1

IT 24632

Aminul Hasan

Solution of a)

$$x \equiv 1 \pmod 3,\ x \equiv 2 \pmod 5,\ x \equiv 3 \pmod 7$$

1. Start with first two congruences:

Let $x = 3a + 1$. Subsititute into the second congruence

$$3a + 1 \equiv 2 \pmod 5$$

$$\Rightarrow 3a = 1 \pmod 5$$

The inverse of 3 module 5 is 2, so:

$$a \equiv 2 \pmod 5$$

$$\Rightarrow a = 5b + 2$$

Then $x = 3(5b+2) + 1 = 15b + 7$

2. Now use third congruence

$$15b + 7 = 3 \pmod 7$$

since $15 \equiv 1 \pmod 7$ & $7 \equiv 0 \pmod 7$ then simplifies to

$$b \equiv 3 \pmod 7 \Rightarrow b = 7c + 3$$

Then $x = 15(7c+3) + 7 = 105c + 52$

$$x \equiv 52 \pmod{15} \Rightarrow 52 \text{ (Ans)}$$

2

Solution of b:

$x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$, $x \equiv 15 \pmod{31}$

1. Combine the first two congruence

$x = 11a + 5$. Subsititue into the second ingruence

$11a + 5 \equiv 14 \pmod{29}$

$11a = 9 \pmod{29}$

The inverse of 11 module 29.

That means we want $11 \cdot x = 1 \mod 29$

$\therefore 11 \cdot 8 = 1 \mod 29$

So the inverse is 8. Multiply both sides:

$a = 9 \times 8 = 72 \equiv 14 \pmod{29}$

$a = 29b + 14$

Subsititute back:

$x = 11(29b + 14) + 5 = 319b + 159$

So ~~159~~ $x \equiv 159 \pmod{319}$

2. Combine with third congruence

We have

$319b + 159 = 15 \pmod{31}$

$319 \equiv 9 \pmod{31}$ & $159 \equiv 4 \pmod{31}$

So: $9b + 4 \equiv 15 \pmod{31} \Rightarrow 9b \equiv 11 \pmod{31}$

(3)

Find the inverse of 9 module 31:

$9 \times 7 = 63 \equiv 1 \pmod{31}$

So the inverse is 7. Multiply both sides:

$b = 11 \times 7 = 77 \equiv 15 \pmod{31} \Rightarrow b = 31c + 15$

Substitute back

$x = 319(31c + 15) + 159$

$= 9889c + 4944$

$\therefore \quad x = 4944 \pmod{9889} \Rightarrow 4944$

Ans.

Solution of c)

$x \equiv 5 \pmod 6,\ x \equiv 4 \pmod{11},\ x \equiv 3 \pmod{17}$

1. Combine the first two ingruence

$x = 6a + 5$. Substitute into the second congruence:

$6a + 5 = 4 \pmod{11}$

$6a \equiv -1 \equiv 10 \pmod{11}$

The inverse of 6 module 11 is 2

Since $6 \times 2 = 12 \equiv 1 \pmod{11}$:

$a = 2 \times 10 = 20 \equiv 9 \mod (11)$

$\Rightarrow a = 11b + 9$

Then $x = 6(11b+9)+5 = 66b+59$

2. Now use the 3rd congruence

$$66b + 59 \equiv 3 \pmod{17}$$

Reduce mod 17

$$66 \equiv 15 \pmod{17}, \quad 59 \equiv 8 \pmod{17}$$

So, $15b + 8 \equiv 3 \pmod{17} \Rightarrow 15b \equiv -5 \equiv 12 \pmod{17}$

The inverse of 15 mod 17 is 8 (since $15 \times 8 = 120$)

$\therefore 15 \times 8 = 120 \equiv 1 \pmod{17}$:

$$b = 8 \times 12 = 96 \equiv 11 \pmod{17}$$

$$\Rightarrow b = 17c + 11$$

Then $x = 66(17c + 11) + 59$

$= 1122c + 785$

$\therefore x \equiv 785 \pmod{1122}$

$\Rightarrow 785$

Ans.