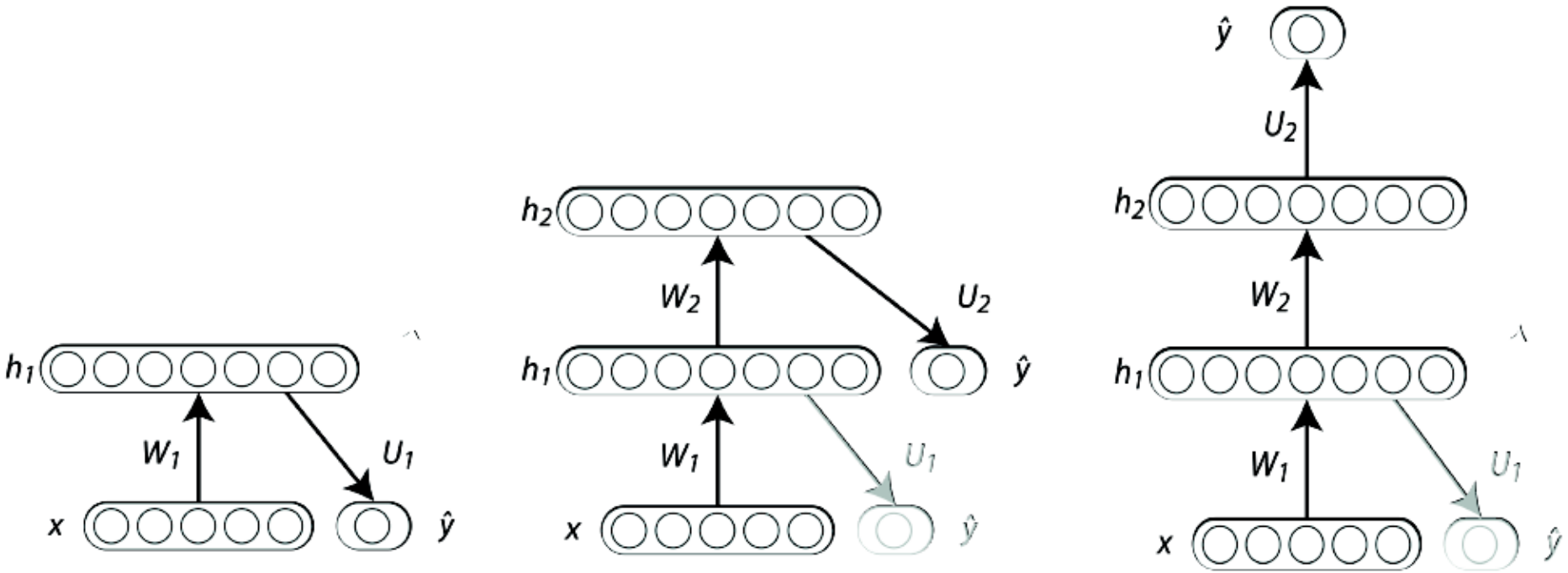








Supervised Pretraining



- Generally performs worse than unsupervised pretraining, but better than random initialization of DNN

Derivatives of Activation Functions

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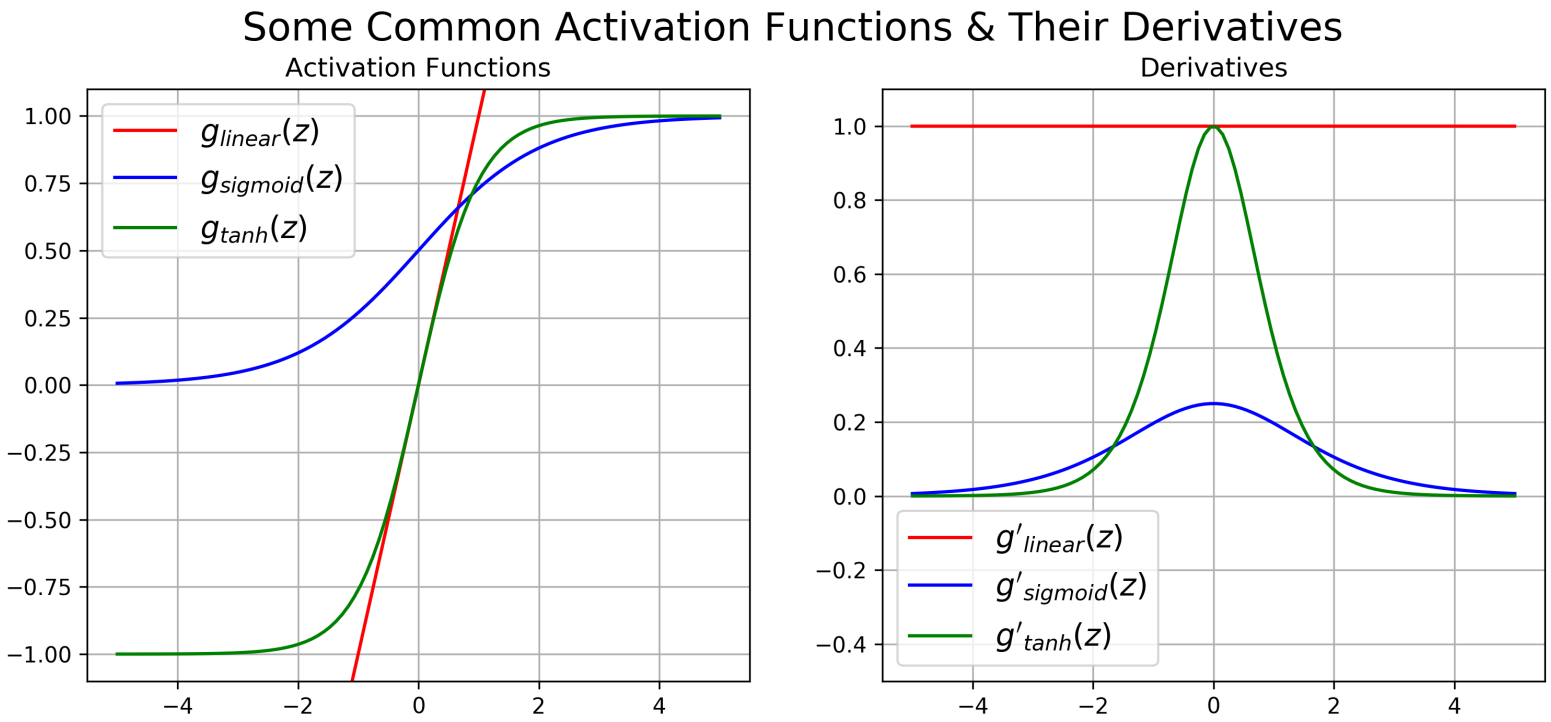

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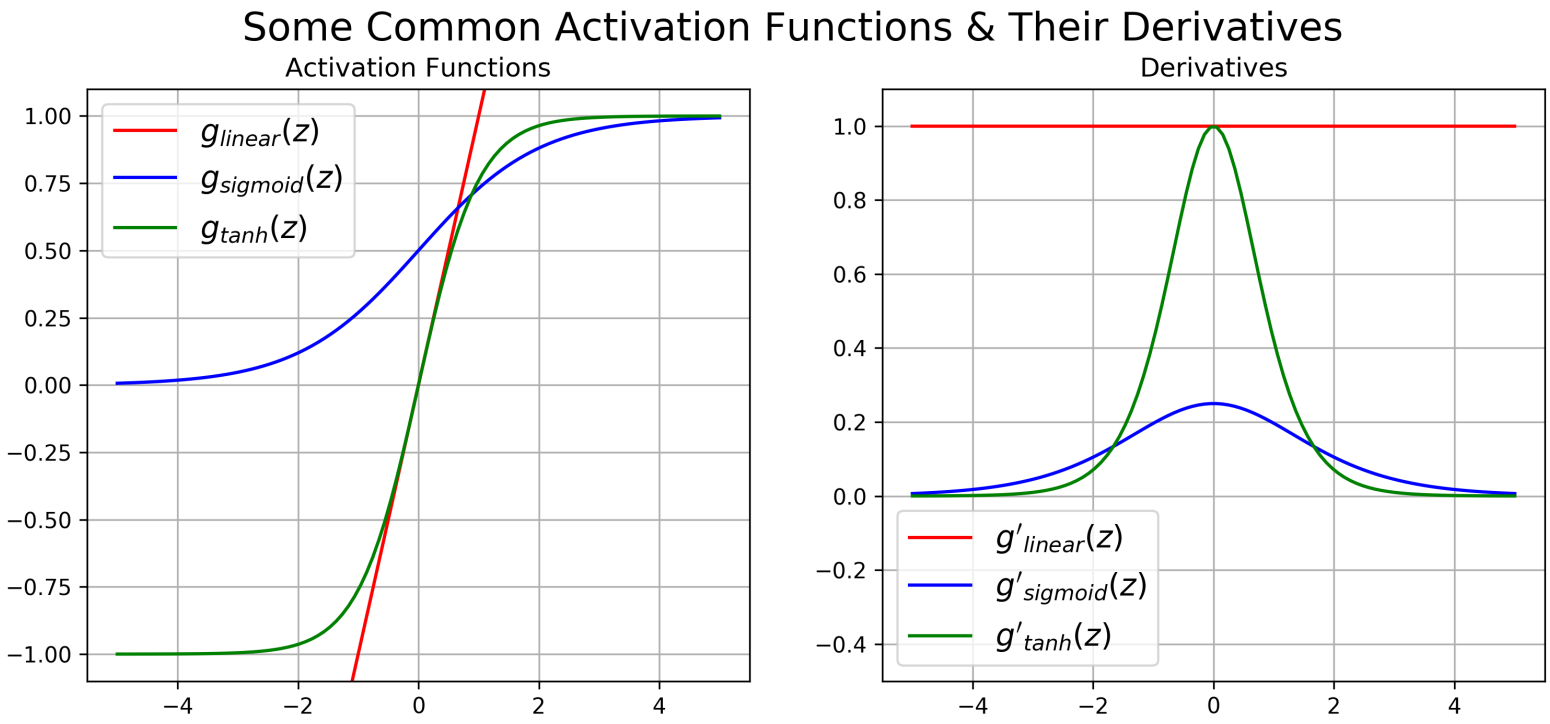
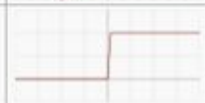
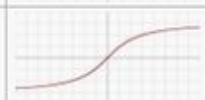
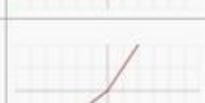
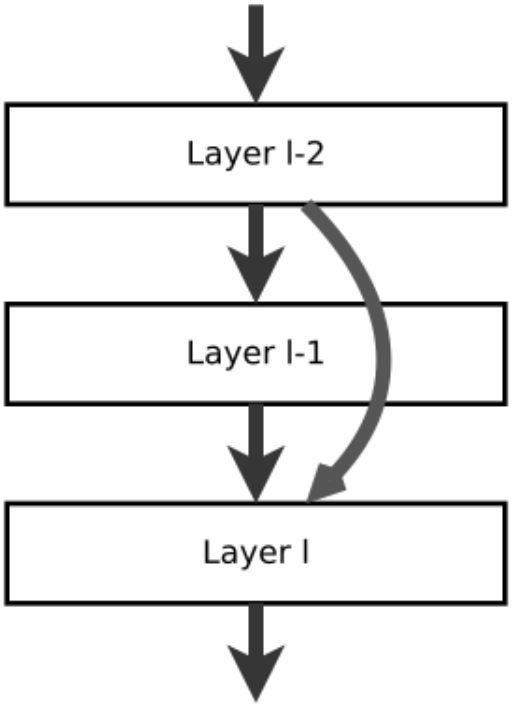


- The magnitude of derivatives of tanh(.) and $\sigma(.)$ are less than 1
- Deep cascade of such activation layers lead to vanishing gradients
- ReLU address this issue as it offers unity gradient for +ve values

Choice of Nonlinearity

Name	Plot	Equation	Derivative
Identity		$f(x) = x$	$f'(x) = 1$
Binary step		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ ? & \text{for } x = 0 \end{cases}$
Logistic (a.k.a Soft step)		$f(x) = \frac{1}{1+e^{-x}}$	$f'(x) = f(x)(1-f(x))$
TanH		$f(x) = \tanh(x) = \frac{2}{1+e^{-2x}} - 1$	$f'(x) = 1 - f(x)^2$
ArcTan		$f(x) = \tan^{-1}(x)$	$f'(x) = \frac{1}{x^2+1}$
Rectified Linear Unit (ReLU)		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Parameteric Rectified Linear Unit (PReLU)[2]		$f(x) = \begin{cases} \alpha x & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} \alpha & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Exponential Linear Unit (ELU)[3]		$f(x) = \begin{cases} \alpha(e^x - 1) & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} f(x) + \alpha & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
SoftPlus		$f(x) = \log_e(1+e^x)$	$f'(x) = \frac{1}{1+e^{-x}}$

Residual Networks (ResNet)



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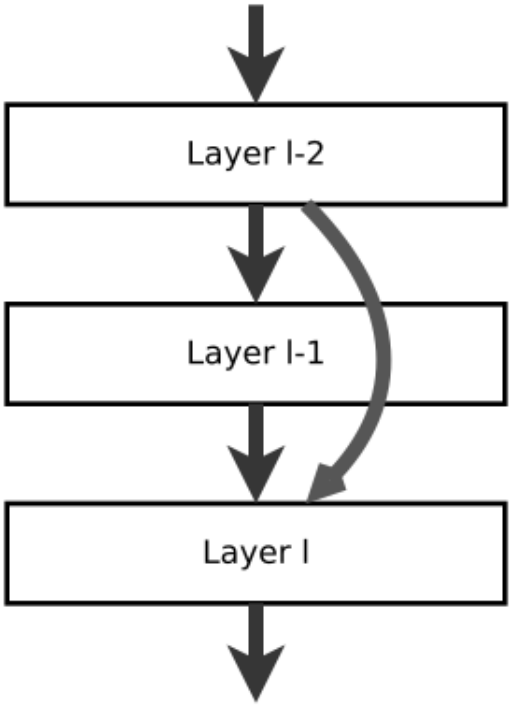

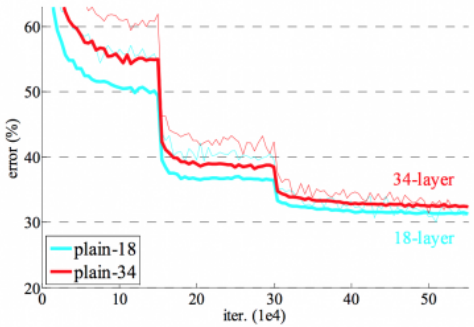

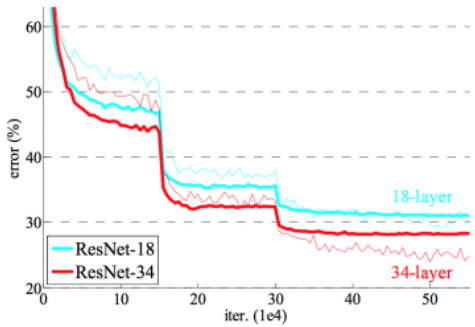

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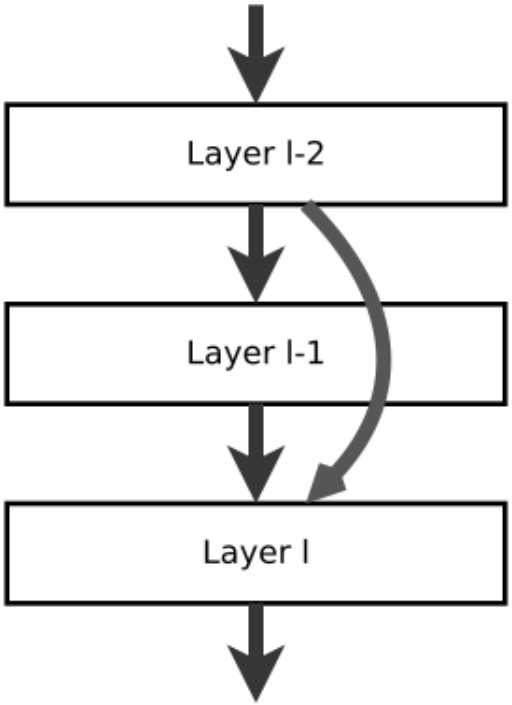
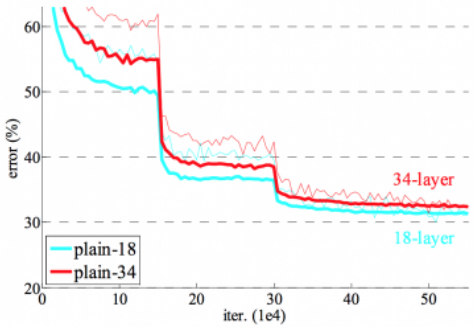
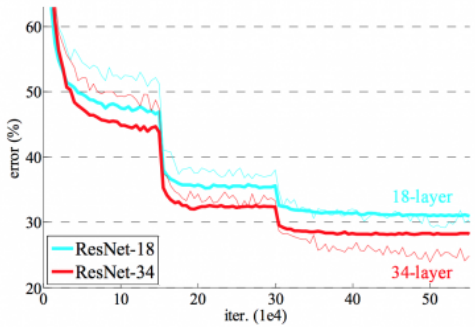


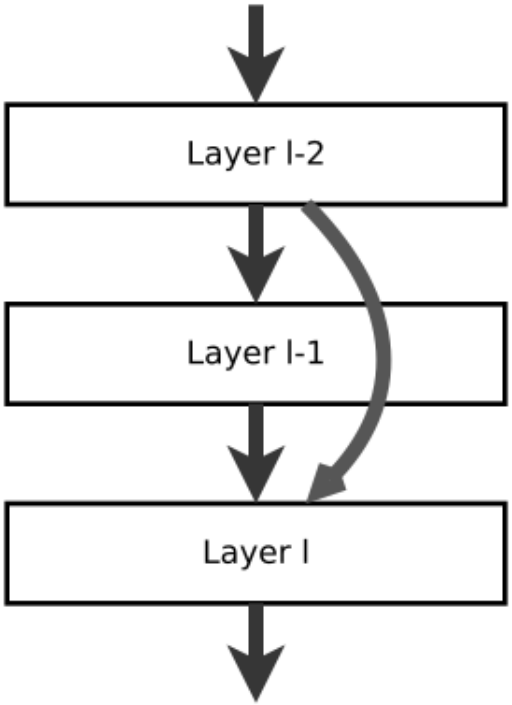
- 18 layer net has lower error than 34 layer net!
- Skip-connections to overcome vanishing grd.

$$\mathbf{z}^{(l)} = h\left(\mathbf{W}^{(l-1,l)}\mathbf{z}^{(l-1)} + \mathbf{W}^{(l-2,l)}\mathbf{z}^{(l-2)}\right)$$





Residual Networks (ResNet)



- 18 layer net has lower error than 34 layer net!
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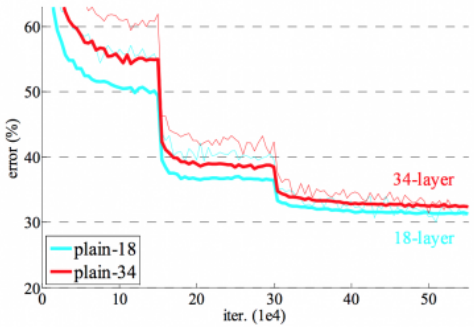

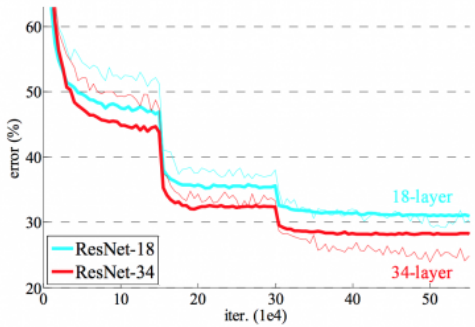

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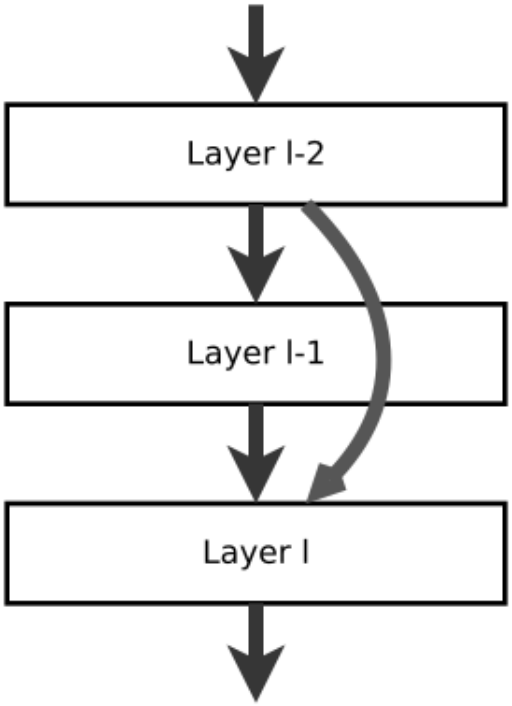

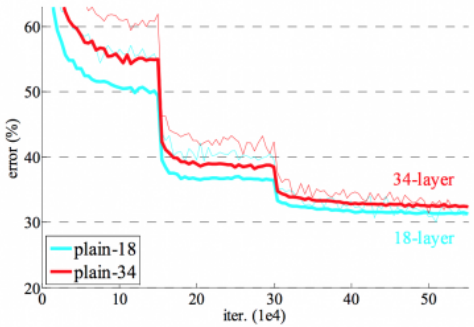

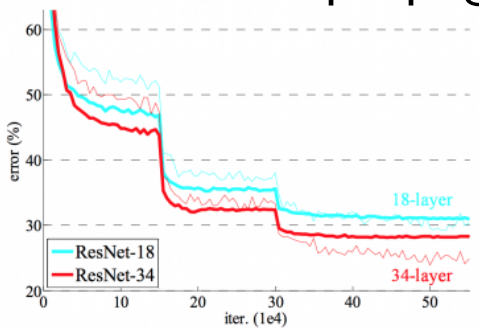

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Sensitivity of Error to Changes in Input

Forward pass input $\mathbf{x}$

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$$= \left(h'(a_j) \sum_{k=1}^{K} w_{kj}^{(2)} \delta_k \right) w_{ji}^{(1)}$$

- Certain features may not influence the error
- Such features may be pruned to reduce input dimensions!

Feature Selection / Adversarial Examples

- Feature selection (pruning): Sensitivity of error to changes in inputs

$$\nabla_{\mathbf{x}} J(\mathbf{W}) = \nabla_{\mathbf{a}_1} J(\mathbf{w}) \nabla_{\mathbf{x}} (\mathbf{a}_1)$$

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$$\tilde{\mathbf{x}} = \mathbf{x} - \epsilon \, \nabla_{\mathbf{x}} J(\mathbf{W})$$

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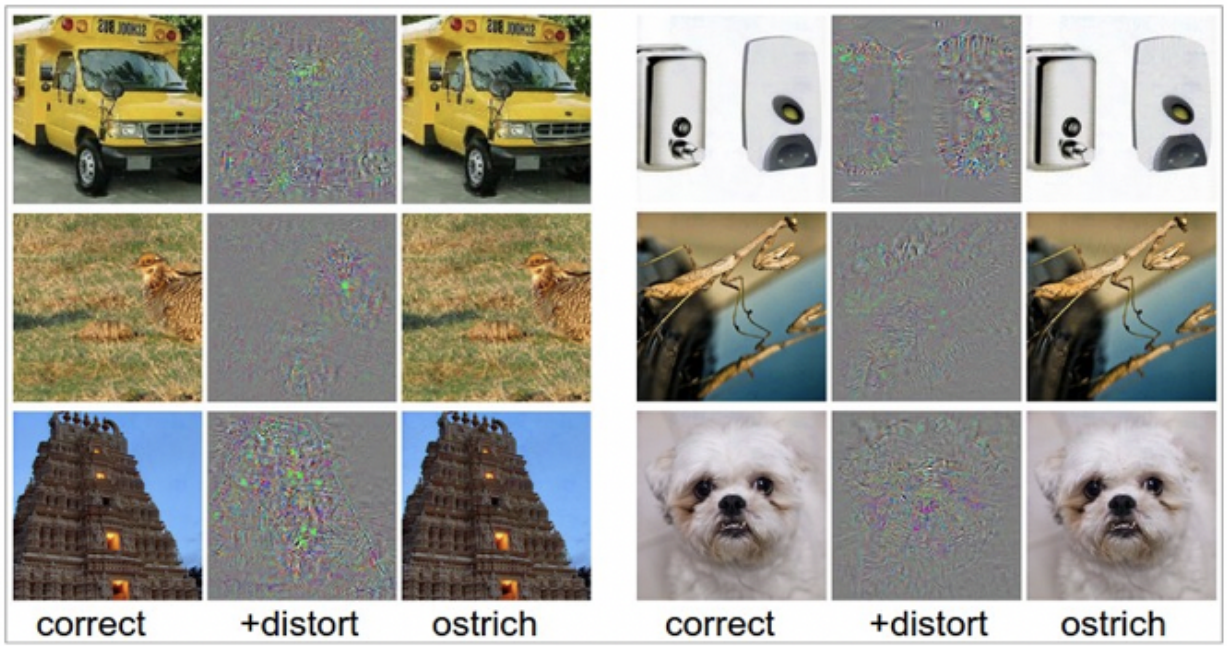

Deep Fool

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- Estimate the smallest possible perturbation needed to alter the inferred class

$$\Delta \mathbf{x} = \arg\min_{\epsilon} \|\boldsymbol{\epsilon}\|_2 \ni f(\mathbf{x} + \boldsymbol{\epsilon}) \neq f(\mathbf{x})$$

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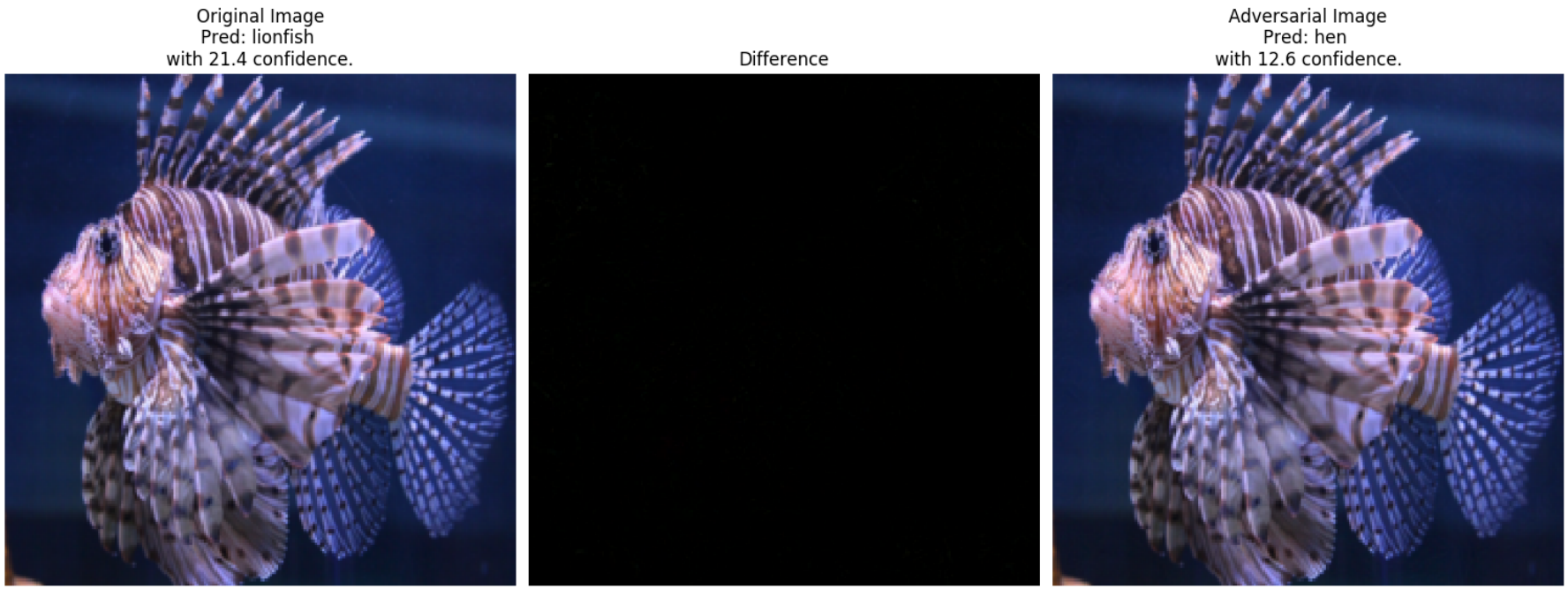
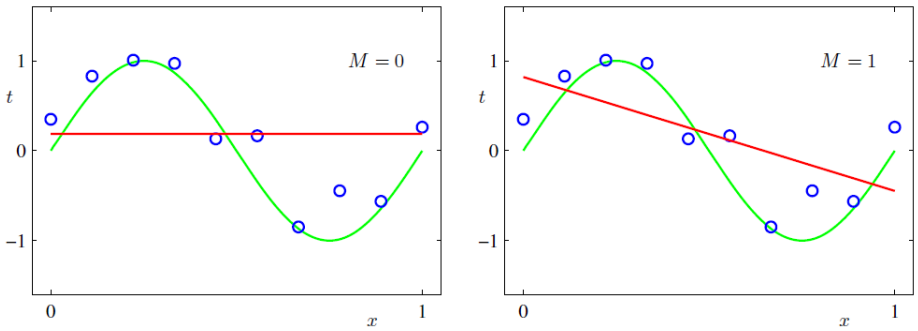


Overfitting



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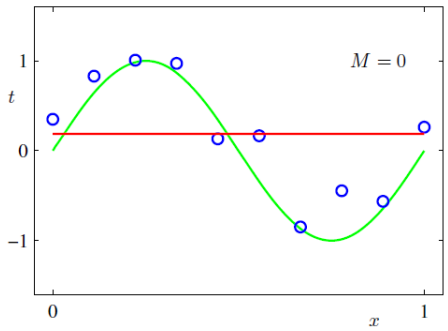

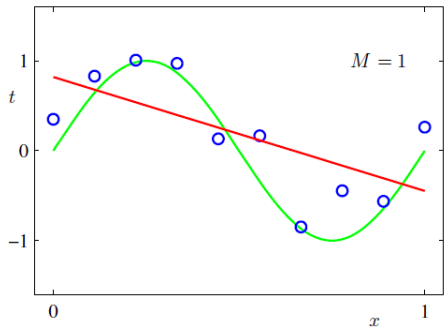

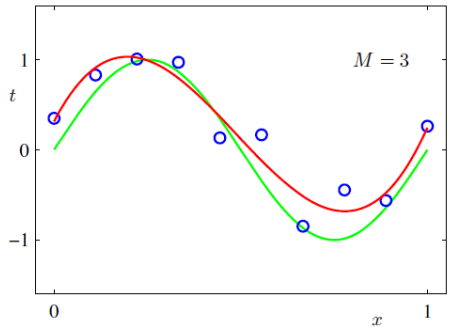

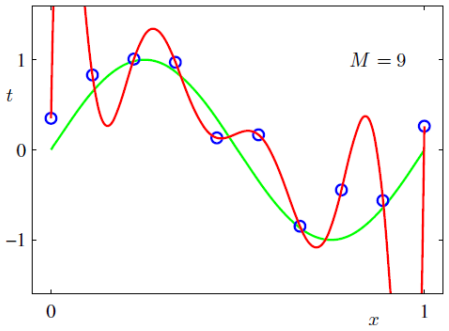

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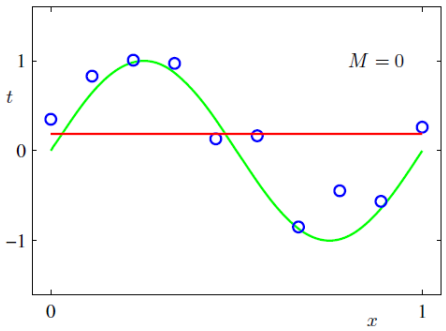
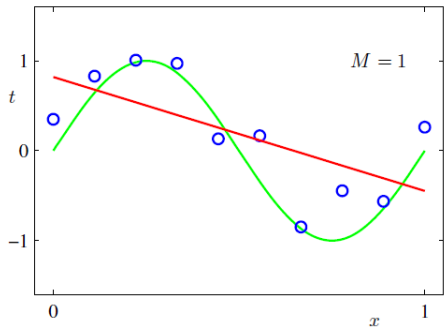
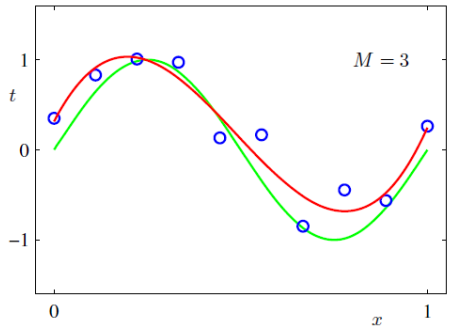
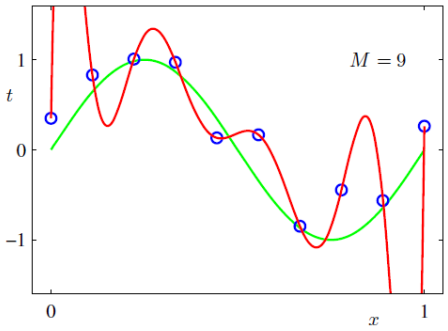
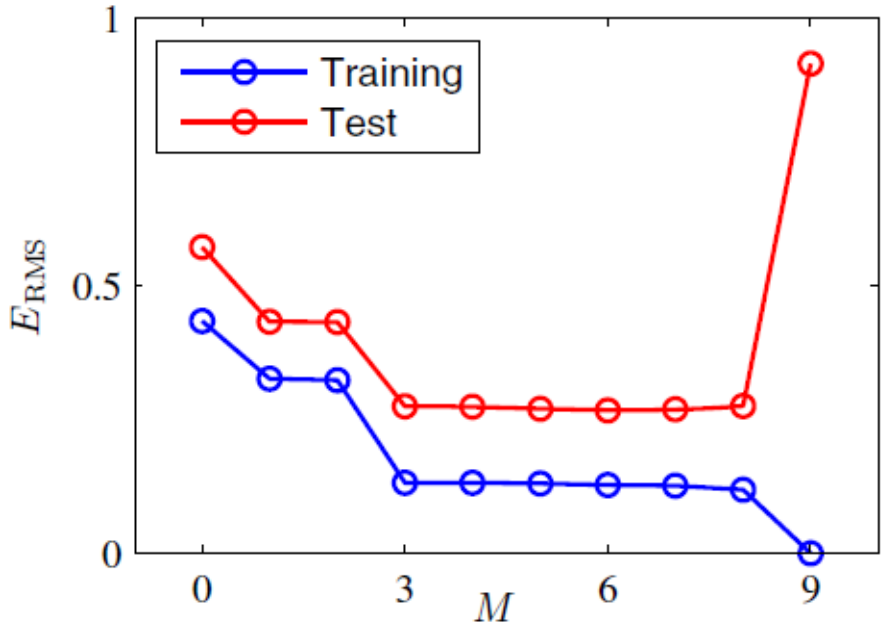


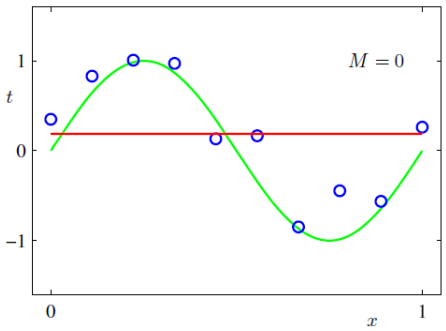


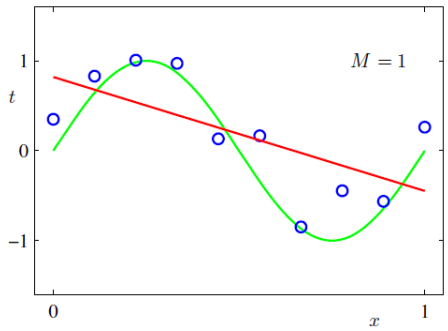


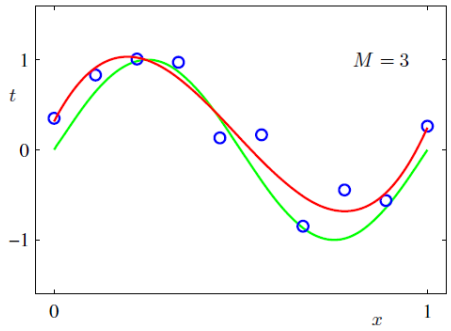


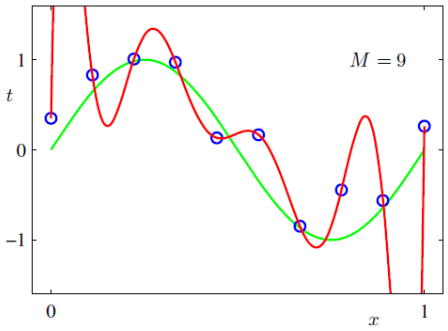


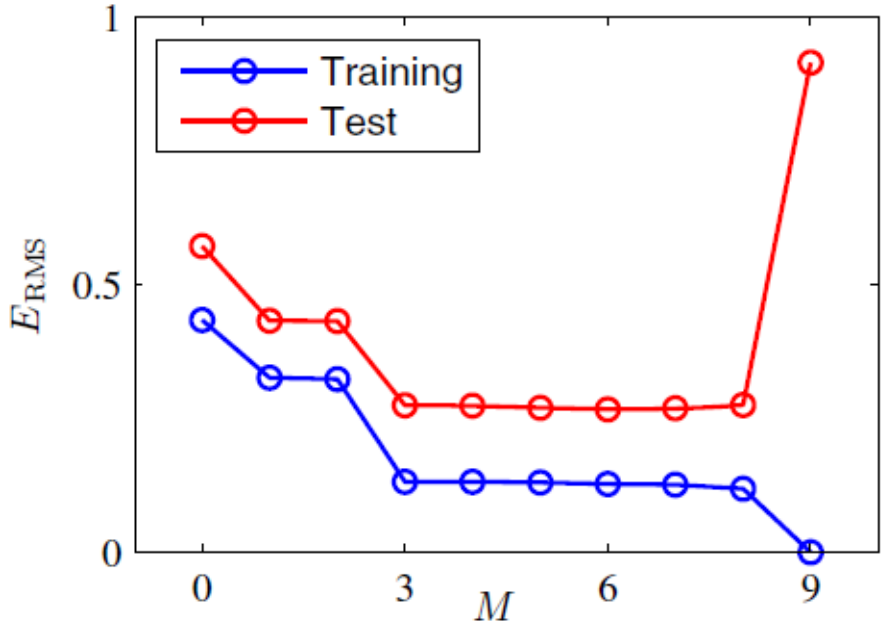
Overfitting











	$M=0$	$M=1$	$M=6$	$M=9$
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$w_1^\star$		-1.27	7.99	232.37
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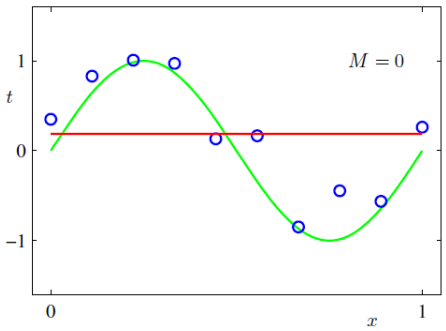

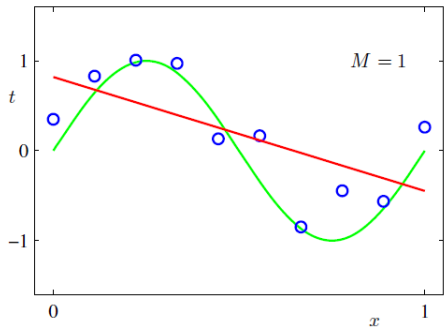

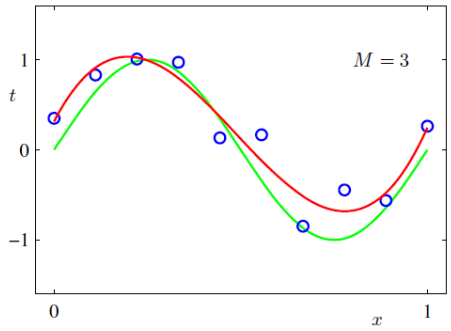

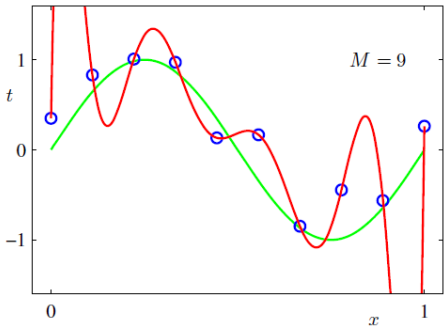

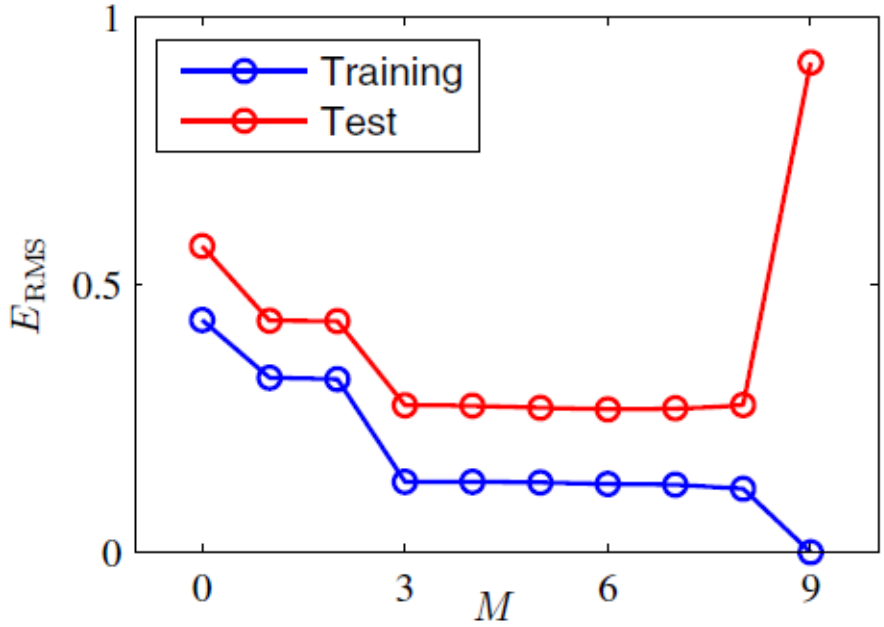

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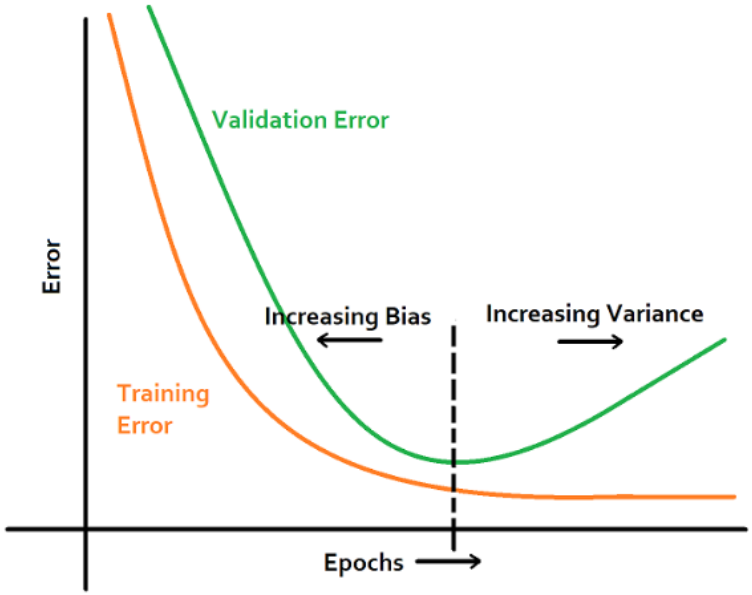
Regularization Techniques

- DNNs with large number of parameters tend to overfit
  - Error on training data reduces, but not on validation/test data
  - Performance on test data could be worse than smaller models (paradox)
  - With larger parameters, the model gets tuned to noise in training data
  - Weights blow up to capture the noisy fluctuations
- Arrest the growth of the weights to avoid overfitting to training data
- Finding the right balance between bias and variance trade-off.
- Common regularization techniques for DNNs include:
  - Early Stopping, explicit weight regularization, activity regularization/constraints, dropout, input corruption with noise

Early Stopping

- Initialize the weights with small random values
  - Glorot Normal - $w_{ji} \in \mathcal{N}\left(0, \frac{\sqrt{2}}{f_{in}+f_{out}}\right)$
  - Update the weights using backpropagation algorithm
- Monitor training and validation errors after every epoch.
- Stop the training when validation error starts to increase

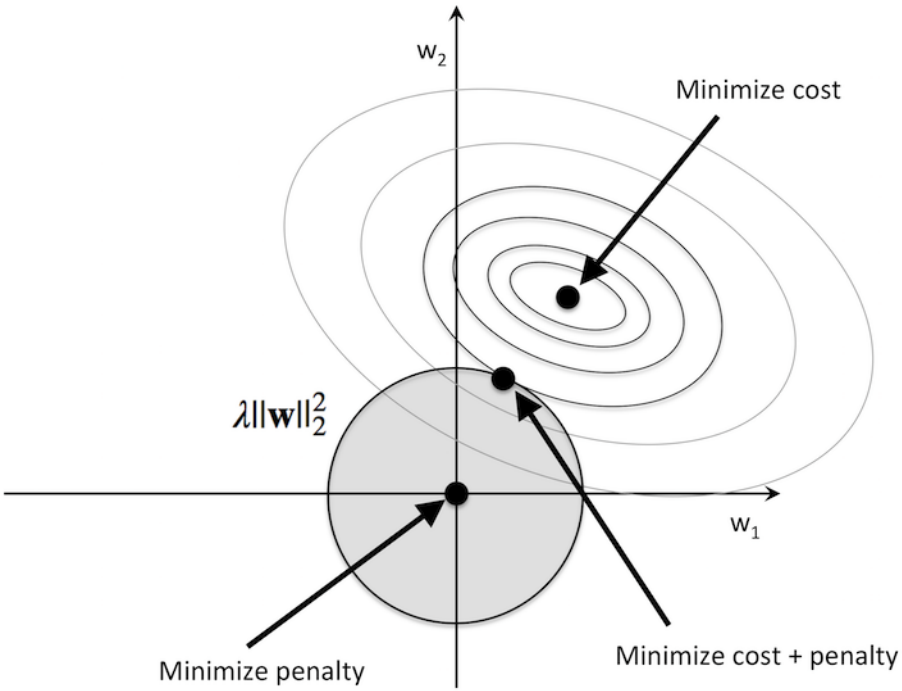


Weight Regularization

- Add a penalty term to the error function to discourage weight growth

$$J(\mathbf{W}) = \frac{1}{2}\sum_{n=1}^{N}\sum_{k=1}^{K}(y_{nk} - d_{nk})^2 + \lambda\|\mathbf{W}\|_p$$

- λ controls the trade-off between bias and variance



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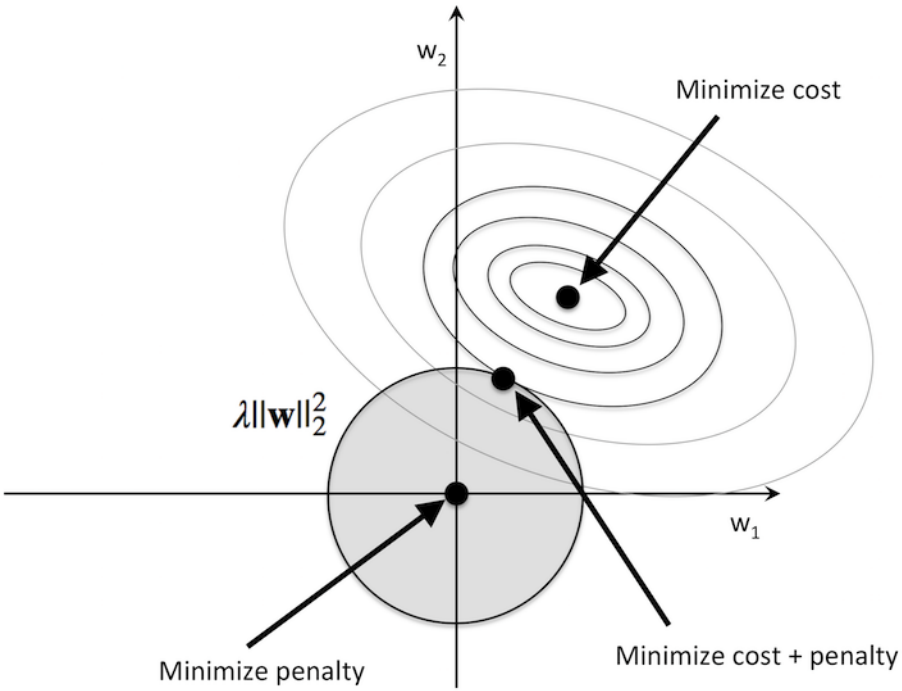
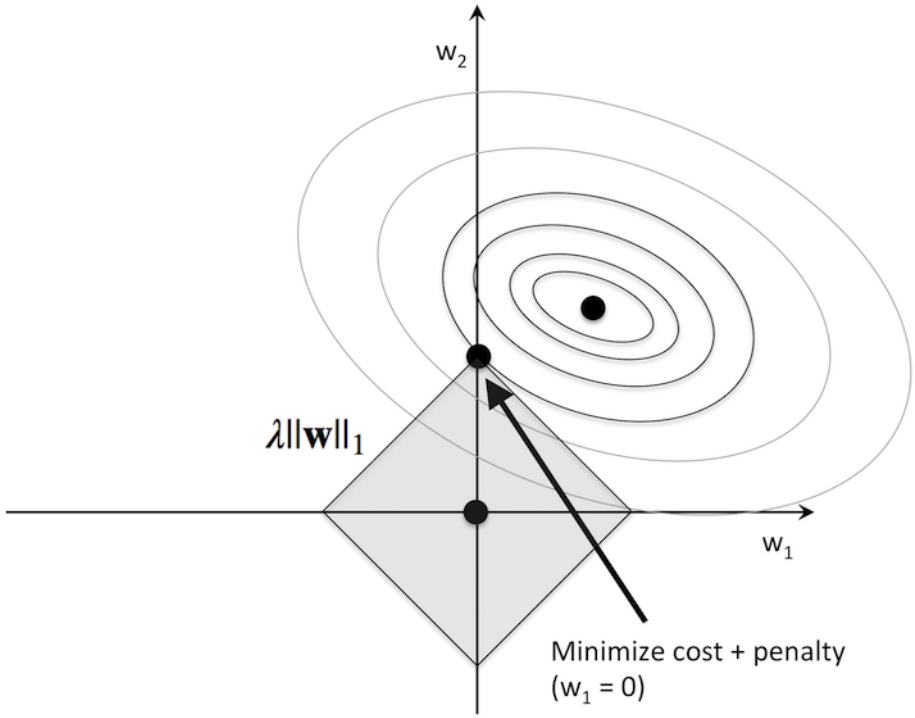
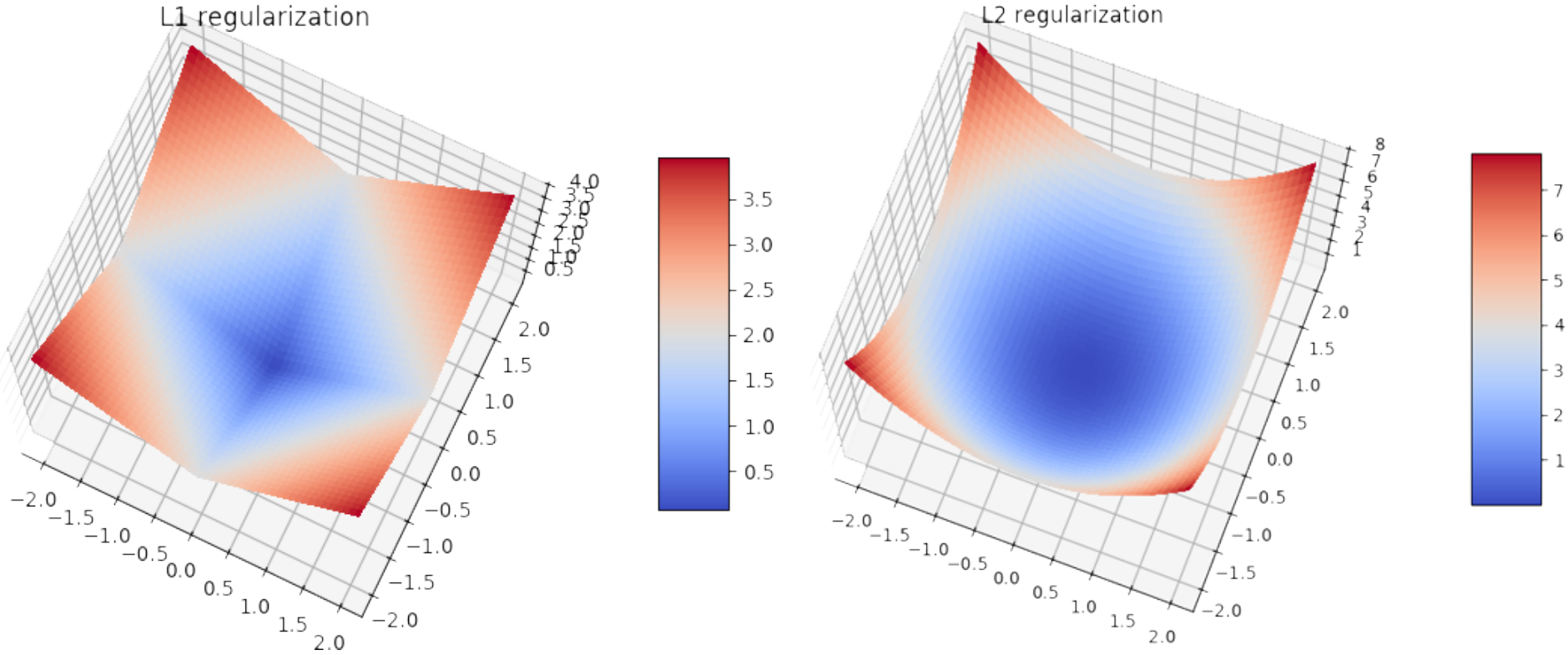




L_1 vs L_2



- L_1 regularization promotes sparser solutions
- L_1 regularization $\Longrightarrow$ Laplacian priors
- L_2 regularization $\Longrightarrow$ Gaussian priors

Dropout

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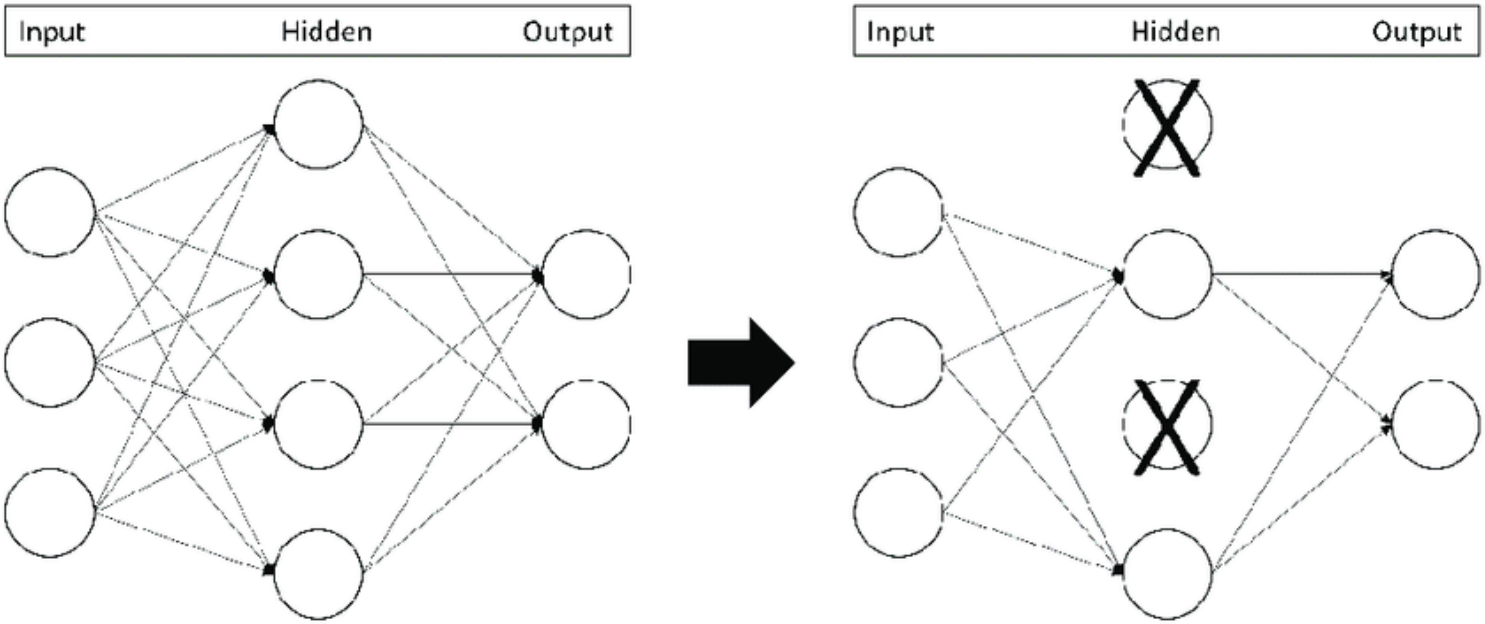

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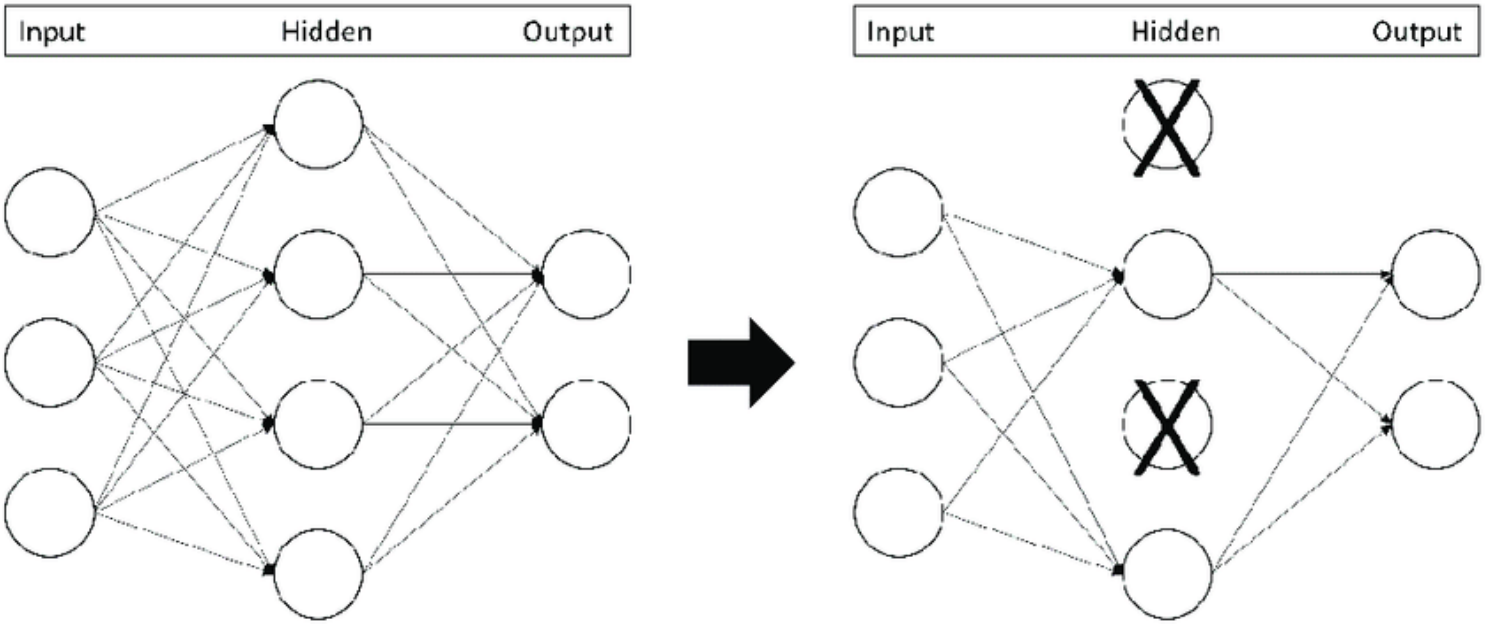

- Some nodes may tend to be too critical in network operation
- Avoid such situation by sharing the responsibility across the nodes
- Drop nodes in the hidden layers with a probability p=(0.5)
- With M hidden units, it can create 2^M different network architectures

Dropout Implementation

- Network training
  - Forwardpass the input to evaluate neuronal outputs in the hidden layer

$$a_j = \sum_{i=0}^{D} w_{ji}^{(1)} x_i \qquad z_j = h(a_j)\, m_j$$

  - m_j is the mask drawn from a Bernoulli distribution with parameter p
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- Interpretation of Dropout
  - Dropout can be interpreted as mixture of experts (novel combination)
  - A node is expected to perform in different configurations: Regularization

Limitations for Fully Connected (Dense) Networks

- FFNN offers data-dependent nonlinear transformation

$$\mathbf{z}[n] = \phi(\mathbf{x}[\boldsymbol{n}]) = g(\mathbf{W}^{(1)}\mathbf{x}[n]) \qquad \mathbf{y}[n] = f(\mathbf{W}^{(2)}\mathbf{z}[n])$$

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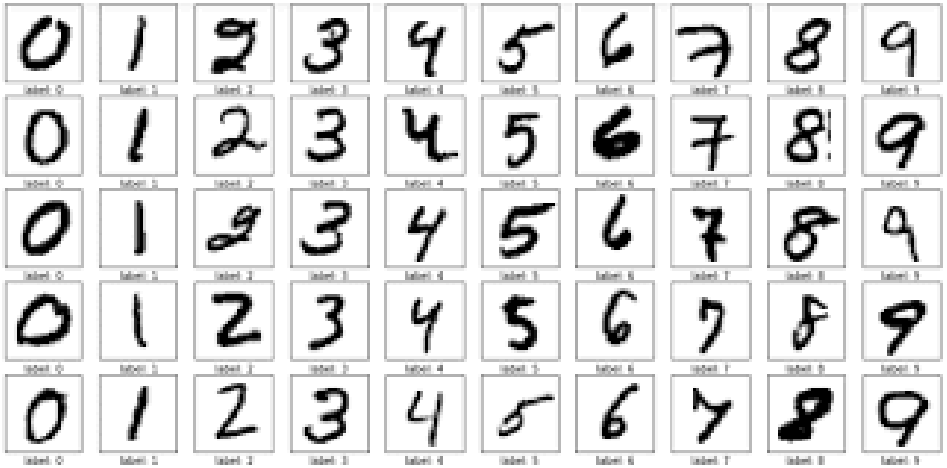
- Incorporate sequential information in the transformed representation
  - Finite memory: $\mathbf{z}[n] = \phi(\mathbf{x}[n-k] \ : \ \mathbf{x}[n+k])$ CNNs
  - Infinite memory: $\mathbf{z}[n] = \phi(\mathbf{x}[-\infty] \ : \ \mathbf{x}[\infty])$ RNNs

Motivation for CNNs



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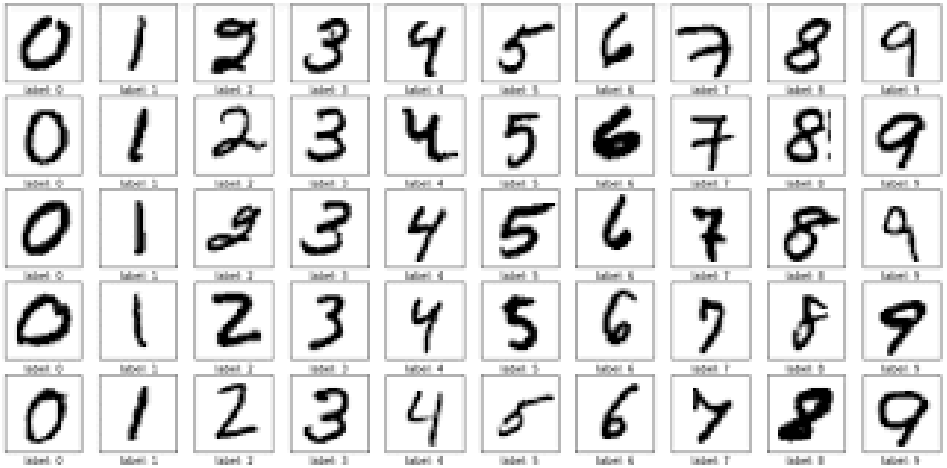

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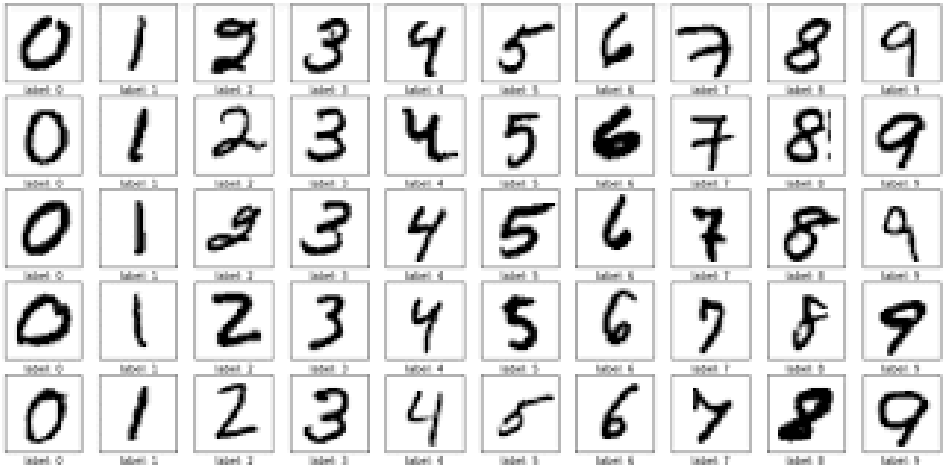
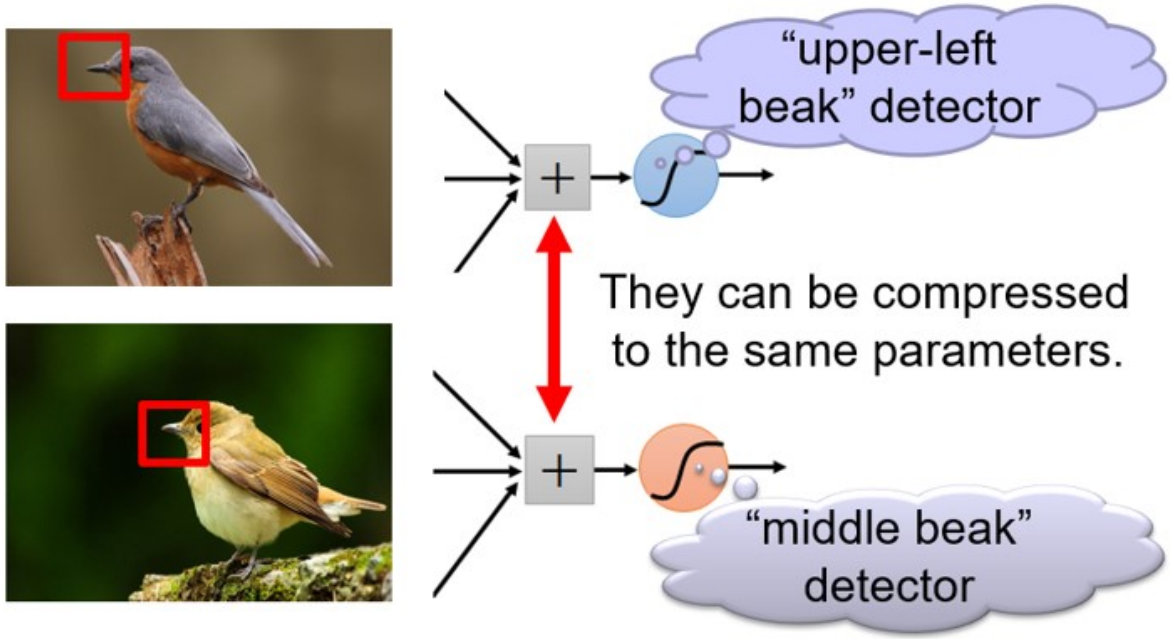


- There could be natural variations among handwritten digits
  - Identity of a digit should be invariant to translation, scaling and (small) rotation
  - Further, the network should be robust to elastic deformations
- One approach is vectorize the input, and train a dense (FC) network
  - Given a large dataset, the network in principle yields good solution
  - However, the key property of the images is ignored - nearby pixels are strongly correlated.

Local Feature Detectors



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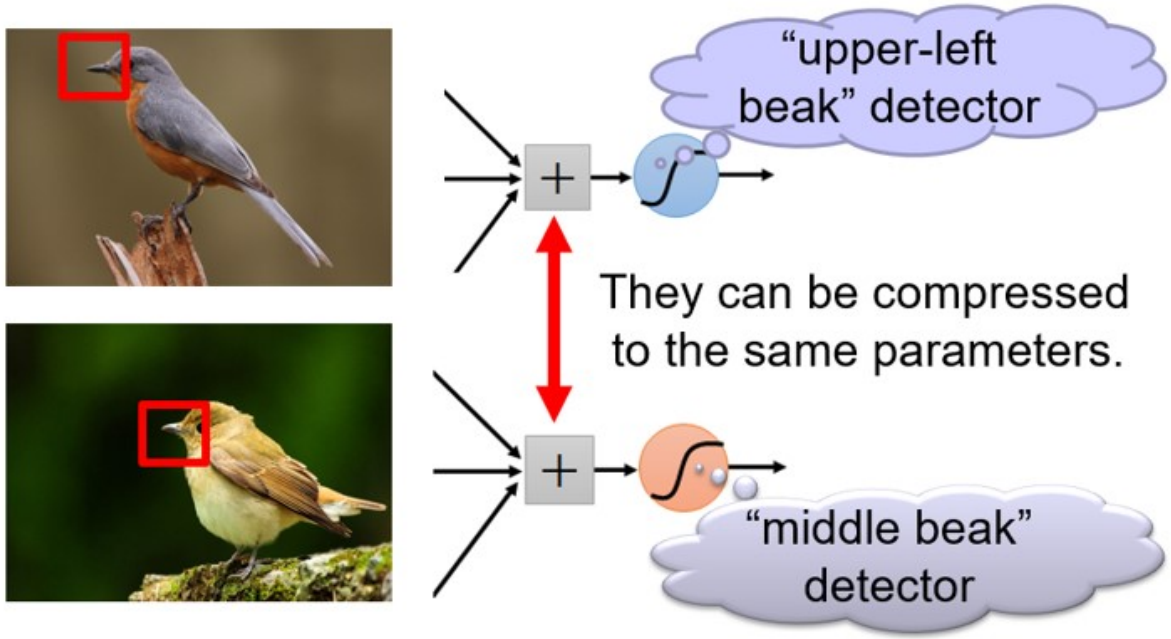
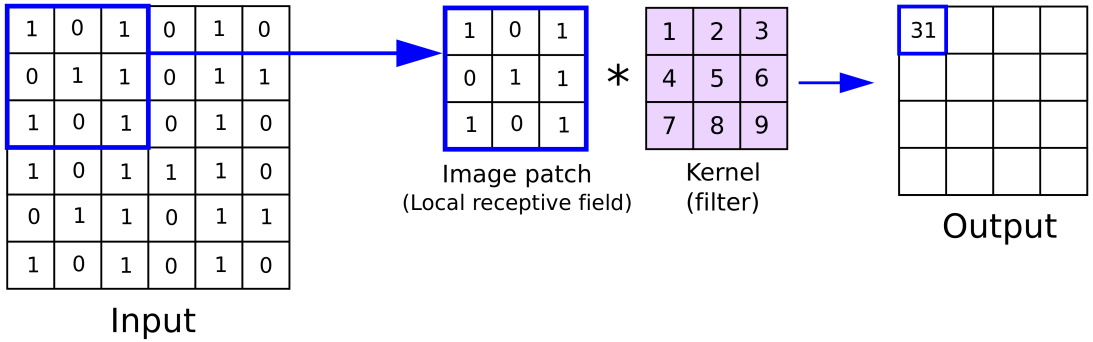


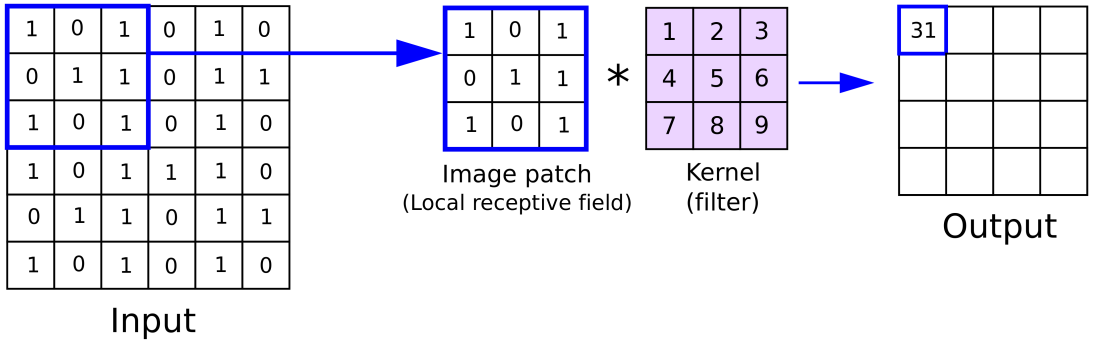
- Some patterns are localized to small regions in the image
- Some patterns can appear in different regions of the image
  - They can be detected using the same kernel (shared parameters)
- Consider a large number of 'small' detectors moving around the image

$$\mathbf{Z}_k[i,j] = \phi\left(\mathbf{X}[i,j] * \mathbf{W}_k[i,j]\right), \qquad k = 1, 2, 3, \ldots K$$

CNN


1 0 1 0 1 0
0 1 1 0 1 1
1 0 1 0 1 0
1 0 1 1 1 0
0 1 1 0 1 1
1 0 1 0 1 0
Input
1 0 1
0 1 1
1 0 1
Image patch
(Local receptive field)
*
1 2 3
4 5 6
7 8 9
Kernel
(filter)
31
Output

CNN



- Extract local features that depend on small subregions of the image

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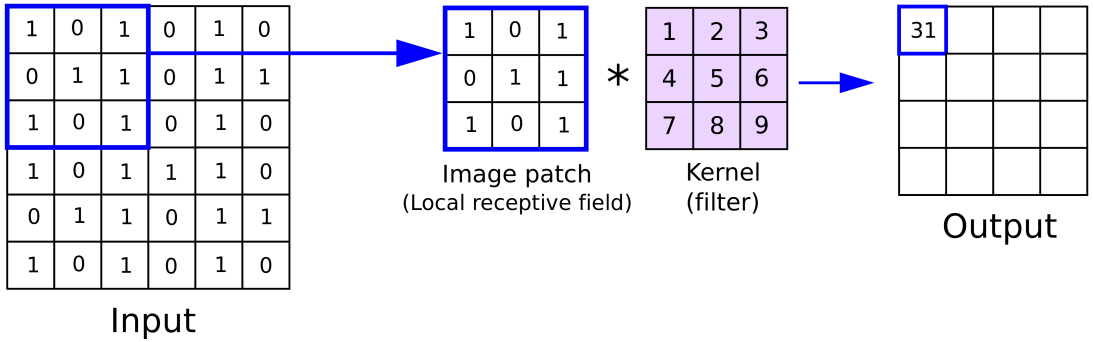

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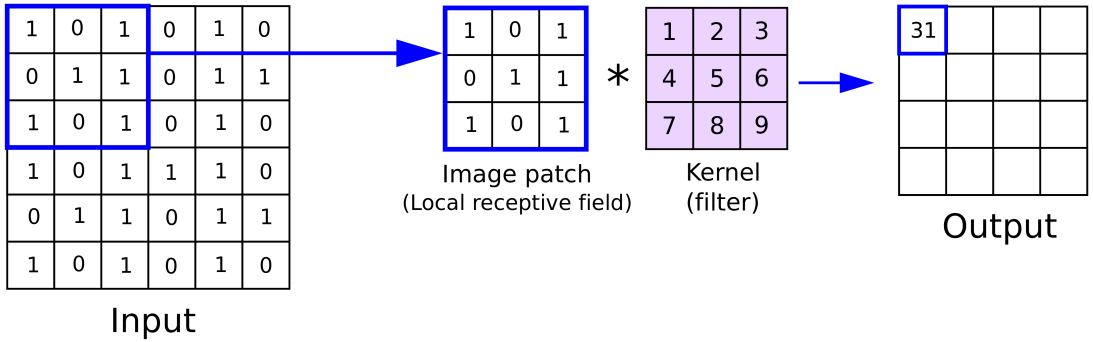

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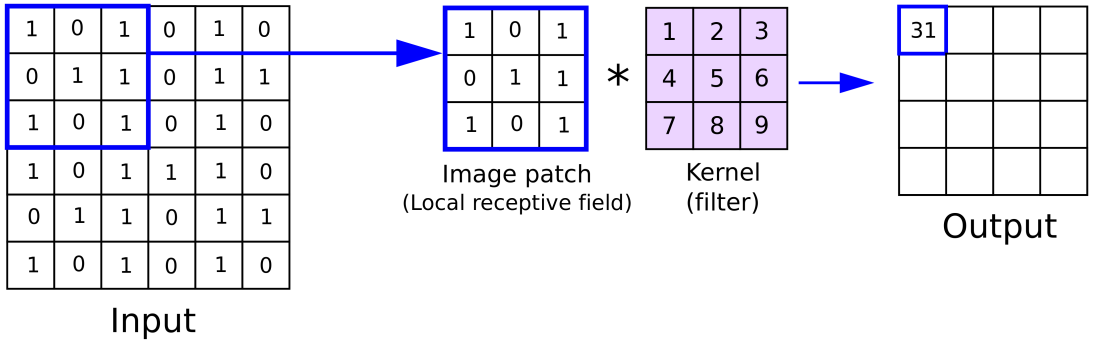

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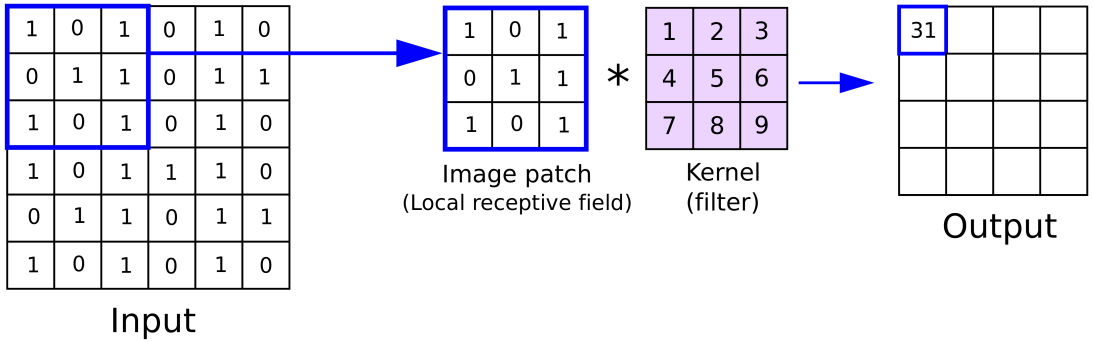

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- These notions are incorporated in CNN though following mechanisms
  - Local receptive field (kernel-size), weight sharing (stride), subsampling (pooling)

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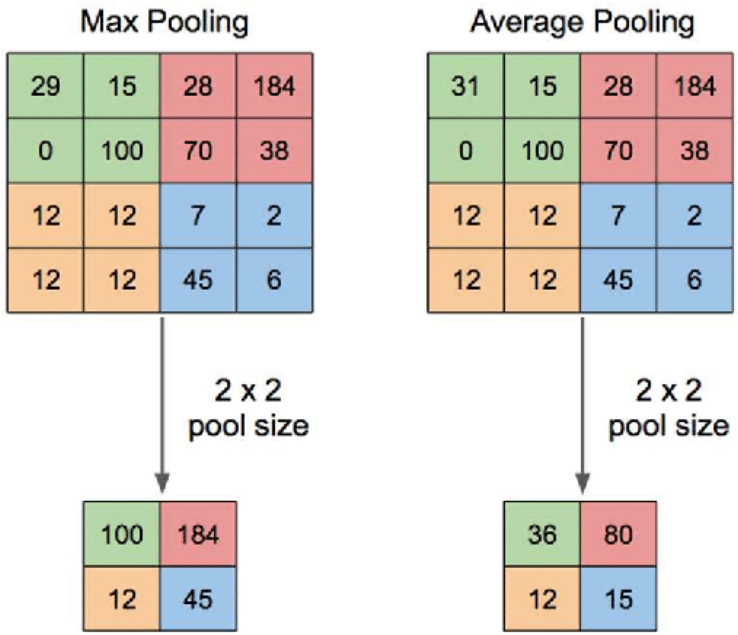
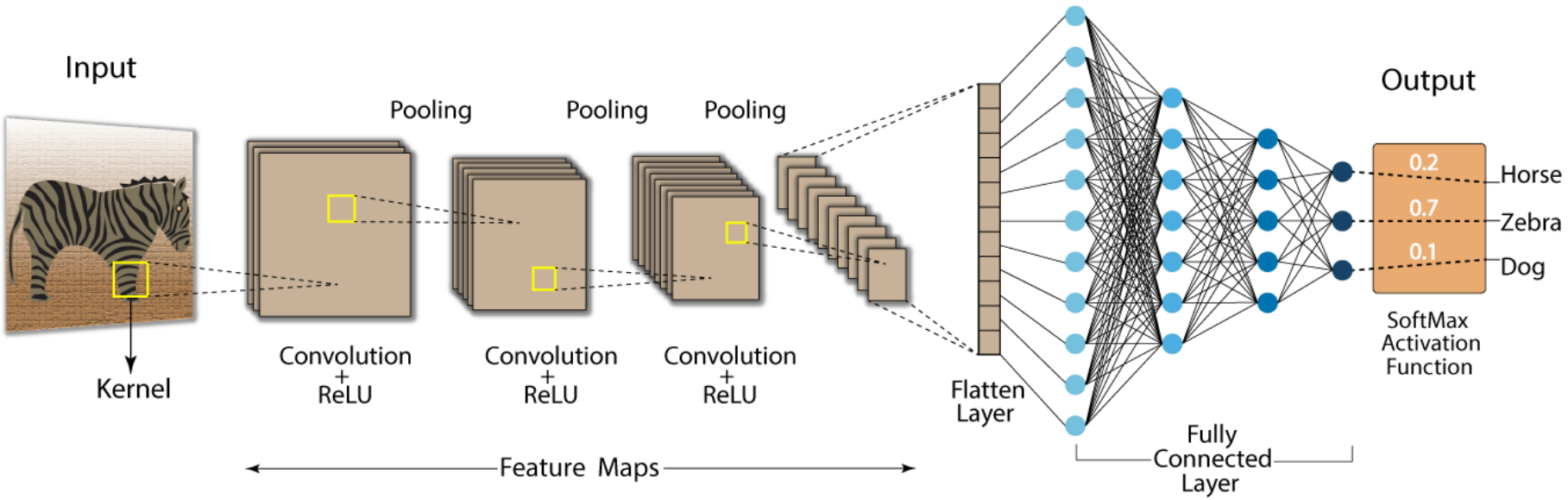


CNN Classifier Architecture



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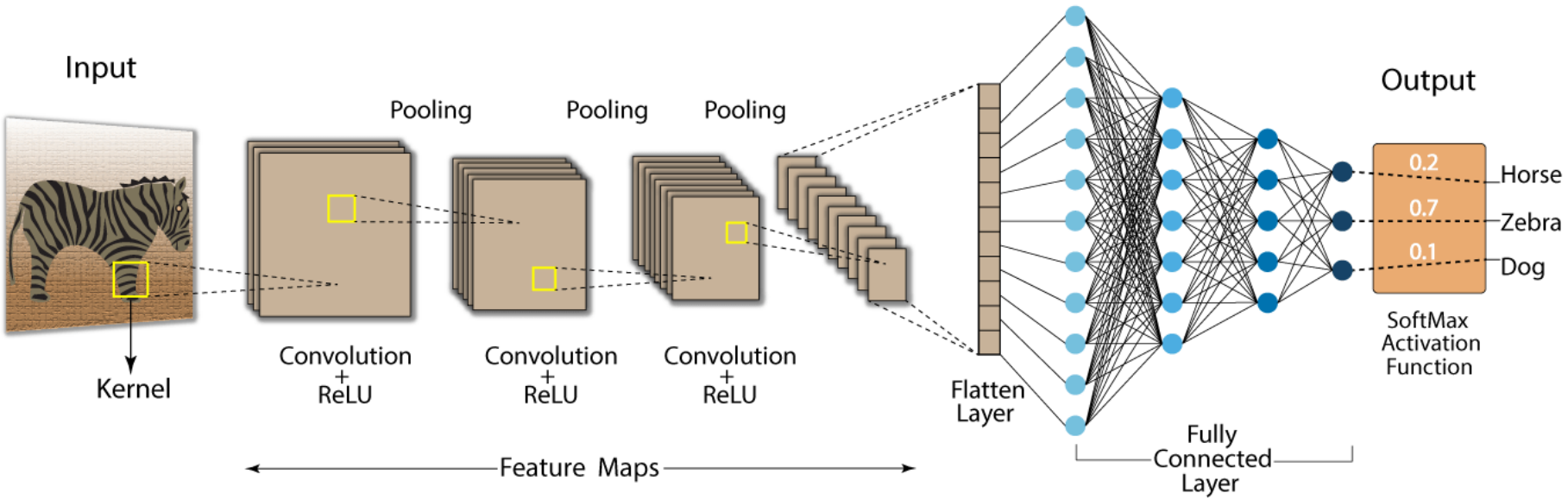

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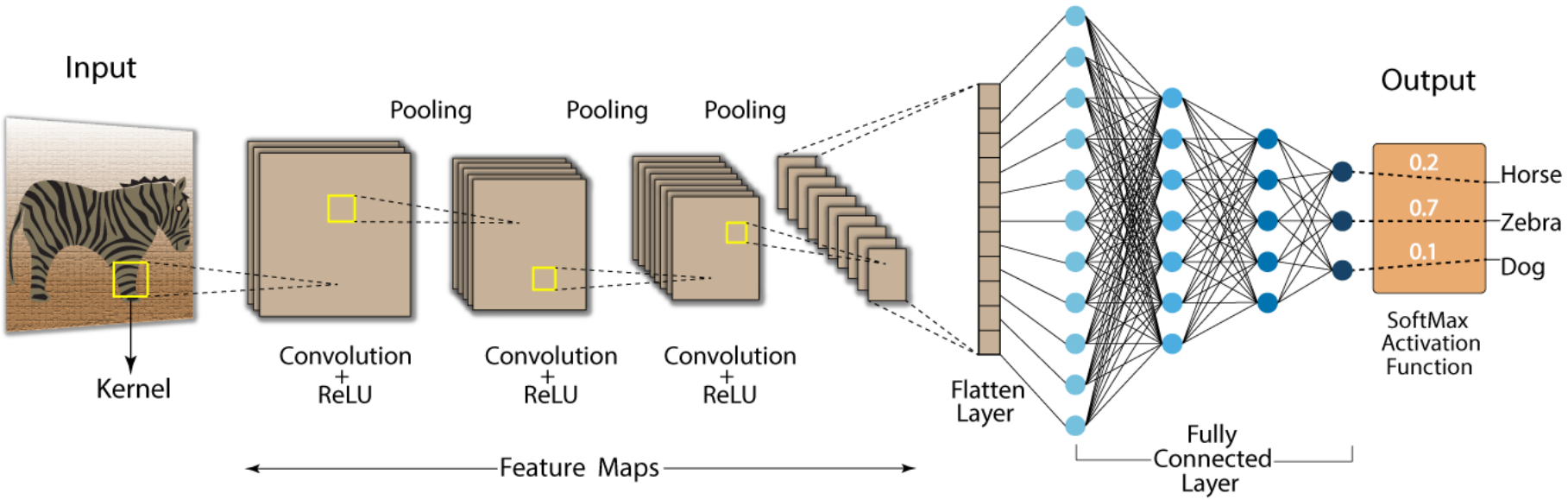

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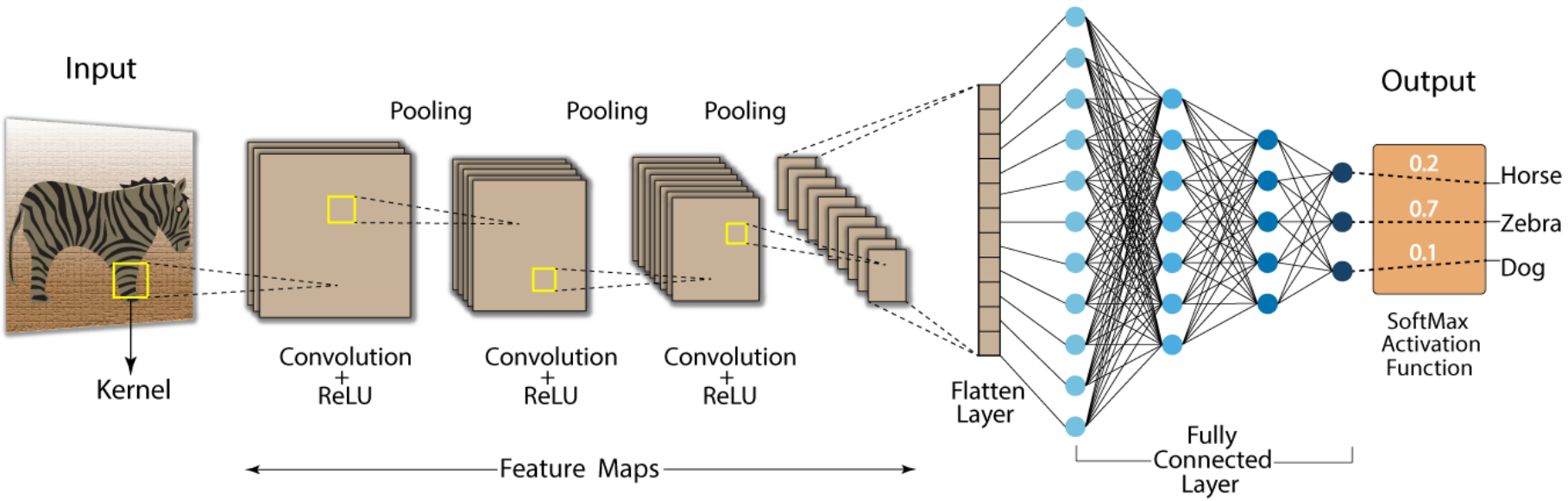

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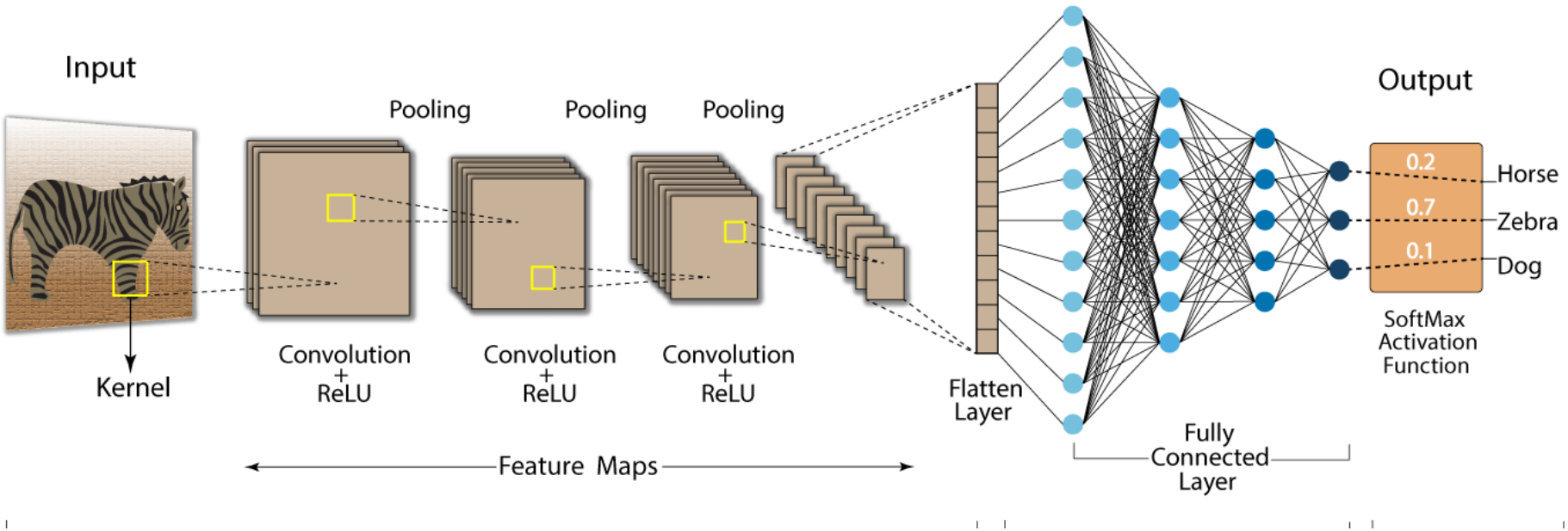

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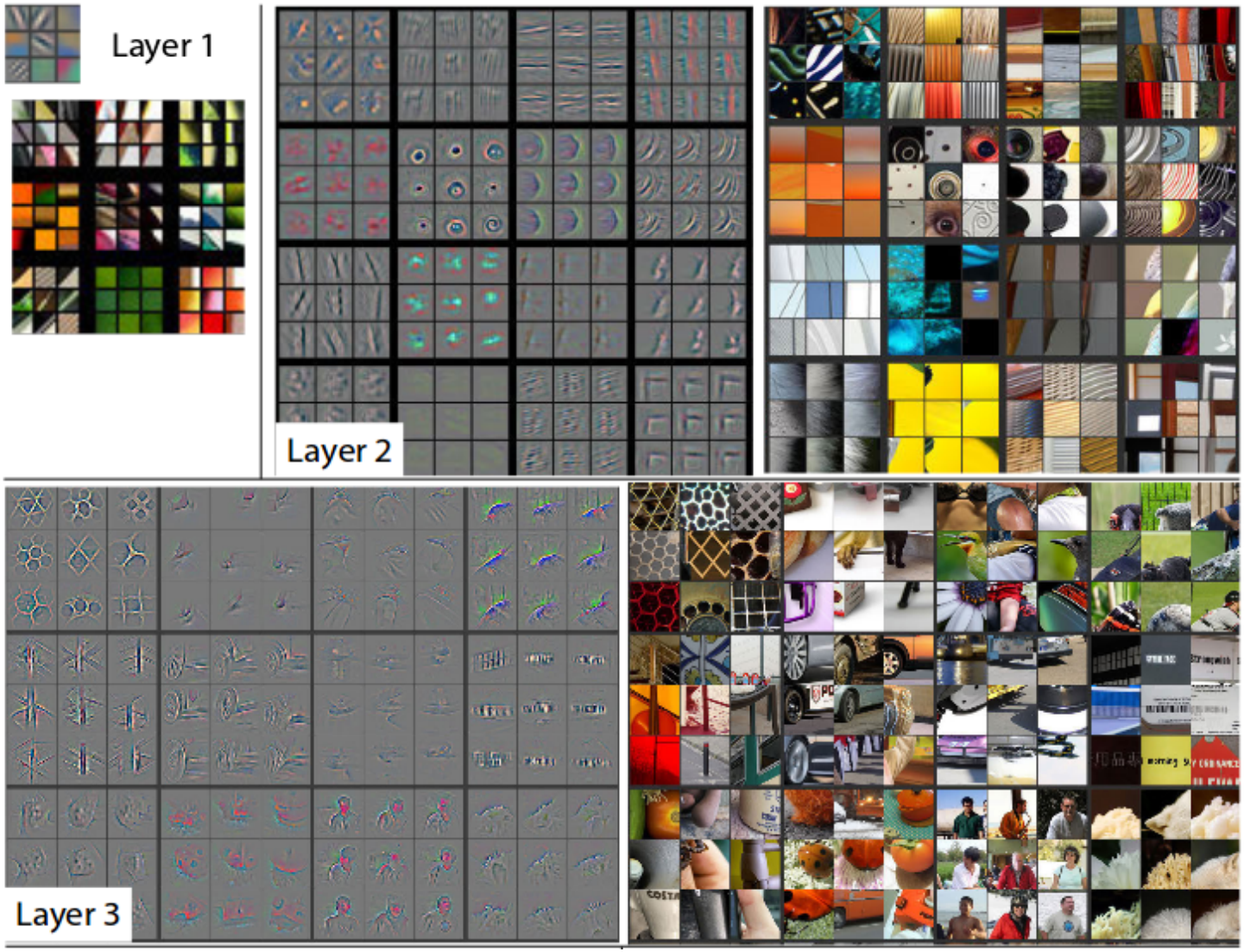
- CNNs replaced the hand-crafted features with self-learned features

CNN Feature Map Visualization



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- How do you modify it to obtain gradients w.r.t the input $x[n]$?

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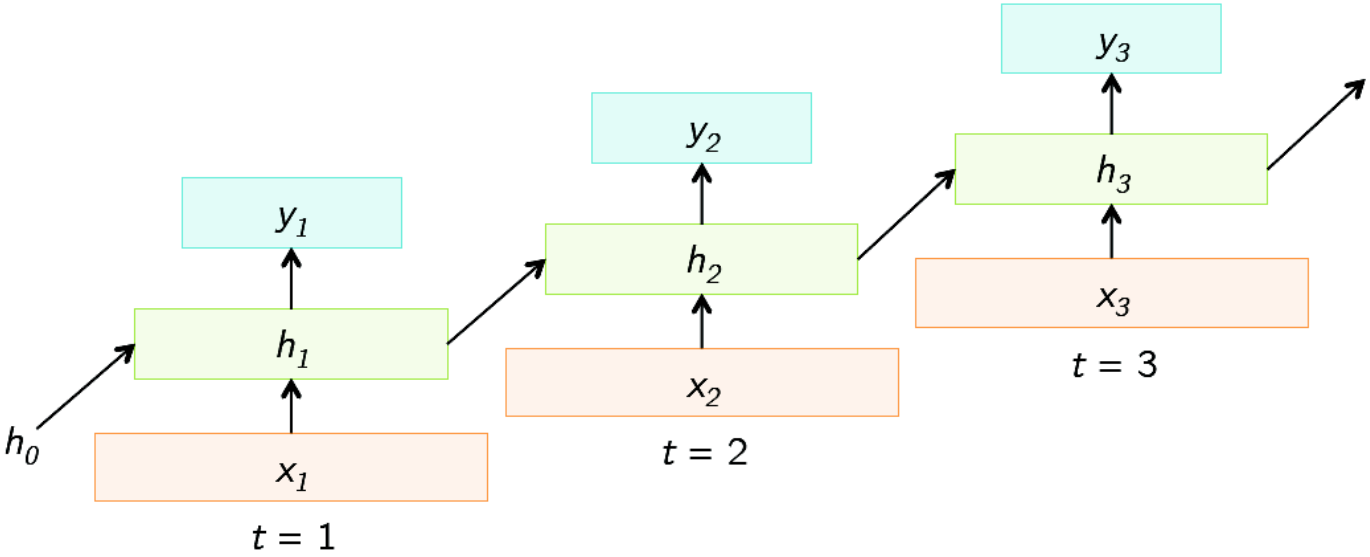
  - Variants of RNN: LSTMs and GRU
- Transformers - Explicit incorporation of context

$$\mathbf{z}[n] = g\left(\sum_{k=n-N}^{n} a_{nk}\mathbf{W}^{(1)}\mathbf{x}[k]\right)$$

a_{nk} is the attention of the k^{th} frame at the n^{th} instant (similarity)

Recurrent Neural Networks



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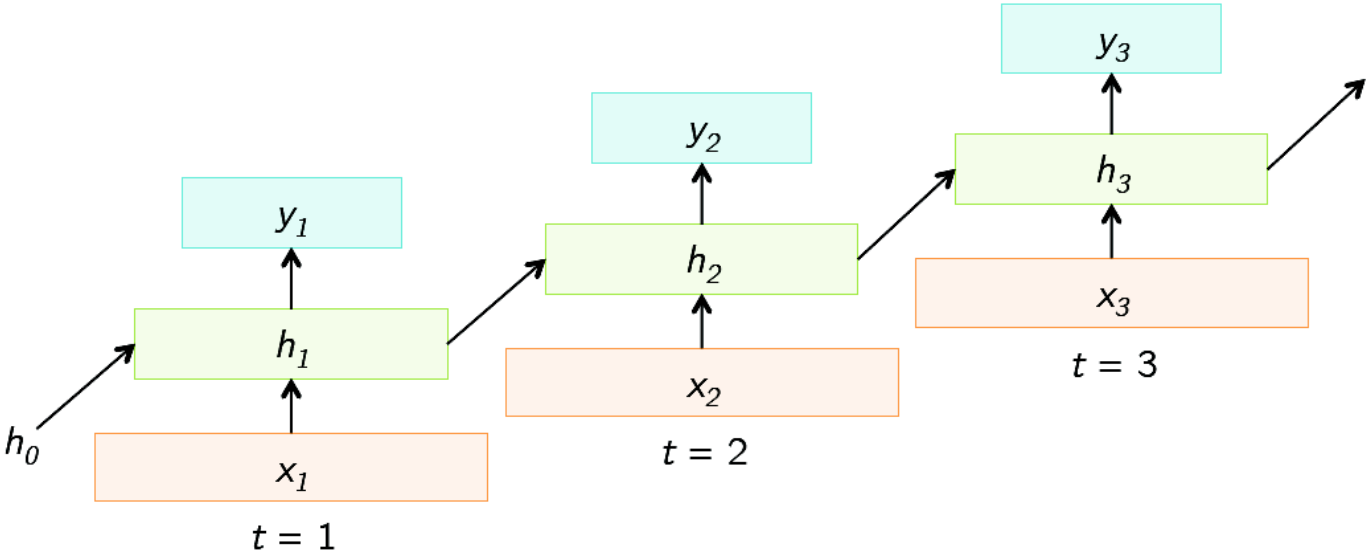

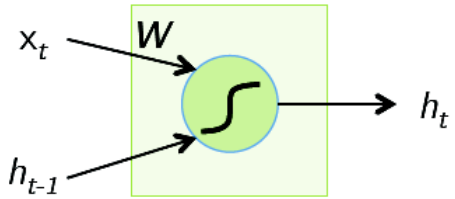

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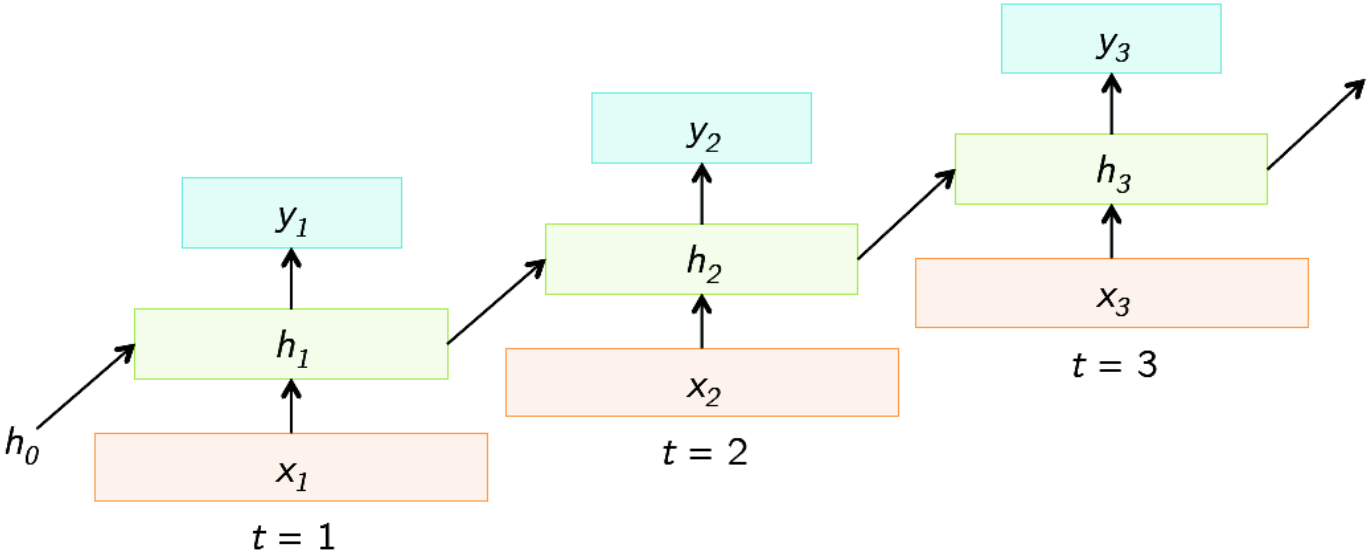

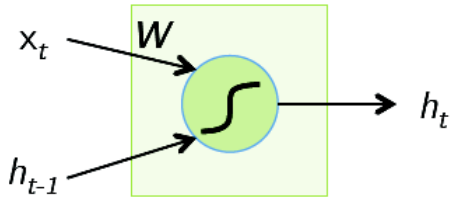

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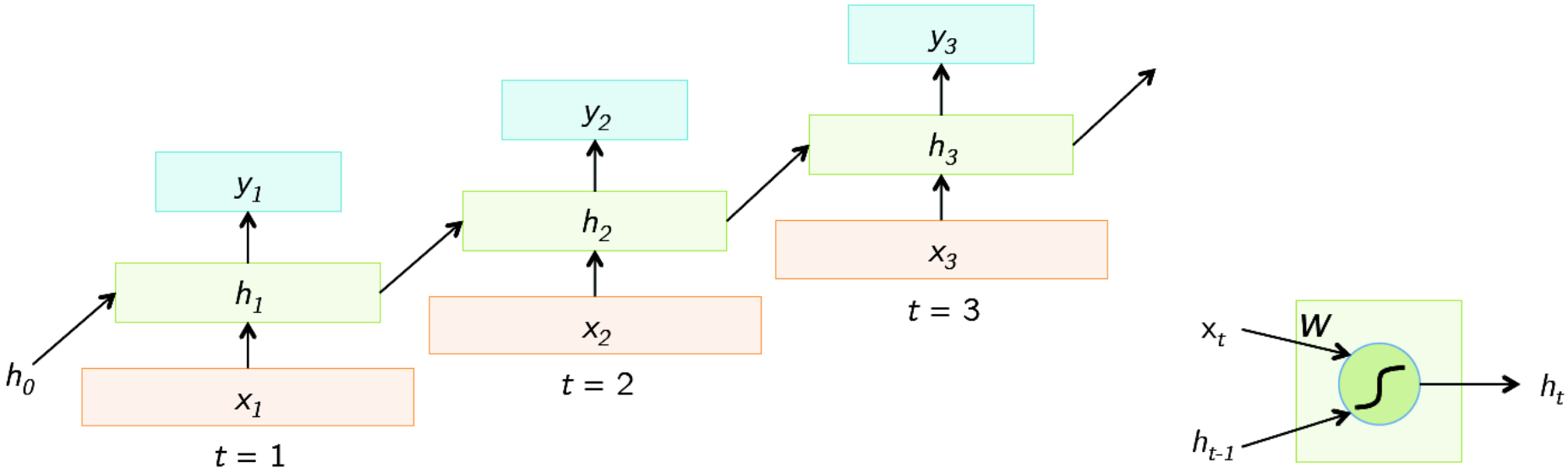

- RNN cell takes current input $\mathbf{x}[n]$ and previous state $\mathbf{h}[n-1]$ as inputs to produce the current state

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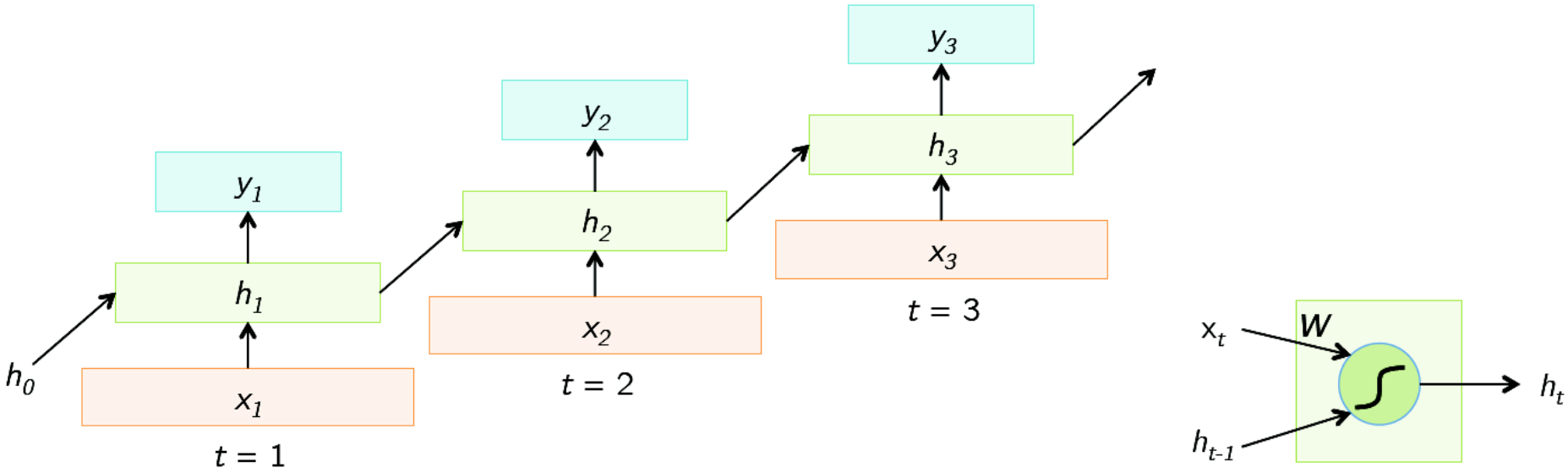

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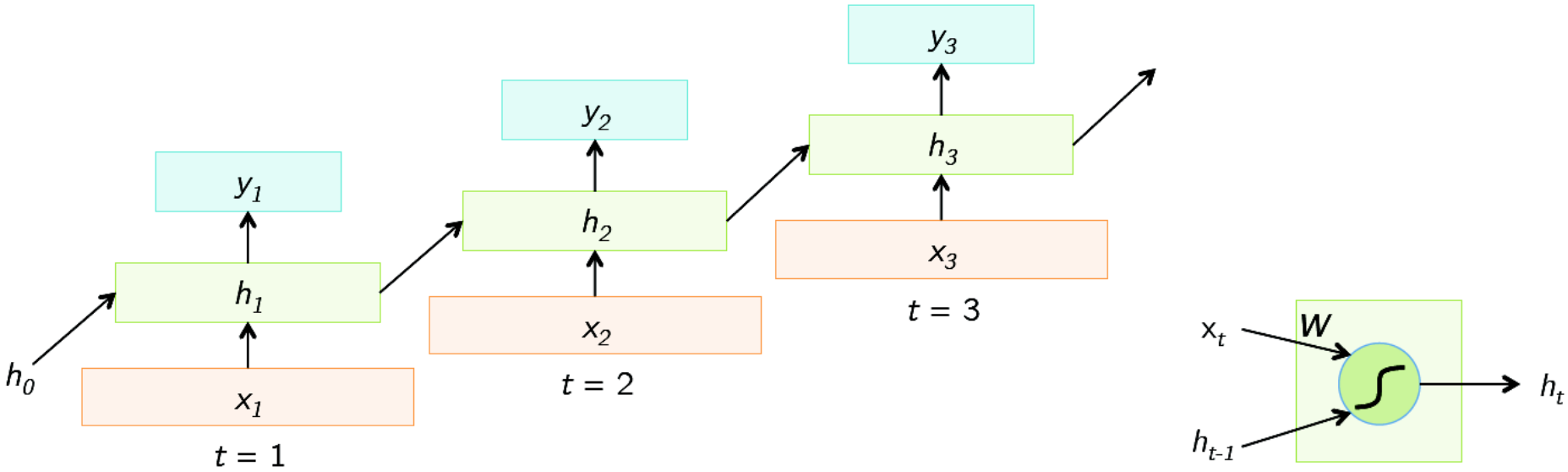
Recurrent Neural Networks

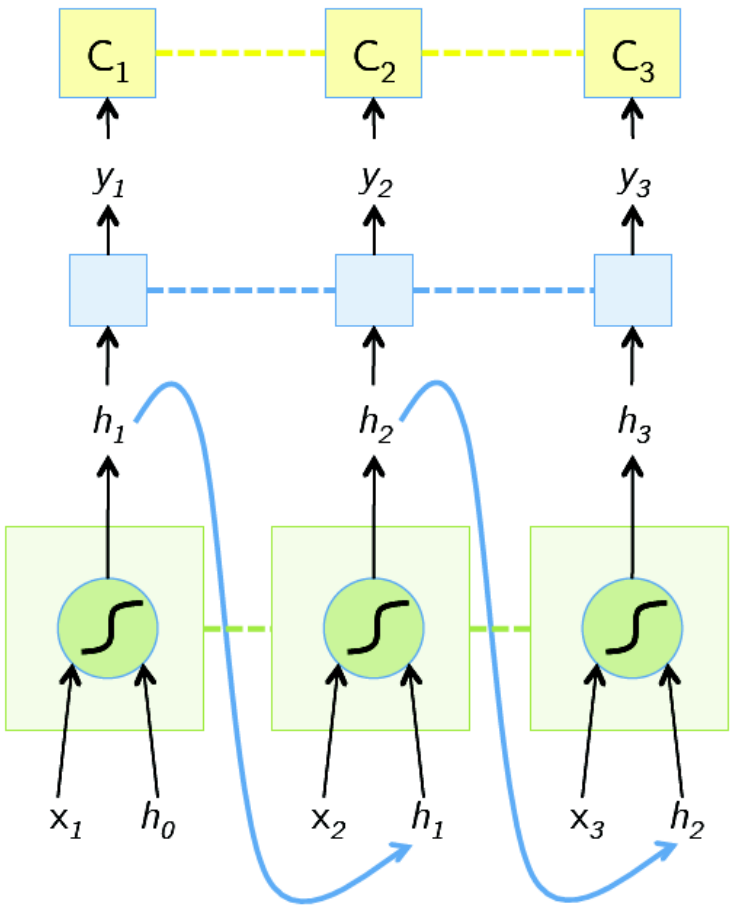


- RNN cell takes current input $\mathbf{x}[n]$ and previous state $\mathbf{h}[n-1]$ as inputs to produce the current state

$$\mathbf{h}[n] = g(\mathbf{W}_{hh}\,\mathbf{h}[n-1] + \mathbf{W}_{xh}\,\mathbf{x}[n]) = g\left([\mathbf{W}_{hh}\,\mathbf{W}_{xh}]\begin{bmatrix}\mathbf{h}[n-1]\\ \mathbf{x}[n]\end{bmatrix}\right)$$

- The weights $\mathbf{W} = [\mathbf{W}_{hh}\,\mathbf{W}_{xh}]$ are shared across the time-steps

RNN Forward Propagation



- Forward propagate the inputs

$$\mathbf{h}[n] = g(\mathbf{W}_{hh}\,\mathbf{h}[n-1] + \mathbf{W}_{xh}\,\mathbf{x}[n])$$

$$\mathbf{y}[n] = f(\mathbf{W}_{hy}\,\mathbf{h}[n])$$

$$J_n(\mathbf{W}) = L(\mathbf{y}[n],\,\mathbf{d}[n])$$

$$J(\mathbf{W}) = \sum_n J_n(\mathbf{W})$$

- Estimating $\mathbf{W} = [\mathbf{W}_{hh}\,\mathbf{W}_{xh}\,\mathbf{W}_{hy}]$

$$\mathbf{W}_* = \arg\min_{\mathbf{W}} J(\mathbf{W})$$

- Backpropagate the error to adjust the parameters

RNN Application - Sentiment Classification

- Classify a user review as 'positive' or 'negative'
  - Movie review from IMDB
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  - **Input:** A sequence of words in the review (one or more sentences)
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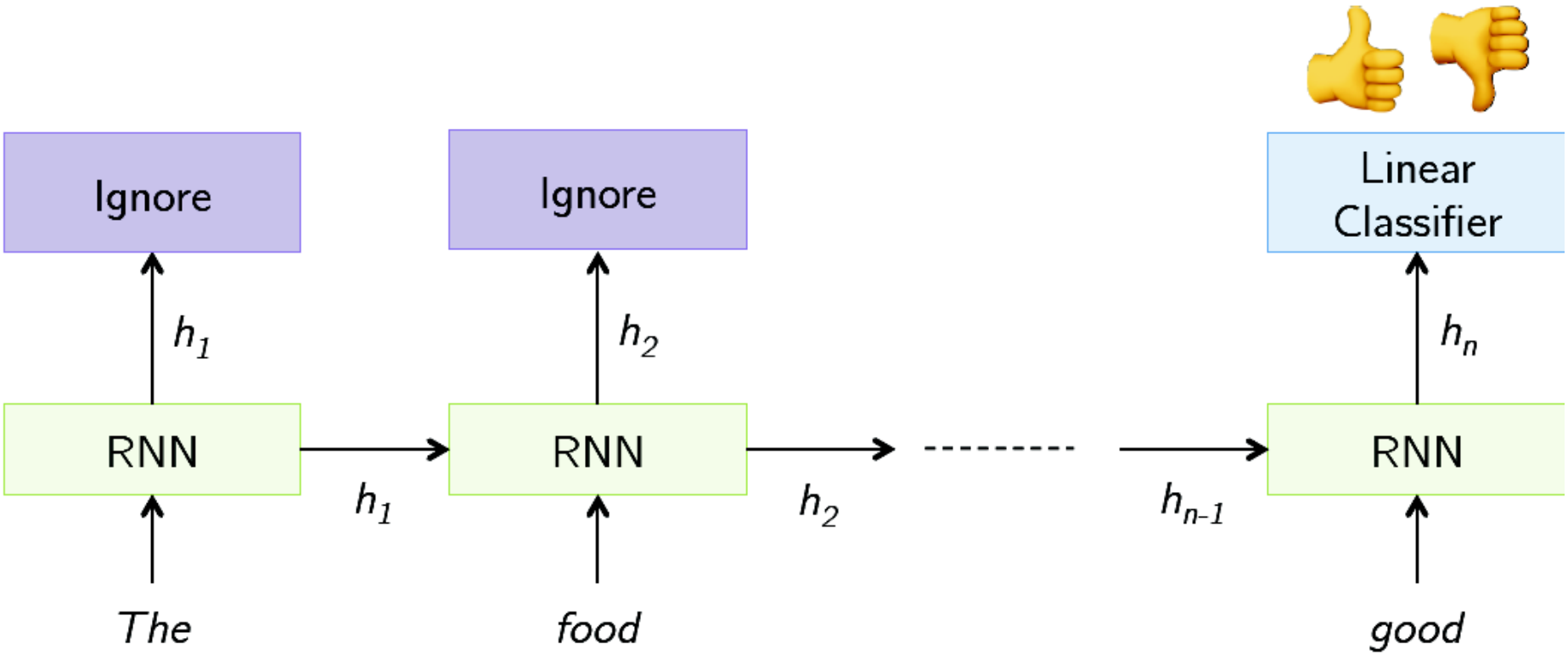
- Build a binary classifier for sentiment analysis
  - **Input:** A sequence of words in the review (one or more sentences)
  - **Output:** Positive or Negative review
  - "The food was really good!"
  - "The chicken crossed the road because it was uncooked"
- Need to analyze the sequence of words to make a decision
- FFNN are not useful as they consider the review as a set of words.

Sentiment Analysis



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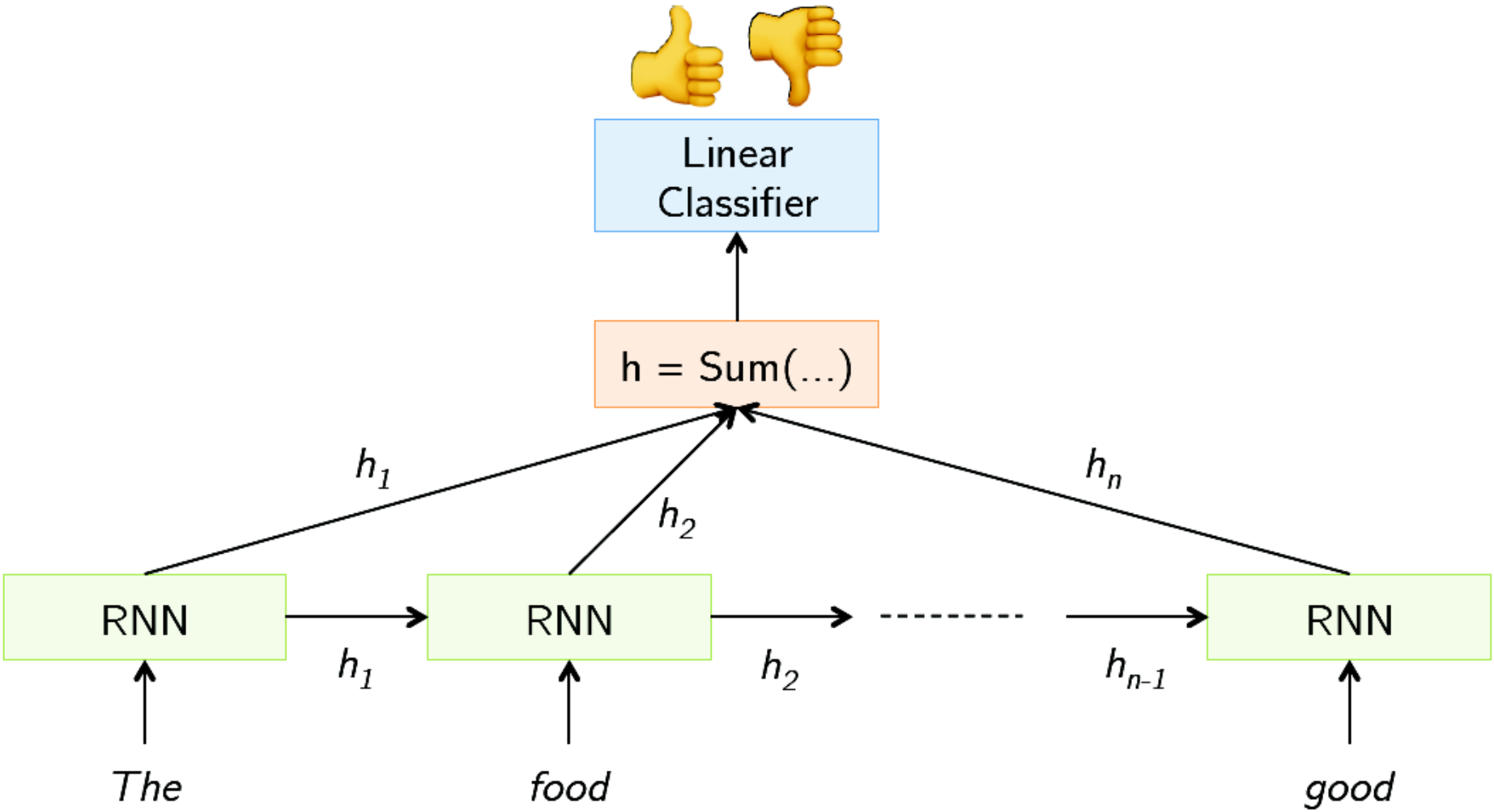

Image Captioning

- Given an image, generate a sentence describing the image content
  - **Input:** Image
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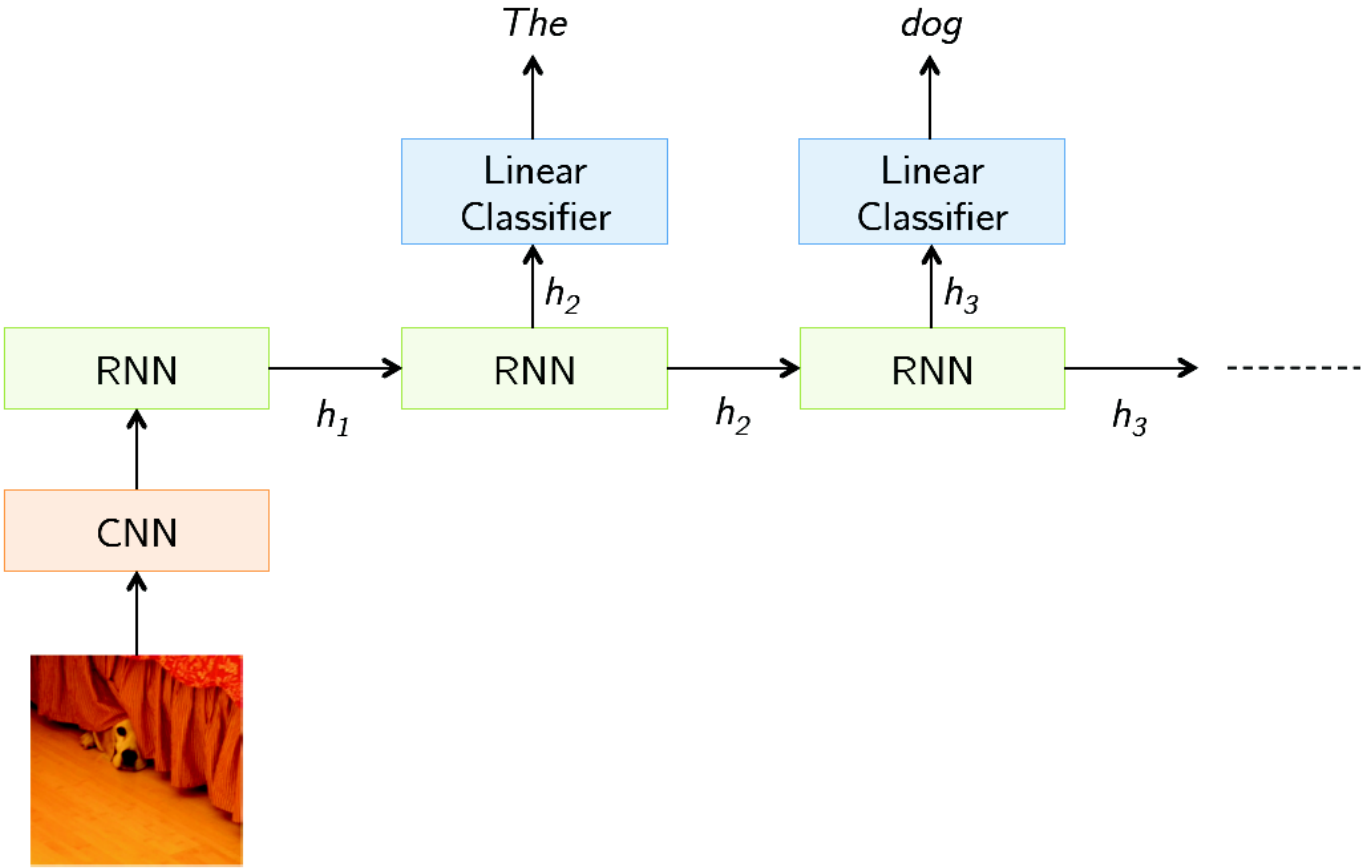

Neural Image Caption Generator

A person riding a motorcycle on a dirt road.

Two dogs play in the grass.

A herd of elephants walking across a dry grass field.

A group of young people playing a game of frisbee.

Two hockey players are fighting over the puck.

A close up of a cat laying on a couch.

Language Modelling

- Predict the next word in a sentence from the previous words
  - **Input:** Last K words in a sentence: $W_{N-K}, \cdots W_N$
  - Output: Predict W_{N+1}

$$P[W_{N+1}/W_N, W_{N-1}, \cdots W_{N-K}]$$

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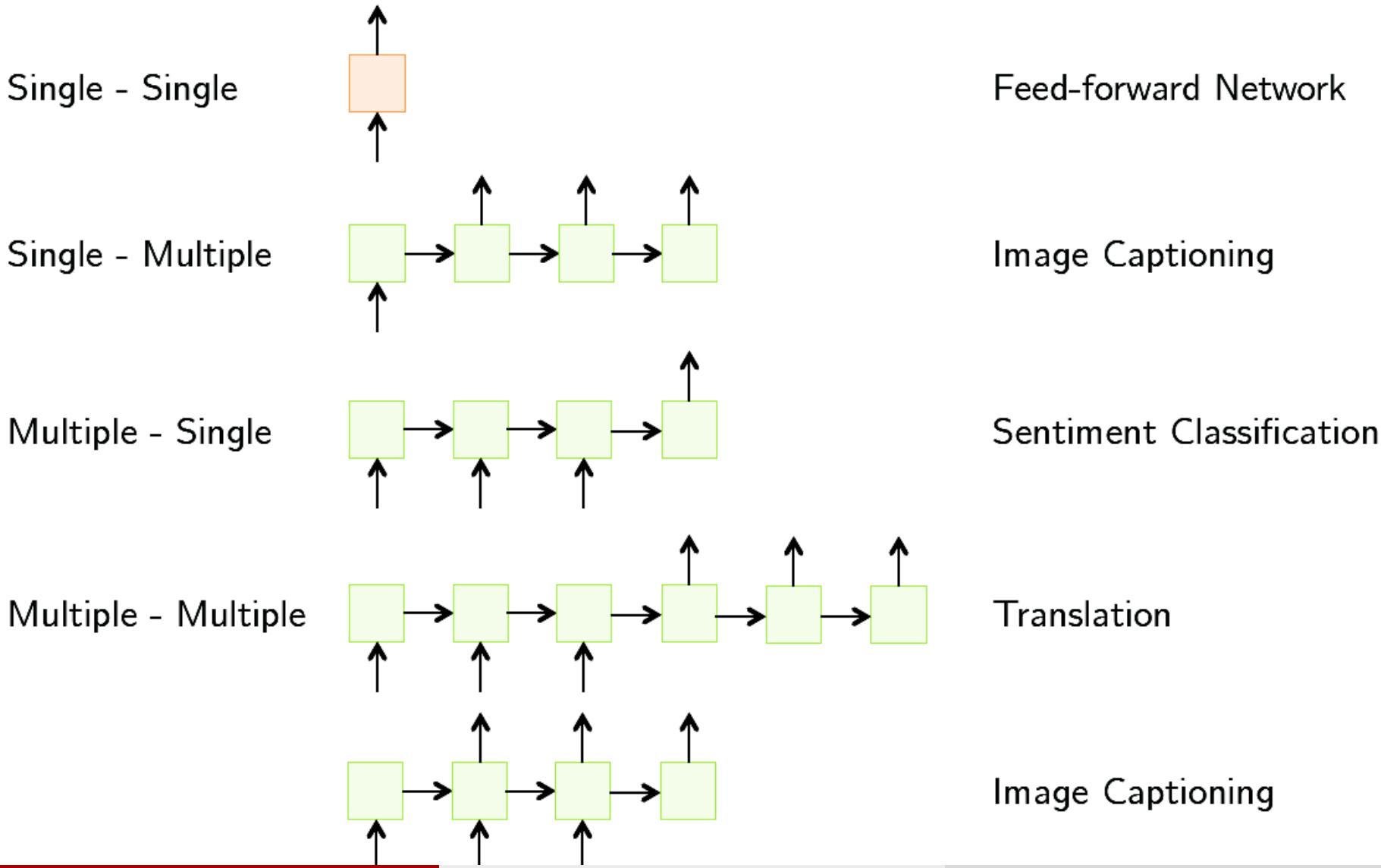
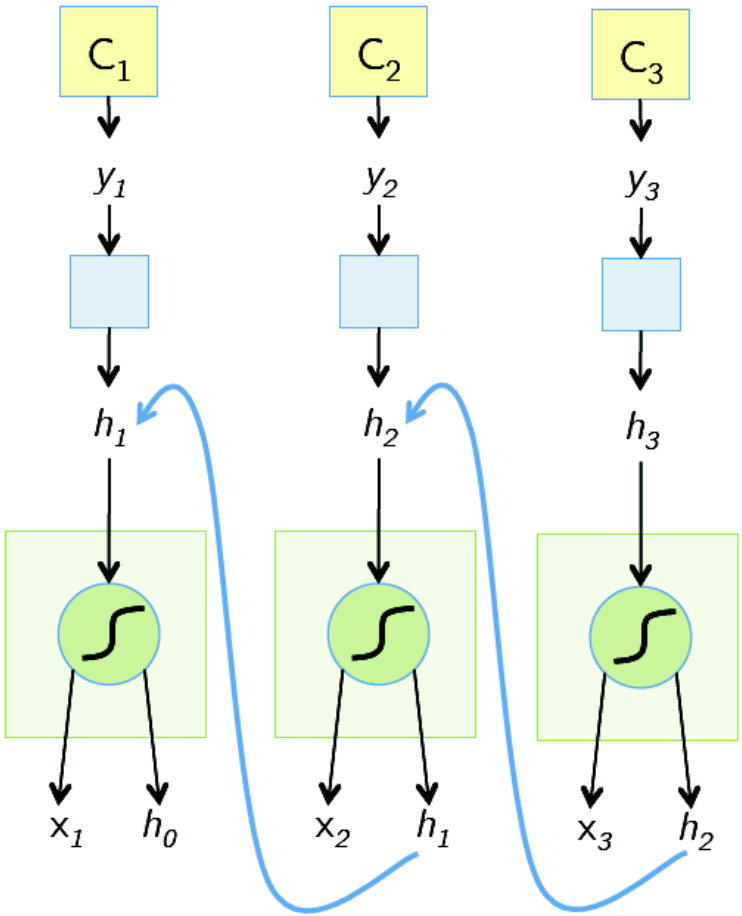
Why, Salisbury must find his flesh and thought. That which I am not aps, not a man and in fire. To show the reining of the raven and the wars. To grace my hand reproach within, and not a fair are hand. That Caesar and my goodly father's world;! When I was heaven of presence and our fleets. We spare with hours, but cut thy council I am great,! Murdered and by thy master's ready there. My power to give thee but so much as hell Some service in the noble bondman here, Would show him to her wine.

RNN Architectures



Temporal Backpropagation



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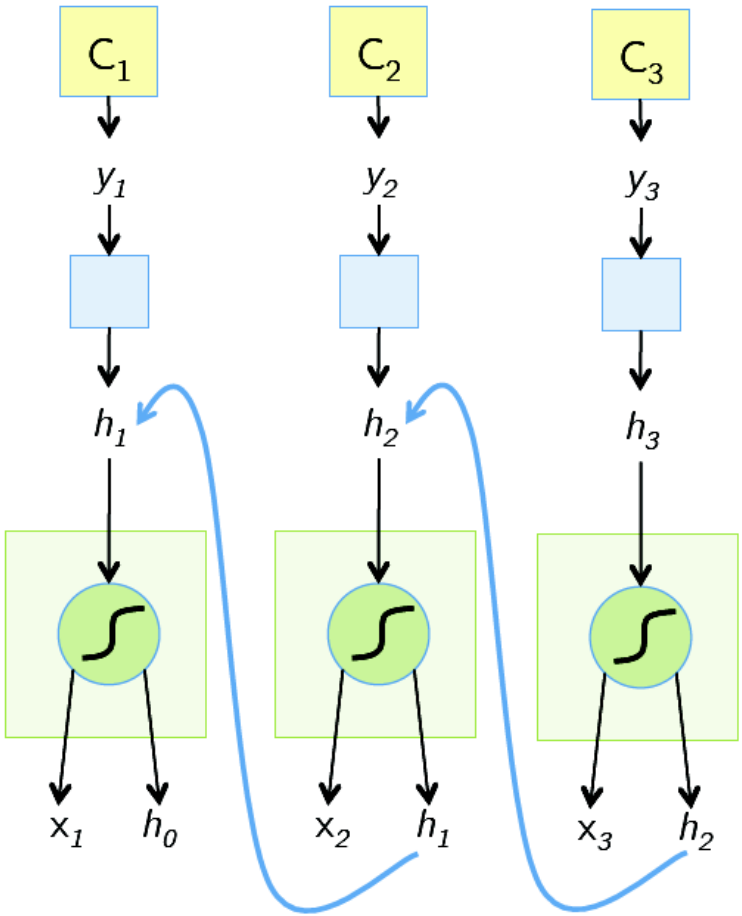

- Unfold the network during forward pass
- Treat unfolded network as a big FFN
- This big FFN takes entire time-series as input
- Compute gradients through usual backpropagation
- Update the shared weights

$$\mathbf{W}^{new} = \mathbf{W}^{old} - \eta \nabla_{\mathbf{W}} J(\mathbf{W})$$

- Error at n^{th} instant depends on all the previous hidden states $\mathbf{h}[k]$, $k <= n$

Issues with RNNs

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Effect of 1^{st} hidden state on error at the n^{th} instant

$$\frac{\partial J_n(\mathbf{W})}{\partial \mathbf{h}[1]} = \frac{\partial J_n(\mathbf{W})}{\partial \mathbf{y}[n]} \frac{\partial \mathbf{y}[n]}{\partial \mathbf{h}[1]}$$

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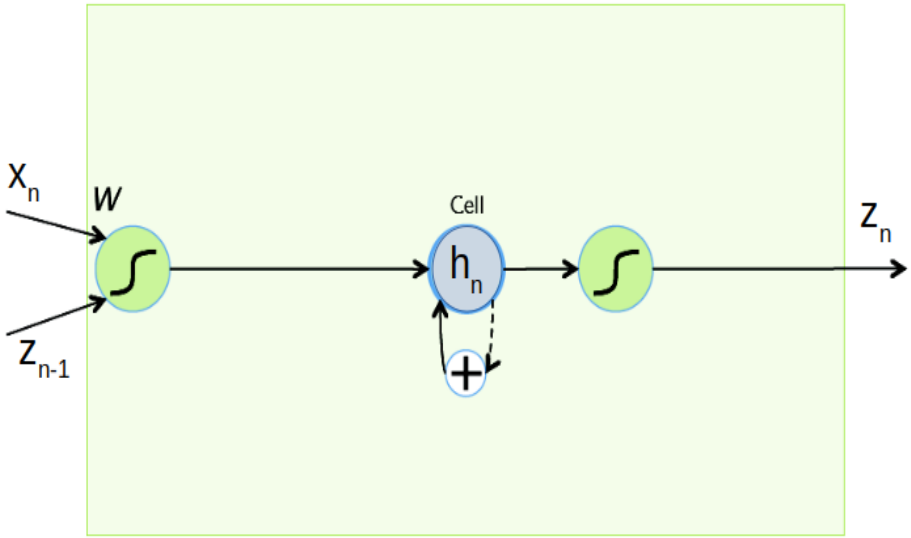
- RNNs suffer from either vanishing gradients or exploding gradients
- Introduce identity relation between the hidden states

$$\mathbf{h}[n] = \mathbf{h}[n-1] + g(\mathbf{x}[n]) \qquad \frac{\partial \mathbf{h}[n]}{\partial \mathbf{h}[n-1]} = 1$$

LSTM Idea



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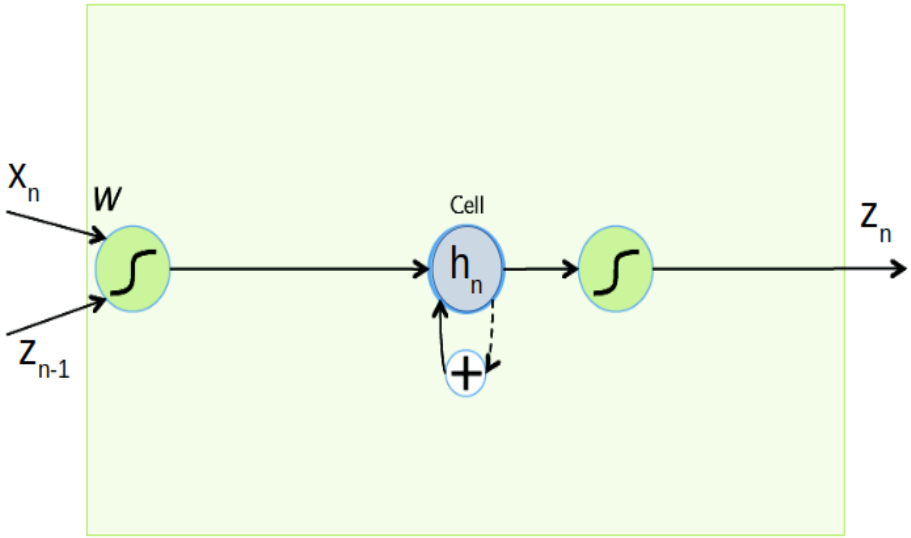

- $\mathbf{x}_n$ - current input
- $\mathbf{z}_{n-1}$ - previous output
- $\mathbf{h}_n$ - current cell state

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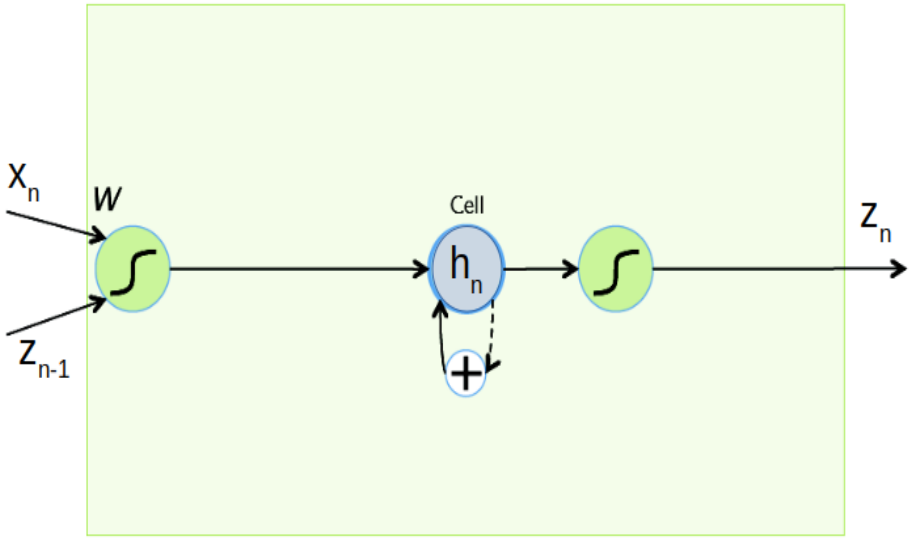

$$\mathbf{h}_n = \mathbf{h}[n-1] + \tanh(\mathbf{W}_{xh}\mathbf{x}_n + \mathbf{W}_{zh}\mathbf{z}_{n-1})$$

$$\mathbf{z}_n = \tanh(\mathbf{h}_n)$$

- $\mathbf{x}_n$ - current input
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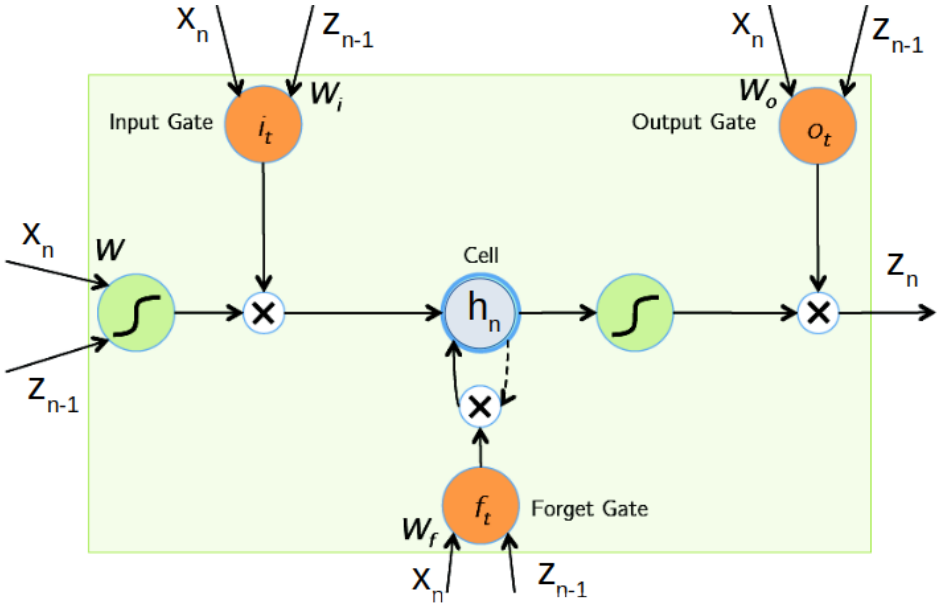
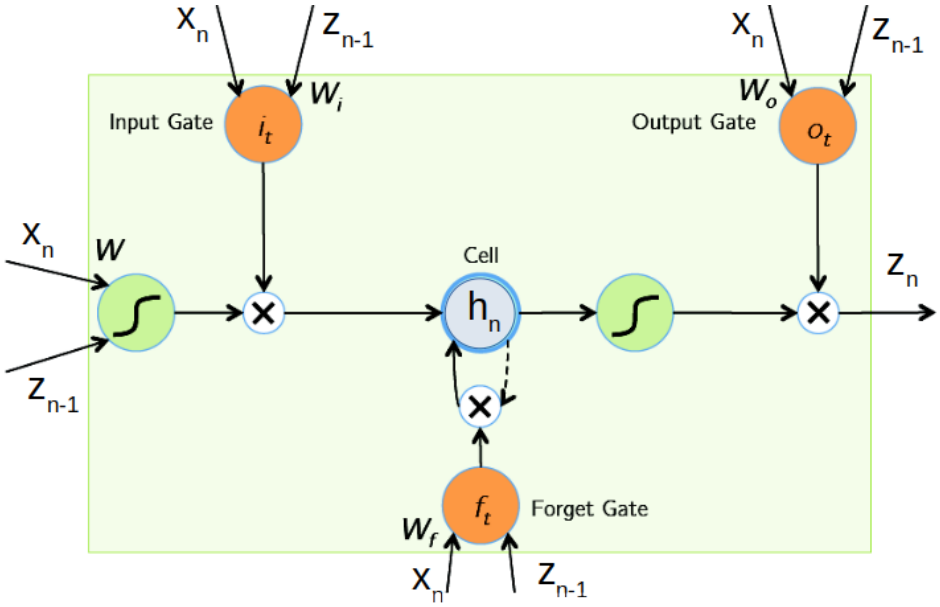
- State and output are different
- Creates highway path for state (to ensure gradient flow)
- State acts like an accumulator over time (memory)
- Additive update for the state

LSTM Cell

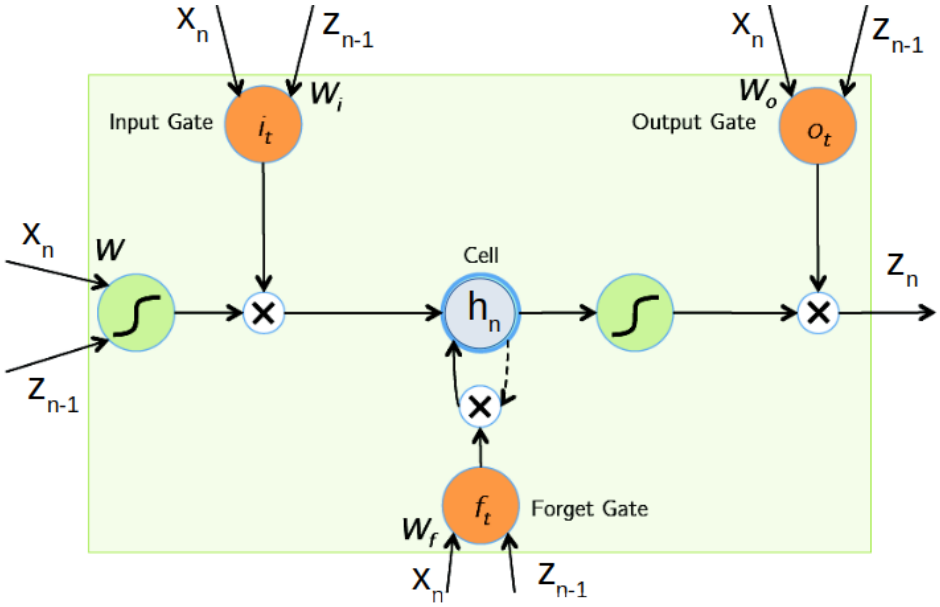

x_n
z_{n-1}
W_i
Input Gate
i_t
x_n
z_{n-1}
W_o
Output Gate
o_t
x_n
W
Cell
h_n
z_n
z_{n-1}
f_t
Forget Gate
W_f
x_n
z_{n-1}

LSTM Cell



- Gates control flow of info.
- Sigmoid functions for gating

LSTM Cell



- Gates control flow of info.
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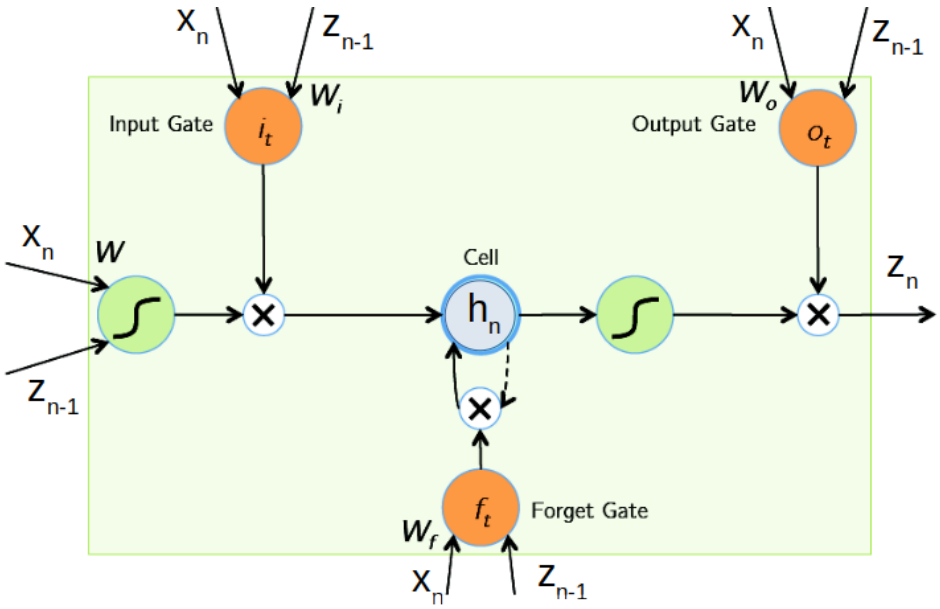
- Forget, input and output gates
- 3-fold increase in parameters

LSTM Cell



- Gating functions

$$\mathbf{f}_n = \sigma(\mathbf{W}_x^f \mathbf{x}_n + \mathbf{W}_z^f \mathbf{z}_{n-1})$$

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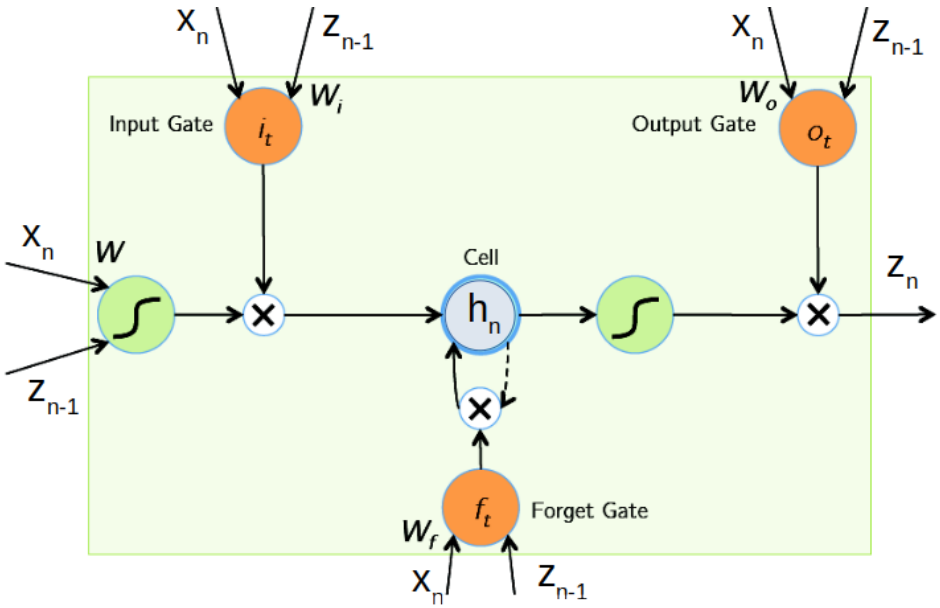

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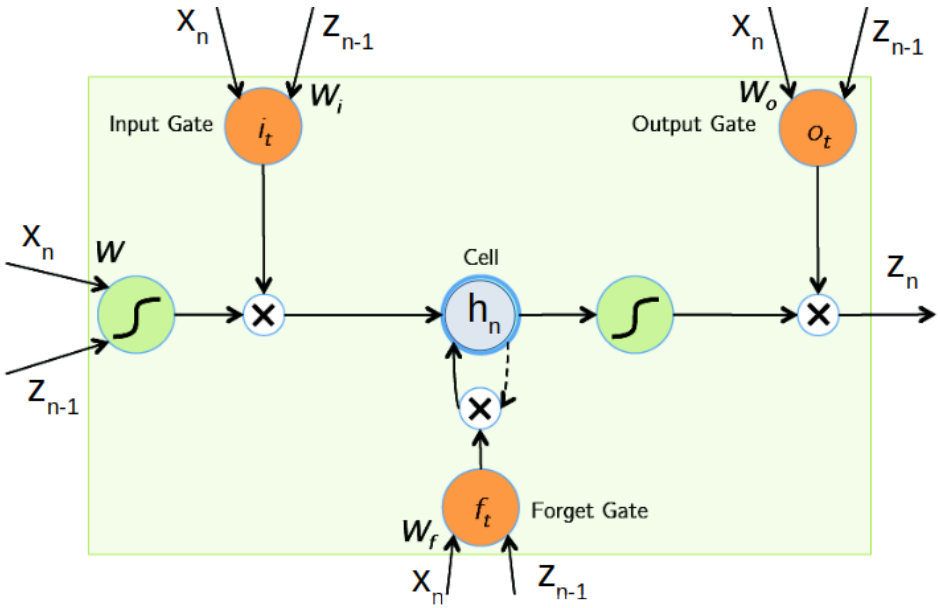

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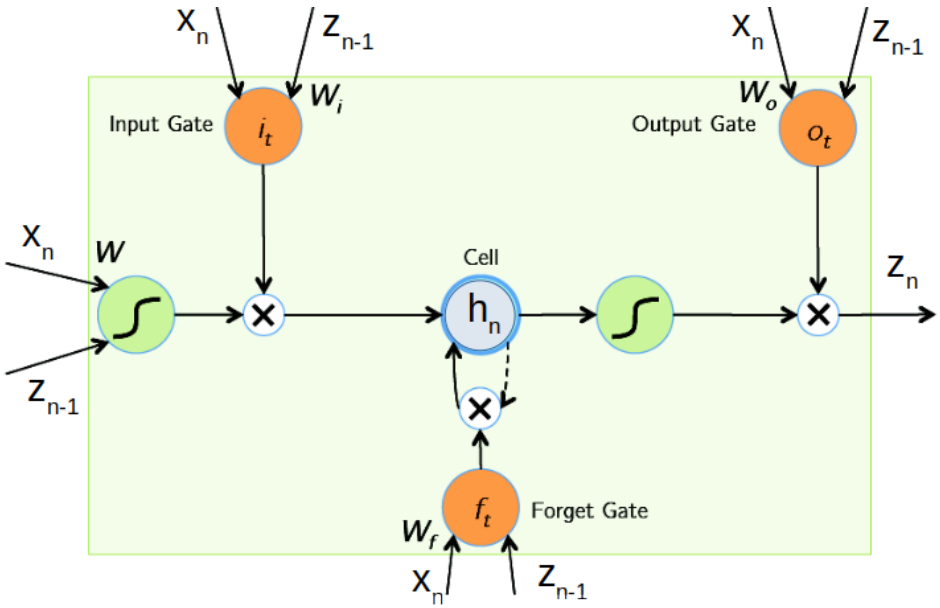

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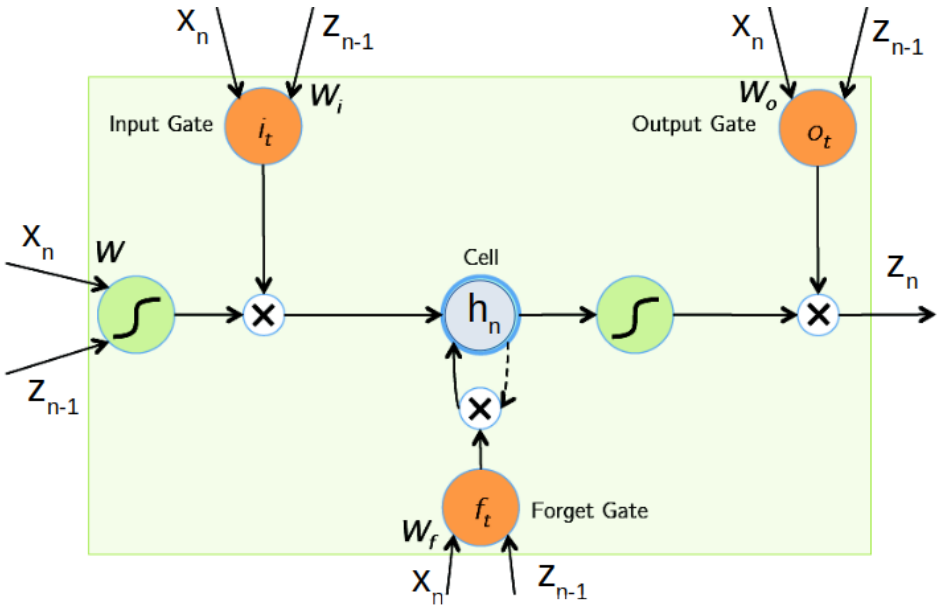

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- Cell output

$$\mathbf{z}_n = \tanh(\mathbf{h}[n]) \odot \mathbf{o}_n$$

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- Several LSTM variants were proposed in the literature
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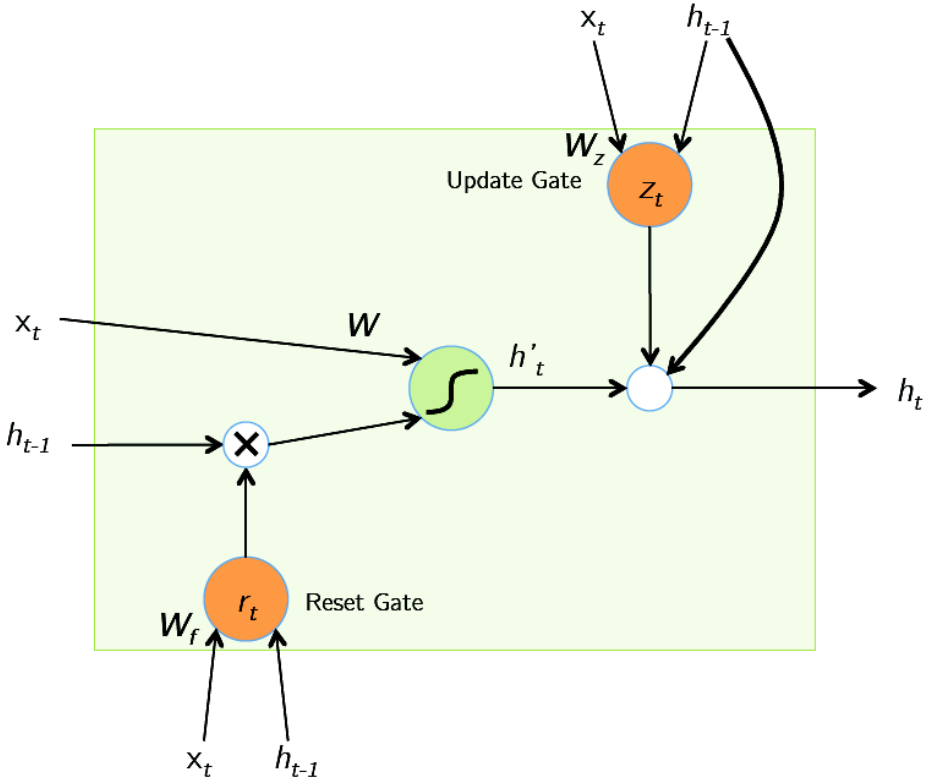
Gated Recurrent Unit (GRU)



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- Merges input and forget gates
- Merges cell state and output

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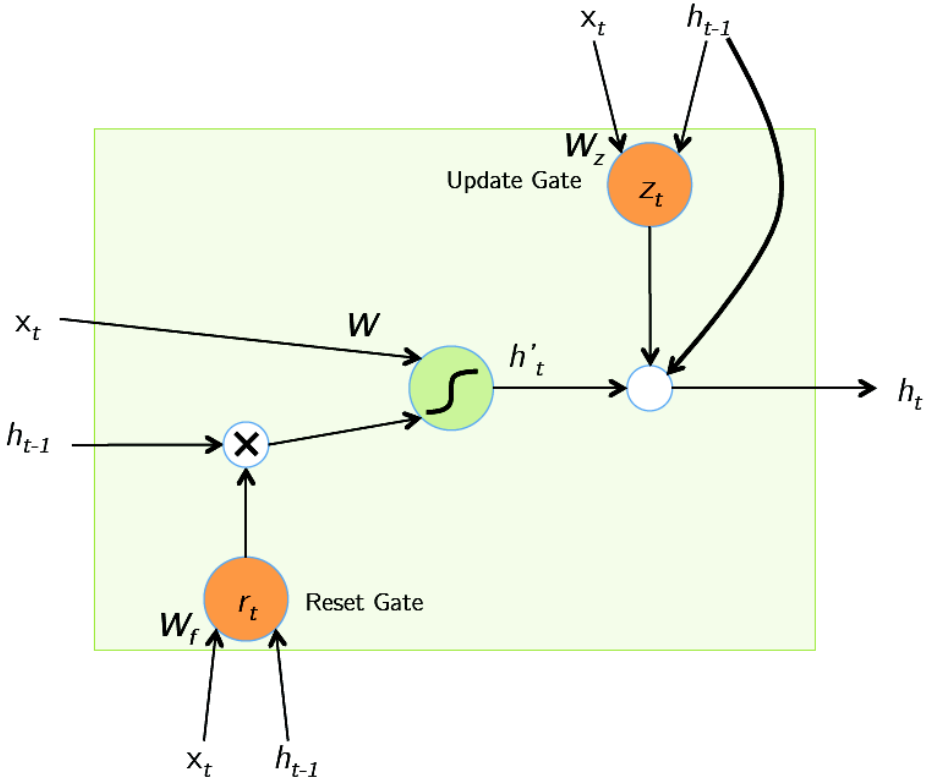

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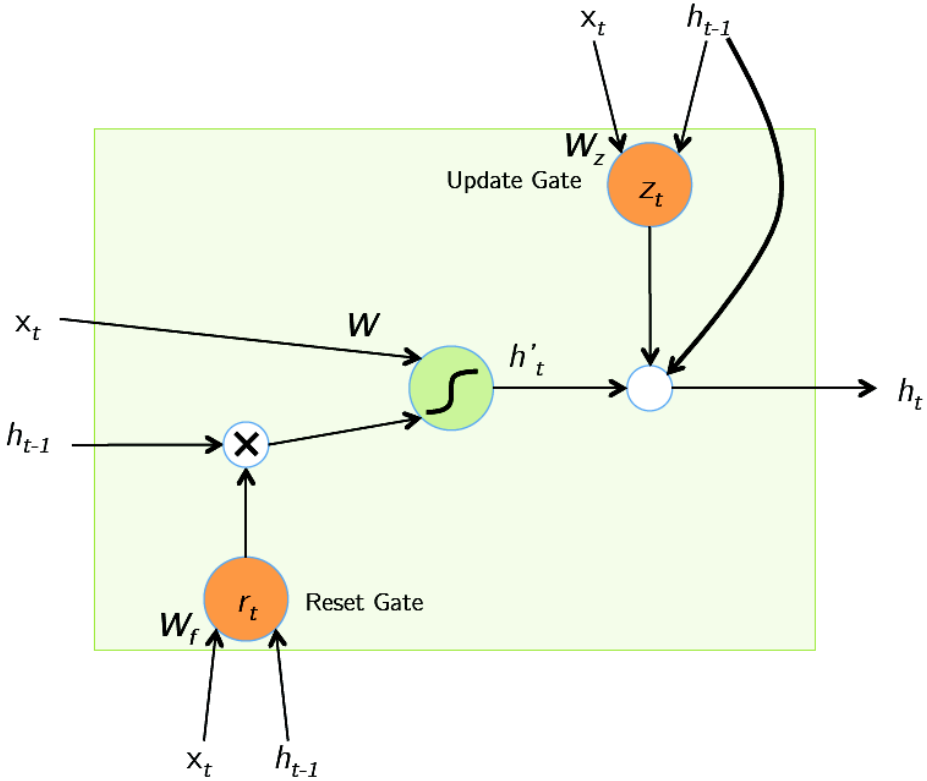

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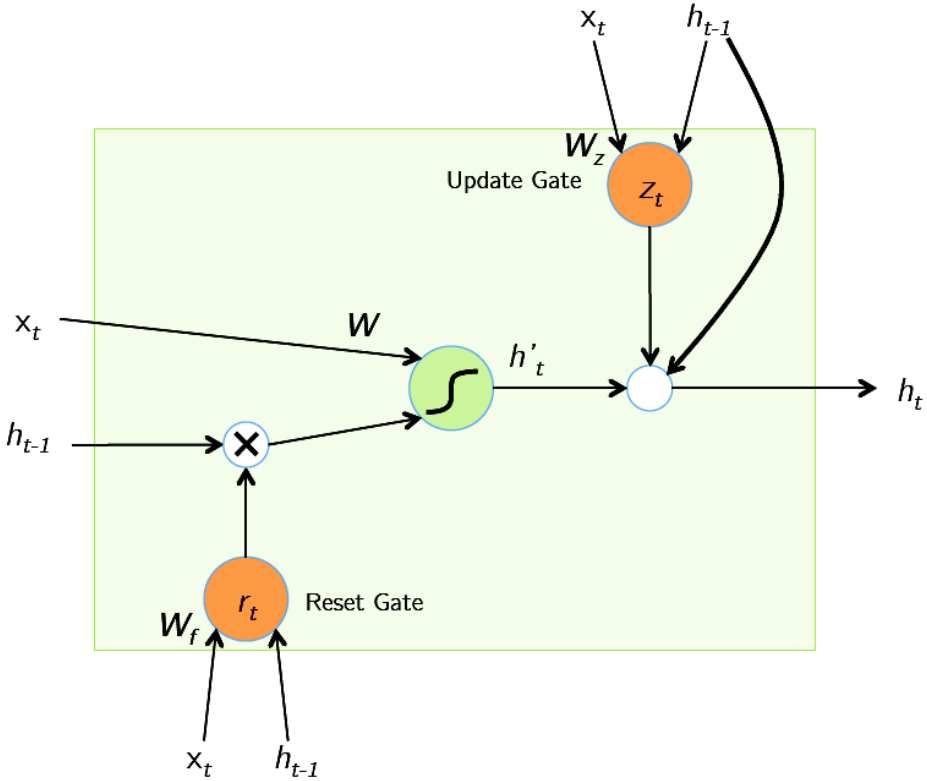

- Simplified version of LSTM
- Merges input and forget gates
- Merges cell state and output

- Gating functions

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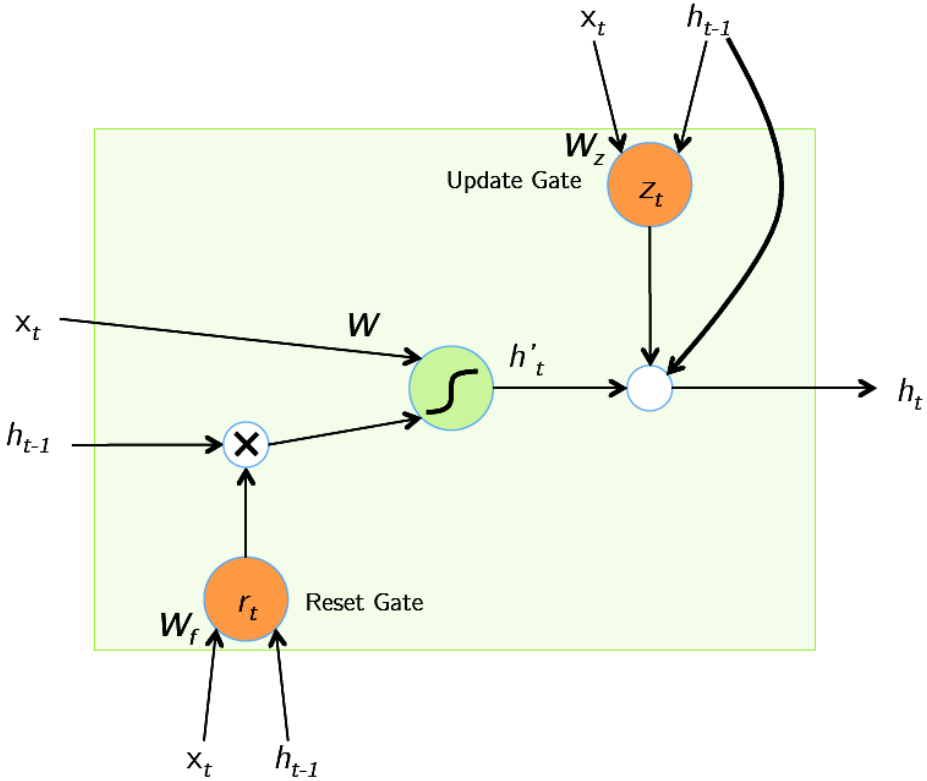

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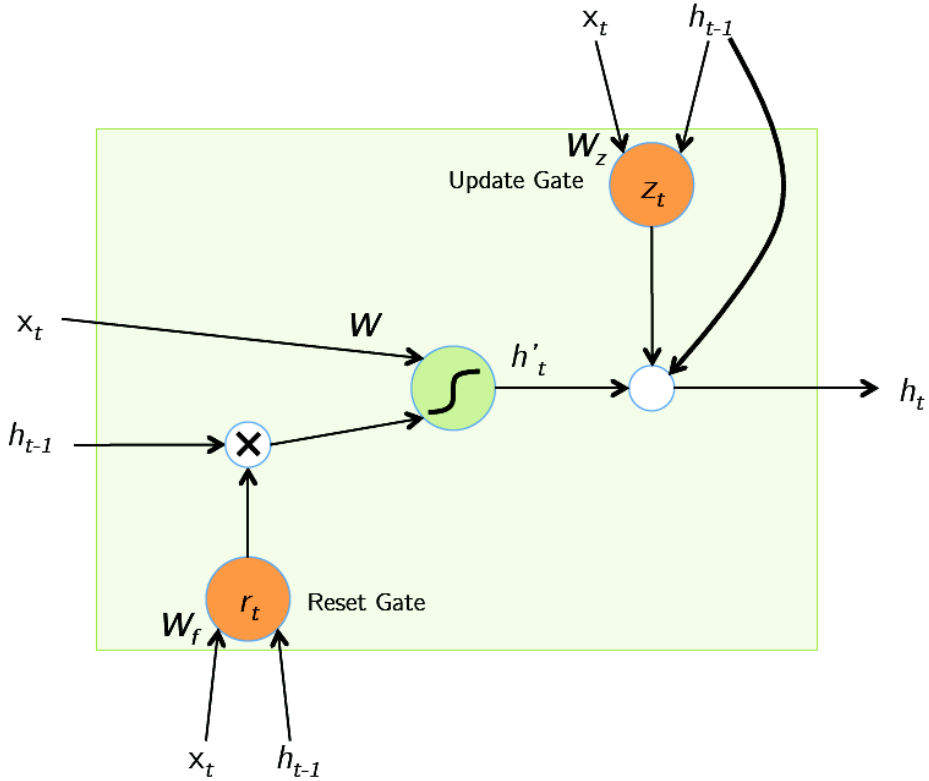

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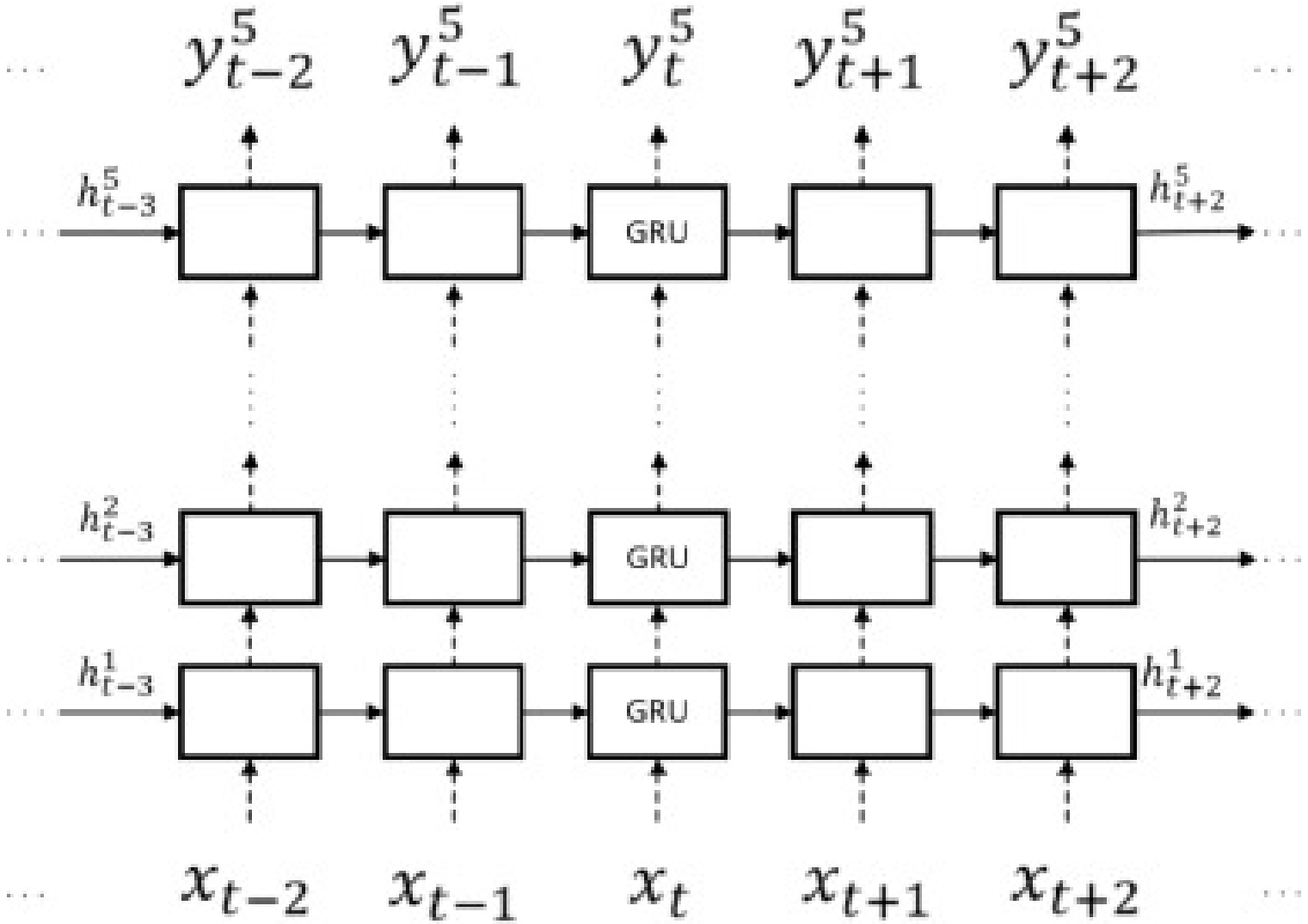
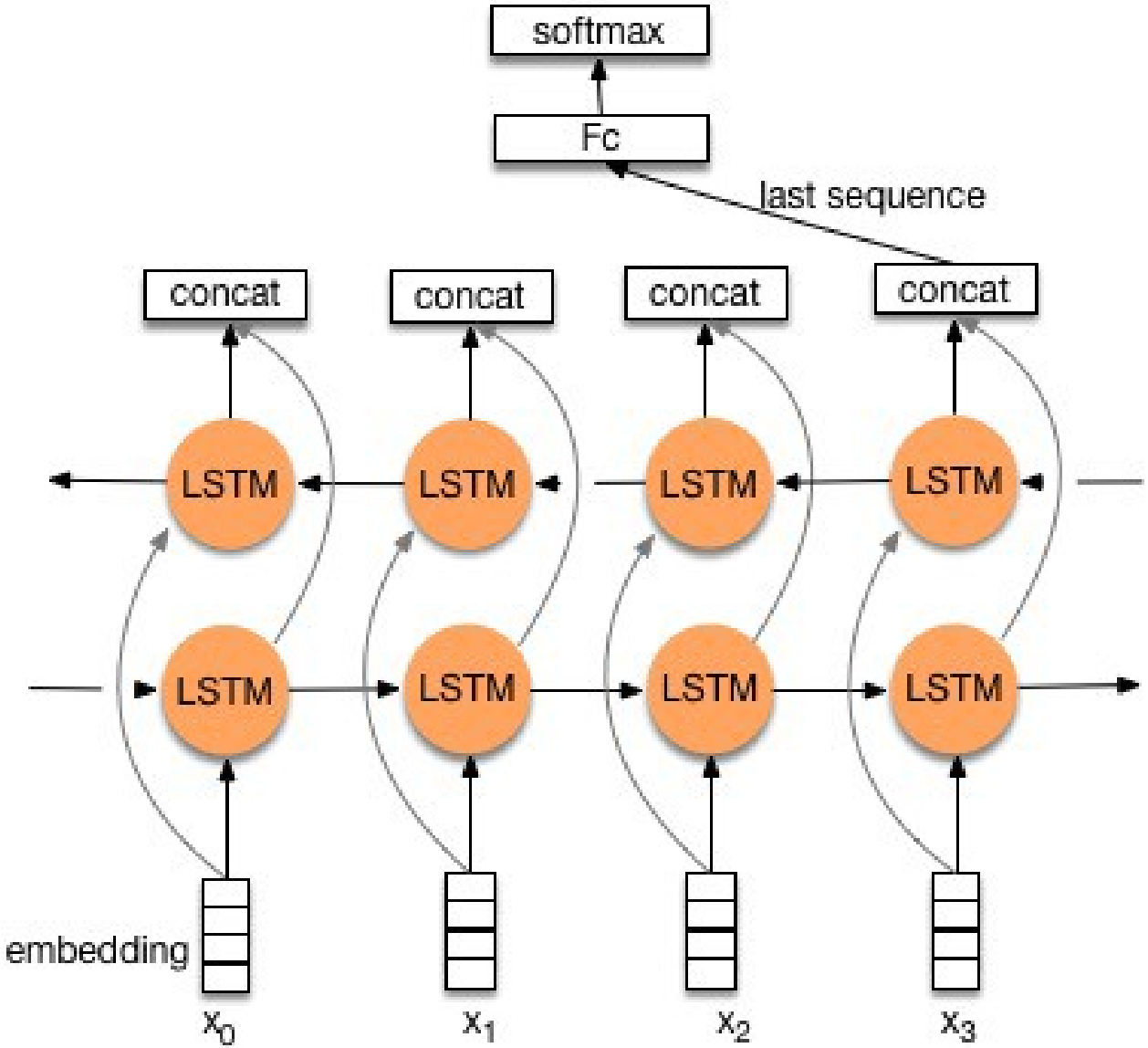
- GRUs were shown to outperform LSTMs in machine translation

Stacked RNN Layers



Bidirectional RNNs



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- Feed-forward alternates for sequence-to-sequence modeling
  - Autoregressive feed-forward models and Transformers
  - Advantages: Parallelization, Training stability, faster inference
  - FFNNs and stable RNNs offer similar performance

Autoregressive Feedforward Models

- Feedforward models to predict next sample from the past k samples
- Autoregressive models are generative models and capture the joint pdf

$$p(x_0, x_1, x_2, \cdots, x_n) = \prod_{k=0}^{n} p(x_k / x_0, x_1, x_2, \cdots x_{k-1})$$

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- AR models are amenable to conditioning - local ($\mathbf{v}_l$) and global ($\mathbf{v}_g$)

$$p(x_k / x_0, x_1, x_2, \cdots x_{k-1}, \mathbf{v}_g, \mathbf{v}_l)$$

- AR models can model multiple time scales (ms to minutes)
- Slow inference is their main drawback (Pixelnets and Wavenets)

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- The context-dependent vector x_n^c is added to context-independent vector x_n and processed further

Thank You!