

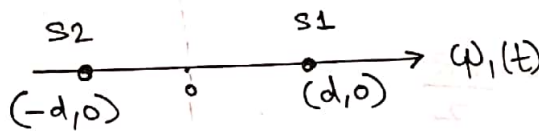
1. Problem Statement

Bit Error Rate (BER) and Symbol Error Rate (SER) performance evaluation of BPSK and QPSK.

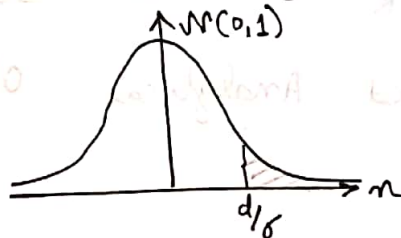
2. Theory

BPSK

The constellation diagram of BPSK can be drawn as



$$\begin{aligned} P_E(\text{error} | S_2) &= P(S_2 + n > 0) \\ &= P(n > d) \\ &= P\left(\frac{n}{\sigma} > \frac{d}{\sigma}\right) = Q\left(\frac{d}{\sigma}\right) \end{aligned}$$



Similarly $\Rightarrow P_E(\text{error} | S_1) = Q\left(\frac{d}{\sigma}\right)$

$$P_E = \frac{1}{2} Q\left(\frac{d}{\sigma}\right) + \frac{1}{2} Q\left(\frac{d}{\sigma}\right) = Q\left(\frac{d}{\sigma}\right)$$

$$d = \|S_1\| = \|S_2\| = \sqrt{E_s};$$

$$\sigma = \sqrt{\frac{N_0}{2}}$$

$$P_E = Q\left(\sqrt{\frac{2E_s}{N_0}}\right) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_s}{N_0}}\right)$$

[here $E_s = E_b$]

QPSK

P_E for MPSK is given by,

$$P_E = 2Q \left(\sqrt{\frac{2 \log_2 M E_b}{N_0}} \sin \frac{\pi}{M} \right)$$

For QPSK $M=4$

$$P_E = 2Q \left(\sqrt{\frac{4 E_b}{N_0}} \frac{1}{\sqrt{2}} \right) = 2Q \left(\sqrt{\frac{2 E_b}{N_0}} \right) = 2Q \left(\sqrt{\frac{E_s}{N_0}} \right)$$

$$\Rightarrow P_E = \frac{1}{2} \cdot 2 \operatorname{erfc} \left(\sqrt{\frac{E_s}{2 N_0}} \right) = \operatorname{erfc} \left(\sqrt{\frac{E_s}{2 N_0}} / 2 \right)$$

$$\Rightarrow P_E = \operatorname{erfc} \left(\sqrt{\text{SNR}} / 2 \right) \quad (\text{SER})$$

BER is given by $P_e = \frac{P_E}{2}$

$$\Rightarrow P_e = \frac{1}{2} \operatorname{erfc} \left(\frac{\sqrt{\text{SNR}}}{2} \right)$$

Observation from MATLAB plot

Modulation Scheme	SNR(dB)	SNR	SER		BER	
			Observed	Analytical	Observed	Analytical
BPSK $P_E = Q(\sqrt{SNR})$ $= \frac{1}{2} \text{erfc}(\sqrt{\frac{SNR}{2}})$	0	1	0.1587	0.1587	0.1587	0.1587
	2	1.585	0.104	0.1040	0.104	0.104
	5	3.162	0.03768	0.0377	0.03768	0.0377
	10	10	0.000782	0.0007827	0.000782	0.0007827
QPSK $P_E = \text{erfc}(\sqrt{\frac{SNR}{2}})$ $P_E = \frac{1}{2} \text{erfc}(\sqrt{\frac{SNR}{2}})$	0	1	0.4222	0.4795	0.2395	0.2398
	2	1.585	0.3391	0.3173	0.1865	0.1687
	5	3.162	0.1933	0.1138	0.105	0.061
	10	10	0.02535	0.0253	0.01325	0.01299