

VECTORS

12th Math - Chapter 10

This is Problem-16 from Exercise 10.5

1. If θ is the angle between two vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$, then $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \geq 0$.

- (a) $0 < \theta < \frac{\pi}{2}$
- (b) $0 \leq \theta \leq \frac{\pi}{2}$
- (c) $0 < \theta < \pi$
- (d) $0 \leq \theta \leq \pi$

Solution: Given \mathbf{a}, \mathbf{b} are two vectors

We know that (1)

$$\theta = \cos^{-1} \left(\frac{\mathbf{a}^\top \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} \right) \quad (2)$$

$$\implies \mathbf{a}^\top \mathbf{b} = \cos \theta \|\mathbf{a}\| \|\mathbf{b}\| \quad (3)$$

$$\mathbf{a}^\top \mathbf{b} \geq 0 \quad (4)$$

$$\cos \theta \|\mathbf{a}\| \|\mathbf{b}\| \geq 0 \quad (5)$$

$$\implies \cos \theta \geq 0 \quad (6)$$

$$\therefore \theta \text{ lies between } 0 \leq \theta \leq \frac{\pi}{2} \text{ and } \frac{3\pi}{2} \leq \theta \leq 2\pi \quad (7)$$

- (a) $0 < \theta < \frac{\pi}{2}$

Comparing with (7), option(a) is incorrect.

- (b) $0 \leq \theta \leq \frac{\pi}{2}$

Comparing with (7), option(b) is correct.

- (c) $0 < \theta < \pi$

Comparing with (7), option(c) is incorrect.

- (d) $0 \leq \theta \leq \pi$

Comparing with (7), option(d) is incorrect.