Fuzzy C-Means Clustering and Cluster Quality Evaluation

GNR602- ADVANCED METHODS IN SATELLITE IMAGE PROCESSING

Introduction

- Fuzzy C-Means (FCM) is a clustering algorithm that assigns each data point to a cluster with a degree of membership.
- The primary objective of this project is to implement the FCM clustering algorithm and evaluate the cluster quality for different values of C (number of clusters).
- m is a fuzziness parameter that controls the degree of overlap between the clusters.
- The minimum and maximum values of C will be specified by the user.

Introduction

- In this project, we will first discuss the theoretical background of the FCM algorithm and its various parameters.
- Next, we will implement the algorithm using Python programming language
- We will then evaluate the cluster quality for different values of C using Davies-Bouldin Index
- The results of this project will provide insights into the effect of the number of clusters on the quality of clusters obtained by the FCM algorithm.

Problem

- Label each pixel of the image into clusters based on a fuzzy membership of that pixel into the cluster

The objective of the FCM algorithm is to minimize the following objective function:

$$J = \sum_i \sum_j w_{ij}^m ||x_i - c_j||^2$$

J is the objective function to be minimized.
w_{ij} is the degree of membership of data point x_i in cluster c_j.
m is a fuzziness parameter that controls the degree of overlap between the clusters.
x_i is the i-th pixel's feature vector.
c_j is the centroid of the j-th cluster.

Algorithm

Fix	Initialize	Update	Evaluate	Repeat	Determine
Fix a C in the range of C specified by the user	Initialize the membership matrix via K-Means (K = C)	Iteratively update the membership and cluster centroids until convergence	Calculate the Davies Bouldin Index (DBI) for the clustering	Repeat these steps for all values of C specified by user	Determine the optimal value of C by the minimum DBI value

Initialize

Initialize the cluster memberships by running a few iterations (≈10) of the K-means algorithm (K = C).

Initialize the fuzzy memberships from the K Mean cluster vectors according to:

$$w_{ij} = \frac{\frac{1}{d(x_i, c_j)}}{\sum_{p=1}^{K} \frac{1}{d(x_i, c_p)}}$$

w_{ij} is the fuzzy membership of pixel i into cluster j
$d(x_i, c_j)$ is the Euclidean distance between feature vector of pixel i from mean vector of cluster j

$$\sum_{j=1}^{K} w_{ij} = 1$$

Update

Update memberships and cluster centres in each iteration as:

$$\forall_{i,j} \quad w_{ij} = \left[\sum_{k=1}^{C} \left(\frac{||x_i - c_j||}{||x_i - c_k||} \right)^{\frac{2}{m-1}} \right]^{-1}$$

$$\forall_j \quad c_j = \frac{\sum_i w_{ij}^m x_i}{\sum_i w_{ij}^m}$$

Evaluate Cluster Quality

- The Davies Bouldin Score (DBI) is a measure of cluster quality that evaluates the ratio of within-cluster similarity to between-cluster dissimilarity.
- The DBI is defined as the average similarity measure of each cluster with its most similar cluster, where similarity is the ratio of within-cluster distances to between-cluster distances. Thus, clusters which are farther apart and less dispersed will result in a better score.
- A lower DBI score indicates better cluster quality, as it means that the clusters are more separated and have a higher degree of similarity within the clusters.

Silhouette Score

- The Silhouette Score is calculated for each data point in the dataset by computing the mean intra-cluster distance (a) and the mean nearest-cluster distance (b), and then subtracting them and dividing by the maximum of the two (max(a, b)).
- The Silhouette Score ranges from -1 to 1, where a score closer to 1 indicates that the data point is well-matched to its assigned cluster and that the clusters are well-separated from each other, and a score closer to -1 indicates that the data point may be better matched to a different cluster.
- But this has high computational time complexity (more than the clustering algorithm itself). So, we didn't use this quality metric

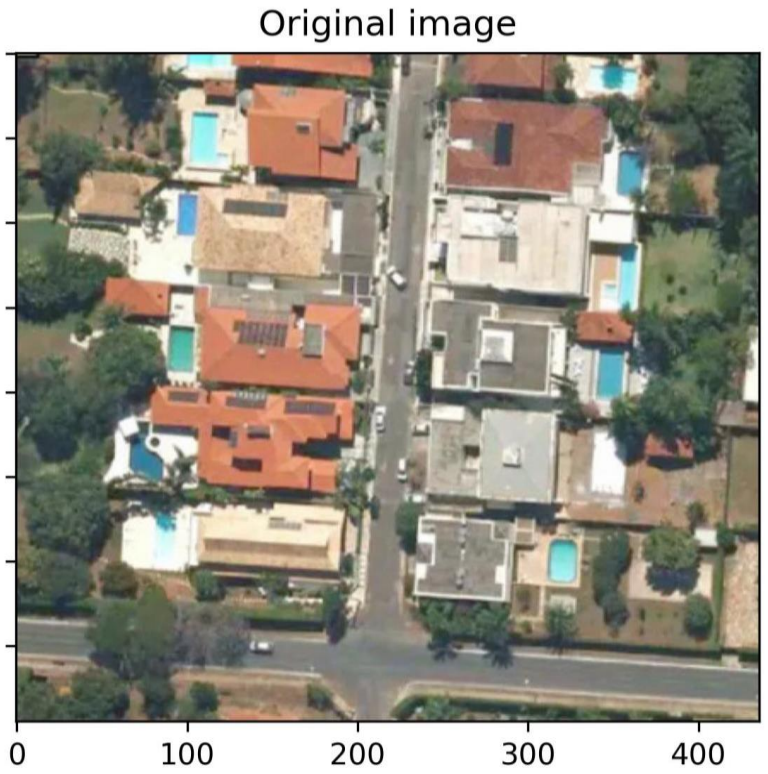
Repeat & Determine

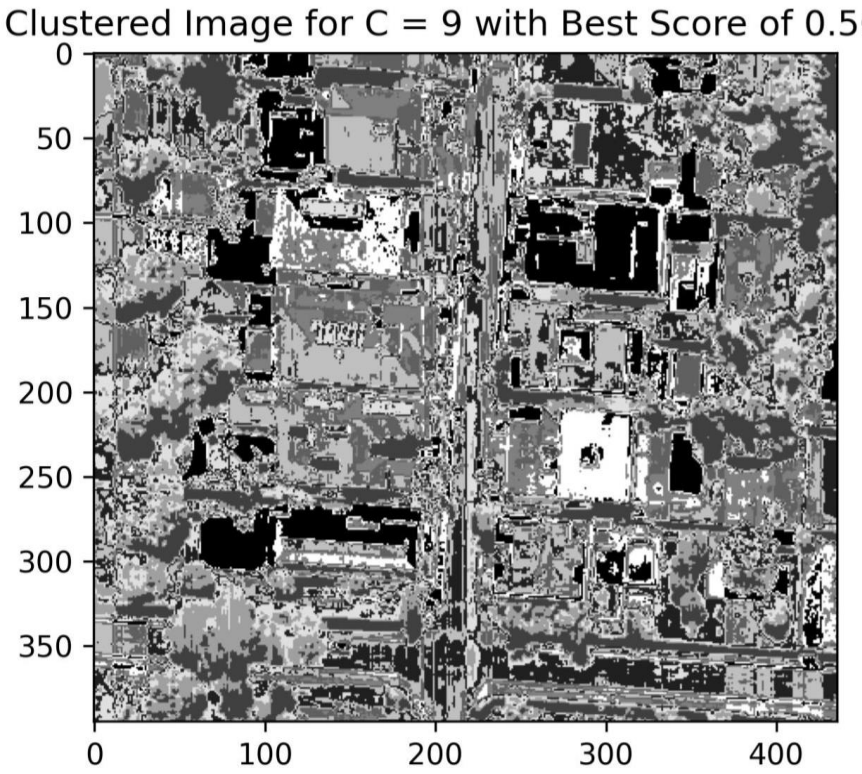
- Repeat the previous steps for all values in the range of C
- The optimal value of C is determined by the clustering which has the minimum Davies Bouldin Score

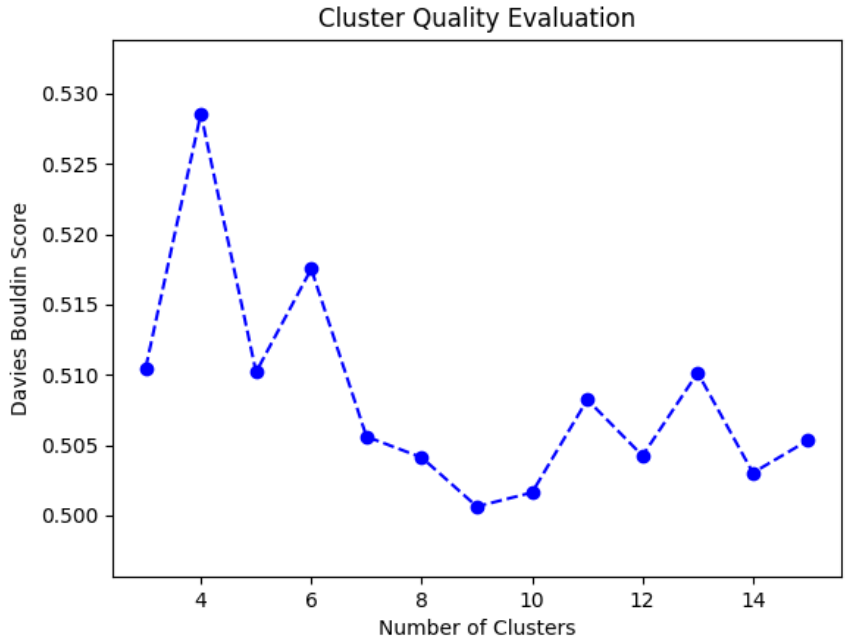
Dataset

Louisiana 2016 Flood Satellite Images
https://www.kaggle.com/datasets/rahultp97/louisiana-flood-2016

Results


Original image
0
100
200
300
400

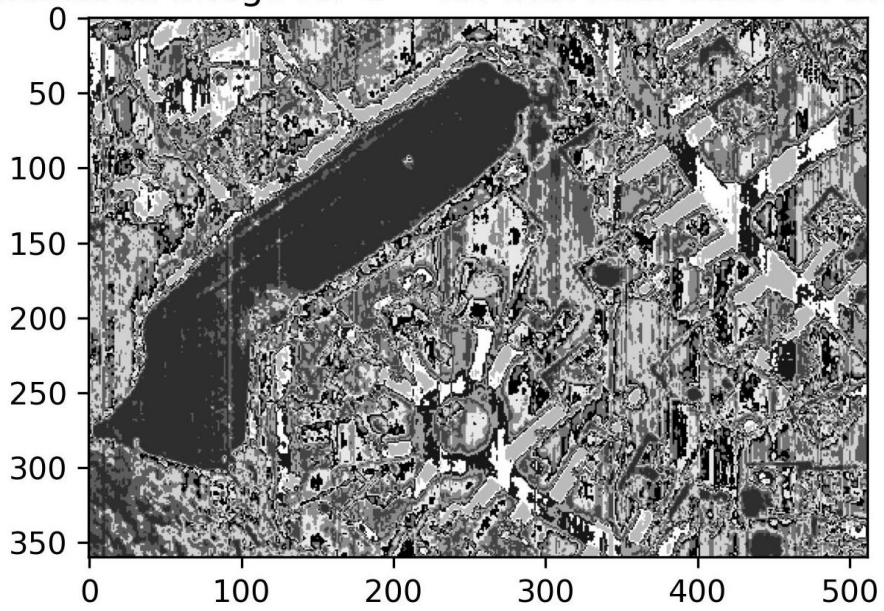

Clustered Image for C = 9 with Best Score of 0.5006
0
50
100
150
200
250
300
350
0
100
200
300
400

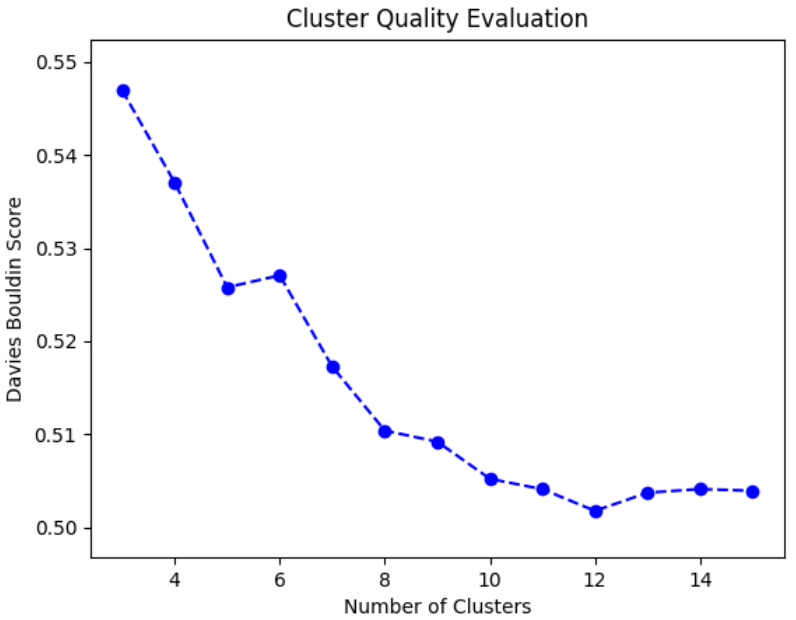

Cluster Quality Evaluation
Davies Bouldin Score
0.530
0.525
0.520
0.515
0.510
0.505
0.500
4
6
8
10
12
14
Number of Clusters

Results

Original image

Clustered Image for C = 12 with Best Score of 0.5018

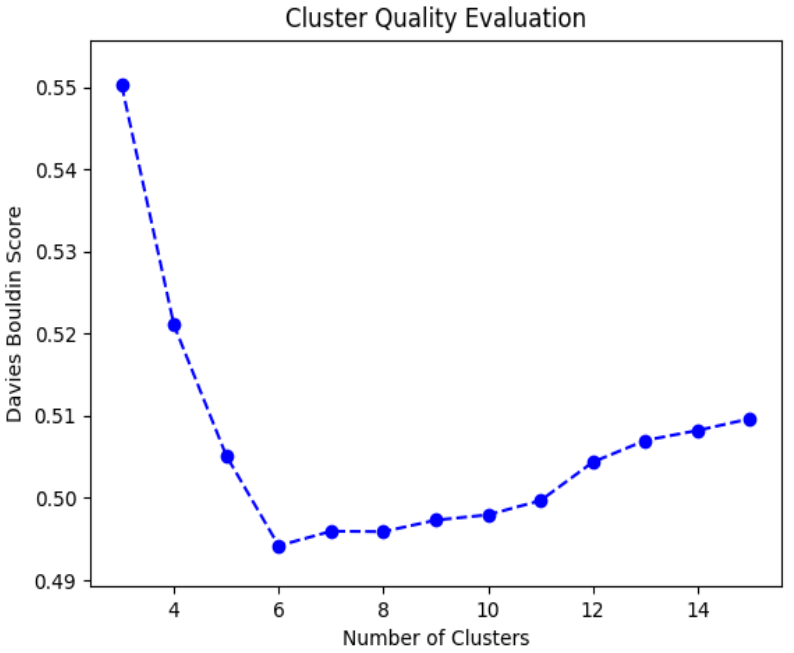




Results

Original image

Clustered Image for C = 6 with Best Score of 0.4942



Conclusion

- A low number of clusters can result in poor cluster quality because the clusters may be too large and heterogeneous, while a high number of clusters can result in poor cluster quality because the clusters may be too small and similar.
- Therefore, it is crucial to find the optimal number of clusters, which lies in the intermediate range and results in well-separated, homogeneous, and meaningful clusters.
- An optimal number of clusters are needed to model various aspects captured in a satellite image such as forests, roads, buildings, water bodies, vegetation, fields, etc.

Thankyou

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Topic 3. Implement FCM clustering algorithm and evaluate the cluster quality for different values of ‘C’. (Minimum and maximum values of C to be specified by user)