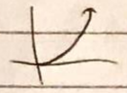
CS 205

Discrete Structures I

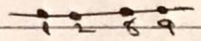
06/27 Discrete = Discontinuous (This is what comp. deal with)

$f(x) = x^2$



→ Binary

→ numbers (integers)



→ Logic

→ Sets — infinite

→ functions, relations

→ models of computation ← basic mathematical description of what computer does

↳ DFA ↳ "machines"

↳ Compilers

Propositional Logic:

· Propositions - declarative statement (sentence)

↳ Declare a fact

↳ two values (possible) < True / False — cannot be both at the same time → paradox

ex) "All cows are brown" → False

ex) " x+2=x " → Not a proposition (b/c there is a variable)

↳ needs to be a number

ex) " x+2 = 2x when x=2 " → True

↳ variable's value is defined = proposition

ex) "Is paradox a word?" → Not a proposition (Not a statement)

ex) " The universe is infinite " → is a proposition

→ BUT! We don't know!

ex) "There are several universes" → Proposition

→ We don't know

→ Not scientific ← Computer answers scientific propositions

ex) "Pluto is a planet" → False

↳ Definition

ex) "This sentence is false" → Not a proposition (Paradox)

→ can be True & False

Operations:

p, q, r, s ... are propositions

1) **not** $\neg p$ (negation)

← not sign

p	$\neg p$
T	F
F	T

2) **and** $p \wedge q$ (conjunction) + BUT

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

3) **or** $p \vee q$ (Disjunction)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

4) **Xor** $p \oplus q$ (exclusive or)

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

True only when one of them is true

Necessary 5) **Conditional** $p \rightarrow q$ → If it rains, → I bring umbrella

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Since this wasn't mentioned, it's not lying.

6) Converse $q \to p$ (not necessarily true)

Contrapositive $\neg q \to \neg p$ (true)

Ex) p = rains

q = is cloudy

$p \to q$ (true)

Converse, $q \to p$ (if it's cloudy, then it rains)

Contrapositive, $\neg q \to \neg p$ (if it's not cloudy, then it doesn't rain)

7) Biconditional $p \leftrightarrow q$ (if and only if)

↳ $(p \to q) \wedge (q \to p)$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

*Compound Propositions precedence:

1. ( )
2. $\neg$
3. $\wedge$
4. $\vee$, $\oplus$
5. $\to$
6. $\leftrightarrow$

Ex) Find truth table of $(p \vee \neg q) \to (p \wedge q)$

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \to (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

☆ Natural language → Propositions

Ex) "You can access the internet on campus only if you are a CS major or you are not a freshman."

p = you can access the internet on campus
q = you are a CS major
r = " " a freshman

$\boxed{\therefore p \to q \vee \neg r}$

$[p \to q]$

- if p then q
- p is sufficient for q
- q is a necessary condition for p, q whenever p
- p implies q
- p only if q
- q follows from p, q if p, q unless $\neg p$

☆ know words that mean '→'

☆ Study this!

ex) "You cannot ride the roller coaster if you are less than 4ft tall unless you are older than 16."

p = You can ride rollercoaster
q = less than 4ft tall
r = you are older than 16.

? $\boxed{q \vee \neg r \to \neg p}$ ×

Contrapositive → $\boxed{p \to \neg(q \wedge \neg r)}$ ✓

✱ know how to check if the proposition is true or false!

Applications:

- System Specifications
  - we need all specs to be consistent
  - There must be at least a case where all are true

Ex) 1. Diagnostic message is stored or transmitted

2. Diagnostic message is not stored.
3. If diagnostic " is stored, then it is transmitted.

Are the specs consistent?

p= diagnostic message is stored

q= " " is transmitted

1. $p \vee q$
2. $\neg p$
3. $p \rightarrow q$

p	q	① $p \vee q$	② $\neg p$	③ $p \rightarrow q$
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	T

☆ A row is all TRUE!

System is consistent!

Ex) 1. If the user answers "yes" then do not execute process A

2. Process A is not executed and the user answers "yes"
3. User answers "no"

assume "yes" or "no"

a = User answers "yes"

b = Process A is executed

1. $a \to \neg b$
2. $\neg b \wedge a$
3. $\neg a$

a	b	$\neg b$	$a \to \neg b$	$\neg b \wedge a$	$\neg a$
T	T	F	F	F	F
T	F	T	T	T	F
F	T	F	T	F	T
F	F	T	T	F	T

No row is all True.
↳ System is NOT Consistent!

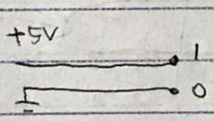
SAT (Satisfiability Problem)

$\emptyset(a,b) = (a \to \neg b) \wedge (\neg b \to a) \wedge (\neg a)$

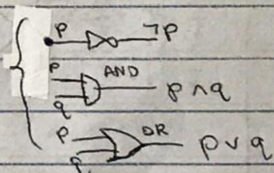
Is T only if each one is T.

Given $\emptyset(a_1, a_2, a_3, \dots, a_n)$
Is there a truth assignment to the variables that would make <u>all</u> of the propositions true?
↳ $O(2^n)$

Applications:
- Boolean Circuits
- Logic circuits



Gates



↳ Add in binary
- operations in binary

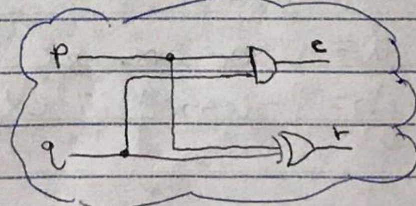
↓

BCD (Binary Coded Decimal) ↗ 7 Segments (decimal)

$p + q =$

p	q	c ($p \wedge q$)	r ($p \oplus q$)
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$* 1+1 = 2 = 10$ Binary

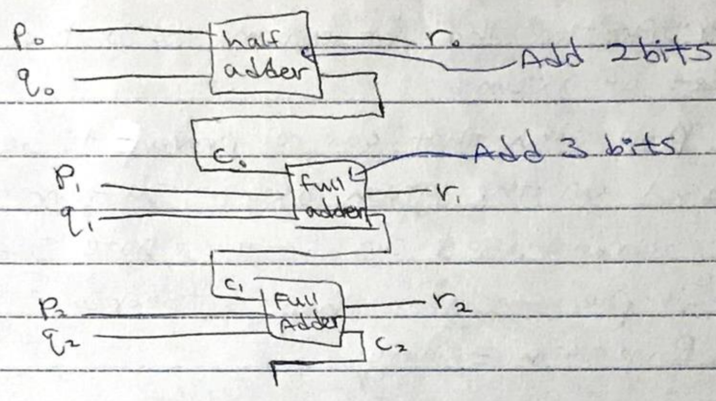


↙ Circuit for adding one bit

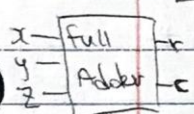
✡ know how to draw circuits

ex)

$$
\begin{array}{rrrr}
 & & & c \\
p & 1 & 0 & 1 \\
q \;+ & 1 & 1 & 0 \\
\hline
 & & & r
\end{array}
$$



Truth table for full adder:



x	y	z	c	r
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1
			↑ $c_0 \vee c_1$	↑ r_0

⇒ Circuit?

$c = 2^1$
$r = 2^0$

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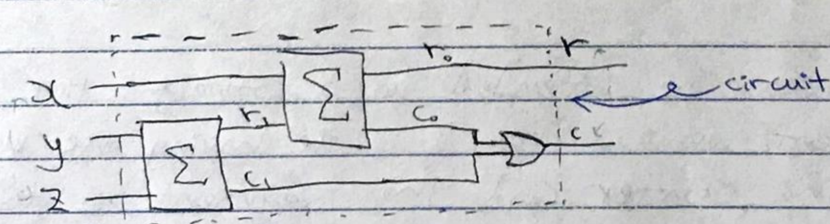

* If 'x' & 'z' flipped, then 'c' and 'r' flips.

*checking

② $x + r_1$		① $y + z$		③
C_0	r_0	C_1	r_1	$C_0 \vee C_1$
0	0	0	0	0
0	1	0	1	0
0	1	0	1	0
0	0	1	0	1
0	1	0	0	0
1	0	0	1	1
1	0	0	1	1
0	1	1	0	1

→ exactly same as r above

→ exactly same as c above