AI 1103 Assignment-3

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PROBLEM

Gate 2010 (MA): QUESTION-48
Let X and Y be continuous random variables with joint probability density function

$$f(x,y) = \begin{cases} a \times e^{-2y} & 0 < x < y < \infty \\ 0 & otherwise \end{cases} \quad (1)$$

The value of a is
(A) 4
(B) 2
(C) 1
(D) 0.5

SOLUTION

Theorem 1: The integral of Probability Density Function over the continuous random variable is equal to 1.

Using This, $(\Pr(-\infty < X < \infty)) = 1 \quad (2)$

$$\iint_{-\infty}^{+\infty} f(x,y)\,\mathrm{d}x\,\mathrm{d}y = 1 \quad (3)$$

$$0 + \int_{y=0}^{y=\infty} \int_{x=0}^{x=y} a \times e^{-2y}\,\mathrm{d}x\,\mathrm{d}y = 1 \quad (4)$$

$$a \times \int_{y=0}^{y=\infty} \left(ye^{-2y}\right)\,\mathrm{d}y = 1 \quad (5)$$

$$a \times \frac{1}{4} = 1 \quad (6)$$

$$\text{So, } a = 4 \quad (7)$$

Therefore, the correct option is **(A).**

Marginal PDF of X is,

$$f_X(X) = \int_{y=-\infty}^{y=\infty} f_X(x,y)\,\mathrm{d}y \quad (8)$$

$$= 0 + \int_{y=0}^{y=\infty} 4 \times e^{-2y}\,\mathrm{d}y \quad (9)$$

$$= \left(-2e^{-2y}\right)\Big|_0^{+\infty} \quad (10)$$

$$= 0 - \left(-2e^0\right) \quad (11)$$

$$= 2 \quad (12)$$

Marginal PDF of Y is,

$$f_Y(Y) = \int_{x=-\infty}^{x=\infty} f_Y(x,y)\,\mathrm{d}x \quad (13)$$

$$= 0 + \int_{x=0}^{x=y} 4 \times e^{-2y}\,\mathrm{d}x \quad (14)$$

$$= \left(4e^{-2y}\right) \times (x)\Big|_0^y \quad (15)$$

$$= 4ye^{-2y} \quad (\text{here:}0 < y < \infty) \quad (16)$$