EE5802: DSP Lab
(Jan – April 2021)

Lecture - 4

Today's Topic

- Bit error rate (BER) Analysis of OFDM system in AWGN channel.

System model

- Wired channel or AWGN channel

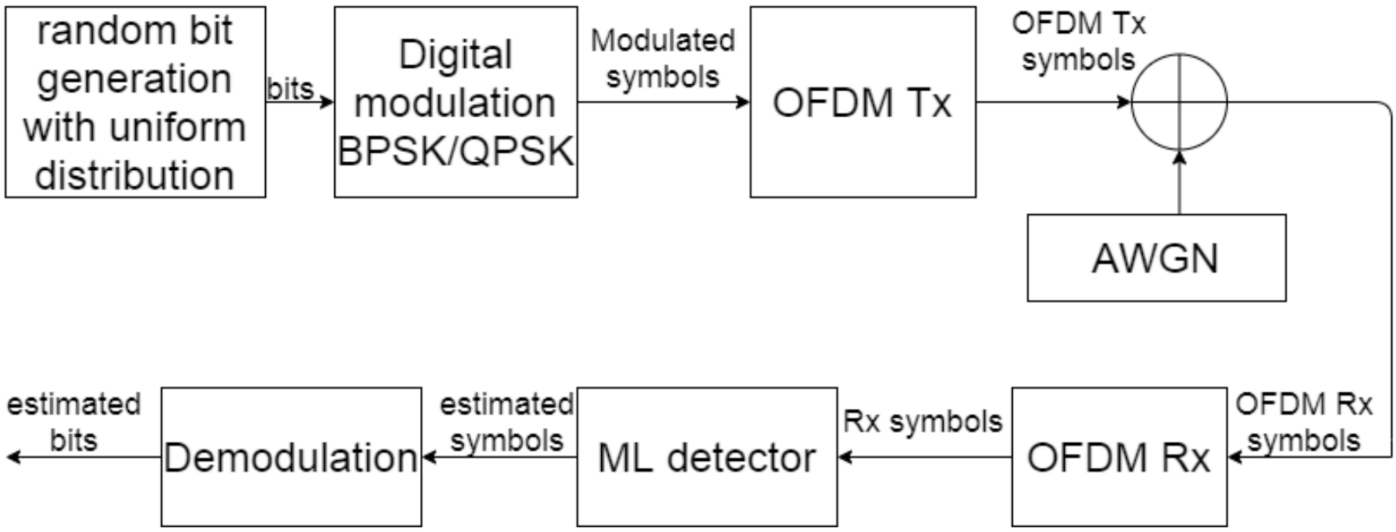
$$y = x + n$$

where, $\mathrm{x} = Transmited\ symbol$

$\mathrm{y} = Recived\ symbol\ at\ reciever$

$\mathrm{n} = AWGN(Adittive\ White\ Gaussian\ Noise)$

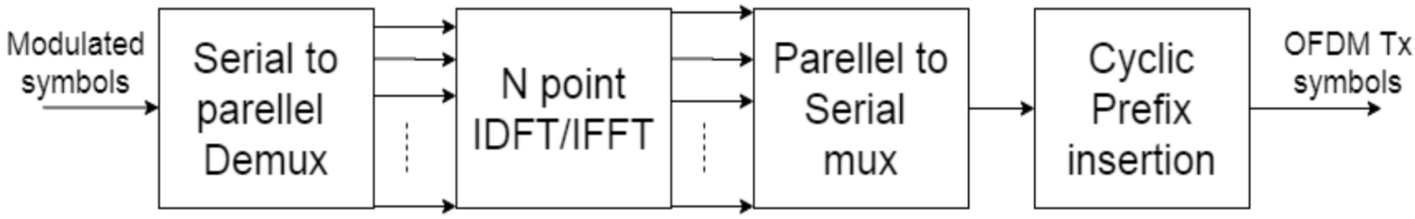
Block diagram of OFDM Tx and Rx with AWGN



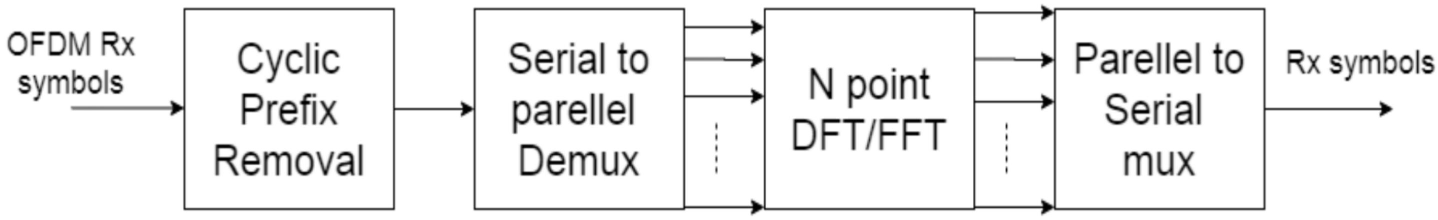
Explanation of each blocks

- Generate random bits with uniform distribution.
- Map bits to symbols
  - **BPSK** symbol set = {1,-1};
    Ex: $0 \rightarrow 1\mathbf{,}\ 1 \rightarrow -1$
  - **QPSK** symbol set = {1+1i,1-1i,-1+1i,-1-1i};
    Ex: $00 \rightarrow 1 + 1i$
- Complex AWGN noise generation(Device noise at receiver)
  - $n = sd * (randn + 1i\ randn)$

OFDM Tx



OFDM Rx



Explanation of each blocks(contd…)

- ML detector(minimum distance decoder)

$$estimated\ symbol(\hat{x}) = \min_{x \in C} |y - x|^2$$

- Ex: For BPSK, $C = \{1, -1\} = \{x_1, x_2\}$

  1. find $d_1 = y - x_1$ and $d_2 = y - x_2$
  2. if $d_1 > d_2$, then $\hat{x} = x_2$

     else if $d_1 < d_2$, then $\hat{x} = x_1$

- In demodulation estimate the bits from symbol

Normalization of digital symbols

- Average Energy of a $Symbol\ set(C)$ is

  $E_s = \frac{1}{M}\sum_{i=1}^{M}|x_i|^2$ where $x_i \in C$ and M=length of C
- BPSK symbol set={1,-1}

  Average energy: $E_s = 1$
- QPSK symbol set={1+1i,1-1i,-1+1i,-1-1i}

  Average energy: $E_s = 2$

  To make total energy of symbol set=1, we need to multiply the set with a factor of $1/\sqrt{2}$

Normalization of OFDM symbols

- IDFT: $x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j2\pi nk/N}$, $n = 0,1, \dots \dots, N-1$
- Matrix form: $\begin{bmatrix} x[0] \\ \vdots \\ x[N-1] \end{bmatrix} = \frac{1}{\mathrm{N}} \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix} \begin{bmatrix} X[0] \\ \vdots \\ X[N-1] \end{bmatrix}$
- If we have data in all subcarrier, then $\|\bar{X}\|_2^2 = N$, but $\|\bar{x}\|_2^2 = 1$
- To make energy of $\bar{x}$ same as $\bar{X}$ multiply $\bar{x}$ with a factor of $\sqrt{N}$.

Cyclic prefix (CP) insertion

- After ifft : $\underbrace{[x_1 \quad x_2 \quad \cdots \quad x_{N-L+1} \quad \cdots \quad x_N]}$

L samples

- After CP insertion :
$[x_{N-L+1} \quad \cdots \quad x_N \quad x_1 \quad x_2 \quad \cdots \quad x_{N-L+1} \quad \cdots \quad x_N]$

L samples

- Ex: In 4G, IFFT/FFT size is N = 2048 and length of CP is L = 144

Complex Gaussian noise

- $SNR\_lin = \frac{E_s}{N_0}$ where N_0 is variance of Complex Gaussian Noise.

  Var(real part)= Var(img part)= $N_0/2$
- Since we are multiplying σ with real and imaginary part of Complex Gaussian Noise, so we need to find standard deviation of real Gaussian.

$$SNR_{dB} = 10 * \log_{10} SNR_{lin}$$
$$SNR_{dB} = 10 * \log_{10} \frac{E_s}{2 * \sigma^2}$$
$$\Rightarrow \frac{E_s}{2 * \sigma^2} = 10^{SNR_{dB}/10} = SNR\_lin$$

$$\Rightarrow \sigma = \sqrt{\frac{E_s}{2} * 10^{-\frac{SNR_{dB}}{20}}} = \sqrt{\frac{E_s}{2 * SNR\_lin}}$$

Symbol energy and bit energy

- $E_s = nE_b$, where n is no. of bits per symbol,

  E_s is Symbol enegy and E_b is bit energy.
- $\left(\frac{E_S}{N_0}\right)_{dB} = \left(\frac{E_S}{N_0}\right)_{dB} + 10\log_{10} n$

Pseudo code for BER analysis of BPSK/QPSK

```
for  Eb/N0 = 0:10 dB
          Es/N0_BPSK = ?
          sd_BPSK = ?
          for iter = 1:N
                    1. bit generation
                    2. mapping to symbol(BPSK/QPSK)
                    3.  AWGN
                    4. ML detector
                    5. estimate bits
                    6. find no. of symbols in error
                    7. find no. of bits in error
          end
BER(Eb/N0) = no. of bits in error/total no. of bits
end
Plot BER vs SNR(Eb/N0)
```

Pseudo code for OFDM system?

you need to write and submit it.

Thank you