

VECTORS

12th Math - Chapter 10

This is Problem-16 from Exercise 10.5

1. If θ is the angle between two vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$, then $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \geq 0$.

(a) $0 < \theta < \frac{\pi}{2}$

(b) $0 \leq \theta \leq \frac{\pi}{2}$

(c) $0 < \theta < \pi$

(d) $0 \leq \theta \leq \pi$

Solution: Given

$$\mathbf{a}, \mathbf{b} \text{ are two vectors} \quad (1)$$

$$\mathbf{a}^\top \mathbf{b} \geq 0 \quad (2)$$

Consider,

$$\vec{\mathbf{a}} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad (3)$$

$$\vec{\mathbf{b}} = \begin{pmatrix} 5 \\ 12 \end{pmatrix} \quad (4)$$

We know that

$$\theta = \cos^{-1} \left(\frac{\mathbf{a}^\top \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} \right) \quad (5)$$

$$\mathbf{a}^\top \mathbf{b} = \cos \theta \|\mathbf{a}\| \|\mathbf{b}\| \quad (6)$$

$$\|\mathbf{a}\| = \sqrt{\mathbf{a}_1^2 + \mathbf{a}_2^2} \quad (7)$$

Then,

$$\|\mathbf{a}\| = \sqrt{4^2 + 3^2} \quad (8)$$

$$\|\mathbf{a}\| = 5 \quad (9)$$

$$\|\mathbf{b}\| = \sqrt{5^2 + 12^2} \quad (10)$$

$$\|\mathbf{b}\| = 13 \quad (11)$$

$$(12)$$

for $\theta = 0$

$$\mathbf{a}^\top \mathbf{b} = \cos(0)(5)(13) \quad (13)$$

$$= 65 \quad (14)$$

which is $\mathbf{a}^\top \mathbf{b} \geq 0$

for $\theta = \frac{\pi}{2}$

$$\mathbf{a}^\top \mathbf{b} = \cos\left(\frac{\pi}{2}\right)(5)(13) \quad (15)$$

$$= 0 \quad (16)$$

which is $\mathbf{a}^\top \mathbf{b} \geq 0$

for $\theta = \pi$

$$\mathbf{a}^\top \mathbf{b} = \cos(\pi)(5)(13) \quad (17)$$

$$= -65 \quad (18)$$

which is $\mathbf{a}^\top \mathbf{b} < 0$

Therefore, the θ is $0 \leq \theta \leq \frac{\pi}{2}$. So, option (b) is correct.