TRIANGLES

9^{th}Math - Chapter 7

This is Problem-8 from Exercise 7.1

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Figure 1). Show that:

(i) $\triangle AMC \cong \triangle BMD$

(ii) $\angle DBC$ is a right angle.

(iii) $\triangle DBC \cong \triangle ACB$

(iv) $CM = \frac{1}{2}AB$

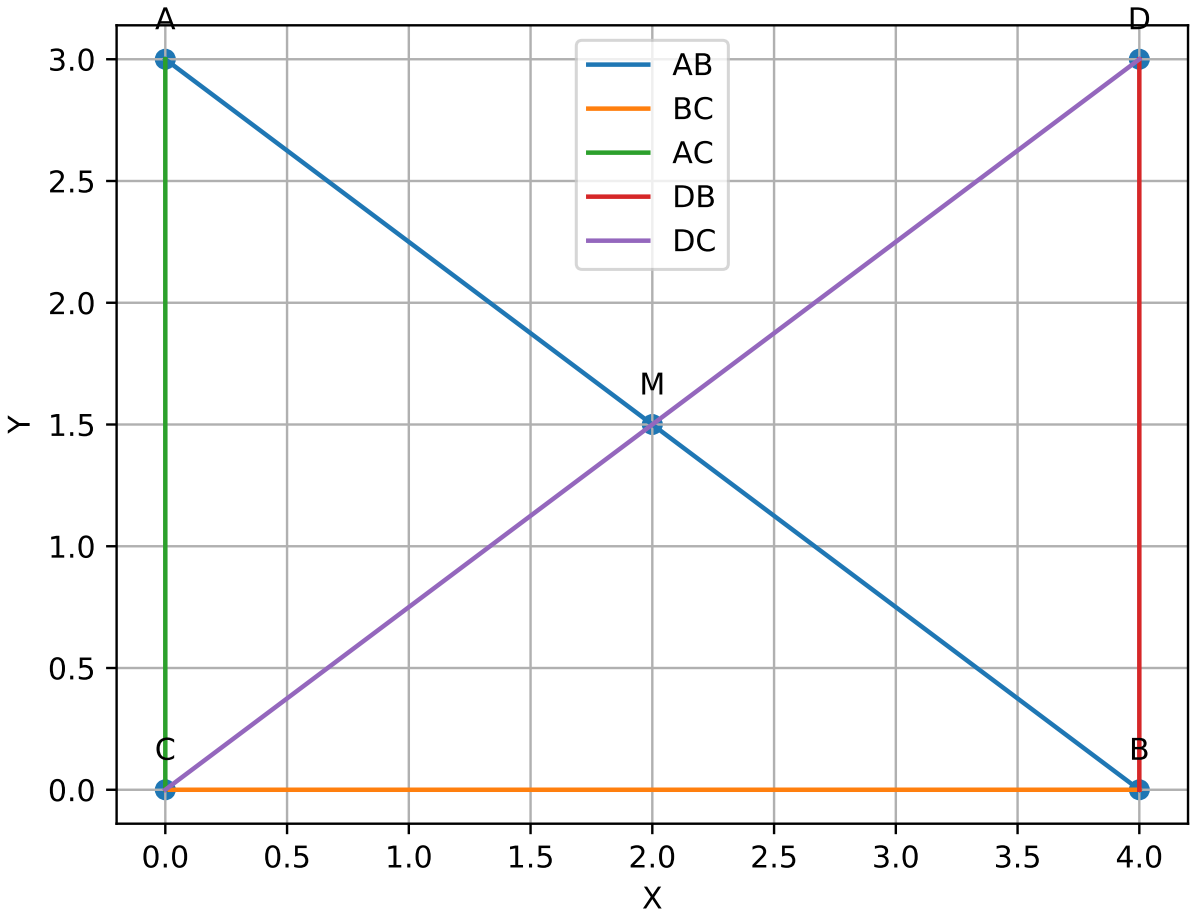

Figure 1

Construction:

The input parameters for construction

Symbol	Values	Description
a	4	CB
b	3	AC

$$\mathbf{A} = \begin{pmatrix} 0 \\ b \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3}$$

from given $\mathbf{M}$ is mid point of AB and CD

$$\mathbf{M} = \frac{A+B}{2} = \frac{1}{2}\begin{pmatrix} a \\ b \end{pmatrix} \tag{4}$$

$$\mathbf{M} = \frac{C+D}{2} \tag{5}$$

$$\implies \mathbf{D} = 2\mathbf{M} - \mathbf{C} \tag{6}$$

Solution: Given

$$\mathbf{M} = \frac{A+B}{2} \tag{7}$$

$$\|\mathbf{D} - \mathbf{M}\| = \|\mathbf{C} - \mathbf{M}\| \tag{8}$$

$$\angle ACB = 90^\circ \tag{9}$$

Proof: From Figure 1

$$(\mathbf{D} - \mathbf{B})^\top (\mathbf{C} - \mathbf{B}) = \begin{pmatrix} 0 & b \end{pmatrix}\begin{pmatrix} a \\ 0 \end{pmatrix} = 0 \tag{10}$$

$$\implies BD \perp BC \tag{11}$$

$$\implies \angle DBC = 90^\circ \tag{12}$$

$$\|\mathbf{A} - \mathbf{B}\| = \left\| \begin{pmatrix} -a \\ b \end{pmatrix} \right\| \tag{13}$$

$$\|\mathbf{C} - \mathbf{D}\| = \left\| \begin{pmatrix} -a \\ -b \end{pmatrix} \right\| \tag{14}$$

$$\implies \|\mathbf{A} - \mathbf{B}\| = \|\mathbf{C} - \mathbf{D}\| \tag{15}$$

$$\text{or, } AB = CD \tag{16}$$

from (7) and (8), $\mathbf{M}$ is midpoint of both AB and CD
from (16)

$$\implies CM = \frac{1}{2}CD = \frac{1}{2}AB \tag{17}$$