AI 1103 Assignment-3

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Download all latex-tikz codes from

https://github.com/Shantanu467/AI1103/blob/main/Assigment_3/Assignment3.tex

Problem

Gate 2010 (MA): QUESTION-48

Let X and Y be continuous random variables with joint probability density function

$$f(x,y) = \begin{cases} a \times e^{-2y} & 0 < x < y < \infty \\ 0 & otherwise \end{cases}$$

The value of a is

(A) 4
(B) 2
(C) 1
(D) 0.5

Solution

Theorem 1: The integral of Probability Density Function over the continuous random variable is equal to 1.

$$\text{Using This, } \left(\Pr\left(-\infty < X < \infty\right)\right) = 1 \quad (1)$$

$$\int_{y=\infty}^{-\infty}\int_{x=y}^{+\infty} f(x,y)\,\mathrm{d}x\,\mathrm{d}y = 1 \quad (2)$$

$$0 + \int_{y=0}^{} \int_{x=0}^{} a \times \mathrm{e}^{-2y}\,\mathrm{d}x\,\mathrm{d}y = 1 \quad (3)$$

$$a \times \int_{y=0}^{y=\infty} \left(y\mathrm{e}^{-2y}\right)\,\mathrm{d}y = 1 \quad (4)$$

$$a \times \frac{1}{4} = 1 \quad (5)$$

$$\text{So, } a = 4 \quad (6)$$

Therefore, the correct option is **(A).**