Time Complexity - 1

- → Time & Space Complexity
- → Asymptotic Analysis
- → Big O notation
- → TLE - Time Limit Exceeded

} Not Today, next class

Today: How to calculate number of iterations ?

Quiz1 : Sum of first N natural no. =

$(N \times (N+1))/2$ $\quad \frac{N^2}{2} + \frac{N}{2} = O(N^2)$

Quiz 2 : How many numbers are there in range [3,10] ?

[ ] → closed bracket / inclusive

( ) → open bracket / exclusive

[3,10] → 3, 4, 5, 6, 7, 8, 9, 10 → 8 numbers

[3,8) → 3, 4, 5, 6, 7, ~~8~~ → 5 numbers

[a,b] → b - a + 1

[a,b) → b - a

(a,b) → b - a - 1

what if a > b ?

↳ invalid input

$[a, b] \rightarrow b - a$

$[3, 8] \rightarrow 4, 5, 6, 7, 8$ → 5 numbers

$(3, 8) \rightarrow 4, 5, 6, 7$ → 4 numbers

Quiz3 How many times do we need to divide N by 2 to reduce it 1 ?

if $N = 10$

$10 \xrightarrow{/2} 5 \xrightarrow{/2} 2 \xrightarrow{/2} 1 \Rightarrow$ 3 times

$N \xrightarrow{/2} N/2 \xrightarrow{/2} N/4 \xrightarrow{/2} N/8 \rightarrow \dots \rightarrow 1$

assume no. of times need to divide $= K$

$N \xrightarrow{1^{st}} N/2 \xrightarrow{2^{nd}} N/2^2 \xrightarrow{3^{rd}} N/2^3 \xrightarrow{4^{th}} N/2^4 \rightarrow$

$\dots \xrightarrow{K^{th}} N/2^K$

after K times, N becomes 1.

$\boxed{N/2^K = 1} \Rightarrow N = 2^K$

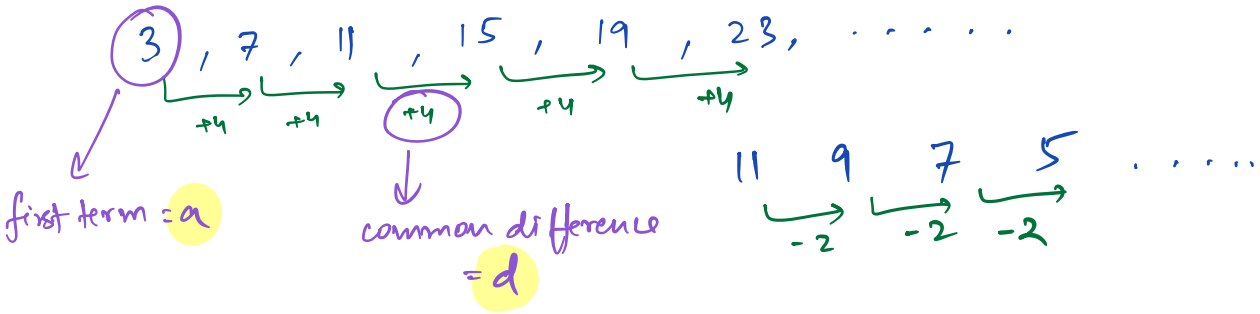
$\log_2 N = \log_2 2^K$

$= K$

$\boxed{K = \log_2 N}$

$\boxed{\log_a a^b = b}$

Arithmetic Progressions (AP)



N^{th} term of this AP ?

a	a+d	a+2d	a+3d	.....
1st	2nd	3rd	4th	

$a+(n-1)d$ — n^{th}

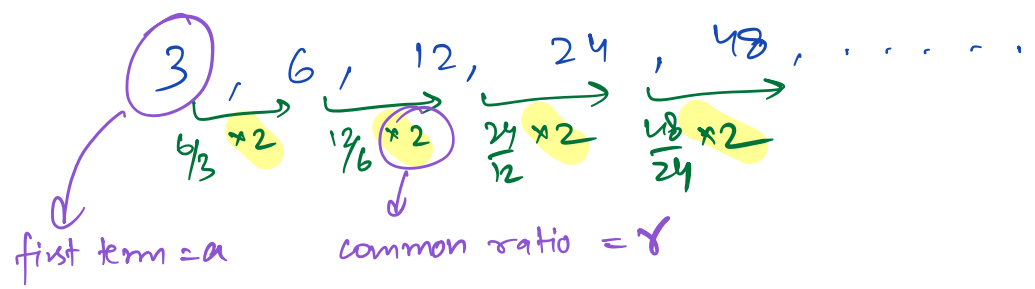
Sum of first N terms of an AP ?

$$= \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{n}{2}[a + \underbrace{a+(n-1)d}_{n^{th}\text{ term (last term)} = l}]$$

$$= \frac{n}{2}[a+l]$$

Geometric Progressions (GP)



N^{th} term of a GP ?

8 4 2 1

$4/8 = 1/2$ $2/4 = 1/2$ $1/2$

$a \quad ar \quad ar^2 \quad ar^3 \quad \ldots$

$1^{st} \quad 2^{nd} \quad 3^{rd} \quad 4^{th}$

ar^{n-1} n^{th} term

Sum of first N terms of a GP ?

$a + ar + ar^2 + \ldots + ar^{n-1}$

$$a\left(\frac{r^n - 1}{r - 1}\right) \qquad r != 0, 1, -1$$

Quiz 4 : $\log_a a^x = ?$

$\rightarrow x$

Quiz 5

```
for [i=1 ; i<=N ; ++i) {
    s = s + i;
}
```

$i = 1, 2, 3, \ldots, N$

$i = [1, N]$

count = $N - 1 + 1 = N$ $O(N)$

```
func (N, M) {
    for (i=1; i<=N; ++i) {          // N iterations
        if (i%2 == 0)
            print(i)
    }
    for (i=1; i<=M; ++i) {          // M iterations
        if (i%2 == 1)
            print(i)
    }
}
```

$N/2 + M/2$

$N + M/2$

$N/2 + M$

$N + M$

$N \times M$

$O(N+M)$

Quiz 6:

```
for (i=0; i<=100; ++i) {
    s=s+i
}
```

$i = 0, 1, 2, \ldots, 100$

$= [0, 100]$

$count = 100 - 0 + 1 = 101$

$O(1)$

Quiz 7:

```
for (i=1; i*i<=N; ++i) {
    s=s+i
}
```

$i*i <= N \Rightarrow i^2 <= N \Rightarrow i <= \sqrt{N}$

$i = 1, 2, 3, \ldots, \sqrt{N}$

$= [1, \sqrt{N}]$

$count = \sqrt{N} - 1 + 1 = \sqrt{N}$

$O(\sqrt{N})$

Quiz 8:

```
i = N
while (i > 1) {
    i = i/2
}
```

$i = N, N/2, N/4, \ldots\ldots, 1$

count = no. of times taken to divide N by 2 to reach 1.

count = $\log_2 N$

$O(\log_2 N)$

Quiz 9:

```
for (i=0; i<=N; i=i*2) {
    s = s+i;
}
```

$i = 0 \xrightarrow{*2} 0 \xrightarrow{*2} 0 \ldots \rightarrow 0$

Infinite

```
for (i=1; i<N; i=i*2) {
}
```

$i = 1 \xrightarrow{*2} 2 \xrightarrow{*2} 4 \rightarrow \ldots\ldots \rightarrow N$

$i = 1, 2, 4, 8, 16, \ldots, N$

if this is K^{th} term

iterations = K

eg: $a = 1, r = 2$

$ar^{K-1} = N$

$1 \times 2^{K-1} = N \Rightarrow 2^{K-1} = N$

$\log_2 2^{K-1} = \log_2 N$

$K-1 = \log_2 N \Rightarrow K = \log_2 N + 1$

$K \approx \log_2 N$ $O(\log_2 N)$

Quiz 10

```
for ( i=1; i<=10; ++i ) {
    for ( j=1; j<=N; ++j ) {
        S = S+i
    }
}
```

$O(N)$

i	j	iteration
1	[1,N]	N
2	[1,N]	N
3		N
⋮		⋮
10	[1,N]	N

$\Rightarrow 10N$

Quiz 11

```
for ( i=0; i<N; ++i ) {
    for ( j=0; j<N; ++j ) {
        S = S+i
    }
}
```

$O(N^2)$

i	j	iteration
0	[0,N-1]	N-1-0+1 = N
1	[0,N-1]	N
2		N
⋮		⋮
N-1	[0,N-1]	N

$\Rightarrow N \times N$

N^2

Quiz12

```
for(i=0; i<N; ++i) {
    for(j=0; j<=i; ++j) {
    }
}
```

i	j : [0, i]	iteration
0	[0,0]	0-0+1 = 1
1	[0,1]	2
2	[0,2]	3
⋮		⋮
N-1	[0,N-1]	N

$1 + 2 + 3 + \dots + N$

$= \frac{N \times (N+1)}{2} = \frac{N^2 + N}{2}$

$O(N^2)$

Quiz13

```
for(i=1; i<=N; ++i) {
    for(j=1; j<=N; j=j*2) {
    }
}
```

i	j	iteration
1	[1, 2, 4, …, N]	$\log_2 N$
2		$\log_2 N$
⋮		⋮
N		$\log_2 N$

$N \times \log_2 N$

$O(N \log_2 N)$

Quizly

```
for(i=1; i<=N; ++i) {
    for(j=1; j<=2^i ; ++j)) {
    }
}
```

(1<<i)

i	j : [1, 2^i]	iterations
1	[1, 2^1]	2^1
2	[1, 2^2]	2^2
3		2^3
⋮		⋮
N	[1, 2^N]	2^N

~~N+1~~

$2^1 + 2^2 + 2^3 + \dots + 2^N$

e.g.: $a = 2$, $r = 2$, terms = N

$$a\left(\frac{r^N - 1}{r - 1}\right) = 2\left(\frac{2^N - 1}{2 - 1}\right) = 2(2^N - 1)$$

$O(2^N)$

Some basics

$\log_2 N < \text{sqrt}(N)$

$N = 8 \quad \log_2 N = 3 \quad \text{sqrt}(N) = 2$

$N = 2^{10} \quad \log_2 N = 10 \quad \text{sqrt}(N) = \sqrt{2^{10}} = 2^5 = 32$

$N = 2^{20} \quad \log_2 N = 20 \quad \text{sqrt}(N) = 2^{10} = 1024$

$N \qquad N \log_2 N$

$N \times 1 < \log_2 N \times N \Rightarrow N < N \log_2 N$

$N > Sqrt(N)$

$N^2 < 2^N$

$$1 < \log_2 N < \sqrt{N} < N < N\log_2 N < N\sqrt{N} < N^2 < N^3 < 2^N$$

How to write Big O ?

~~what?~~ ~~why?~~

1. Calculate iterations based on input.
2. Neglect lower order terms.
3. Neglect constant coeffient term

$N^2 + N \Rightarrow O(N^2)$

$10N^2 + 2N\log N + 5 \Rightarrow O(N^2)$

Quiz 15

$f(N) = 4N + 3N\log N + 10^6$

$O(f(N)) = ?$ $O(N\log N)$