CHAPTER-11
THREE DIMENSIONAL GEOMETRY

January 31, 2023

EXERCISE-11.3

Short Answer(S.A)

1. Find the position vector of a point A in space such that $\overrightarrow{OA}$ is inclined at 60 ° to OX and at 45 ° to OY and $|\overrightarrow{OA}| = 10$ units.

2. Find the vector equation of the line which is parallel to the vector $3\hat{i} - 2\hat{j} + 6\hat{k}$ and which passes through the point $(1, -2, 3)$.

3. Show the lines
$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$
$$and \quad \frac{x-4}{5} = \frac{y-1}{2} = z \quad intersect.$$
Also, find their point of intersection.

4. Find the angle between the lines
$$\overrightarrow{r} = 3\hat{i} - 2\hat{j} + 6\hat{k} + \lambda(2\hat{i} + \hat{j} + 2\hat{k}) \text{ and } \overrightarrow{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$$

5. Prove that the line through A$(0,-1,-1)$ and B$(4,5,1)$ intersects the line through C$(3,9,4)$ and D$(-4,4,4)$.

6. Prove that the lines $x = py + q, z = ry + s$ and $x = p'y + q', z = r'y + s'$ are perpendicular if $pp' + rr' + 1 = 0$.

7. Find the equation of a plane which bisects perpendicularly the line joining the points A$(2,3,4)$ and B$(4,5,8)$ at right angles.

8. Find the equation of a plane which is at a distance $3\sqrt{3}$ units from origin and the normal to which is equally inclined to coordinate axis.

9. If the line drawn from the point $(-2,-1,-3)$ meets a plane at right angle at the point $(1,-3,3)$, find the equation of the plane.

10. Find the equation of the plane through the points $(2,1,0)$, $(3,-2,-2)$ and $(3,1,7)$.

11. Find the equations of the two lines through the origin which intersect the line $\dfrac{x-3}{2} = \dfrac{y-3}{1} = \dfrac{z}{1}$ at angles of $\dfrac{\pi}{3}$each.

12. Find the angle between the lines whose direction cosines are given by the equations $l + m + n = 0$, $l^2 + m^2 - n^2 = 0$.

13. If a variable line in two adjacent positions has directions cosines l, m, n and $l + \delta l, m + \delta m, n + \delta n$, show that the small angle $\delta\theta$ between the two positions is given by
$$\delta\theta^2 = \delta l^2 + \delta m^2 + \delta n^2$$

14. O is the origin and A is (a, b, c).Find the direction cosines of the line OA and the equation of plane through A at right angle at OA.

15. Two systems of rectangular axis have the same origin. If a plane cuts them at distances a, b, c and a', b', c', respectively, from the origin, prove that
$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}$$
.

Long Answer(L.A)

16. Find the foot of perpendicular from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.Also, find the perpendicular distance from the given point to the line.

17. Find the distance of a point $(2, 4, -1)$ from the line
$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$$

18. Find the length and the foot of perpendicular from the point $\left(1, \frac{3}{2}, 2\right)$ to the plane $2x - 2y + 4z + 5 = 0$.

19. Find the equations of the line passing through the point $(3, 0, 1)$ and parallel to the planes $x + 2y = 0$ and $3y - z = 0$.

20. Find the equation of the plane through the points $(2, 1, -1)$ and $(-1, 3, 4)$, and perpendicular to the plane $x - 2y + 4z = 10$.

21. Find the shortest distance between the lines given by $\overrightarrow{r} = (8 + 3\lambda\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k}$ and $\overrightarrow{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

22. Find the equation of the plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$.

23. The plane $ax + by = 0$ is rotated about its line of intersection with the plane $z = 0$ through an angle α. Prove that the equation of the plane in its new position is $ax + by \pm (\sqrt{a^2 + b^2} tan\alpha)z = 0$.

24. Find the equation of the plane through the intersection of the planes $\overrightarrow{r} \cdot (\hat{i} + 3\hat{j}) - 6 = 0$ and $\overrightarrow{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$, whose perpendicular distance from origin is unity.

25. Show that the points $(\hat{i} - \hat{j} + 3\hat{k})$ and $3(\hat{i} + \hat{j} + \hat{k})$ are equidistant from the plane $\overrightarrow{r} \cdot (5\hat{i} + 2\hat{j} - 7\hat{k}) + 9 = 0$ and lies on opposite side of it.

26. $\overrightarrow{AB} = 3\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{CD} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ are two vectors. The position vectors of the points A and C are $6\hat{i}+7\hat{j}+4\hat{k}$ and $-9\hat{j}+2\hat{k}$, respectively. Find the position vector of a point P on the line AB and a point Q on the line CD such that $\overrightarrow{PQ}$ is perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{CD}$ both.

27. Show that the straight lines whose direction cosines are given by $2l + 2m - n = 0$ and $mn + nl + lm = 0$ are at right angles.

28. If $l_1, m_1, n_1; l_2, m_2, n_2; l_3, m_3, n_3$ are the direction cosines of the three mutually perpendcular lines, prove that the line whose direction cosines are propotional to $l_1 + l_2 + l_3, m_1 + m_2, m_3, n_1 + n_2 + n_3$ make angles with them.

Objective Type Questions

Choose the correct answer from the given four options in each of the Exercises from 29 to 36.

29. Distance of the point $(\alpha\beta\gamma)$ from y-axis is

a) β b) $|\beta|$ c) $|\beta| + |\gamma|$ d) $\sqrt{\alpha^2 + \gamma^2}$

30. If the directions cosines of a line are k, k, k, then

a) $k > 0$ b) $0 < k < 1$ c) $k = 1$ d) k=$\frac{1}{\sqrt{3}}$ or -$\frac{1}{\sqrt{3}}$

31. The distance of the plane $\overrightarrow{r} \cdot \left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = 1$ from the origin is

a) 1 b) 7 c) $\frac{1}{7}$ d) None of these

32. The sine of the angle between the straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and the plane $2x - 2y + z = 5$ is

a) $\frac{10}{6\sqrt{5}}$ b) $\frac{4}{5\sqrt{2}}$ c) $\frac{2\sqrt{3}}{5}$ d) $\frac{\sqrt{2}}{10}$

33. The reflection of the point $(\alpha\beta\gamma)$ in the xy-plane is

a) $\alpha, \beta, 0)$ b) $(0, 0, \gamma)$ c) $(-\alpha, -\beta, \gamma)$ d) $(\alpha, \beta, -\gamma)$

34. The area of the quadrilateral ABCD, where A$(0, 4, 1)$, B$(2, 3, -1)$, C$(4, 5, 0)$ and D$(2, 6, 2)$, is equal to

a) 9 sq. units b) 18 sq. units c) 27 sq. units d) 81 sq. units

35. The locus represented by $xy + yz = 0$ is

a) A pair of perpendicular lines b) A pair of parallel lines

c) A pair of parallel planes d) A pair of perpendicular planes

36. The plane $2x - 3y + 6z - 11 = 0$ makes an angle $sin^{-1}(\alpha)$ with x-axis. The value of α is equal to

a) $\frac{\sqrt{3}}{2}$ b) $\frac{\sqrt{2}}{3}$ c) $\frac{2}{7}$ d) $\frac{3}{7}$

Fill in the blanks in each of the Exercises 37 to 41.

37. A plane passes through the points $(2, 0, 0)(0, 3, 0)$ and $(0, 0, 4)$.The equation of plane is ________.

38. The direction cosines of the vector $(2\hat{i} + 2\hat{j} - \hat{k})$ are ________.

39. The vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ is ________.

40. The vector equation of the line through the points $(3, 4, -7)$ and $(1, -1, 6)$ is ________.

41. The cartesian equation of the plane $\overrightarrow{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$ is ________.

State **True** or **False** for the statements in each of the Exercises 42 to 49.

42. the unit vector normal to the plane $x + 2y + 3z - 6 = 0$ is $\frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$.

43. The intercepts made by the plane $2x-3y+5z+4=0$ on the co-ordinate axis are -2,$\frac{4}{3}, -\frac{4}{5}$.

44. The angle between the line $\overrightarrow{r} = (5\hat{i} - \hat{j} - 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and the plane $\overrightarrow{r} \cdot (3\hat{i} - 4\hat{j} - \hat{k}) + 5 = 0$ is $sin^{-1}\left(\frac{5}{2\sqrt{91}}\right)$.

45. The angle between the planes $\overrightarrow{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k}) = 1$ and $\overrightarrow{r} \cdot (\hat{i} - \hat{j}) = 4$ is $cos^{-1}\left(\frac{-5}{\sqrt{58}}\right)$.

46. The line $\overrightarrow{r} = 2\hat{i}-3\hat{j}-\hat{k}+\lambda(\hat{i}-\hat{j}+2\hat{k})$ lies in the plane $\overrightarrow{r}\cdot(3\hat{i}+\hat{j}-\hat{k})+2 = 0$.

47. The vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ is

$$\overrightarrow{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k}).$$

48. The equation of a line, which is parallel to $2\hat{i}+\hat{j}+3\hat{k}$ and which passes through the point $(5,-2,4)$ is $\frac{x-5}{2} = \frac{y+2}{-1} = \frac{z-4}{3}$.

49. If the foot of perpendicular drawn from the origin to a plane is $(5,-3,-2)$, then the equation of plane is $\overrightarrow{r} \cdot (5\hat{i} - 3\hat{j} - 2\hat{k}) = 38$.