TRIANGLES

9^{th}Math - Chapter 7

This is Problem-8 from Exercise 7.1

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Figure 1). Show that:

(i) $\triangle AMC \cong \triangle BMD$

(ii) $\angle DBC$ is a right angle.

(iii) $\triangle DBC \cong \triangle ACB$

(iv) $CM = \frac{1}{2}AB$

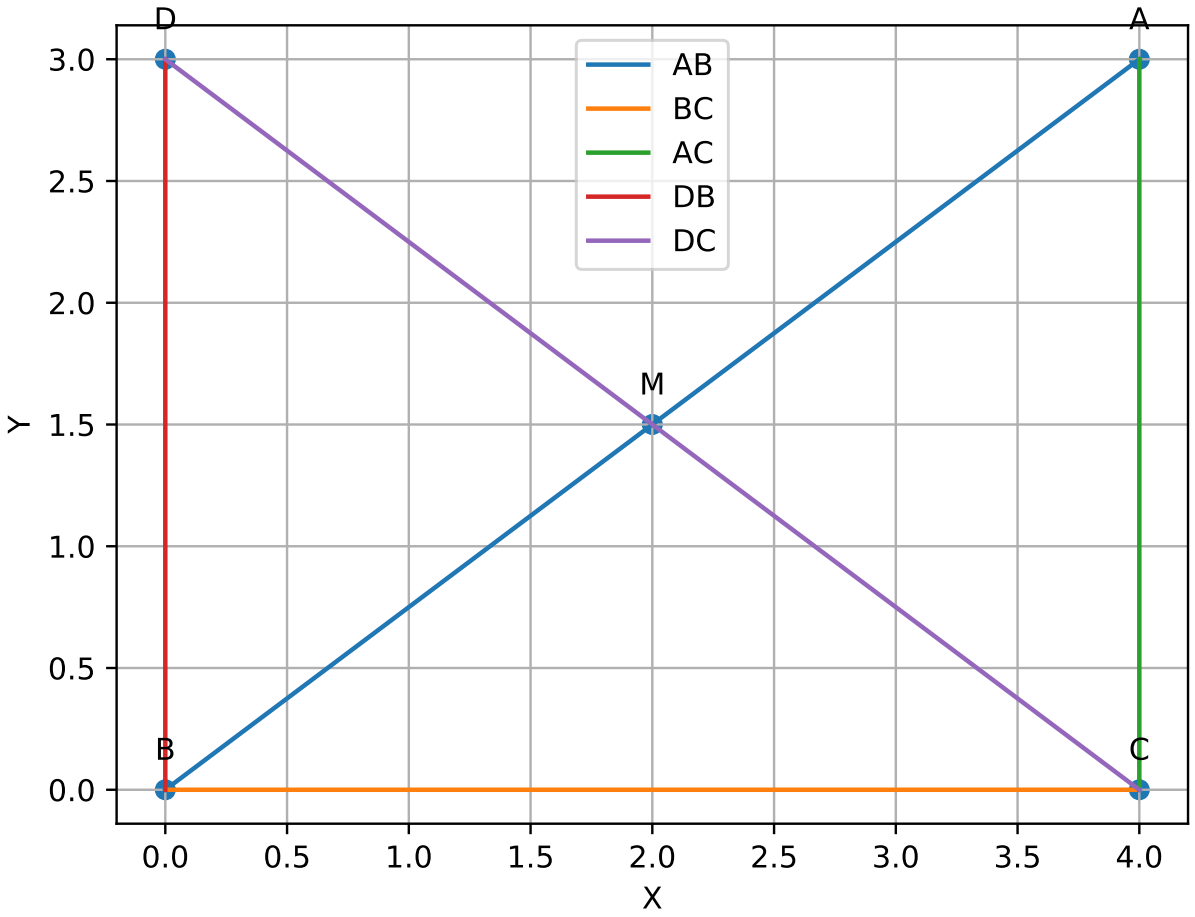

Figure 1

Construction:

The input parameters for construction

Symbol	Values	Description
a	4	BC
b	3	AC=BD

$$\mathbf{A} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{3}$$

$$\mathbf{D} = \begin{pmatrix} 0 \\ b \end{pmatrix} \tag{4}$$

$$\mathbf{M} = \frac{A+B}{2} = \frac{1}{2}\begin{pmatrix} a \\ b \end{pmatrix} \tag{5}$$

Solution: Given

$$\mathbf{M} = \frac{A+B}{2} \tag{6}$$

$$\|\mathbf{D}-\mathbf{M}\| = \|\mathbf{C}-\mathbf{M}\| \tag{7}$$

$$\angle ACB = 90^\circ \tag{8}$$

(i) $\triangle AMC \cong \triangle BMD$

from (6) we can write as, (9)

$$\|\mathbf{A}-\mathbf{M}\| = \|\mathbf{B}-\mathbf{M}\| \tag{10}$$

$$\angle AMC = \cos^{-1}\left(\frac{(\mathbf{A}-\mathbf{M})^\top(\mathbf{C}-\mathbf{M})}{\|\mathbf{A}-\mathbf{M}\|\|\mathbf{C}-\mathbf{M}\|}\right) \tag{11}$$

$$\angle DMB = \cos^{-1}\left(\frac{(\mathbf{D}-\mathbf{M})^\top(\mathbf{B}-\mathbf{M})}{\|\mathbf{D}-\mathbf{M}\|\|\mathbf{B}-\mathbf{M}\|}\right) \tag{12}$$

$$\angle AMC = \angle DMB \approx 74^\circ \tag{13}$$

from (7), (10) and (13),

$$\triangle AMC \cong \triangle BMD \tag{14}$$

from (14) we can say that, (15)

$$\|\mathbf{D}-\mathbf{B}\| = \|\mathbf{A}-\mathbf{C}\| \tag{16}$$

(ii) $\angle DBC$ is a right angle

$$(\mathbf{D}-\mathbf{B})^\top(\mathbf{C}-\mathbf{B}) = \begin{pmatrix} 0 & b \end{pmatrix}\begin{pmatrix} a \\ 0 \end{pmatrix} \tag{17}$$

$$= 0 \tag{18}$$

$$\implies BD \perp BC \tag{19}$$

$$\implies \angle DBC = 90^\circ \tag{20}$$

(iii) $\triangle DBC \cong \triangle ACB$

from (8) and (20) (21)

$$\angle ACB = \angle DBC = 90^\circ \tag{22}$$

$$\|\mathbf{B}-\mathbf{C}\| = \|\mathbf{C}-\mathbf{B}\| \tag{23}$$

from (16), (22) and (23),

$$\triangle DBC \cong \triangle ACB \tag{24}$$

(iv) $CM = \dfrac{1}{2}AB$

$$\|\mathbf{A} - \mathbf{B}\| = \left\| \begin{pmatrix} a \\ b \end{pmatrix} \right\| \tag{25}$$

$$\|\mathbf{C} - \mathbf{D}\| = \left\| \begin{pmatrix} a \\ -b \end{pmatrix} \right\| \tag{26}$$

$$\implies \|\mathbf{A} - \mathbf{B}\| = \|\mathbf{C} - \mathbf{D}\| \tag{27}$$

$$\text{or, } AB = CD \tag{28}$$

from (6) and (7), $\mathbf{M}$ is midpoint of both AB and CD
from (28)

$$CM = \frac{1}{2}CD = \frac{1}{2}AB \tag{29}$$