VECTORS

12^{th}Math - Chapter 10

This is Problem-16 from Exercise 10.5

1. If θ is the angle between two vectors $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$,then$\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}} \geq 0$.

   (a) $0 < \theta < \frac{\pi}{2}$

   (b) $0 \leq \theta \leq \frac{\pi}{2}$

   (c) $0 < \theta < \pi$

   (d) $0 \leq \theta \leq \pi$

   Solution: Given

$$\mathbf{a}, \mathbf{b} \text{ are two vectors} \tag{1}$$

$$\mathbf{a}^\top \mathbf{b} \geq 0 \tag{2}$$

Assume that $\mathbf{a}$,$\mathbf{b}$ are

$$\mathbf{a} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \tag{3}$$

$$\mathbf{b} = \begin{pmatrix} 5 \\ 12 \end{pmatrix} \tag{4}$$

We know that

$$\theta = \cos^{-1}\left(\frac{\mathbf{a}^\top \mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|}\right) \tag{5}$$

$$\implies \mathbf{a}^\top \mathbf{b} = \cos\theta \|\mathbf{a}\|\|\mathbf{b}\| \tag{6}$$

$$\|\mathbf{a}\| = \sqrt{\mathbf{a}_1{}^2 + \mathbf{a}_2{}^2} \tag{7}$$

Verification:

$$\|\mathbf{a}\| = \sqrt{4^2+3^2} \tag{8}$$
$$\implies = 5 \tag{9}$$
$$\|\mathbf{b}\| = \sqrt{5^2+12^2} \tag{10}$$
$$\implies = 13 \tag{11}$$

for $\theta = 0$

$$\mathbf{a}^\top\mathbf{b} = \cos(0)(5)13) \tag{12}$$
$$\implies = 65 \tag{13}$$

$\implies \mathbf{a}^\top\mathbf{b} \geq 0$

for $\theta = \dfrac{\pi}{2}$

$$\mathbf{a}^\top\mathbf{b} = \cos(\frac{\pi}{2})(5)(13) \tag{14}$$
$$\implies = 0 \tag{15}$$

$\implies \mathbf{a}^\top\mathbf{b} \geq 0$

for $\theta = \pi$

$$\mathbf{a}^\top\mathbf{b} = \cos(\pi)(5)(13) \tag{16}$$
$$\implies = -65 \tag{17}$$

$\implies \mathbf{a}^\top\mathbf{b} < 0$

Therefore, the θ is $0 \leq \theta \leq \frac{\pi}{2}$ .So, option (b) is correct.