

VECTORS

12th Math - Chapter 10

This is Problem-16 from Exercise 10.5

1. If θ is the angle between two vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$, then $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \geq 0$.

(A) $0 < \theta < \frac{\pi}{2}$

(B) $0 \leq \theta \leq \frac{\pi}{2}$

(C) $0 < \theta < \pi$

(D) $0 \leq \theta \leq \pi$

Solution: Given \mathbf{a} \mathbf{b} are two vectors

We know that

$$\theta = \cos^{-1} \left(\frac{\mathbf{a}^\top \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} \right) \quad (1)$$

$$\implies \mathbf{a}^\top \mathbf{b} = \cos(\theta) \|\mathbf{a}\| \|\mathbf{b}\| \quad (2)$$

(a) for $\theta = 0$

$$\mathbf{a}^\top \mathbf{b} = \cos(0) \|\mathbf{a}\| \|\mathbf{b}\| \quad (3)$$

$$= \|\mathbf{a}\| \|\mathbf{b}\| \quad (4)$$

$$\implies \mathbf{a}^\top \mathbf{b} \geq 0 \quad (5)$$

(b) for $\theta = \frac{\pi}{2}$

$$\mathbf{a}^\top \mathbf{b} = \cos\left(\frac{\pi}{2}\right) \|\mathbf{a}\| \|\mathbf{b}\| \quad (6)$$

$$\mathbf{a}^\top \mathbf{b} = 0 \quad (7)$$

$$\implies \mathbf{a}^\top \mathbf{b} \geq 0 \quad (8)$$

(c) for $\theta = \pi$

$$\mathbf{a}^\top \mathbf{b} = \cos(\pi) \|\mathbf{a}\| \|\mathbf{b}\| \quad (9)$$

$$= -\|\mathbf{a}\| \|\mathbf{b}\| \quad (10)$$

$$\implies \mathbf{a}^\top \mathbf{b} < 0 \quad (11)$$

$\therefore \theta$ is $0 \leq \theta \leq \frac{\pi}{2}$. So, Option (B) is correct.