

Bit Manipulation - I

Decimal Number System ? \rightarrow digits $[0, 9]$

$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ \rightarrow 10 digits
 \hookrightarrow base 10

$$342 \rightarrow 300 + 40 + 2$$

$$\rightarrow 3 \times 100 + 4 \times 10 + 2 \Rightarrow 3 \times 10^2 + 4 \times 10^1 + 2 \times 10^0$$

$\begin{matrix} 2 & 5 & 3 & 6 \\ 3 & 2 & 1 & 0 \end{matrix}$

$$\rightarrow 2 \times 10^3 + 5 \times 10^2 + 3 \times 10^1 + 6 \times 10^0$$

Binary Number System \rightarrow 2 digits $\{0, 1\}$
 \Rightarrow base 2

$\begin{matrix} 1 & 1 & 0 \\ 2 & 1 & 0 \end{matrix}$

$$\rightarrow 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

$$= 4 + 2 + 0 = 6$$

$$(110)_2 = (6)_{10}$$

$\begin{matrix} 1 & 0 & 1 & 1 & 0 \\ 4 & 3 & 2 & 1 & 0 \end{matrix}$

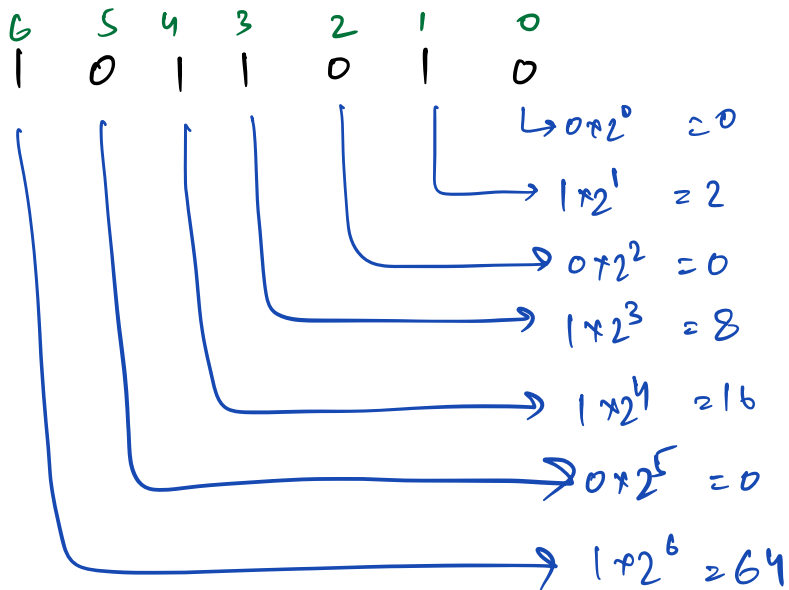
$$\rightarrow 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

$$= 16 + 0 + 4 + 2 + 0$$

$$= 22$$

$$(10110)_2 = (22)_{10}$$

Decimal representation of



$1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $2^6 + 2^4 + 2^3 + 2^1$
 $64 + 16 + 8 + 2$
 $= 90$

90

Decimal to Binary

$(20)_{10}$

remainder

2	20	0
2	10	0
2	5	1
2	2	0
2	1	1
	0	

↑

$\Rightarrow (10100)_2$
 $\downarrow \quad \downarrow$
 $2^4 + 2^2 = 16 + 4 = 20$

Binary representation of $(45)_{10}$

2	45	1
2	22	0
2	11	1
2	5	1
2	2	0
2	1	1
0		

$$= (101101)_2$$

$\swarrow \quad \swarrow \quad \swarrow \quad \downarrow$
 $2^5 \quad 2^3 \quad 2^2 \quad 2^0$

$$32 + 8 + 4 + 1 = 45$$

Addition in Decimal

$$\begin{array}{r}
 10 \quad 10 \\
 3 \quad 6 \quad 8 \\
 + 4 \quad 3 \quad 5 \\
 \hline
 8 \quad 10 \quad 13 = 803
 \end{array}$$

$$9 + 9 = 18$$

$$1 + 9 + 9 = 19$$

Addition in Binary

$$\begin{array}{r}
 10 \quad 10 \\
 1 \quad 0 \quad 1 \\
 + 0 \quad 1 \quad 1 \\
 \hline
 10 \quad 10 \quad 10 = 1000
 \end{array}$$

$$(1000)_2 = (8)_{10}$$

$$(2)_{10} = (10)_2$$

$$\begin{array}{r}
 10110 \\
 + 00111 \\
 \hline
 11101
 \end{array}
 \quad (3)_{10} = (11)_2 = (11101)_2$$

'+' → arithmetic operator

+, -, *, / ⇒ arithmetic operators

Bitwise Operations →

AND, OR, XOR, NOT, Left Shift and Right Shift
 & , | , ^ , ! / ~ / << , >>

0 → off → false
 1 → on → true

A	B	A & B	A B	A ^ B
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

→ addition without carry

both are same → false
 both are different → true

5 & 6

$5 \& 6 = 4$

⇒

&

1	0	1
1	1	0
1	0	0

= (4)₁₀

$$7 \& 8 \Rightarrow \begin{array}{cccc} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array} = 0$$

$$7 \& 8 = 0$$

$$20 | 45 \Rightarrow \begin{array}{cccccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ \hline 1 & 1 & 1 & 1 & 0 & 1 \end{array}$$

$\downarrow 2^5 \quad \downarrow 2^4 \quad \downarrow 2^3 \quad \downarrow 2^2 \quad \downarrow 2^1 \quad \downarrow 2^0$
 $= 32 + 16 + 8 + 4 + 1$
 $= 61$

$$20 | 45 = 61$$

$$20 \wedge 45 \Rightarrow \begin{array}{cccccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ \hline 1 & 1 & 1 & 0 & 0 & 1 \end{array}$$

$$20 \wedge 45 = 57$$

$$= 61 - 2^2$$

$$= 61 - 4 = 57$$

Properties

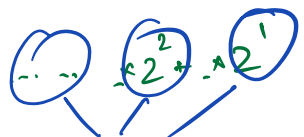
1. $A \& 1 = \text{last bit of } A$

$$A = \begin{array}{ccc} 1 & 0 & 1 \\ 12 & 0 & 0 & 1 \\ \hline 0 & 0 & 1 & = 1 \end{array}$$

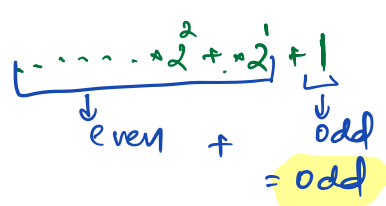
$$A = \begin{array}{ccc} 1 & 0 & 1 \\ 1 = & 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 1 & = 1 \end{array}$$

$$A = \begin{array}{ccc} 1 & 1 & 0 \\ 1 = & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & = 0 \end{array}$$

$$A = \dots + x2^2 + x2^1 + x2^0$$

if $x=0 \Rightarrow$ 

sum of even = even

if $x=1 \Rightarrow$ 

$$A \cdot 2 == A \& 1$$

2. $A \& 0 = 0$

$1 \& 0 = 0$
 $0 \& 0 = 0$

3. $A \& A = A$

$A = 101$
 $A = 101$
 $\hline 101 = A$

$1 \& 1 = 1$
 $0 \& 0 = 0$

4. $A | 0 = A$

$A = 101$
 $0 = 000$
 $\hline 101 = A$

$1 | 0 \rightarrow 1$
 $0 | 0 \rightarrow 0$

5. $A | A = A$

$1 | 1 \rightarrow 1$
 $0 | 0 \rightarrow 0$

6. $A \wedge 0 = A$

$A = 1010$
 $0 = 0000$
 $\hline 1010 = A$

$1 \wedge 0 \rightarrow 1$
 $0 \wedge 0 \rightarrow 0$

$$7. A \wedge A = 0$$

$$\begin{aligned} 1 \wedge 1 &\rightarrow 0 \\ 0 \wedge 0 &\rightarrow 0 \end{aligned}$$

$$8. A | 1 =$$

A if $(A \text{ is odd})$

$A+1$ else

$$\begin{array}{r} A = 1010 \\ 1 = 0001 \\ \hline 1011 = A+1 \end{array}$$

$$\begin{array}{r} A = 1001 \\ 1 = 0001 \\ \hline 1001 = A \end{array}$$

$$\begin{aligned} 1|0 &\rightarrow 1 \\ 1|1 &\rightarrow 1 \end{aligned}$$

Commutative Property

$$A \& B = B \& A$$

$$A | B = B | A$$

$$A \wedge B = B \wedge A$$

$$\begin{aligned} A \& B \& C &= C \& A \& B \\ &= C \& B \& A \end{aligned}$$

Associative Property

$$(A \& B) \& C = A \& (B \& C)$$

$$\begin{array}{l} A \quad 101101 \\ B \quad 001011 \\ C \quad 111001 \end{array} \left. \begin{array}{l} \text{---} \\ \text{---} \end{array} \right\} \begin{array}{l} A \& B \rightarrow 001001 \\ B \& C \rightarrow 001001 \end{array} \xrightarrow{(A \& B) \& C} 001001$$

$$A \& (B \& C) \rightarrow 001001$$

$$(A|B) | C = A | (B | C)$$

$$(A \wedge B) \wedge C = A \wedge (B \wedge C)$$

Question: $a \wedge b \wedge a \wedge d \wedge b$

$$\Rightarrow a \wedge a \wedge b \wedge b \wedge d$$

$$\Rightarrow (a \wedge a) \wedge (b \wedge b) \wedge d$$

$$\Rightarrow (0 \wedge 0) \wedge d$$

$$\Rightarrow 0 \wedge d \Rightarrow d$$

Question 1

Given N elements, every element repeats twice except one. Find the unique element?

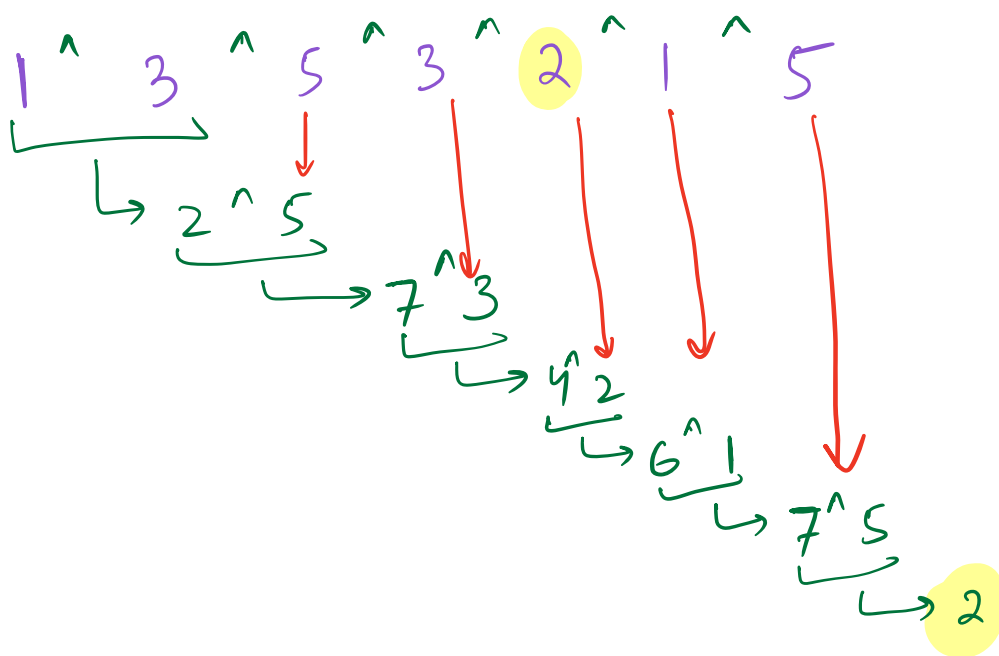
XOR all since order doesn't matter

Code

```
int unique( a[] ) {
    n = a.length
    ans = 0;
    for (i=0; i<n; ++i)
        ans = ans ^ a[i]
    return ans
}
```

TC: $O(N)$

SC: $O(1)$



$$\begin{array}{r} 01 \\ 11 \\ \hline 10 \end{array}$$

Left Shift

int \rightarrow 4 Bytes \rightarrow 32 bits

assume 8 bit number

$A = 45$

$A \ll 1$ give $\begin{array}{ccccccc} 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \end{array}$ $= 90$

$A \ll 2$ $\begin{array}{ccccccc} 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{array}$ $= 180$

$A \ll 3$ $\begin{array}{ccccccc} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \end{array}$ $= 360 \rightarrow 104$

$\begin{array}{ccccccc} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \end{array}$

Max value of 8 bit number:

$$11111111 \Rightarrow 255$$

$$A \ll \textcircled{1} = A \times 2^{\textcircled{1}}$$

$$A \ll \textcircled{2} = A \times 2 \times 2 = A \times 2^{\textcircled{2}}$$

$$A \ll \textcircled{n} = A \times 2^{\textcircled{n}}$$

if $A \geq 1 \Rightarrow \boxed{1 \ll n = 2^n}$

If you left shift many times your number will overflow.

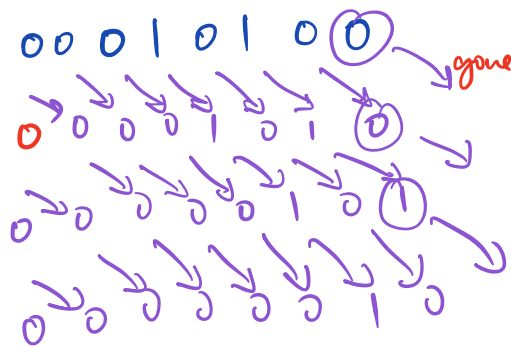
Right Shift

$$A = 20$$

$$A \gg 1$$

$$A \gg 2$$

$$A \gg 3$$



$$= 10$$

$$= 5$$

$$= 2$$

$$= 20/2 = 20/2^1$$

$$= 20/4 = 20/2^2$$

$$= 20/8 = 20/2^3$$

$$= 5/2 = 2$$

$$\boxed{A \gg n = \frac{A}{2^n}}$$

NO OVERFLOW