**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans = B = 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans = the Z-scores for both 44 years and 38 years using the formula:

Z = (X - μ) / σ

For X = 44: Z1 = (44 - 38) / 6 = 1

For X = 38: Z2 = (38 - 38) / 6 = 0

Now, we can use a standard normal distribution table or calculator to find the probabilities associated with these Z-scores.

P(X > 44) corresponds to the area to the right of Z1, and P(X > 38) corresponds to the area to the right of Z2.

Since Z1 = 1, we find P(X > 44) is the probability that an employee is older than 44.

Since Z2 = 0, we find P(X > 38) is the probability that an employee is older than 38.

The probability that an employee is older than 44 is greater than the probability that an employee is older than 38, which means there are more employees older than 44 than between 38 and 44. Therefore, statement A is True.

B. True.

To find the expected number of employees under the age of 30, we can use the properties of the normal distribution. We are given that the mean age (μ) is 38 years, and the standard deviation (σ) is 6 years.

Let X represent the age of an employee. We want to find the probability that an employee is under the age of 30, which is P(X < 30). To find this probability, we can calculate the Z-score for X = 30:

Z = (X - μ) / σ Z = (30 - 38) / 6 Z = -8 / 6 Z = -4/3

Now, we can use a standard normal distribution table or calculator to find P(X < 30), which is the probability that an employee is under the age of 30.

P(X < 30) is the probability that an employee is younger than 30, and to find the expected number of employees, we can multiply this probability by the total number of employees (400):

Expected number of employees under 30 = P(X < 30) \* 400

Using the standard normal distribution table or calculator, you can find the corresponding probability and calculate the expected number.

So, statement B is True, and you can calculate the expected number using the above method.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.
2. Ans = **2X1**:
   * Distribution: Since X1 is a normal random variable, 2X1 is also a normal random variable.
   * Parameters:
     + Mean (μ\_2X1) = 2μ (The mean of 2X1 is twice the mean of X1)
     + Variance (σ^2\_2X1) = 4σ^2 (The variance of 2X1 is four times the variance of X1)
3. **X1 + X2**:
   * Distribution: The sum of two independent normal random variables is also a normal random variable.
   * Parameters:
     + Mean (μ\_X1+X2) = μ + μ = 2μ (The mean of X1 + X2 is the sum of the means of X1 and X2)
     + Variance (σ^2\_X1+X2) = σ^2 + σ^2 = 2σ^2 (The variance of X1 + X2 is the sum of the variances of X1 and X2)
4. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
5. 90.5, 105.9
6. 80.2, 119.8
7. 22, 78
8. 48.5, 151.5
9. 90.1, 109.9

Ans = Since your random variable X follows N(100, 202), we need to find the values a and b in terms of standard deviations from the mean.

1. Find the standard deviation of X: σ = √202.
2. Determine the value of ±2.576 standard deviations from the mean: ±2.576 \* σ = ±2.576 \* √202 ≈ ±28.54 (rounded to two decimal places).
3. Now, find a and b symmetrically around the mean:
   * a = Mean - 28.54 = 100 - 28.54 ≈ 71.46
   * b = Mean + 28.54 = 100 + 28.54 ≈ 128.54

So, the values a and b, symmetric about the mean, such that the probability of X falling between them is approximately 0.99, are approximately a ≈ 71.46 and b ≈ 128.54.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans = Given: $1 = Rs. 45

A. To specify a Rupee range centered on the mean that contains 95% probability for the annual profit of the company, we'll calculate a confidence interval.

For Profit1 ~ N(5, 32):

* Mean (μ1) = $5 million
* Standard Deviation (σ1) = √32 million

For Profit2 ~ N(7, 42):

* Mean (μ2) = $7 million
* Standard Deviation (σ2) = √42 million

Now, let's calculate the combined profit distribution for the company:

Total Profit = Profit1 + Profit2

* Mean of Total Profit (μ\_total) = μ1 + μ2 = $5 million + $7 million = $12 million
* Variance of Total Profit (σ\_total^2) = σ1^2 + σ2^2 = 32 million + 42 million = 74 million

Standard Deviation of Total Profit (σ\_total) = √74 million ≈ $8.60 million

Now, we need to find the z-score for a 95% confidence interval (which corresponds to ±1.96 standard deviations for a normal distribution):

Z = 1.96

Now, we can calculate the Rupee range:

Lower Limit = μ\_total - Z \* σ\_total Lower Limit = $12 million - 1.96 \* $8.60 million ≈ $12 million - $16.856 million ≈ -$4.856 million

Upper Limit = μ\_total + Z \* σ\_total Upper Limit = $12 million + 1.96 \* $8.60 million ≈ $12 million + $16.856 million ≈ $28.856 million

So, the Rupee range (centered on the mean) that contains 95% probability for the annual profit of the company is approximately -Rs. 218.52 million to Rs. 656.52 million.

B. To specify the 5th percentile of profit for the company (in Rupees), we can find the z-score corresponding to the 5th percentile and then calculate the profit value:

Z\_5th percentile = -1.645 (you can find this value from a standard normal distribution table)

Profit\_5th percentile = μ\_total + Z\_5th percentile \* σ\_total Profit\_5th percentile = $12 million - 1.645 \* $8.60 million ≈ $12 million - $14.167 million ≈ -$2.167 million

So, the 5th percentile of profit for the company is approximately -Rs. 97.52 million.

C. To determine which of the two divisions has a larger probability of making a loss in a given year, we need to calculate the probability that each division has a negative profit.

For Profit1 ~ N(5, 32), we want to find P(Profit1 < 0).

First, calculate the z-score for this:

Z1 = (0 - 5) / √32 ≈ -1.118

Using a standard normal distribution table or calculator, find P(Z1 < -1.118). This is the probability of Profit1 being negative.

For Profit2 ~ N(7, 42), we want to find P(Profit2 < 0).

Calculate the z-score for this:

Z2 = (0 - 7) / √42 ≈ -1.451

Using a standard normal distribution table or calculator, find P(Z2 < -1.451). This is the probability of Profit2 being negative