Proof (by Strong Induction) Basis: same as before I.H.: given a fixed LEN, LZ12 4125rsk -> r=5a+46 for some a, b EN I.S.: Show that for K+1, it is true. we need to show: kt1 = 5 a' + 46' for some a', b' EN Strong Induction: use Several base cases to show that everything up to k works I.S .: Show that the property is true for k+1 4 Postage 212 with 4c &5c Idea: any postage is (property true) I can make ktl if I Subtract 3 from k ex) 4-3+4c=4+1 Base cases: Postage 12 | 13 | 14 | 15 IH: Assume that for a fixed 12 Sk, we can generate the # 12 STER using the stamps and 50 stamps. I.S.: We need to Show that we can generate postage of · Ketl using to and 5c.

Cases: 1) Lt1 515 -> k+1 is one of base cases 2) N+1 >15 > K+1 ≥16 Subtract 4 on both sides K+1-4=16-4 (K-3≥12) K-3≤K from I.H. 4 12 5 K-3 5 K -> k-3 satisfies I.H. -> k-3 can be made with 4c and 5c Stamps we can make k+1 just by adding a 4c stamp of ex) Polygons: (Simple and closed) Not simple we can always divide a Peryan Livides Polygon into triangles. the plane SHENI (1 Property: any polygon with n sides 2) Outside can be divided into n-2 triangles. Property: Any polygon with n sides can be divided into N-2 triangles. (n≥3) Proof (by Strong induction on 1) Bas;s: ~=3 & number of triangles = 1 I.H.: given a fixed 123, every polygon with 35r5k Sides can be divided into v-2 sides. I.S .: We need to show that a polygon with k+1 sides can be divided into k+1-2= k-1 triangles. K+1 ≥4 -> has at least one interior diagonal A we know lath= let1

5ides of P.: a+1 sides of Pai b+1 Since we found an interior diagonal > a 22 > a+123 b=2 -> b+1=3 clearly at 15k) 6+1 5h tormal. I Let P be a polygon with ket sides we know that k+124 1 sal -> 3 at least one interior diagonal d -> P is divided into two polygons by o P. has at 1 sides Pz has b+1 sides and at b= k+1 since dis an interior diagonal they a 2 2 and b 2 2 \* has at least 2 sides → (a+1≥3) and (b+1≥3) mot including diagonal we can also see that (atisk) and (btisk) Satisfy the conditions of the I.H. Because of J.H. SP, can be triangulated with at 1-2= a-1 triangles LP2 " " b+1-2=b-1 " -> P can be triangulated with (a-1) + (b-1) triangles. atb-2 triangles → k+1-2 triangles. Since we know atb=k+1 → K-1 triangles 1

Sets (naive set theory): What is a Set? · Collection of elements (no restriction of elements) Yincludes sets Define a set: · Provide a 1954 of all the elements · Provide a property that all elements wust satisfy {x IP(x)} ex) x={1,3,9} (1:5+) = {NEN | Nisodd \ Neg ~ N +7 ~ N +5} Y= { N E N | N ? S even} · Universal Set V all the elements in the domain. · Ø = {} empty set · a is an element of Set A ex) 3 EX++rue 3 EY + folse (3 does not sotisfy QEA · a is not an element of set A 7 (aEA) = a &A · A (E)B (A is a subset of B) ↔ YXEA → XEB Venn Diagrams 7 = {x en |xisodb} can we prove that x 52? Proof 1 Ex, 115 odd → 1 EZ 3EX, 3 " " -> 3 EZ gex quu -> gez ·. + a e X -> a e Z -> X E Z ISH true A SA? · ACB A is a proper subset of B Yes: YOEA + aEA. ↔ YXEA → XEB NA +B

\$ must prove that A \displays

ex) Is it true that 4 CM? True: Let XEV -> XEN A X is even -> XEN To show that A +B. We can either a) show a EA but a &B or b) Show a EB but a &A to show that y = N, we find an element aEN St. a & y Has E=DE a & y be cause 3 is not even. : 4 × M -> 4 CM ·A=B () A SB N B SA · \$ + ¢ ex) Show that  $\Phi \subseteq S$  (4 sets S)

(everyelement in  $\Phi$  is in S) [ we reed to show that & a ED - a ES. To show that the property is false, we need an element exists, then the property is vacuously true ex) Show that & & S is true \*7 (05S) = \$\$5 + we need on element in S S.t. it is not in \$ -> any element in S will do as long as 5 = 0 ex) { 2} E A 1) A= fx EIR | x 15 on integer > 1} {2,3,4,...} (2CA = {23CA) | .: False! 2) A = {2, [2]} True 3) A= ESS2373 False

PER EA

(ridden) $S$ & $S$		
Thus  5) $\phi \in \{i\}\}$ True  (i) $\phi \in \{o\}$ True  7) $\{o\} \in \{o\}$ Folse  8) $\phi \in \{x\}$ Folse  9) $\phi \in \{i\}, x\}$ True  Condinsity of a set: IAI  Number of elements  Theorem:  A $\subseteq B \rightarrow  A  \leq  B $ A $\subset B \rightarrow  A  \leq  B $ Thus sets have the form condity $ A  =  B $ if there is a one-to-one motoding of A to B.  ex) $A = \{a, b, c\}$ $\theta = \{2, 3, 4\}$ $A = \{a, b, c\}$ $A = \{a, b, c$	7	11) \$ E \$ B ? There
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1/303.		there is a one-to-one matching of A to B.  ex) $A = \{a, b, c\}$ , $B = \{2, 3, 4\}$ A  B $A = \{a, b, c\}$ , $A $

Classify Sets: · Normal Sets: A is a normal set if A & A (ex) 5 is not normal set) ex Set of all normal sets N= EALA &A3 we arready said S € NK {a,b,c} ∈ N REN unes & Question: NEN? 1) Assume true NEN -> NEN faise 2) Assume false NEN - NEN true => Paradox (Proposition true & false at same time) know this Recap: ASB acA ACB A= {x1P(x)} A=BlaEA, Pla) Normal Sets: N= {A \$ A | A } € 10 Myan Si U C= {C, 1, 2, 3, 21, 3}} Cis not normal NEN? ASSUME NEN -> N EN ASSUME NEW JUEN + NEN Russell's Paradox \* fears - formatize of Cratural numbers)