

# VECTORS

## 12<sup>th</sup> Math - Chapter 10

This is Problem-16 from Exercise 10.5

1. If  $\theta$  is the angle between two vectors  $\vec{\mathbf{a}}$  and  $\vec{\mathbf{b}}$ , then  $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \geq 0$ .

(a)  $0 < \theta < \frac{\pi}{2}$

(b)  $0 \leq \theta \leq \frac{\pi}{2}$

(c)  $0 < \theta < \pi$

(d)  $0 \leq \theta \leq \pi$

**Solution:** Given  $\mathbf{a}, \mathbf{b}$  are two vectors

We know that

$$\theta = \cos^{-1} \left( \frac{\mathbf{a}^\top \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} \right) \quad (1)$$

$$\implies \mathbf{a}^\top \mathbf{b} = \cos(\theta) \|\mathbf{a}\| \|\mathbf{b}\| \quad (2)$$

(a)  $0 < \theta < \frac{\pi}{2}$

Assume,  $\theta = \frac{\pi}{3}$

$$\mathbf{a}^\top \mathbf{b} = \cos\left(\frac{\pi}{3}\right) \|\mathbf{a}\| \|\mathbf{b}\| \quad (3)$$

$$= \frac{1}{2} \|\mathbf{a}\| \|\mathbf{b}\| \quad (4)$$

$$\implies \mathbf{a}^\top \mathbf{b} > 0 \quad (5)$$

$$\implies \mathbf{a}^\top \mathbf{b} \neq 0 \quad (6)$$

$$(b) \ 0 \leq \theta \leq \frac{\pi}{2}$$

$$\text{for } \theta = 0 \quad (7)$$

$$\mathbf{a}^\top \mathbf{b} = \cos(0) \|\mathbf{a}\| \|\mathbf{b}\| \quad (8)$$

$$= \|\mathbf{a}\| \|\mathbf{b}\| \quad (9)$$

$$\implies \mathbf{a}^\top \mathbf{b} > 0 \quad (10)$$

$$\text{for } \theta = \frac{\pi}{2} \quad (11)$$

$$\mathbf{a}^\top \mathbf{b} = \cos\left(\frac{\pi}{2}\right) \|\mathbf{a}\| \|\mathbf{b}\| \quad (12)$$

$$= 0 \quad (13)$$

$$\implies \mathbf{a}^\top \mathbf{b} = 0 \quad (14)$$

$$(c) \ 0 < \theta < \pi$$

$$\text{Assume, } \theta = \frac{2\pi}{3}$$

$$\mathbf{a}^\top \mathbf{b} = \cos\left(\frac{2\pi}{3}\right) \|\mathbf{a}\| \|\mathbf{b}\| \quad (15)$$

$$= -\frac{1}{2} \|\mathbf{a}\| \|\mathbf{b}\| \quad (16)$$

$$\implies \mathbf{a}^\top \mathbf{b} < 0 \quad (17)$$

$$(d) \ 0 \leq \theta \leq \pi$$

$$\text{for } \theta = 0 \quad (18)$$

$$\mathbf{a}^\top \mathbf{b} = \cos(0) \|\mathbf{a}\| \|\mathbf{b}\| \quad (19)$$

$$= \|\mathbf{a}\| \|\mathbf{b}\| \quad (20)$$

$$\implies \mathbf{a}^\top \mathbf{b} > 0 \quad (21)$$

$$\text{for } \theta = \pi \quad (22)$$

$$\mathbf{a}^\top \mathbf{b} = \cos(\pi) \|\mathbf{a}\| \|\mathbf{b}\| \quad (23)$$

$$= -\|\mathbf{a}\| \|\mathbf{b}\| \quad (24)$$

$$\implies \mathbf{a}^\top \mathbf{b} < 0 \quad (25)$$

$\therefore \theta$  is  $0 \leq \theta \leq \frac{\pi}{2}$ . So, Option (B) is correct.