CS103 Unit 8b

Intro to Recursion

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Recursion

- Defining an object, mathematical function, or computer function in terms of *itself*

GNU

- Makers of gedit, g++ compiler, etc.
- GNU = GNU is Not Unix

GNU is Not Unix

GNU is Not Unix

… is Not Unix is not Unix is Not Unix

Recursion

- Problem in which the solution can be expressed in terms of ***smaller versions of itself*** and ***a base/terminating case***
- Can often take the place of a loop and can lead to much more elegant coding solutions that if we were too use a loop
  - More guidance on this later
- Input to the problem must be categorized as a:
  - **Base case**: Small version of problem whose known beforehand or easily computable (no recursion needed)
  - **Recursive case**: Solution can be described using solutions to smaller problems of the same type
    - Keeping putting in terms of something smaller until we reach the base case
- Factorial: `n! = n * (n-1) * (n-2) * … * 2 * 1`
  - `n! = n * (n-1)!`
  - Base case: `n = 1`
  - Recursive case: `n > 1 =>  n*(n-1)!`

Recursive Function Mechanics

- Avoid infinite recursions by
  - Put the base case check first
  - Ensuring you recurse on a smaller problem [e.g. f(n) should NOT recurse on f(n)] ***but a smaller version of the problem***
- The system stack provides separate areas of memory for each ***instance*** of a recursive function
  - Thus each local variable and actual parameter of a function is separate and isolated from other recursive function instances [e.g. each fact(n) has it's own version of n]

C Code:

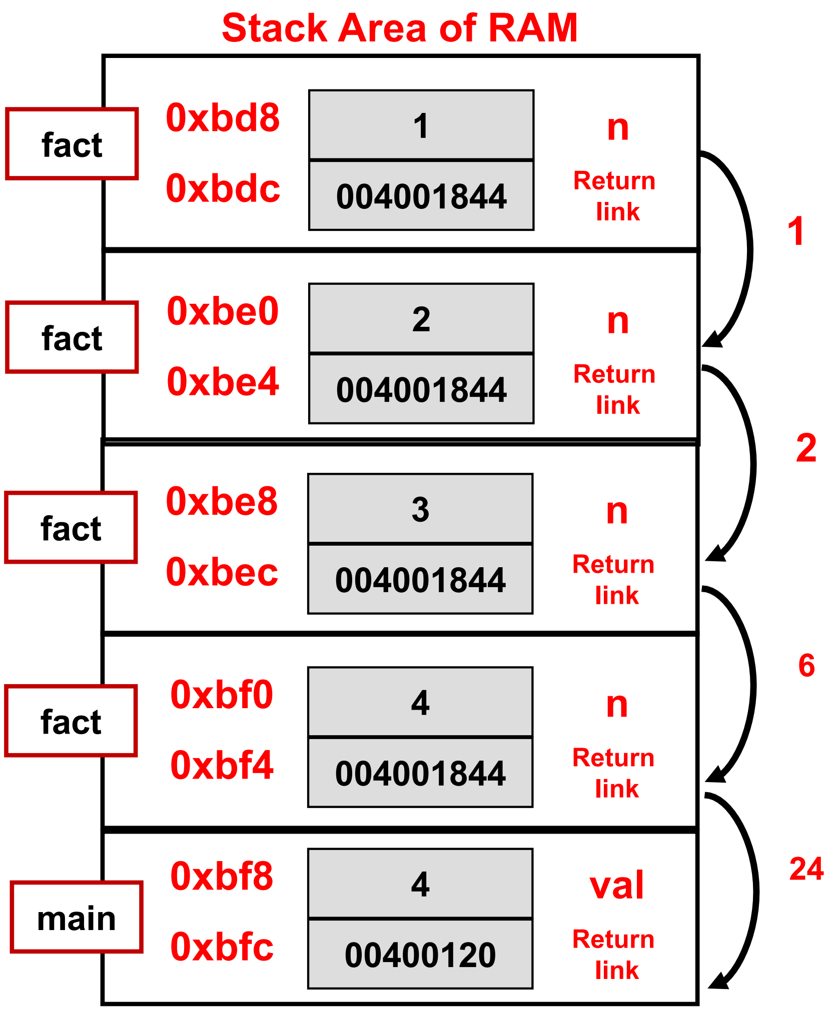
```
int fact(int n)
{
  if(n == 1){
     // base case
     return 1;
  }
  else {
     // recursive case
     return n * fact(n-1);
  }
}
```

Factorial:

- n! = n * (n-1) * (n-2) * … * 2 * 1
- n! = n * (n-1)!
  - Base case: n = 1
  - Recursive case: n > 1 => n*(n-1)!

Recursion & the Stack

- Must return back through the each call



Value/version of n is implicitly "saved" and "restored" as we move from one instance of the 'fact' function to the next

```
int fact(int n)
{
  if(n == 1){
     // base case
     return 1;
  }
  else {
     // recursive case
     return n * fact(n-1);

  }
}


int main()

{

  int val = 4;

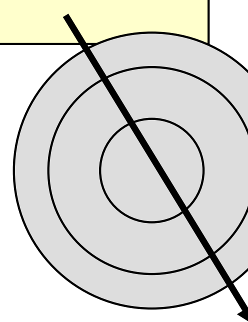
  cout << fact(val) << endl;

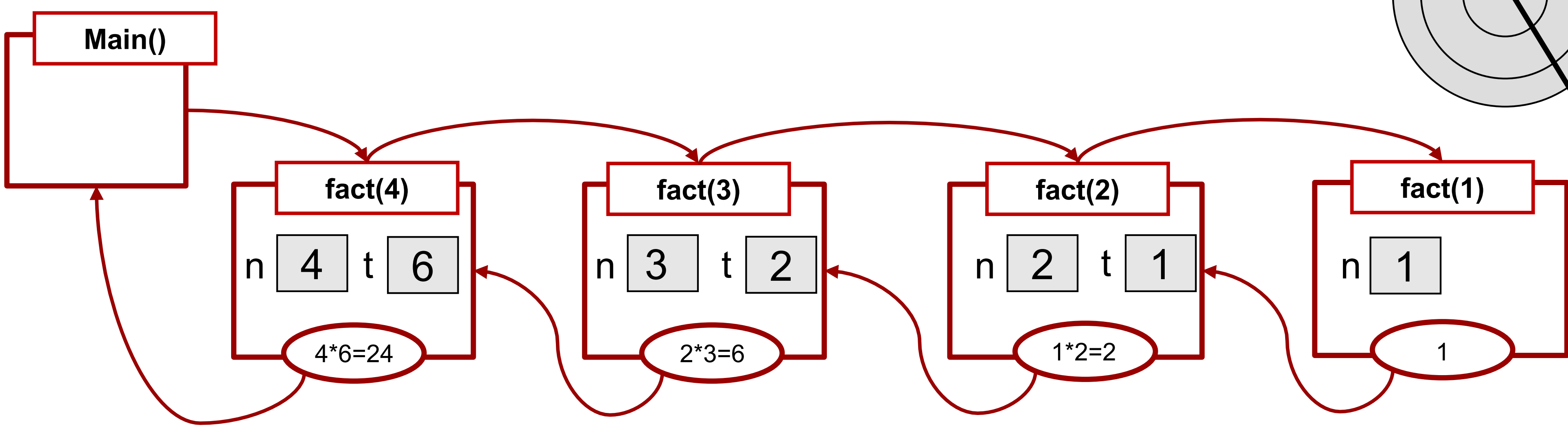
}
```

Recursive Analysis Tip: Box Diagrams

- Just for example's sake let's introduce a temporary local variable, t
- To analyze recursive functions draw a box diagram which is…
  - A simplified view of each function instance on the stack
- One box per function
  - Show arguments, pertinent local variables, and return values

```
int fact(int n)
{
  if(n == 1){
     return 1;
  }
  else {
     int t = fact(n-1);
     return n * t;
  }
}
```





Steps to Formulating Recursive Solutions

1. Write out some solutions for a few input cases to discover how the problem can be decomposed into smaller problems of the same form
   – Does solving the problem on an input of smaller value or size help formulate the solution to the larger
2. Identify the base case
   – An input for which the answer is trivial
3. Identify how to combine the small solution(s) to solve the larger problem

Put another way, there are often 2 principles to finding recursive solutions

- Principle 1: Generally recursive functions are "responsible" for ONLY 1 value / element / item and use recursion to handle all remaining items
- Principle 2: Determine how to combine the 1 element you are responsible for and the answer returned by recursion
  – Assume via the "magic of recursion" you get the answer for all remaining elements. How can you combine that with your 1 element to form the large solution

Recursive Functions

- **Many loop/iteration based approaches can be defined recursively as well**
- How could **summing an array** be implemented recursively?
  - Question to answer:
    - What 1 thing could each recursion be responsible for?
    - How could we combine that 1 thing with the solution from the recursive call?
- Once you have a recursive formulation all that's left is the mechanics of the implementation (i.e. arguments & return values)

800

	8	6	7	9
data[4]:	0	1	2	3

C Code:

```
int main()
{
  int data[4] = {8, 6, 7, 9};
  int size=4;
  int sum1 = isum_it(data, size);
  int sum2 = rsum_it(data, size);
}

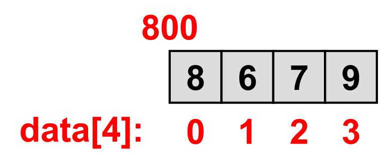
int isum_it(int data[], int len)
{
  int sum = data[0];
  for(int i=1; i < len; i++){
    sum += data[i];
  }
}

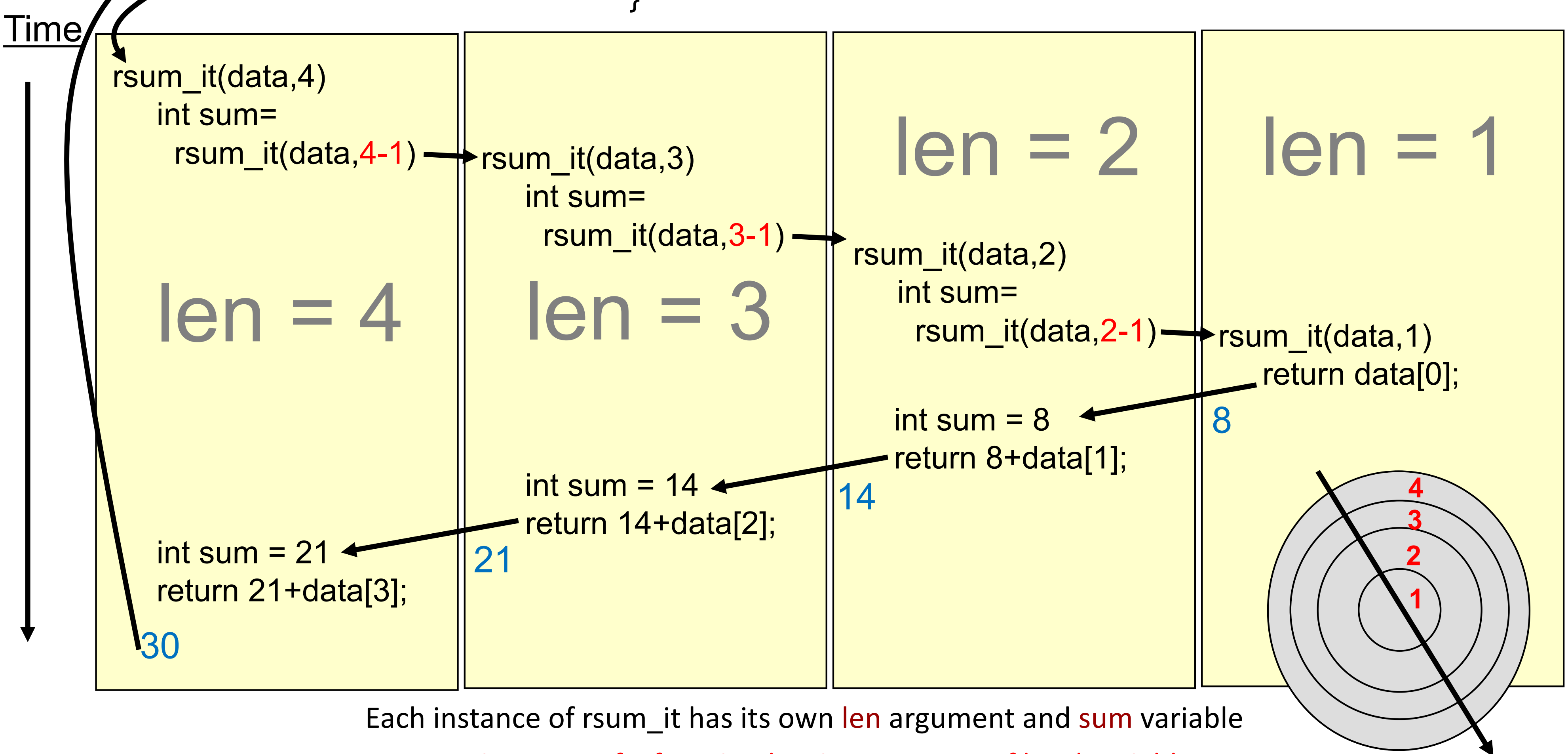
int rsum_it(int data[], int len)
{
  if(len == 1)
    return _______________;
  else
    ______________________________
    return __________________;
}
```

Recursive Call Timeline

```
int main(){
  int data[4] = {8, 6, 7, 9};
  int size=4;
  cout << rsum_it(data, size);
  ...
}
```

```
int rsum_it(int data[], int len)
{
  if(len == 1)
    return data[0];
  else
    int sum = rsum_it(data, len-1);
    return sum + data[len-1];
}
```





Each instance of rsum_it has its own len argument and sum variable

Every instance of a function has its own copy of local variables

System Stack & Recursion

- The system stack makes recursion possible by providing separate memory storage for the local variables of each running instance of the function

```
int main()
{
  int data[4] = {8, 6, 7, 9};
  int sum2 = rsum_it(data, 4);
}

int rsum_it(int data[], int len)
{
  if(len == 1)
    return data[0];
  else
    int sum =
       rsum_it(data, len-1);
    return sum + data[len-1];
}
```

System Memory

(RAM)

Code for all functions
Data for rsum_it (data=800, len=1, sum=??) and return link
Data for rsum_it (data=800, len=2, sum=8) and return link
Data for rsum_it (data=800, len=3, sum=14) and return link
Data for rsum_it (data=800, len=4, sum=21) and return link
Data for main (data=800,sum2=??) and return link
System stack area

800

data[4]:	8	6	7	9
	0	1	2	3

Recursion Double Check

- When you write a recursive routine:
  - Check that you have appropriate base cases
    - Need to check for these first before recursive cases
  - Check that each recursive call makes progress toward the base case
    - Otherwise you'll get an infinite loop and stack overflow
  - Check that you use a 'return' statement at each level to return appropriate values back to each recursive call
    - You have to return back up through every level of recursion, so make sure you are returning something (the appropriate thing)

Head vs. Tail Recursion

- Head Recursion: Recursive call is made before the real work is performed in the function body
- Tail Recursion: Some work is performed and then the recursive call is made

Tail Recursion

```
void doit(int n)
{
  if(n == 1) cout << "Stop" << endl;
  else {
   cout << "Go" << endl;
   doit(n-1);
  }
}
```

Head Recursion

```
void doit(int n)
{
  if(n == 1) cout << "Stop" << endl;
  else {
   doit(n-1);
   cout << "Go" << endl;
  }
}
```

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Tail Recursion

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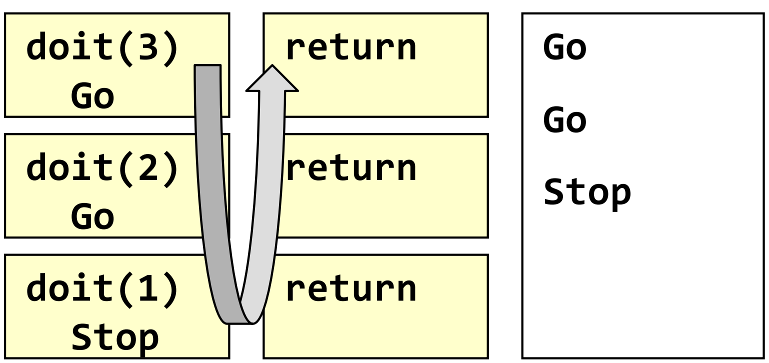

Head Recursion

```
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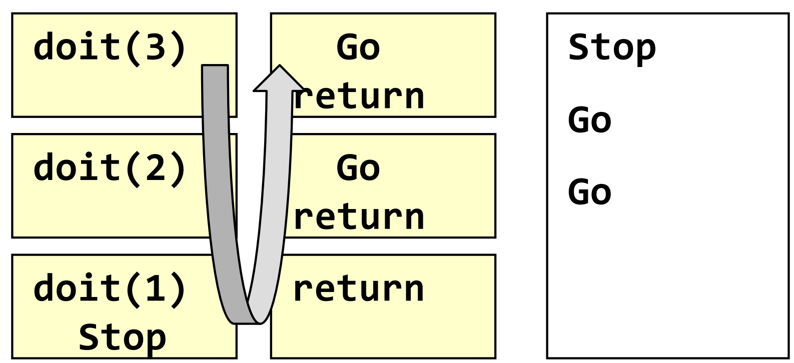


Exercise

- Exercises
  - Count-down
  - Count-up

Loops & Recursion

- Is it better to use recursion or iteration?
  - ANY problem that can be solved using recursion can also be solved with iteration and other appropriate data structures
    - Often 1 recursive call per function can be implemented with a loop
- Why use recursion?
  - Usually clean & elegant.
  - For some problems an iterative approach would be very difficult to formulate but the recursive approach is small and elegant
    - Often occurs when the function instance makes ***multiple (more than 1)*** recursive calls
  - Can lead to stack overflow if recursive depth is too large
- How do you choose?
  - Iteration is usually faster and uses less memory and should be used when possible
  - However, if iteration produces a very complex solution, consider recursion

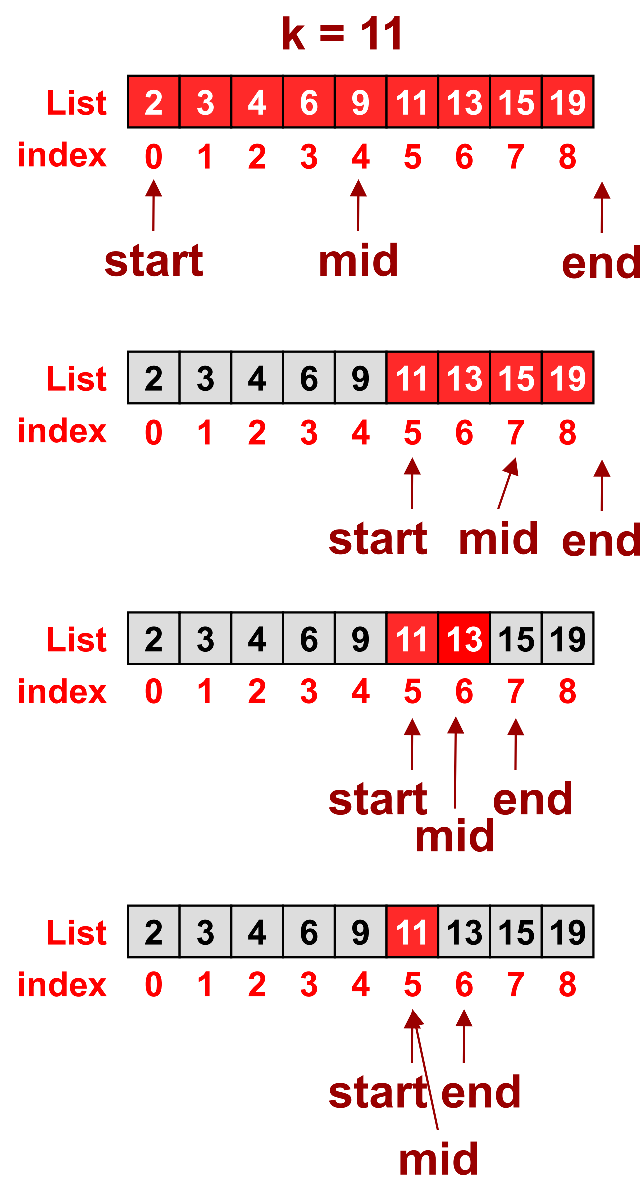
```
void rfunc1(int n)
{
  ...
  rfunc1(n-1);
  ...
}
```

```
void rfunc2(int n)
{
  ...
  t = rfunc2(n-1);
  s = rfunc2(n-2);
  ...
}
```

MORE EXAMPLES

Recursive Binary Search

- Assume remaining items = [start, end)
  - start is inclusive index of start item in remaining list
  - End is exclusive index of start item in remaining list
- binSearch(target, list[], start, end)
  - Perform base check (empty list)
    - Return NOT FOUND (-1)
  - Pick mid item
  - Based on comparison of k with List[mid]
    - EQ => Found => return mid
    - LT => return answer to binSearch[start,mid)
    - GT => return answer to binSearch[mid+1,end)



Sorting

- If we have an unordered list, sequential search becomes our only choice
- If we will perform a lot of searches it may be beneficial to sort the list, then use binary search
- Many sorting algorithms of differing complexity (i.e. faster or slower)
- Bubble Sort (simple though not terribly efficient)
  - On each pass through thru the list, pick up the maximum element and place it at the end of the list. Then repeat using a list of size n-1 (i.e. w/o the newly placed maximum value)

List	7	3	8	6	5	1
index	0	1	2	3	4	5

Original

List	3	7	6	5	1	8
index	0	1	2	3	4	5

After Pass 1

List	3	6	5	1	7	8
index	0	1	2	3	4	5

After Pass 2

List	3	5	1	6	7	8
index	0	1	2	3	4	5

After Pass 3

List	3	1	5	6	7	8
index	0	1	2	3	4	5

After Pass 4

List	1	3	5	6	7	8
index	0	1	2	3	4	5

After Pass 5

Exercise

- Exercises
  - Text-based fractal

Flood Fill Exercise

- Use Vocareum
  - sandbox-floodfill
- Or download the code:
  - wget http://ee.usc.edu/~Redekopp/cs103/floodfill.tar

Flood Fill

- Imagine you are given an image with outlines of shapes (boxes and circles) and you had to write a program to shade (make black) the inside of one of the shapes. How would you do it?
- Flood fill is a recursive approach
- Given a pixel
  - Base case: If it is black already, stop!
  - Recursive case: Call floodfill on each neighbor pixel
  - Hidden base case: If pixel out of bounds, stop!