

```

% Helper function to plot the signal, pole-zero plot & FFT of a given
% *real* signal.
% Parameters:
%   n - Sample number
%   x - Signal value (real signals only)
%   z - Zeroes of the z-transform (decreasing powers of z^-1)
%   p - Poles of the z-transform (decreasing powers of z^-1)
%   main_title - String for figure title
%   dtft_exists - Boolean parameter to indicate whether Discrete Time
%                 Fourier Transform exists for the signal (DTFT)
function plot_sig_pz_and_fft(n, x, z, p, main_title, dtft_exists)
    arguments
        n
        x
        z
        p
        main_title
        dtft_exists = true
    end

    figure("Position", [0 0 900 950]);
    sgtitle(main_title);

    if (dtft_exists)
        subplot(3, 1, 1);
    else
        subplot(2, 1, 1);
    end
    stem(n, x, "Marker", ".", "LineWidth", 2, "MarkerSize", 15);
    xlabel("Sample"); ylabel("Amplitude");
    title(sprintf("%s, n = %d:%d", main_title, min(n), max(n)));
    grid on; grid minor;
    axis on;
    axis([min(n) - 0.5, max(n) + 0.5, min(x) - 0.2, max(x) + 0.2]);

    if (dtft_exists)
        subplot(3, 1, 2);
    else
        subplot(2, 1, 2);
    end
    % Plot the unit circle
    plot(exp(1j * 2 * pi * (0 : 0.01 : 1)), "LineWidth", 2);
    hold on;
    [hz, hp, ht] = zplane(z, p);
    % Set the zeros & poles to be bolder and in red
    set(findobj(hz, "Type", "line"), "LineWidth", 2, "Color", "r");
    set(findobj(hp, "Type", "line"), "LineWidth", 2, "Color", "r");
    % Make the default zplane circle, axis, etc. faint
    set(findobj(ht, "Type", "line"), "LineWidth", 0.01);
    hold off;

```

```

title("Poles and zeros, X(z)");
grid on;
grid minor;
axis("square");
axis([-1.2 1.2 -1.2 1.2]);

if (dtft_exists)
    subplot(3, 1, 3);
    plot(... [-0.5 : 1/200 : 0.5-1/200], fftshift(abs(fft(x/sum(x), 200))), ...
        "LineWidth", 2);
    grid on; grid minor;
    axis([-0.5 0.5 0 1.1]);
    title("Frequency response, X(\theta)");
    xlabel("Normalized frequency (f/f_S)"); ylabel("Magnitude");
end
end

% Helper function to plot the signal, pole-zero plot & FFT of a given
% *complex* signal.
% Parameters:
%   n - Sample number
%   x - Signal value (complex signals only)
%   z - Zeroes of the z-transform (decreasing powers of z^-1)
%   p - Poles of the z-transform (decreasing powers of z^-1)
%   main_title - String for figure title
%   dtft_exists - Boolean parameter to indicate whether Discrete Time
%                 Fourier Transform exists for the signal (DTFT)
function plot_cmplx_sig_pz_and_fft(n, x, z, p, main_title, dtft_exists)
    arguments
        n
        x
        z
        p
        main_title
        dtft_exists = true
    end

    figure("Position", [0 0 900 950]);

    subplot(2, 1, 1);
    stem(n, real(x), "Marker", ".", "LineWidth", 2, "MarkerSize", 15);
    xlabel("Sample"); ylabel("Magnitude");
    title(sprintf("Real part, n = %d:%d", min(n), max(n)));
    grid on; grid minor;
    axis on;
    axis([min(n) - 0.5, max(n) + 0.5, min(real(x)) - 0.2, max(real(x)) + 0.2]);

    subplot(2, 1, 2);

```

```

stem(n, imag(x), "Marker", ".", "LineWidth", 2, "MarkerSize", 15);
xlabel("Sample"); ylabel("Magnitude");
title(sprintf("Imaginary part, n = %d:%d", min(n), max(n)));
grid on; grid minor;
axis on;
axis([min(n) - 0.5, max(n) + 0.5, min(imag(x)) - 0.2, max(imag(x)) +
0.2]);

sgtitle(main_title);

figure("Position", [0 0 900 950]);

subplot(2, 1, 1);
% Plot the unit circle
plot(exp(1j * 2 * pi * (0 : 0.01 : 1)), "LineWidth", 2);
hold on;
[hz, hp, ht] = zplane(z, p);
% Set the zeros & poles to be bolder and in red
set(findobj(hz, "Type", "line"), "LineWidth", 2, "Color", "r");
set(findobj(hp, "Type", "line"), "LineWidth", 2, "Color", "r");
% Make the default zplane circle, axis, etc. faint
set(findobj(ht, "Type", "line"), "LineWidth", 0.01);
hold off;
title("Poles and zeros, X(z)");
grid on; grid minor;
axis("square");
axis([-1.2 1.2 -1.2 1.2]);

if (dtft_exists)
    subplot(2, 1, 2);
    fft_x = fft(x/sum(x), 200);
    plot([-0.5 : 1/200 : 0.5-1/200], fftshift(abs(fft_x)), "LineWidth", 2);
    grid on; grid minor;
    axis([-0.5, 0.5, min(abs(fft_x)) - 1, max(abs(fft_x)) + 1]);
    title("Frequency response, X(\theta)");
    xlabel("Normalized frequency (f/f_S)"); ylabel("Magnitude");
end

sgtitle(main_title);

end
clc; clear; close all;

```

## a) $\delta[n]$

z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$X(z) = x[0]z^{-0}$$

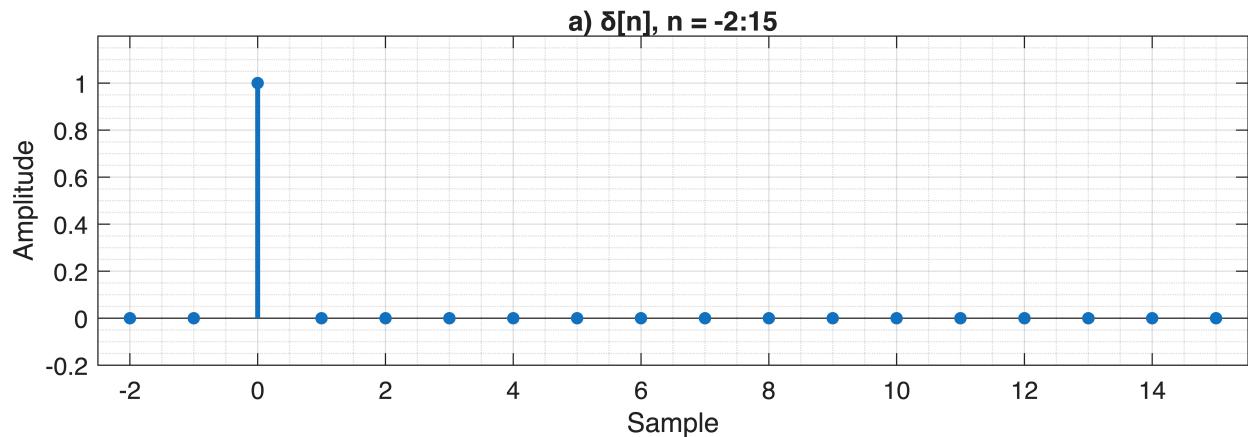
$$X(z) = 1, ROC : z \in \mathbb{C}$$

Discrete Time Fourier Transform (DTFT):

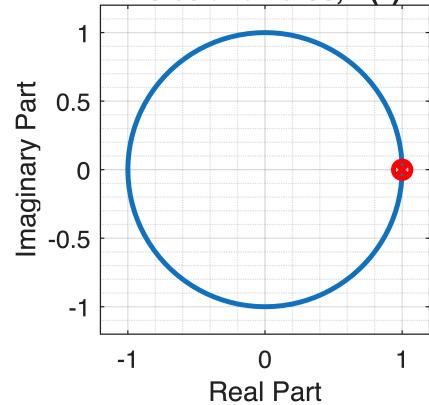
$$X(\theta) = X(z)|_{z=e^{j\theta}} = 1$$

```
n = [-2 : 15];  
x_a = (n == 0);  
% z-transform is 1  
plot_sig_pz_and_fft(n, x_a, 1, 1, "a)  $\delta[n]$ ");
```

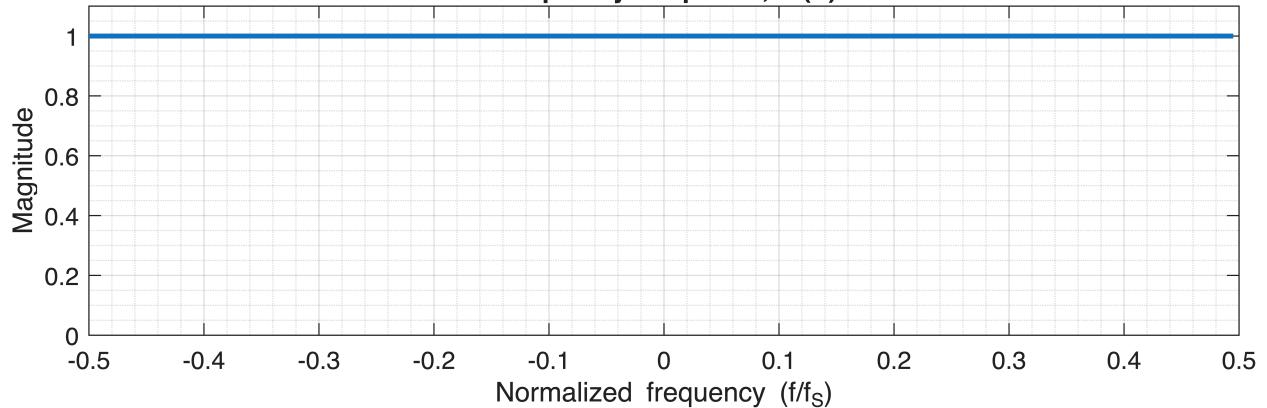
a)  $\delta[n]$



Poles and zeros,  $X(z)$



Frequency response,  $X(\theta)$



## b) $\delta[n-2]$

z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$X(z) = x[2]z^{-2}$$

$$X(z) = z^{-2}, ROC : z \in \mathbb{C}, \text{ except } z = 0$$

or

$$X(z) = \frac{1}{z^2}, ROC : z \in \mathbb{C}, \text{ except } z = 0$$

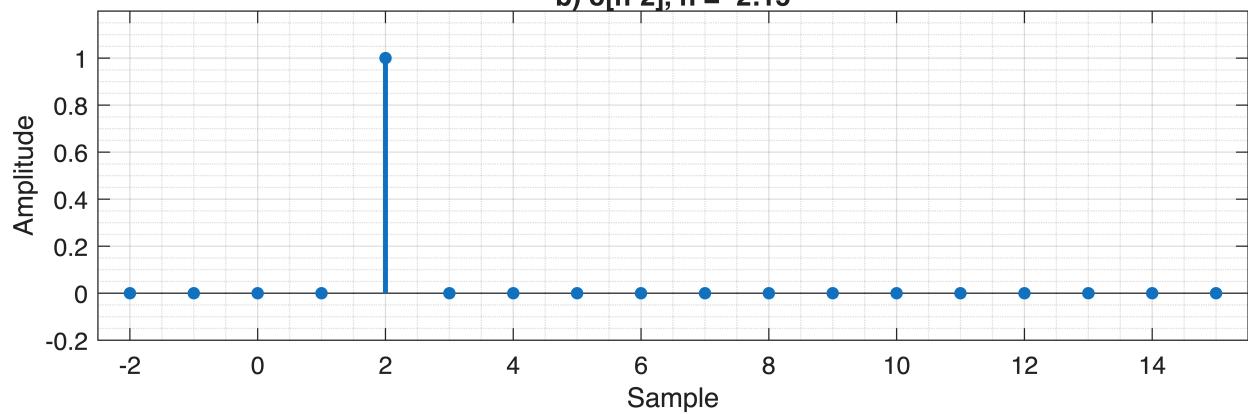
Discrete Time Fourier Transform (DTFT):

$$X(\theta) = X(z)|_{z=e^{j\theta}} = 1$$

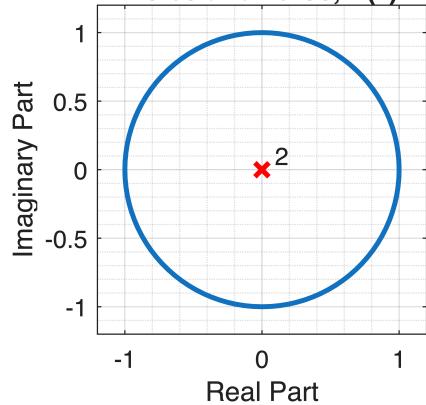
```
x_b = (n == 2);
% z-transform is (z^-2)
plot_sig_pz_and_fft(n, x_b, [0 0 1], 1, "b) δ[n-2]");
```

b)  $\delta[n-2]$

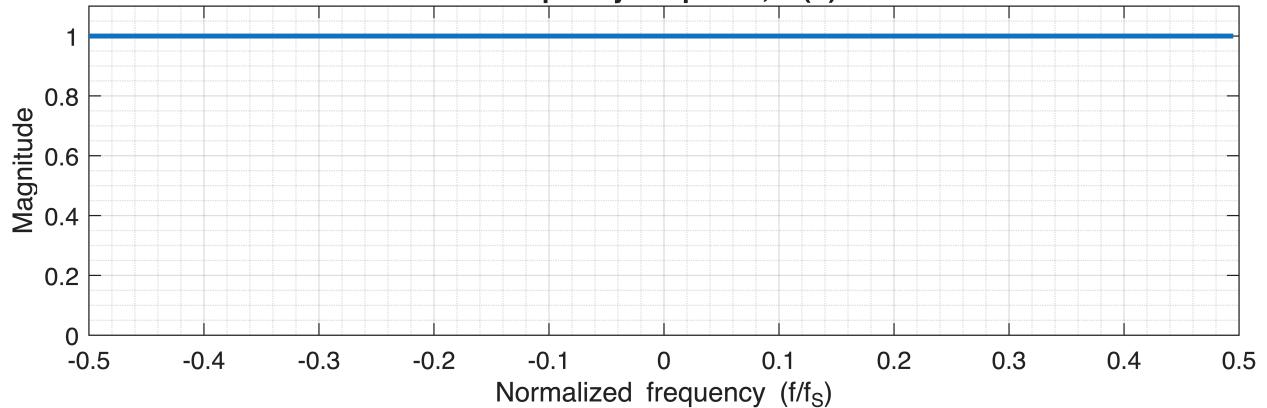
b)  $\delta[n-2], n = -2:15$



Poles and zeros,  $X(z)$



Frequency response,  $X(\theta)$



### c) $\delta[n] + \delta[n-2]$

z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$X(z) = x[0]z^{-0} + x[2]z^{-2}$$

$$X(z) = 1 + z^{-2}, ROC : z \in \mathbb{C}, \text{ except } z = 0$$

or

$$X(z) = \frac{z^2 + z^0}{z^2}, ROC : z \in \mathbb{C}, \text{ except } z = 0$$

This can also be written as

$$X(z) = \frac{z(z^1 + z^{-1})}{z^2} = \frac{z^1 + z^{-1}}{z}$$

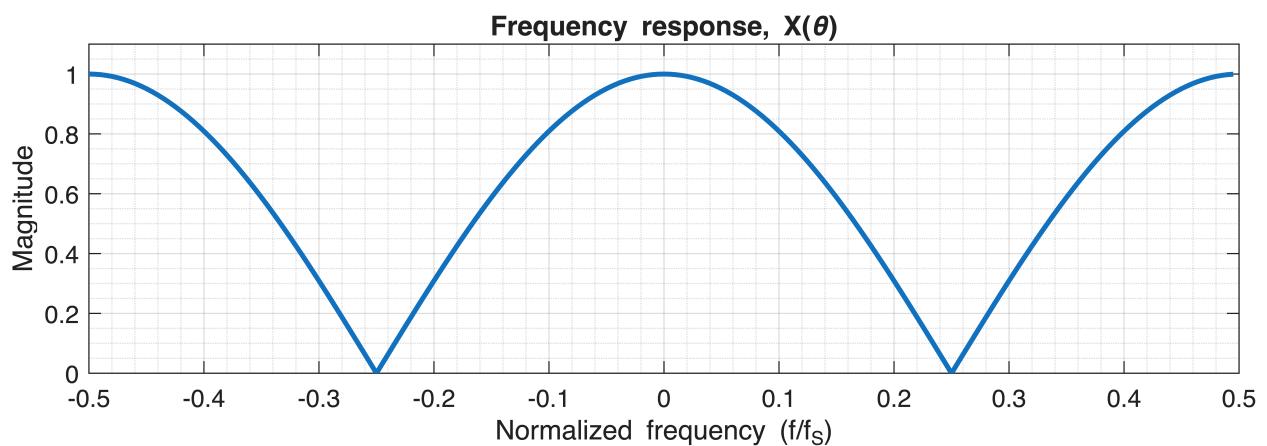
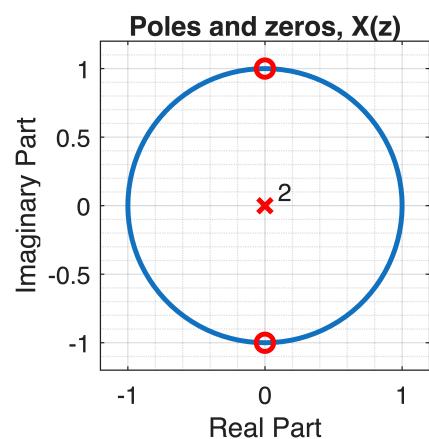
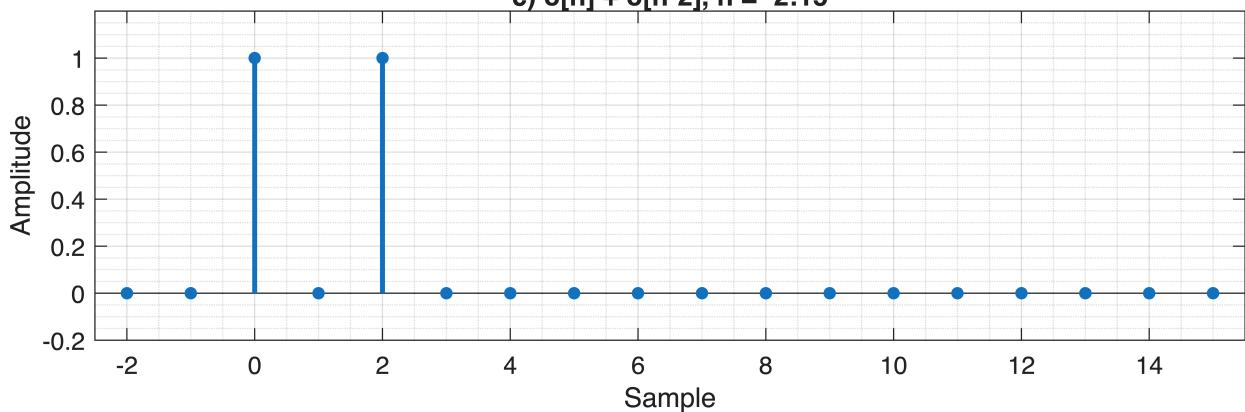
Discrete Time Fourier Transform (DTFT):

$$X(\theta) = X(z)|_{z=e^{j\theta}} = \frac{z^1 + z^{-1}}{z} \Big|_{z=e^{j\theta}}$$

$$X(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{e^{j\theta}} = e^{-j\theta} 2\cos(\theta)$$

```
x_c = (n == 0) + (n == 2);
% z-transform is (1 + z^-2)
plot_sig_pz_and_fft(n, x_c, [1 0 1], 1, "c) δ[n] + δ[n-2]");
```

c)  $\delta[n] + \delta[n-2]$   
 c)  $\delta[n] + \delta[n-2], n = -2:15$



## d) $\delta[n] + \delta[n-1] + \delta[n-2]$

z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$X(z) = x[0]z^{-0} + x[1]z^{-1} + x[2]z^{-2}$$

$$X(z) = 1 + z^{-1} + z^{-2}, ROC : z \in \mathbb{C}, \text{ except } z = 0$$

or

$$X(z) = \frac{z^2 + z^1 + z^0}{z^2}, ROC : z \in \mathbb{C}, \text{ except } z = 0$$

This can also be written as

$$X(z) = \frac{1}{z^2} \cdot \frac{z^3 - 1}{z - 1} = \frac{1}{z^2} \cdot \frac{z^{1.5} - z^{-1.5}}{z^{0.5} - z^{-0.5}} = \frac{1}{z} \cdot \frac{z^{1.5} - z^{-1.5}}{z^{0.5} - z^{-0.5}}$$

Discrete Time Fourier Transform (DTFT):

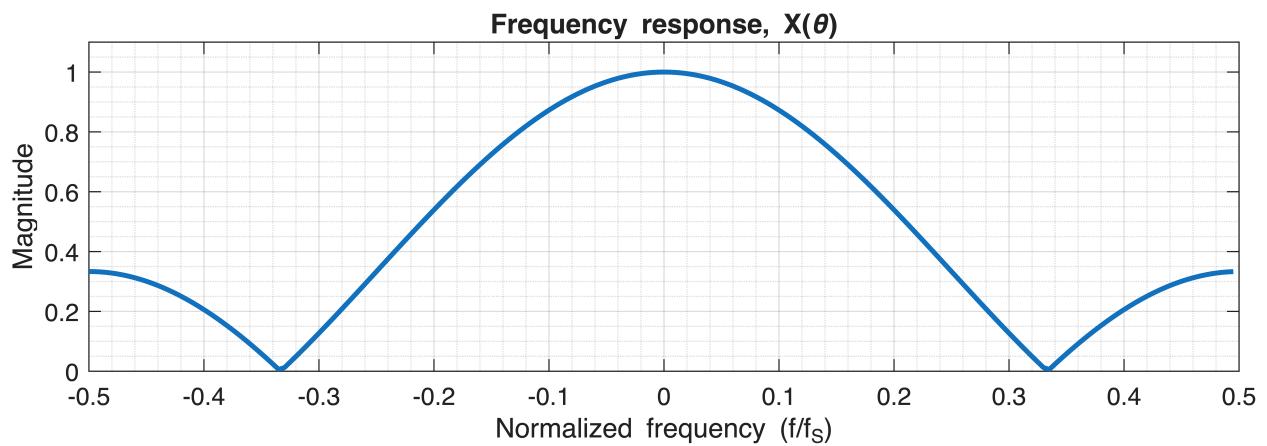
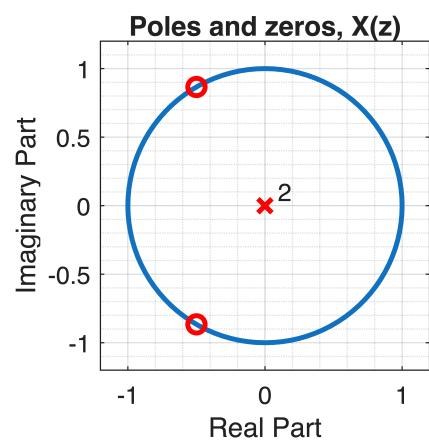
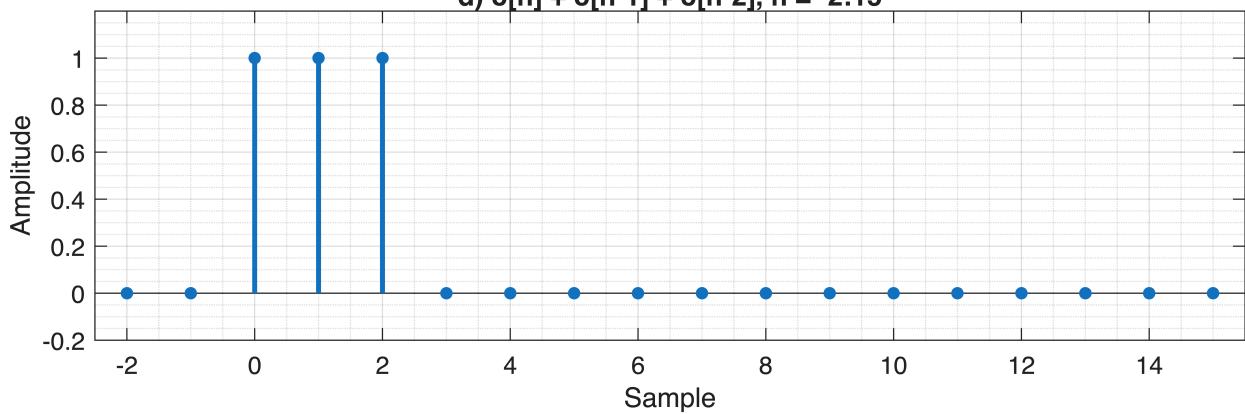
$$X(\theta) = X(z)|_{z=e^{j\theta}} = \frac{1}{z} \cdot \frac{z^{1.5} - z^{-1.5}}{z^{0.5} - z^{-0.5}} \Big|_{z=e^{j\theta}}$$

$$X(\theta) = e^{-j\theta} \cdot \frac{e^{j1.5\theta} - e^{-j1.5\theta}}{e^{j0.5\theta} - e^{-j0.5\theta}} = e^{-j\theta} \cdot \frac{\sin(1.5\theta)}{\sin(0.5\theta)}$$

```
x_d = (n == 0) + (n == 1) + (n == 2);
% z-transform is (1 + z^-1 + z^-2)
plot_sig_pz_and_fft(n, x_d, ones(1, 3), 1, "d) δ[n] + δ[n-1] + δ[n-2]");
```

$$d) \delta[n] + \delta[n-1] + \delta[n-2]$$

$$d) \delta[n] + \delta[n-1] + \delta[n-2], n = -2:15$$



$$e) \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5] + \delta[n-6] + \delta[n-7] + \delta[n-8] + \delta[n-9] + \delta[n-10] + \delta[n-11]$$

z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$X(z) = \sum_{n=0}^{11} x[n]z^{-n}$$

$$X(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6} + z^{-7} + z^{-8} + z^{-9} + z^{-10} + z^{-11}, ROC : z \in \mathbb{C}, \text{ except } z = 0$$

or

$$X(z) = \frac{z^{11} + z^{10} + z^9 + z^8 + z^7 + z^6 + z^5 + z^4 + z^3 + z^2 + z^1 + z^0}{z^{11}}, ROC : z \in \mathbb{C}, \text{ except } z = 0$$

This can also be written as

$$X(z) = \frac{1}{z^{11}} \cdot \frac{z^{12} - 1}{z - 1} = \frac{1}{z^{11}} \cdot \frac{z^6 - z^{-6}}{z^{0.5} - z^{-0.5}} = \frac{1}{z^{5.5}} \cdot \frac{z^6 - z^{-6}}{z^{0.5} - z^{-0.5}}$$

Discrete Time Fourier Transform (DTFT):

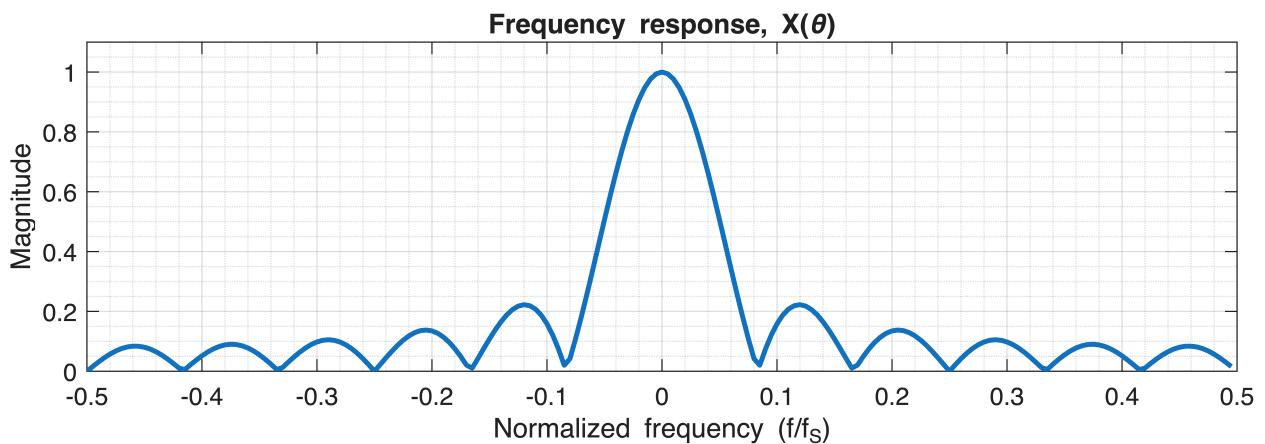
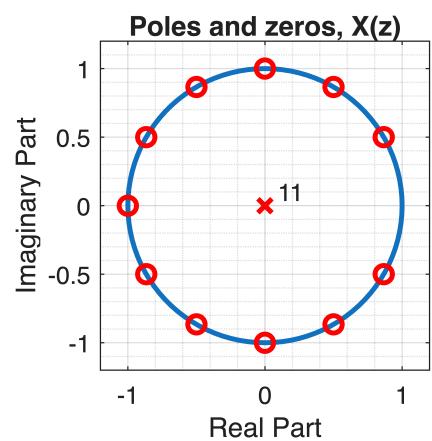
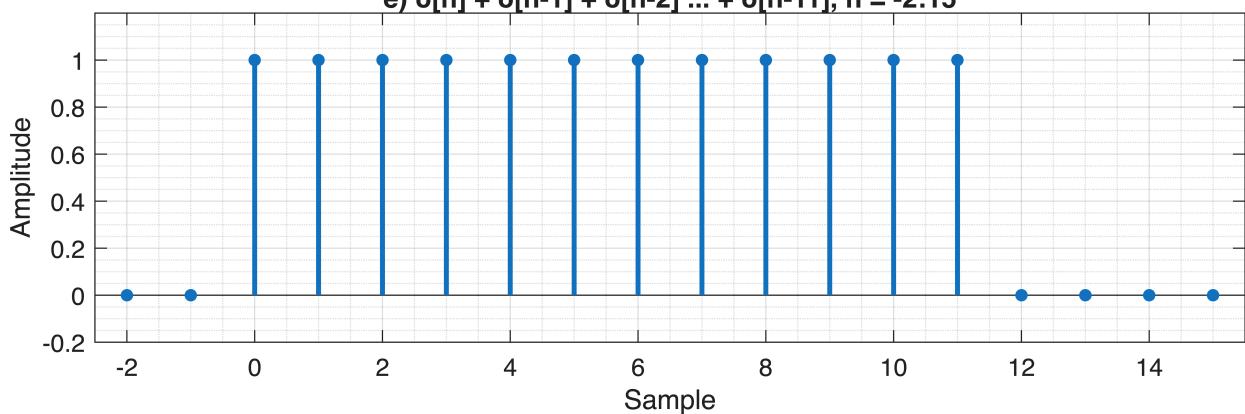
$$X(\theta) = X(z)|_{z=e^{j\theta}} = \frac{1}{z^{5.5}} \cdot \frac{z^6 - z^{-6}}{z^{0.5} - z^{-0.5}}|_{z=e^{j\theta}}$$

$$X(\theta) = e^{-j5.5\theta} \cdot \frac{e^{j6\theta} - e^{-j6\theta}}{e^{j0.5\theta} - e^{-j0.5\theta}} = e^{-j5.5\theta} \cdot \frac{\sin(6\theta)}{\sin(0.5\theta)}$$

```
x_e = (n == 0) + (n == 1) + (n == 2) + (n == 3) + (n == 4) + (n == 5) + ...
      (n == 6) + (n == 7) + (n == 8) + (n == 9) + (n == 10) + (n == 11);
% z-transform is (1 + z^-1 + z^-2 + z^-3 + z^-4 + z^-5 ... z^-11)
plot_sig_pz_and_fft(...
    n, x_e, ones(1, 12), 1, "e) \delta[n] + \delta[n-1] + \delta[n-2] ... + \delta[n-11]");
```

$$e) \delta[n] + \delta[n-1] + \delta[n-2] \dots + \delta[n-11]$$

$$e) \delta[n] + \delta[n-1] + \delta[n-2] \dots + \delta[n-11], n = -2:15$$



## f) $\delta[n+1] + \delta[n-1]$

z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$X(z) = x[-1]z^1 + x[1]z^{-1}$$

$$X(z) = z + z^{-2}, ROC : z \in \mathbb{C}, \text{ except } z = 0$$

or

$$X(z) = \frac{z^3 + z^0}{z^2}, ROC : z \in \mathbb{C}, \text{ except } z = 0$$

This can also be written as

$$X(z) = z^{1.5} \cdot \frac{z^{1.5} + z^{-1.5}}{z^2} = \frac{1}{z^{0.5}} \cdot (z^{1.5} + z^{-1.5})$$

Discrete Time Fourier Transform (DTFT):

$$X(\theta) = X(z)|_{z=e^{j\theta}} = \frac{1}{z^{0.5}} \cdot (z^{1.5} + z^{-1.5})|_{z=e^{j\theta}}$$

$$X(\theta) = e^{-j0.5\theta} \cdot (e^{j1.5\theta} + e^{-j1.5\theta}) = e^{-j0.5\theta} \cos(1.5\theta)$$

```

x_f = (n == -1) + (n == 1);
% z-transform is (z + z^-1), or, (1 + z^-2)/z^-1
% Since this is non-causal, Matlab tf2zp routine called within zplane
% throws an
% error "Denominator must have non-zero leading coefficient". Hence, need
% to do the pole zero plot the hard way.
figure("Position", [0 0 900 950]);
sgtitle("f)  $\delta[n+1] + \delta[n-1]$ ");
subplot(3, 1, 1);
stem(n, x_f, "Marker", ".", "LineWidth", 2, "MarkerSize", 15);
xlabel("Sample"); ylabel("Amplitude");
title(sprintf("%s, n = %d:%d", "f)  $\delta[n+1] + \delta[n-1]$ ", min(n), max(n)));
grid on; grid minor;
axis on;
axis([min(n) - 0.5, max(n) + 0.5, min(x_f) - 0.2, max(x_f) + 0.2]);

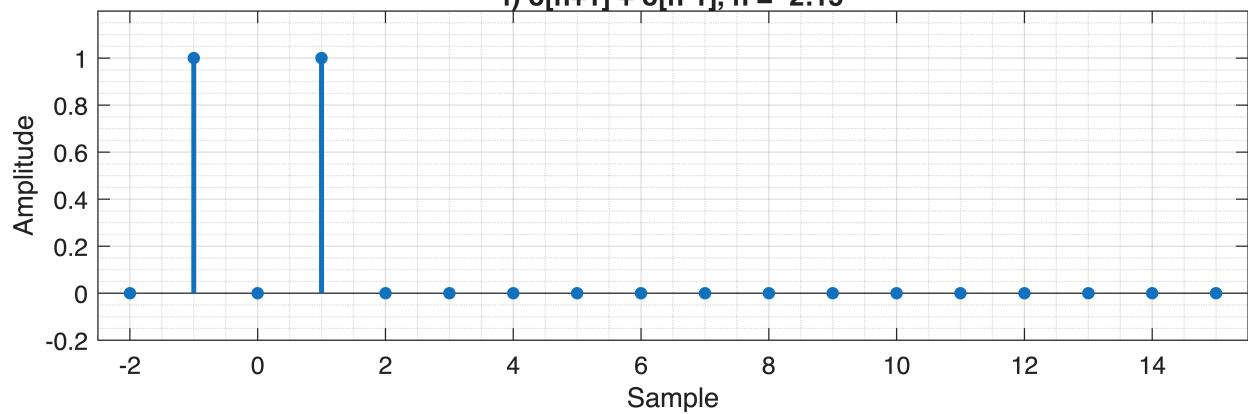
subplot(3, 1, 2);
% Plot the unit circle
plot(exp(1j * 2 * pi * (0 : 0.01 : 1)), "LineWidth", 2);
hold on;
plot(roots([1 0 1]), "ro", "LineWidth", 2, "MarkerSize", 10);
plot(0, 0, "rx", "LineWidth", 2, "MarkerSize", 10);
hold off;
text(0.1, 0.0, "2", "FontSize", 10);
title("Poles and zeros, X(z)");
grid on; grid minor;
axis("square");

```

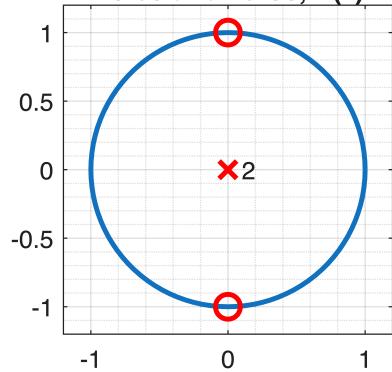
```
axis([-1.2 1.2 -1.2 1.2]);  
  
subplot(3, 1, 3);  
plot(...  
    [-0.5 : 1/200 : 0.5-1/200], fftshift(abs(fft(x_f/sum(x_f), 200))), ...  
    "LineWidth", 2);  
grid on; grid minor;  
axis([-0.5 0.5 0 1.1]);  
title("Frequency response, X(\theta)");  
xlabel("Normalized frequency (f/f_S)"); ylabel("Magnitude");
```

$$f) \delta[n+1] + \delta[n-1]$$

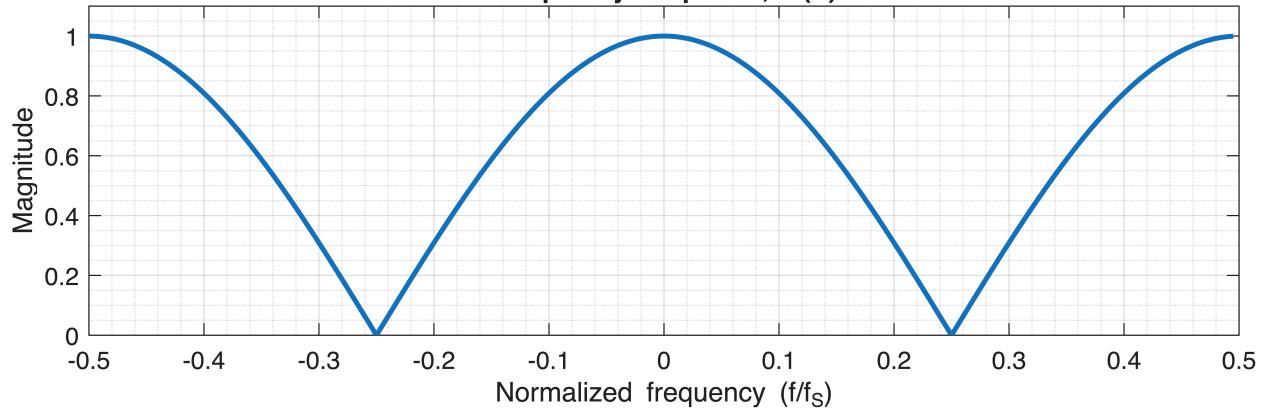
$$f) \delta[n+1] + \delta[n-1], n = -2:15$$



Poles and zeros,  $X(z)$



Frequency response,  $X(\theta)$



### g) $\delta[n] + \delta[n-1] + \delta[n-2] + \dots$ (non-terminating)

z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$X(z) = \sum_{n=0}^{\infty} z^{-n}$$

The above series will converge for  $|z| > 1$ , and the resulting geometric progression's sum would be

$$X(z) = \frac{1}{1 - z^{-1}}, ROC : |z| > 1$$

or

$$X(z) = \frac{z^1}{z^1 - z^0}, ROC : |z| > 1$$

DTFT does not exist for this signal, since it is **not** absolutely summable, i.e.

$$\sum_{n=-\infty}^{\infty} |x[n]| < \infty$$

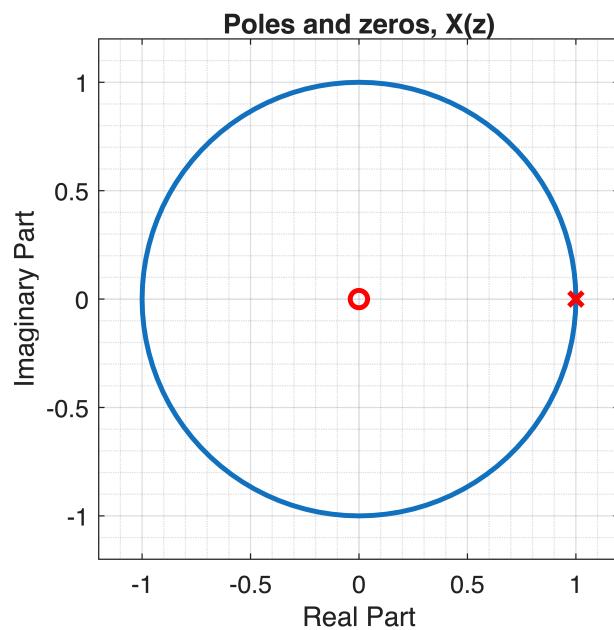
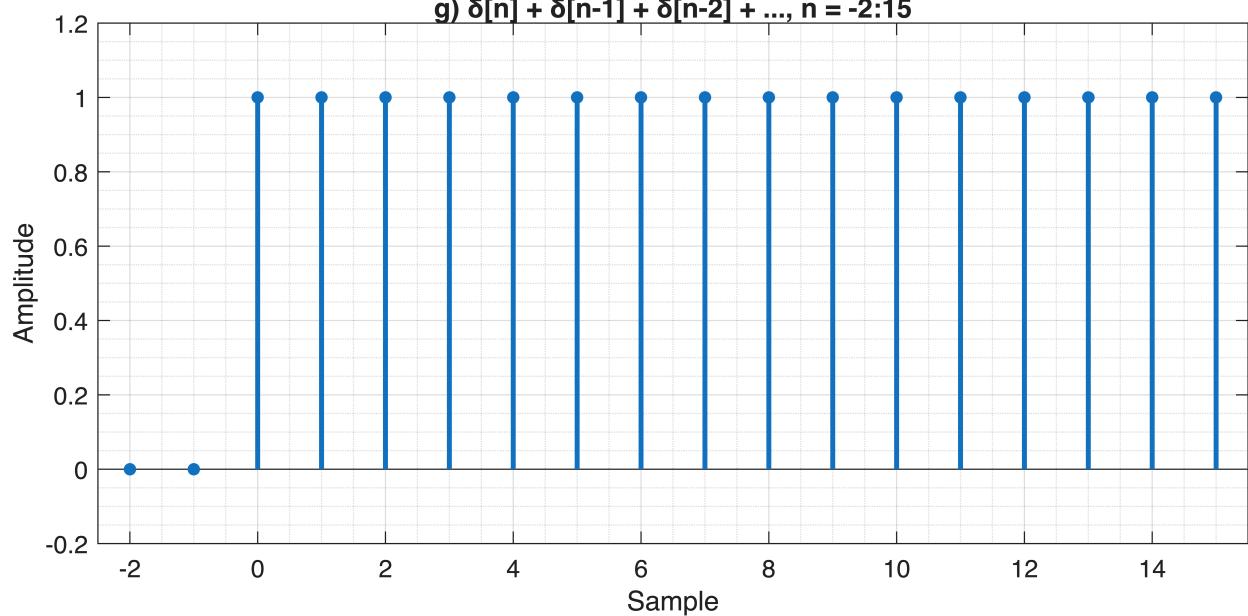
does not hold.

This is also indicated by the ROC not containing the unit circle.

```
x_g = (n >= 0);
% z-transform is (1 + z^-1 + z^-2 + z^-3 + z^-4 + z^-5 ...), or,
% 1/(1 - z^-1) for ROC |z| > 1
plot_sig_pz_and_fft(
    n, x_g, 1, [1 -1], "g)  $\delta[n] + \delta[n-1] + \delta[n-2] + \dots$ ", false);
```

$$g) \delta[n] + \delta[n-1] + \delta[n-2] + \dots$$

$$g) \delta[n] + \delta[n-1] + \delta[n-2] + \dots, n = -2:15$$



**h)  $\delta[n] + (0.9)\delta[n-1] + (0.9^2)\delta[n-2] + \dots$  (non-terminating)**

z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$X(z) = \sum_{n=0}^{\infty} (0.9)^n z^{-n}$$

The above series will converge for  $|z| > 0.9$ , and the resulting geometric progression's sum would be

$$X(z) = \frac{1}{1 - 0.9z^{-1}}, ROC : |z| > 0.9$$

or

$$X(z) = \frac{z^1}{z^1 - 0.9z^0}, ROC : |z| > 0.9$$

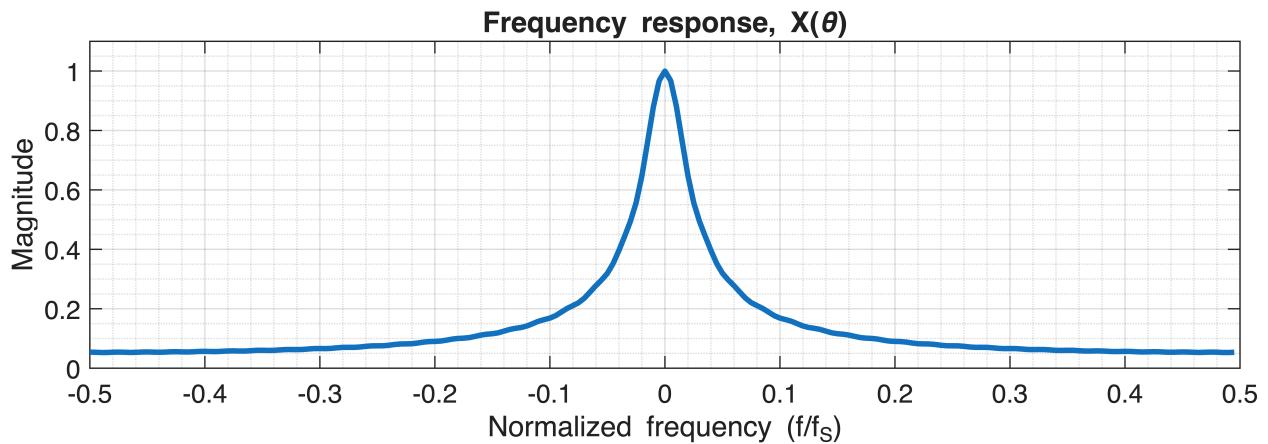
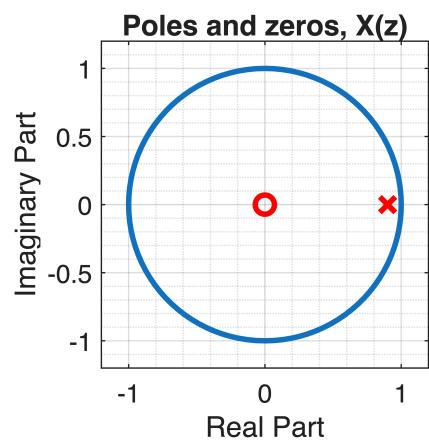
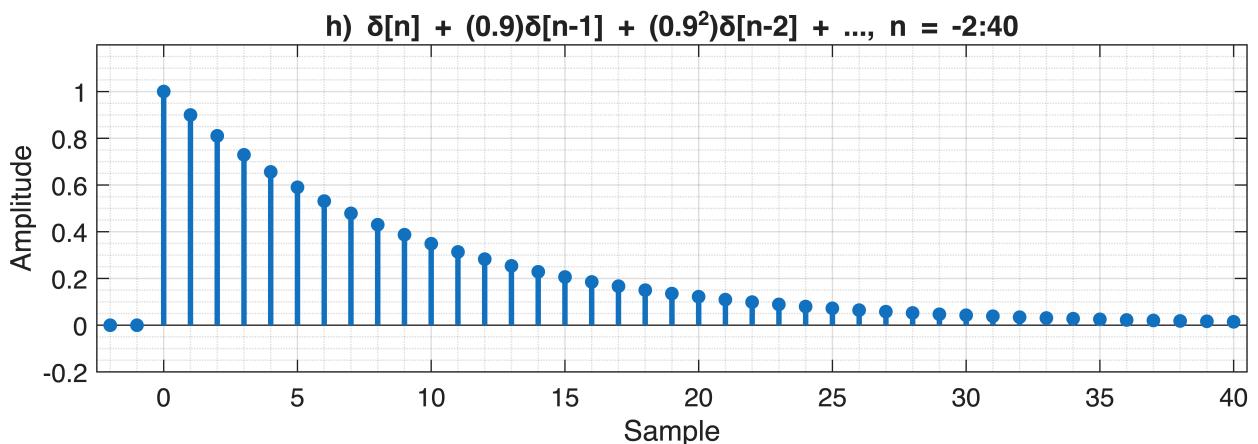
Discrete Time Fourier Transform (DTFT):

$$X(\theta) = X(z)|_{z=e^{j\theta}} = \frac{1}{1 - 0.9z^{-1}} \Big|_{z=e^{j\theta}}$$

$$X(\theta) = \frac{1}{1 - 0.9e^{-j\theta}}$$

```
n = [-2 : 40]; % Increasing the sample size to show the amplitude attenuation

a = 0.9;
x_h = (a.^n) .* (n >= 0);
% z-transform is (1 + z^-1/a + z^-2/a^2 + z^-3/a^3 + ...), or,
% 1/(1 - a*z^-1) for ROC |z| > |a|
plot_sig_pz_and_fft(...  
n, x_h, 1, [1 -a], "h)  $\delta[n] + (0.9)\delta[n-1] + (0.9^2)\delta[n-2] + \dots$ ");
```



i)  $\delta[n] + e^{j2\pi/10}\delta[n-1] + e^{j2\pi/10}\delta[n-2] + \dots$  (non-terminating)

z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$X(z) = \sum_{n=0}^{\infty} (e^{j\frac{2\pi n}{10}})z^{-n}$$

The above series will converge for  $|z| > 1$ , and the resulting geometric progression's sum would be

$$X(z) = \frac{1}{1 - (e^{j\frac{2\pi}{10}})z^{-1}}, ROC : |z| > 1$$

or

$$X(z) = \frac{z^1}{z^1 - (e^{j\frac{2\pi}{10}})z^0}, ROC : |z| > 1$$

DTFT does not exist for this signal, since it is **not** absolutely summable, i.e.

$$\sum_{n=-\infty}^{\infty} |x[n]| < \infty$$

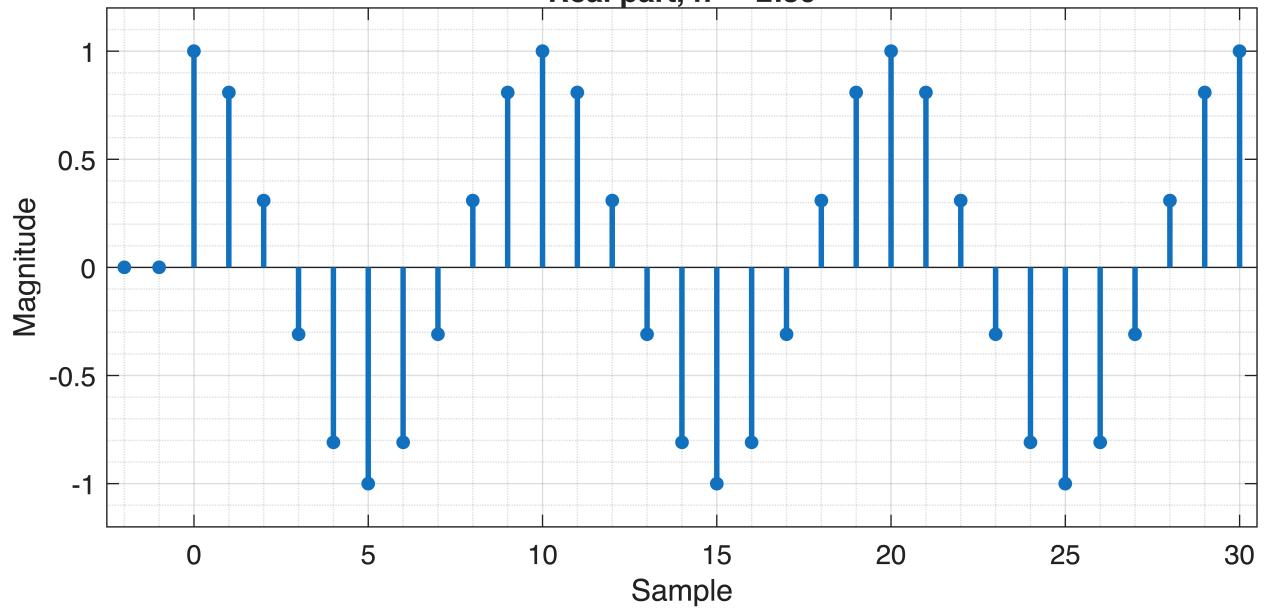
does not hold.

This is also indicated by the ROC not containing the unit circle.

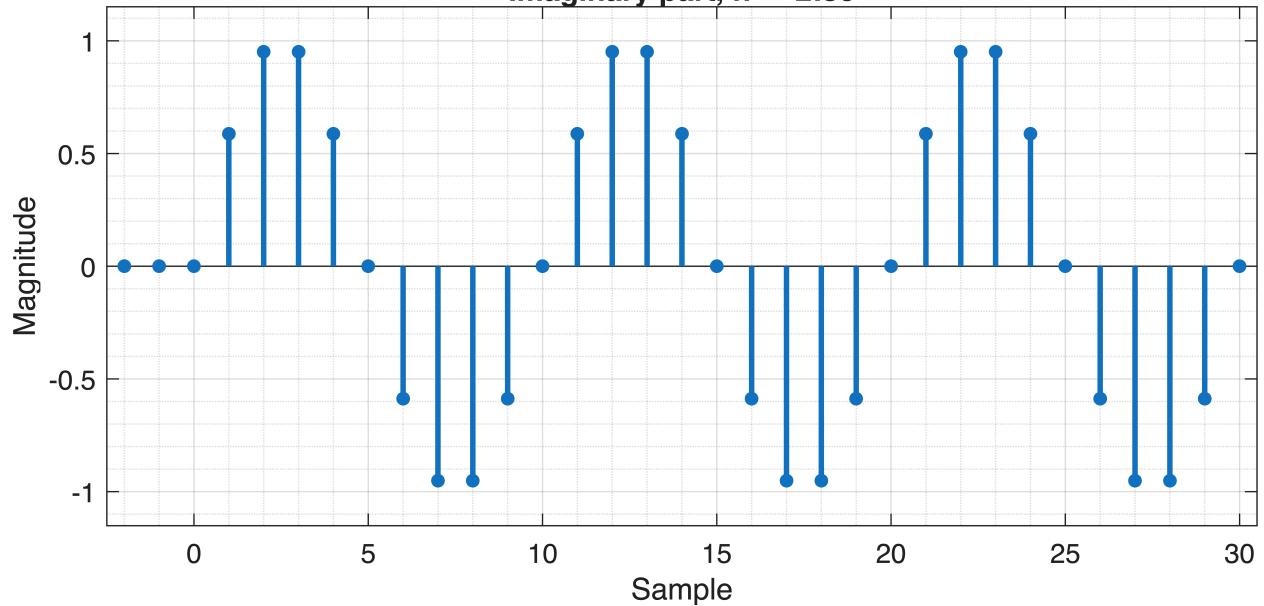
```
n = [-2 : 30];

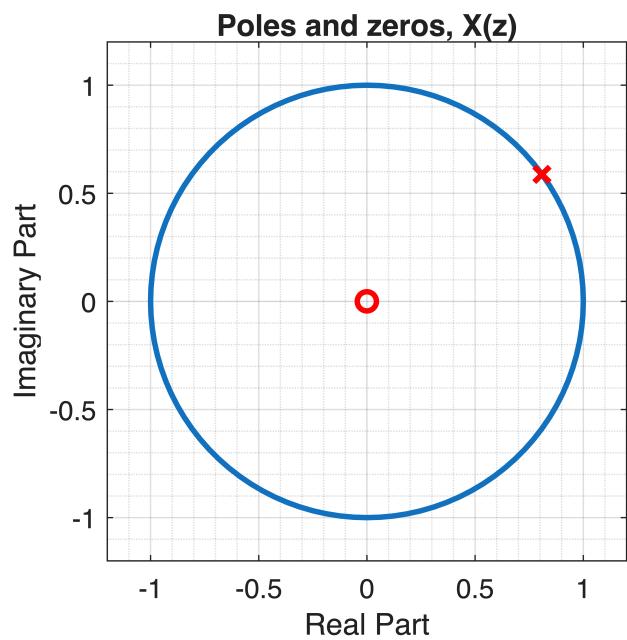
a = exp(1i * 2 * pi / 10);
x_i = (a.^n) .* (n >= 0);
% z-transform is (1 + z^-1/a + z^-2/a^2 + z^-3/a^3 + ...), or,
% 1/(1 - a*z^-1) for ROC |z| > |a|
plot_cmplx_sig_pz_and_fft(
    n, x_i, 1, [1 -a], ...
    "i)  $\delta[n] + e^{j2\pi/10}\delta[n-1] + e^{j2\pi/10}\delta[n-2] + \dots$ ", false);
```

**Real part,  $n = -2:30$**



**Imaginary part,  $n = -2:30$**





j)  $\delta[n] + 0.9e^{j2\pi/10}\delta[n-1] + (0.9e^{j2\pi/10})^2\delta[n-2] + \dots$  (non-terminating)

z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$X(z) = \sum_{n=0}^{\infty} (0.9e^{j\frac{2\pi}{10}})^n z^{-n}$$

The above series will converge for  $|z| > 0.9$

$$X(z) = \frac{1}{1 - (0.9e^{j\frac{2\pi}{10}})z^{-1}}, ROC : |z| > 0.9$$

Discrete Time Fourier Transform (DTFT):

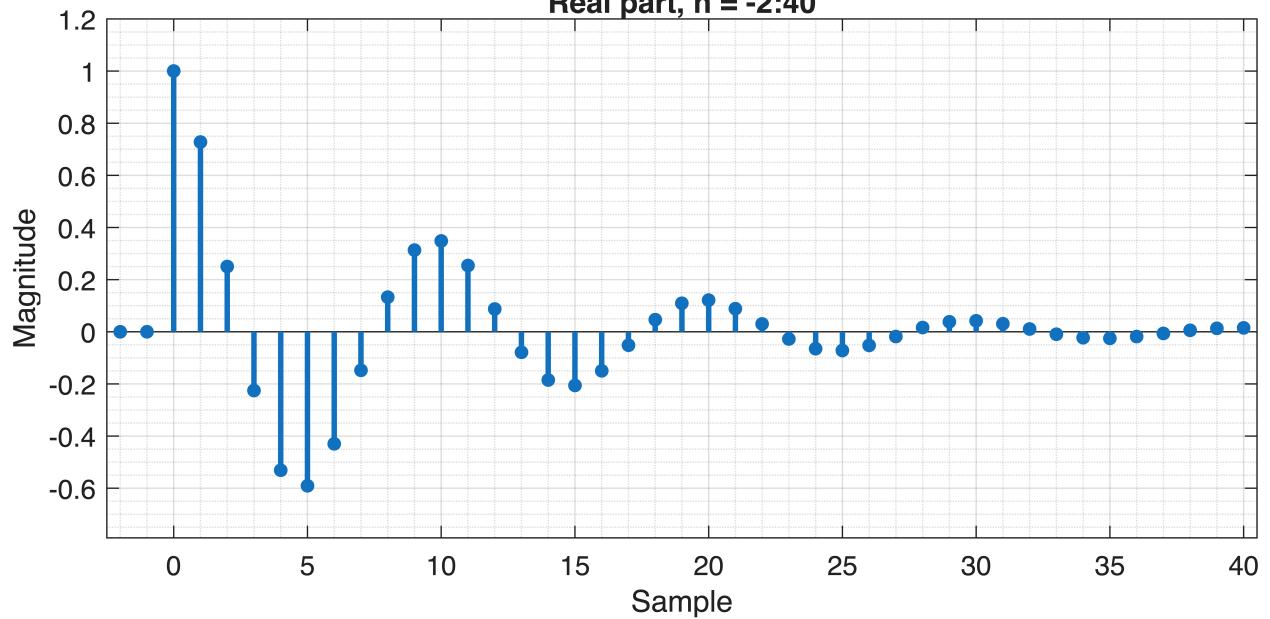
$$X(\theta) = X(z)|_{z=e^{j\theta}} = \frac{1}{1 - (0.9e^{j\frac{2\pi}{10}})z^{-1}} \Big|_{z=e^{j\theta}}$$

$$X(\theta) = \frac{1}{1 - (0.9e^{j\frac{2\pi}{10}})e^{-j\theta}} = \frac{1}{1 - 0.9e^{-j(\theta - \frac{2\pi}{10})}}$$

```
n = [-2 : 40]; % Increasing the sample size to show the amplitude
attenuation

a = 0.9 * exp(1i * 2 * pi / 10);
x_j = (a.^n) .* (n >= 0);
% z-transform is (1 + z^-1/a + z^-2/a^2 + z^-3/a^3 + ...), or,
% 1/(1 - a*z^-1) for ROC |z| > |a|
plot_cmplx_sig_pz_and_fft(
    n, x_j, 1, [1 -a], ...
    "j)  $\delta[n] + 0.9e^{j2\pi/10}\delta[n-1] + (0.9e^{j2\pi/10})^2\delta[n-2] + \dots$ ");
```

**Real part,  $n = -2:40$**



**Imaginary part,  $n = -2:40$**

