STRAIGHT LINES

11^{th}Math - Chapter 10

This is Problem-8 from Exercise 10.4

Find the area of triangle formed by the lines $y-x=0, x+y=0,$ and $x-k=0$.
Solution: Given line equations are

$$y - x = 0 \tag{1}$$
$$x + y = 0 \tag{2}$$
$$x - k = 0 \tag{3}$$

These (1),(2),(3) line equations intersect at points A,B,C.

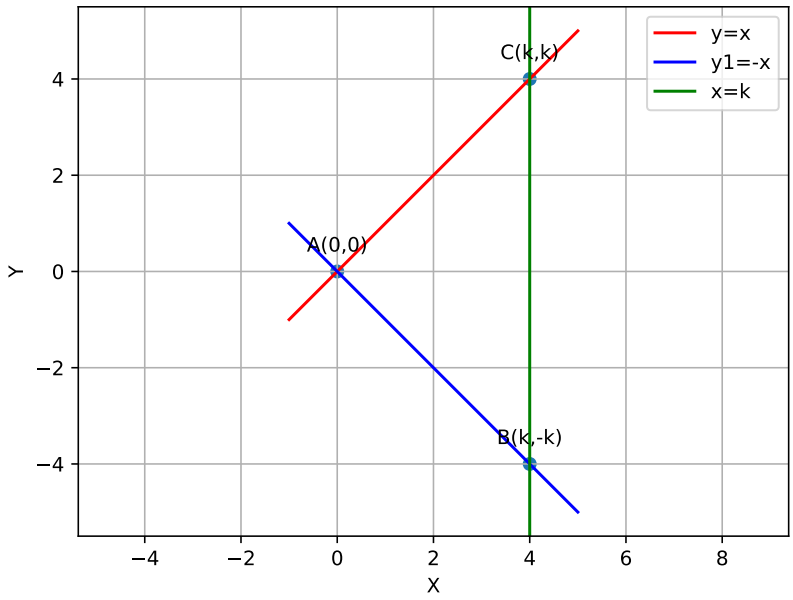

Figure 1

from graph,

$$A = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{4}$$

$$B = \begin{pmatrix} k \\ -k \end{pmatrix} \tag{5}$$

$$C = \begin{pmatrix} k \\ k \end{pmatrix} \tag{6}$$

We know that

$$ar(ABC) = \frac{1}{2}\left\|(A-B)\times(A-C)\right\| \tag{7}$$

$$= \frac{1}{2}\left\|\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} k \\ -k \end{pmatrix}\right) \times \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} k \\ k \end{pmatrix}\right)\right\| \tag{8}$$

$$= \frac{1}{2}\left\|\begin{pmatrix} -k \\ k \end{pmatrix} \times \begin{pmatrix} -k \\ -k \end{pmatrix}\right\| \tag{9}$$

$$= \frac{1}{2}\left\|2k^2\right\| \tag{10}$$

$$\implies = k^2 \tag{11}$$