# 3F1 Lab Worksheet

## 2.1 Simplified Aircraft Model

Transfer function = $\frac{N}{s^2+Ms}$

`num` = [10]  `den` = [1, 10, 0]

## 2.2 Modelling Manual Control

Controller transfer function = $ke^{-sD}$

$k =$ 0.469  $D =$ 0.88

Phase margin = 35° (= 0.612 rad)

Amount of extra time delay which can be tolerated = 1.30 (s)

## 2.3 Pilot Induced Oscillation

Period of oscillation (observed) = 8.30 (s)

Period of oscillation (theoretical) = 6.87 (s)

## 2.4 Sinusoidal disturbances

Maximum stabilising gain = 5.4 dB

Gain at 0.66 Hz = 9.04 dB  Phase at 0.66 Hz = −315°

## 2.5 Unstable Aircraft

Fastest stabilised pole at $T =$ 0.5

## 3.2 Autopilot with Proportional Control

Proportional gain $K_c =$ 16  Period of oscillation $T_c =$ 2.0 (s)

## 3.3 Autopilot with PID Control

Transfer function of PID controller = $K_p + \frac{K_p}{T_i s} + K_p T_d s$

PID constants: $K_p =$ 9.6  $T_i =$ 1.0  $T_d =$ 0.25

Adjusted value of $T_d =$ 0.35

## 3.4 Integrator Wind-up

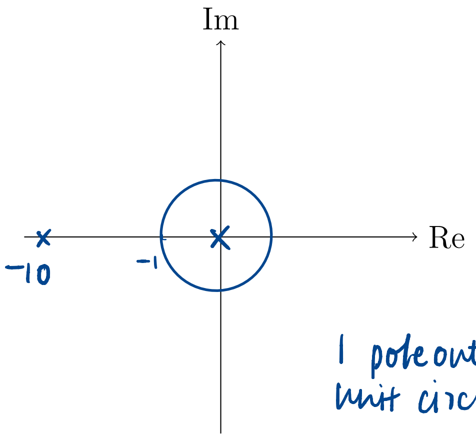
Integrator bound $Q =$ 0.208

# Report Template (3F1 Flight Control Lab)

This report template contains **9 questions** spread over **4 pages**.

**1.** (§2.2 Modelling Manual Control) Nyquist diagram for plant and controller from Bode diagram.

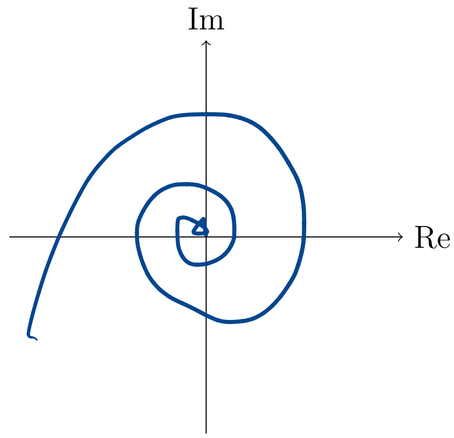
Transfer function: $\dfrac{10ke^{-sD}}{s^2+10s} = \dfrac{4.69e^{-0.88s}}{s^2+10s}$




z-plane or s-plane contour for Nyquist diagram (state which)

S-plane contour

1 pole outside unit circle, C=1




Nyquist diagram

**2.** (§2.2 Modelling Manual Control) Are you using any integral action? Give a brief explanation. What does this imply about the accuracy of the model of the human controller?

No, integral control is not used in this part of the experiment, because the control of input signal u(t) is completed only manually without any autopilot functions. When compared to the control results of an autopilot in later parts of the lab exercise, it is clear that manual piloting is very unstable and hence inaccurate.

**3.** (§2.3 Pilot Induced Oscillation) Explain the oscillation of the feedback loop. How does your observed period of oscillation compare to the theoretical prediction?

The pilot-induced oscillation of the feedback loop is a result of the systematic time delay which results both from the control transfer function and sensitivity of control input device (i.e. mouse in this experiment), and the out-of-phase situation of the pilot and feedback, likely a consequence of unstable manual control.

As observed during the lab exercise, the experimental period of oscillation is slightly higher than the theoretically calculated value, which indicates the part of effect on oscillation that is present due to the manual control.

**4.** (§2.3 Pilot Induced Oscillation) Can you give a rough guideline to the control designer to make PIO less likely?

In order to reduce PIO, which would cause unstable control response, the control designer could tackle the problem both from the control system and the control input signals.

Using systems which are less sensitive and responsive to minor fluctuations in control signals would prevent the system from generating PIO by a small unstableness of input. Meanwhile, it is also more beneficial to have more continuous and smooth input signals so that PIO would be reduced, as observed in later sections of the experiment.

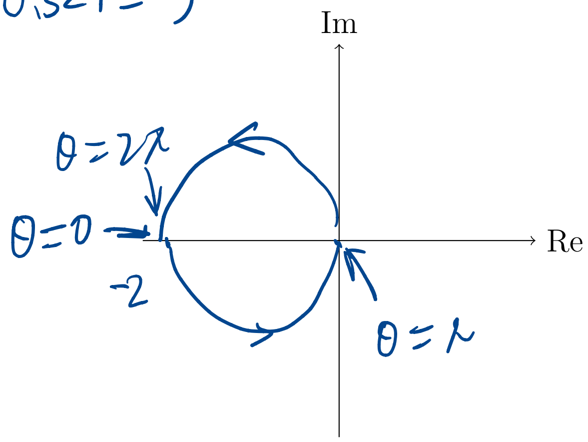
It would also be helpful to apply smoothing functions to input control signals if manual control is inevitable or required.

**5.** (§2.4 Sinusoidal Disturbances) Was your manual input able to reduce the error (as compared to providing no input)?

As observed during the experiment, the error is hardly damped when manual input is present, regardless of the amplitude or stability of manual input. There is a reduction in error oscillation amplitude only when the manual input and response are completely out-of-phase.

**6.** (§2.5 An Unstable Aircraft) Nyquist diagram for $G_2(s)$.

$$G_2(s) = \frac{2}{-1+sT} \quad (0.5 < T \leq 1)$$



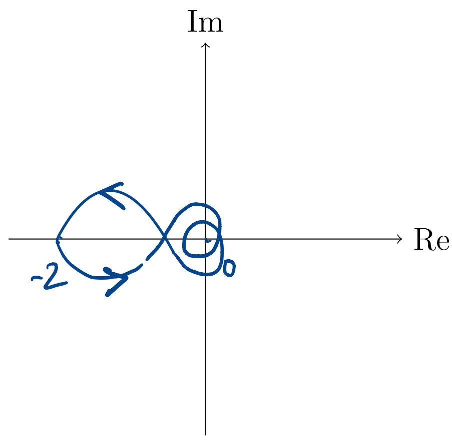

Nyquist diagram of $G_2(s)$

**7.** (§2.5 An Unstable Aircraft) Explain, using the Nyquist criterion, why the feedback system is stable with a proportional gain greater than 0.5.

According to the Nyquist criterion, #unstable poles of L(s) = # unstable poles of 1/(1+L(s)) + # clockwise encirclements of -1. As shown in the Nyquist diagram above, the plant transfer function has 1 anticlockwise encirclement of -1(-1 clockwise encirclement) and 1 unstable pole at 1/T. In order to stabilise the feedback system, the encirclement needs to be remained, so the proportional gain must be greater than 0.5.

**8.** (§2.5 An Unstable Aircraft) Sketch of a Nyquist diagram for $G_2(s)$. with a small time delay $D$.

$$G_2(e^{j\omega}) = \frac{2e^{-j\omega D}}{-1+j\omega T}$$




Nyquist diagram with small time delay

**9.** (§3.4 Integrator Wind-up) Explain how you calculated the bound on $Q$.

The aim of using the integrator limit $Q$ is to limit the value of steady-state response.

Using final-value theorem (FVT),

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

So PID controller in steady-state $= \lim_{s \to 0} \cancel{sK_p}^{0} + \frac{K_p}{K_i} + \cancel{K_p T_d s^2}^{0}$

$$= \frac{K_p}{K_i}$$

With $Q$: $\frac{K_p}{K_i} Q$

Since the step disturbance has magnitude 2, $\frac{K_p}{K_i} Q$ needs to match this magnitude to remove overshoot or undershoot.

So $\frac{K_p}{K_i} Q = 2$

Given $K_p = 9.6$, $K_i = 1.0$

$$\Rightarrow Q = \frac{2K_i}{K_p} = 0.208$$