STRAIGHT LINES

11^{th}Math - Chapter 10

This is Problem-8 from Exercise 10.4

Find the area of triangle formed by the lines $y-x=0, x+y=0,$ and $x-k=0$.
Solution:
Given line equations represented in vector form:

$$\begin{pmatrix} -1 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{1}$$
$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{2}$$
$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = k \tag{3}$$

The coordinates of the intersection of (1),(2)

$$\left(\begin{array}{cc|c} -1 & 1 & 0 \\ 1 & 1 & 0 \end{array}\right) \xleftrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{cc|c} 1 & 1 & 0 \\ -1 & 1 & 0 \end{array}\right) \tag{4}$$
$$\xleftrightarrow{R_2 \to R_2 + R_1} \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 2 & 0 \end{array}\right) \tag{5}$$
$$\xleftrightarrow{R_2 \to \frac{R_2}{2}} \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & 0 \end{array}\right) \tag{6}$$
$$\xleftrightarrow{R_1 \to R_1 - R_2} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right) \tag{7}$$
$$\text{The intersection of lines is} \tag{8}$$
$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{9}$$

The coordinates of the intersection of (2),(3)

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 0 & k \end{array}\right) \xleftrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{cc|c} 1 & 0 & k \\ 1 & 1 & 0 \end{array}\right) \tag{10}$$
$$\xleftrightarrow{R_2 \to R_2 - R_1} \left(\begin{array}{cc|c} 1 & 0 & k \\ 0 & 1 & -k \end{array}\right) \tag{11}$$
$$\text{The intersection of lines is} \tag{12}$$
$$\mathbf{B} = \begin{pmatrix} k \\ -k \end{pmatrix} \tag{13}$$

The coordinates of the intersection of (3),(1)

$$\left(\begin{array}{cc|c} 1 & 0 & k \\ -1 & 1 & 0 \end{array}\right) \xleftrightarrow{R_2 \to R_2 + R_1} \left(\begin{array}{cc|c} 1 & 0 & k \\ 0 & 1 & k \end{array}\right) \tag{14}$$

The intersection of lines is (15)

$$\mathbf{C} = \begin{pmatrix} k \\ k \end{pmatrix} \tag{16}$$

We know that

$$ar(ABC) = \frac{1}{2}\left\|(\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{C})\right\| \tag{17}$$

$$= \frac{1}{2}\left\|\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} k \\ -k \end{pmatrix}\right) \times \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} k \\ k \end{pmatrix}\right)\right\| \tag{18}$$

$$= \frac{1}{2}\left\|\begin{pmatrix} -k \\ k \end{pmatrix} \times \begin{pmatrix} -k \\ -k \end{pmatrix}\right\| \tag{19}$$

$$= \frac{1}{2}\left\|2k^2\right\| \tag{20}$$

$$\implies = k^2 \tag{21}$$

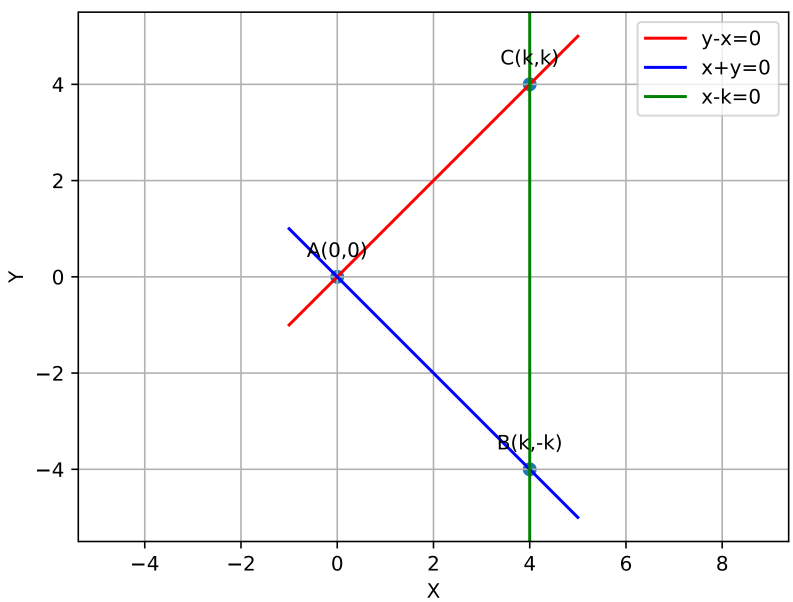

Figure 1