CHAPTER-11
THREE DIMENSIONAL GEOMETRY

January 31, 2023

EXERCISE-11.3

Short Answer(S.A)

1. Find the position vector of a point A in space such that $\overrightarrow{OA}$ is inclined at 60 $^\circ$ to OX and at 45 $^\circ$ to OY and $\left|\overrightarrow{OA}\right| = 10$ units.

2. Find the vector equation of the line which is parallel to the vector $3\hat{i} - 2\hat{j} + 6\hat{k}$ and which passes through the point $(1, -2, 3)$.

3. Show the lines
$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$
$$\text{and } \frac{x-4}{5} = \frac{y-1}{2} = z \text{ intersect .}$$
Also, find their point of intersection.

4. Find the angle between the lines
$$\overrightarrow{r} = 3\hat{i} - 2\hat{j} + 6\hat{k} + \lambda(2\hat{i} + \hat{j} + 2\hat{k}) \text{ and } \overrightarrow{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$$

5. Prove that the line through $\text{A}(0, -1, -1)$ and $\text{B}(4, 5, 1)$ intersects the line through $\text{C}(3, 9, 4)$ and $\text{D}(-4, 4, 4)$.

6. Prove that the lines $x = py + q, z = ry + s$ and $x = p\prime y + q\prime, z = r\prime y + s\prime$ are perpendicular if $pp\prime + rr\prime + 1 = 0$.

7. Find the equation of a plane which bisects perpendicularly the line joining the points $\text{A}(2, 3, 4)$ and $\text{B}(4, 5, 8)$ at right angles.

8. Find the equation of a plane which is at a distance $3\sqrt{3}$ units from origin and the normal to which is equally inclined to coordinate axis.

9. If the line drawn from the point $(-2,-1,-3)$ meets a plane at right angle at the point $(1,-3,3)$, find the equation of the plane.

10. Find the equation of the plane through the points $(2,1,0)$, $(3,-2,-2)$ and $(3,1,7)$.

11. Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at angles of $\frac{\pi}{3}$each.

12. Find the angle between the lines whose direction cosines are given by the equations $l+m+n = 0$, $l^2 + m^2 - n^2 = 0$.

13. If a variable line in two adjacent positions has directions cosines l, m, n and $l+\delta l, m+\delta m, n+\delta n$, show that the small angle $\delta\theta$ between the two positions is given by

$$\delta\theta^2 = \delta l^2 + \delta m^2 + \delta n^2$$

14. O is the origin and A is (a, b, c).Find the direction cosines of the line OA and the equation of plane through A at right angle at OA.

15. Two systems of rectangular axis have the same origin. If a plane cuts them at distances a, b, c and $a\prime, b\prime, c\prime$, respectively, from the origin, prove that

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a\prime^2} + \frac{1}{b\prime^2} + \frac{1}{c\prime^2}$$

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Long Answer(L.A)

16. Find the foot of perpendicular from the point $(2,3,-8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.Also, find the perpendicular distance from the given point to the line.

17. Find the distance of a point $(2,4,-1)$ from the line

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$$

18. Find the length and the foot of perpendicular from the point $\left(1, \frac{3}{2}, 2\right)$ to the plane $2x - 2y + 4z + 5 = 0$.

19. Find the equations of the line passing through the point $(3,0,1)$ and parallel to the planes $x + 2y = 0$ and $3y - z = 0$.

20. Find the equation of the plane through the points $(2,1,-1)$ and $(-1,3,4)$, and perpendicular to the plane $x - 2y + 4z = 10$.

21. Find the shortest distance between the lines given by $\overrightarrow{r} = (8+3\lambda\hat{i} - (9+16\lambda)\hat{j} + (10+7\lambda)\hat{k}$ and $\overrightarrow{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

22. Find the equation of the plane which is perpendicular to the plane $5x+3y+6z+8=0$ and which contains the line of intersection of the planes $x+2y+3z-4=0$ and $2x+y-z+5=0$.

23. The plane $ax+by=0$ is rotated about its line of intersection with the plane $z=0$ through an angle α. Prove that the equation of the plane in its new position is $ax+by\pm(\sqrt{a^2+b^2}tan\alpha)z=0$.

24. Find the equation of the plane through the intersection of the planes $\overrightarrow{r}\cdot(\hat{i}+3\hat{j})-6=0$ and $\overrightarrow{r}\cdot(3\hat{i}-\hat{j}-4\hat{k})=0$, whose perpendicular distance from origin is unity.

25. Show that the points $(\hat{i}-\hat{j}+3\hat{k})$ and $3(\hat{i}+\hat{j}+\hat{k})$ are equidistant from the plane $\overrightarrow{r}\cdot(5\hat{i}+2\hat{j}-7\hat{k})+9=0$ and lies on opposite side of it.

26. $\overrightarrow{AB}=3\hat{i}-\hat{j}+\hat{k}$ and $\overrightarrow{CD}=-3\hat{i}+2\hat{j}+4\hat{k}$ are two vectors. The position vectors of the points A and C are $6\hat{i}+7\hat{j}+4\hat{k}$ and $-9\hat{j}+2\hat{k}$, respectively. Find the position vector of a point P on the line AB and a point Q on the line CD such that $\overrightarrow{PQ}$ is perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{CD}$ both.

27. Show that the straight lines whose direction cosines are given by $2l+2m-n=0$ and $mn+nl+lm=0$ are at right angles.

28. If $l_1, m_1, n_1; l_2, m_2, n_2; l_3, m_3, n_3$ are the direction cosines of the three mutually perpendcular lines, prove that the line whose direction cosines are propotional to $l_1+l_2+l_3, m_1+m_2, m_3, n_1+n_2+n_3$ make angles with them.

Objective Type Questions

Choose the correct answer from the given four options in each of the Exercises from 29 to 36.

29. Distance of the point $(\alpha\beta\gamma)$ from y-axis is

(a) β (b) $|\beta|$ (c) $|\beta+\gamma|$ (d) $\sqrt{\alpha^2+\gamma^2}$

30. If the directions cosines of a line are k, k, k, then

(a) $k>0$ (b) $0<k<1$ (c) $k=1$ (d) $k=\frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

31. The distance of the plane $\overrightarrow{r}\cdot\left(\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}\right)=1$ from the origin is

(a) 1 (b) 7 (c) $\frac{1}{7}$ (d) None of these

32. The sine of the angle between the straight line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ and the plane $2x-2y+z=5$ is

(a) $\frac{10}{6\sqrt{5}}$ (b) $\frac{4}{5\sqrt{2}}$ (c) $\frac{2\sqrt{3}}{5}$ (d) $\frac{\sqrt{2}}{10}$

33. The reflection of the point $(\alpha\beta\gamma)$ in the xy-plane is

(a) $\alpha, \beta, 0)$ (b) $(0, 0, \gamma)$ (c) $(-\alpha, -\beta, \gamma)$ (d) $(\alpha, \beta, -\gamma)$

34. The area of the quadrilateral ABCD, where A$(0, 4, 1)$, B$(2, 3, -1)$, C$(4, 5, 0)$ and D$(2, 6, 2)$, is equal to

(a) 9 sq. units (b) 18 sq. units (c) 27 sq. units (d) 81 sq. units

35. The locus represented by $xy + yz = 0$ is

(a) A pair of perpendicular lines
(b) A pair of parallel lines
(c) A pair of parallel planes
(d) A pair of perpendicular planes

36. The plane $2x - 3y + 6z - 11 = 0$ makes an angle $sin^{-1}(\alpha)$ with x-axis. The value of α is equal to

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{2}{7}$ (d) $\frac{3}{7}$

Fill in the blanks in each of the Exercises 37 to 41.

37. A plane passes through the points $(2, 0, 0)(0, 3, 0)$ and $(0, 0, 4)$.The equation of plane is __________.

38. The direction cosines of the vector $(2\hat{i} + 2\hat{j} - \hat{k})$ are __________.

39. The vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ is __________.

40. The vector equation of the line through the points $(3, 4, -7)$ and $(1, -1, 6)$ is __________.

41. The cartesian equation of the plane $\overrightarrow{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$ is __________.

State **True** or **False** for the statements in each of the Exercises 42 to 49.

42. the unit vector normal to the plane $x + 2y + 3z - 6 = 0$ is $\frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$.

43. The intercepts made by the plane $2x - 3y + 5z + 4 = 0$ on the co-ordinate axis are $\left(-2\frac{4}{3}, -\frac{4}{5}\right)$.

44. The angle between the line $\overrightarrow{r} = (5\hat{i} - \hat{j} - 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and the plane $\overrightarrow{r} \cdot (3\hat{i} - 4\hat{j} - \hat{k}) + 5 = 0$ is $sin^{-1}\left(\frac{5}{2\sqrt{91}}\right)$.

45. The angle between the planes $\overrightarrow{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k}) = 1$ and $\overrightarrow{r} \cdot (\hat{i} - \hat{j}) = 4$ is $cos^{-1}\left(\frac{-5}{\sqrt{58}}\right)$.

46. The line $\overrightarrow{r} = 2\hat{i} - 3\hat{j} - \hat{k} + \lambda(\hat{i} - \hat{j} + 2\hat{k})$ lies in the plane $\overrightarrow{r} \cdot (3\hat{i} + \hat{j} - \hat{k}) + 2 = 0$.

47. The vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ is

$$\overrightarrow{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k}).$$

48. The equation of a line, which is parallel to $2\hat{i} + \hat{j} + 3\hat{k}$ and which passes through the point $(5, -2, 4)$ is $\frac{x-5}{2} = \frac{y+2}{-1} = \frac{z-4}{3}$.

49. If the foot of perpendicular drawn from the origin to a plane is $(5, -3, -2)$, then the equation of plane is $\overrightarrow{r} \cdot (5\hat{i} - 3\hat{j} - 2\hat{k}) = 38$.