IT 24632

- B. Prove that the set ob national numbers B, equipped with the two binarry operations of addition & multiplication, forms a tield.
- set ob equivalence classes ob ordered pairs (a,b) with a, b ∈ 2 and b≠0, where (a,b)~(a'b') the usual traction a Define addition € multiplication in the usual cuap:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + be}{bd}, \frac{ac}{bd} = \frac{a}{b}, \frac{c}{d},$$

- for \$\$\delta 0, \$\delta 40, Below we show these operations

 make \$\mathcal{G}\$ a field.
- 1. The operations are well defined We must check that it $\frac{a}{b} = \frac{a'}{b'} C C = \frac{c'}{d}$ then $\frac{ad+bc}{bd} = \frac{a'd'+b'c'}{b'd'} C = \frac{ac}{bd} = \frac{a'c'}{b'd'}$

From
$$\frac{a}{b} = \frac{a'}{b'}$$
 $\mathcal{C} = \frac{e'}{d'}$ we have $ab' = a'b$ $\mathcal{C}(ad+be)$ $b'd' = (ab')(dd') + (be)(b'd') = (a'b)(dd') + (be)(a'b') = (a'b)(a'b') + (be)(a'b') = (a'b)(a'b') + (a'b')(a'b') + (a'b')($

I similarly expand the right-hand numerator times bdbd'. Rearcreanging and using ab = a'b, ad=cd shows both cross-products are equal therefore the sums represent the same equivalence class. So addition & multiplication are well-defined.

2. (g, +) is an abelian group Take any a, c, e & Q.

Closurce: a + c = ad+be is a rational number since bit Associativity: to Now torm associativity of integer addition

 $\left(\frac{a}{b} + \frac{e}{f}\right) + \frac{e}{f} = \frac{ad + be}{bd} + \frac{e}{f} = \frac{f(ad + be) + e(bd)}{ad + be}$

similar expansion for a + (g+e), both give the same numerator by associativity ob integer operation

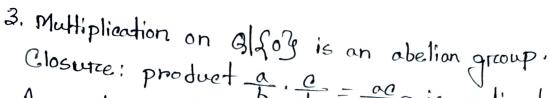
Identity: $0 = \frac{0}{1}$ satisfy $\frac{a}{b} + 0 = \frac{a}{b}$

Inverse: additive inverse of a is-a = -a because

 $\frac{a}{b} + \frac{a}{b} = \frac{0}{b} = 0$

Commutationity: \(\frac{a}{b} \rightarrow \frac{a}{d} + \frac{a}{b} = \frac{ad+be}{db} = \frac{ad+be}{db} = \frac{a}{d} + \frac{a}{b}

Thus (g,+) is on abelian group.



Multiplicative identity:
$$1 = \frac{1}{1}$$
 satisfies $\frac{a}{b}$. $1 = \frac{a}{b}$.

Distributivity: For alily $\frac{a}{b}$ integer multiplication.

Distribution ! For addition le multiplication

$$\frac{a}{b} \cdot \left(\frac{c}{d} + \frac{e}{b}\right) = \frac{a}{b} \cdot \frac{ef}{db} = \frac{ac}{b} \cdot \frac{e}{db} = \frac{ac}{b} + \frac{ae}{b} = \frac{ac}{b} + \frac{ae}{b} = \frac{ac}{b} + \frac{ae}{b} = \frac{ac}{b} \cdot \frac{e}{b} + \frac{ae}{b} = \frac{ac}{b} \cdot \frac{e}{b} = \frac{ac}{b} = \frac{$$

using integer distributivity.

So g is a commutative ring with unity 1.

4. Multiplicative inverse exist for nonzero rationals. Take a nonzero rational a. Its multiplicative invense is to because

$$\frac{a}{b} = \frac{a}{b}, \frac{b}{a} = \frac{ab}{ba} = \frac{1}{1} = 1.$$

We must check this inverse is well-defined: it = a/b. Cato, then ab'=a'b. Multipling both sides by 1/(aa') is informal but the connect check is to = b' ib conly it ba'= b'a but

from ab=ba are get exactly ba=ba, so invense agrees agree for dibberent representative.

5. Non-triviality: 0 \ 1

Clearly $\frac{0}{1} \neq \frac{1}{01}$ because it 0.1 = 1.1 then 0 = 1, contradicting integers properties. So the field is not the zero ring.