VECTORS

12^{th}Math - Chapter 10

This is Problem-16 from Exercise 10.5

1. If θ is the angle between two vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$,then$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \geq 0$ only when

   (a) $0< \theta < \frac{\pi}{2}$

   (b) $0\leq \theta \leq \frac{\pi}{2}$

   (c) $0< \theta < \pi$

   (d) $0\leq \theta \leq \pi$

   Solution: Given $\mathbf{a}, \mathbf{b}$ are two vectors

   We know that (1)

   $$\theta = \cos^{-1}\left(\frac{\mathbf{a}^\top \mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|}\right) \tag{2}$$

   $$\implies \mathbf{a}^\top \mathbf{b} = \cos\theta \|\mathbf{a}\|\|\mathbf{b}\| \tag{3}$$

   $$\mathbf{a}^\top \mathbf{b} \geq 0 \tag{4}$$

   $$\cos\theta \|\mathbf{a}\|\|\mathbf{b}\| \geq 0 \tag{5}$$

   $$\implies \cos\theta \geq 0 \tag{6}$$

   $$\therefore \theta \text{ lies between } 0 \leq \theta \leq \frac{\pi}{2} \text{ and } \frac{3\pi}{2} \leq \theta \leq 2\pi \tag{7}$$

   (a) $0 < \theta < \frac{\pi}{2}$
   Comparing with (7), option(a) is incorrect.

   (b) $0 \leq \theta \leq \frac{\pi}{2}$
   Comparing with (7), option(b) is correct.

   (c) $0 < \theta < \pi$
   Comparing with (7), option(c) is incorrect.

   (d) $0 \leq \theta \leq \pi$
   Comparing with (7), option(d) is incorrect.