

# AI 1103 Assignment-3

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Download all latex-tikz codes from

[https://github.com/Shantanu467/AI1103/blob/main/Assignment\\_3/Assignment3.tex](https://github.com/Shantanu467/AI1103/blob/main/Assignment_3/Assignment3.tex)

## PROBLEM

### Gate 2010 (MA): QUESTION-48

Let  $X$  and  $Y$  be continuous random variables with joint probability density function

$$f(x, y) = \begin{cases} a \times e^{-2y} & 0 < x < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

The value of  $a$  is

- (A) 4
- (B) 2
- (C) 1
- (D) 0.5

## SOLUTION

*Theorem 1:* The integral of Probability Density Function over the continuous random variable is equal to 1.

Using This,  $(\Pr(-\infty < X < \infty)) = 1$  (1)

$$\iint_{-\infty}^{+\infty} f(x, y) dx dy = 1 \quad (2)$$

$$0 + \int_{y=0}^{y=\infty} \int_{x=0}^{x=y} a \times e^{-2y} dx dy = 1 \quad (3)$$

$$a \times \int_{y=0}^{y=\infty} (ye^{-2y}) dy = 1 \quad (4)$$

$$a \times \frac{1}{4} = 1 \quad (5)$$

$$\text{So, } a = 4 \quad (6)$$

Therefore, the correct option is (A).

Marginal PDF of  $X$  is,

$$f_X(X) = \int_{y=-\infty}^{y=\infty} f_X(x, y) dy \quad (7)$$

$$= 0 + \int_{y=0}^{y=\infty} 4 \times e^{-2y} dy \quad (8)$$

$$= (-2e^{-2y}) \Big|_0^{+\infty} \quad (9)$$

$$= 0 - (-2e^0) \quad (10)$$

$$= 2 \quad (11)$$

Marginal PDF of  $Y$  is,

$$f_Y(Y) = \int_{x=-\infty}^{x=\infty} f_Y(x, y) dx \quad (12)$$

$$= 0 + \int_{x=0}^{x=y} 4 \times e^{-2y} dx \quad (13)$$

$$= (4e^{-2y}) \times (x) \Big|_0^y \quad (14)$$

$$= 4ye^{-2y} \quad (\text{here: } 0 < y < \infty) \quad (15)$$