Bit Manipulation - 1

Decimal Number System ? → digits [0, 9]

{0, 1, 2, 3, 4, 5, 6, 7, 8, 9} → 10 digits
↳ base 10

342 → 300 + 40 + 2
→ $3 \times 100 + 4 \times 10 + 2$ ⇒ $3 \times 10^2 + 4 \times 10^1 + 2 \times 10^0$

2536 → $2 \times 10^3 + 5 \times 10^2 + 3 \times 10^1 + 6 \times 10^0$
3 2 1 0

Binary Number System → 2 digits {0, 1}
⇒ base 2

110 → $1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$
2 1 0
= 4 + 2 + 0 = 6

$(110)_2 = (6)_{10}$

1 0 1 1 0 → $1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$
4 3 2 1 0
= 16 + 0 + 4 + 2 + 0
= 22

$(10110)_2 = (22)_{10}$

Decimal representation of

6	5	4	3	2	1	0
1	0	1	1	0	1	0

$0 \times 2^0 = 0$

$1 \times 2^1 = 2$

$0 \times 2^2 = 0$

$1 \times 2^3 = 8$

$1 \times 2^4 = 16$

$0 \times 2^5 = 0$

$1 \times 2^6 = 64$

$\boxed{90}$

1 0 1 1 0 1 0

$2^6 + 2^4 + 2^3 + 2^1$

$64 + 16 + 8 + 2$

$= 90$

Decimal to Binary

$(20)_{10}$

		remainder
2	20	0
2	10	0
2	5	1
2	2	0
2	1	1
	0	

$\Rightarrow (10100)_2$

$2^4 + 2^2 = 16 + 4 = 20$

Binary representation of $(45)_{10}$

2	45	1
2	22	0
2	11	1
2	5	1
2	2	0
2	1	1
	0	

$= (101101)_2$

$2^5 \quad 2^3 \quad 2^2 \quad 2^0$

$32 + 8 + 4 + 1 = 45$

Addition in Decimal

$$
\begin{array}{rccc}
 & {}^{1}\,3 & {}^{1}\,6 & 8 \\
+ & 4 & 3 & 5 \\
\hline
 & 8 & 10 & 13
\end{array}
= 8\;0\;3
$$

$9 + 9 = 18$

$1 + 9 + 9 = 19$

Addition in Binary

$$
\begin{array}{rccc}
 & {}^{1}\,1 & {}^{1}\,0 & 1 \\
+ & 0 & 1 & 1 \\
\hline
 & 10 & 10 & 10
\end{array}
= 1\;0\;0\;0
$$

$(1000)_2 = (8)_{10}$

$(2)_{10} = (10)_2$

$$
\begin{array}{cccccc}
 & & 1 & 1 & & \\
 & 1 & 0 & 1 & 1 & 0 \\
+ & 0 & 0 & 1 & 1 & 1 \\
\hline
 & 1 & 1 & 1 & 0 & 1
\end{array}
= (11101)_2
$$

$(3)_{10} = (11)_2$

'+' → arithmetic operator

+, −, *, / ⇒ arithmetic operators

Bitwise Operations →

AND, OR, XOR, NOT, Left Shift and Right Shift

&, |, ^, ! / ~ / <>, <<, >>

0 → off → false

1 → on → true

A	B	A&B	A|B	A^B
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

A^B → addition without carry

both are same → false

both are different → true

5 & 6 ⇒

$$
\begin{array}{cccc}
 & 1 & 0 & 1 \\
\& & 1 & 1 & 0 \\
\hline
 & 1 & 0 & 0
\end{array}
= (4)_{10}
$$

$\boxed{5 \& 6 = 4}$

$7 \,\&\, 8 \Rightarrow$

$$
\begin{array}{rcccc}
 & 0 & 1 & 1 & 1 \\
\& & 1 & 0 & 0 & 0 \\
\hline
 & 0 & 0 & 0 & 0
\end{array}
= 0
$$

$\boxed{7 \,\&\, 8 = 0}$

$20 \mid 45 \Rightarrow$

$$
\begin{array}{rcccccc}
 & 0 & 1 & 0 & 1 & 0 & 0 \\
\text{OR} & 1 & 0 & 1 & 1 & 0 & 1 \\
\hline
 & 1 & 1 & 1 & 1 & 0 & 1 \\
 & 2^5 & 2^4 & 2^3 & 2^2 & & 2^0
\end{array}
= 32+16+8+4+1
$$

$= 61$

$\boxed{20 \mid 45 = 61}$

$20 \wedge 45 \Rightarrow$

$$
\begin{array}{rcccccc}
 & 0 & 1 & 0 & 1 & 0 & 0 \\
\wedge & 1 & 0 & 1 & 1 & 0 & 1 \\
\hline
 & 1 & 1 & 1 & 0 & 0 & 1
\end{array}
$$

$= 61 - 2^2$

$= 61 - 4 = 57$

$\boxed{20 \wedge 45 = 57}$

Properties

1. $A \,\&\, 1$ = last bit of A

$$
\begin{array}{rccc}
A = & 1 & 0 & 1 \\
1 = & 0 & 0 & 1 \\
\hline
 & 0 & 0 & 1
\end{array}
= 1
\qquad
\begin{array}{rcccc}
A = & 1 & 0 & 1 & 1 \\
1 = & 0 & 0 & 0 & 1 \\
\hline
 & 0 & 0 & 0 & 1
\end{array}
= 1
\qquad
\begin{array}{rccc}
A = & 1 & 1 & 0 \\
1 = & 0 & 0 & 1 \\
\hline
 & 0 & 0 & 0
\end{array}
= 0
$$

$A = \ldots\ldots x2^2 + x2^1 + x*2^0$

if $x = 0 \Rightarrow$ $\underbrace{(\ldots) \quad (x2^2) \quad (x2^1)}_{\text{even no.(s)}}$

sum of even = even

if $x = 1 \Rightarrow$ $\underbrace{\ldots\ldots x2^2 + x2^1}_{\text{even}} + \underbrace{1}_{\text{odd}}$

even + odd

= odd

$$\boxed{A \% 2 == A \& 1}$$

2. $A \& 0 = 0$

$1 \& 0 = 0$
$0 \& 0 = 0$

3. $A \& A = A$

$A = 101$
$A = 101$
$101 = A$

$1 \& 1 = 1$
$0 \& 0 = 0$

4. $A | 0 = A$

$A = 101$
$0 = 000$
$101 = A$

$1 | 0 \rightarrow 1$
$0 | 0 \rightarrow 0$

5. $A | A = A$

$1 | 1 \rightarrow 1$
$0 | 0 \rightarrow 0$

6. $A \wedge 0 = A$

$A = 1010$
$0 = 0000$
$1010 = A$

$1 \wedge 0 \rightarrow 1$
$0 \wedge 0 \rightarrow 0$

7. $A \wedge A = 0$

$1 \wedge 1 \rightarrow 0$

$0 \wedge 0 \rightarrow 0$

8. $A \mid 1 =$

A if (A is odd)

A+1 else

$$\begin{array}{r} A = 1\,0\,1\,0 \\ 1 = 0\,0\,0\,1 \\ \hline 1\,0\,1\,1 \end{array} = A+1$$

$$\begin{array}{r} A = 1\,0\,0\,1 \\ 1 = 0\,0\,0\,1 \\ \hline 1\,0\,0\,1 \end{array} = A$$

$1 \mid 0 \rightarrow 1$

$1 \mid 1 \rightarrow 1$

Commutative Property

$A \& B = B \& A$ $\quad$ $A \mid B = B \mid A$ $\quad$ $A \wedge B = B \wedge A$

$A \& B \& C = C \& A \& B$

$= C \& B \& A$

Associative Property

$(A \& B) \& C = A \& (B \& C)$

A	1 0 1 1 0 1
B	0 0 1 0 1 1
C	1 1 1 0 0 1

$A \& B \rightarrow 0\,0\,1\,0\,0\,1$

$(A \& B) \& C \rightarrow 0\,0\,1\,0\,0\,1$

$B \& C \rightarrow 0\,0\,1\,0\,0\,1$

$A \& (B \& C) \rightarrow 0\,0\,1\,0\,0\,1$

$$(A|B)|C = A|(B|C)$$

$$(A \wedge B) \wedge C = A \wedge (B \wedge C)$$

Question : $a \wedge b \wedge a \wedge d \wedge b$

$\Rightarrow a \wedge a \wedge b \wedge b \wedge d$

$\Rightarrow (a \wedge a) \wedge (b \wedge b) \wedge d$

$\Rightarrow (0 \wedge 0) \wedge d$

$\Rightarrow 0 \wedge d \Rightarrow d$

Question 1

Given N elements, every element repeats twice except one. Find the unique element?

XOR all since order doesn't matter

Code

```
int unique( a[] ) {
    n = a.length
    ans=0;
    for (i=0; i<n; ++i )
        ans = ans^ a[i]
    return ans
}
```

TC : O(N)

SC : O(1)

1 ^ 3 ^ 5 ^ 3 ^ 2 ^ 1 ^ 5

2 ^ 5

7 ^ 3

4 ^ 2

6 ^ 1

7 ^ 5

2

$$\begin{array}{r} 01 \\ \wedge\ 11 \\ \hline 10 \end{array}$$

Left Shift

int → 4 Bytes → 32 bits

assume $\boxed{\text{8 bit}}$ number

A = 45		0	0	1	0	1	1	0	1	
	gone									
A<<1		0	1	0	1	1	0	1	0	= 90
A<<2		1	0	1	1	0	1	0	0	= 180
A<<3		0	1	1	0	1	0	0	0	= ~~360~~ → 104

Max value of 8 bit number :

1111 1111 ⇒ 255

$A << 1 = A \times 2^1$

$A << 2 = A \times 2 \times 2 = A \times 2^2$

$A << n = A \times 2^n$

if $A == 1 \Rightarrow$ $\boxed{1 << n = 2^n}$

If you left shift many times your number will overflow.

Right Shift

$A = 20$ 0 0 0 1 0 1 0 0 → gone

$A >> 1$ 0 0 0 0 1 0 1 0 $= 10 = 20/2 = 20/2^1$

$A >> 2$ 0 0 0 0 0 1 0 1 $= 5 = 20/4 = 20/2^2$

$A >> 3$ 0 0 0 0 0 0 1 0 $= 2 = 20/8 = 20/2^3$

$= 5/2 = 2$

$$\boxed{A >> n = \frac{A}{2^n}}$$

NO OVERFLOW