COMP 2211 Exploring Artificial Intelligence

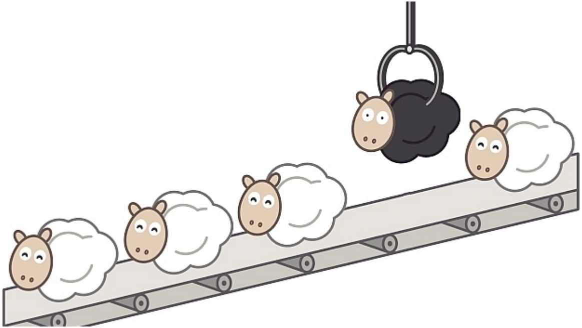
Artificial Neural Network - Perceptron

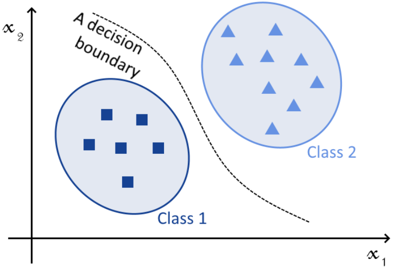
Dr. Desmond Tsoi

Department of Computer Science & Engineering
The Hong Kong University of Science and Technology, Hong Kong SAR, China

Recap: What is Classification?

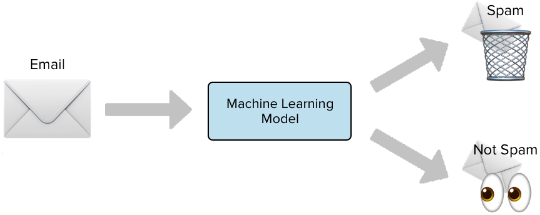
- Classification is the process of predicting the class of given data.
- Classes are sometimes called as labels/categories.



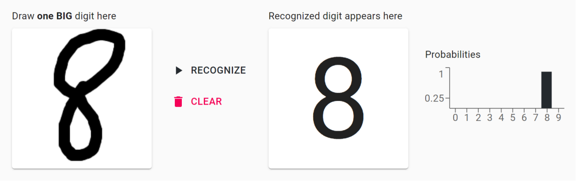


Classification Examples

- Spam detection in email service: 2 classes - spam and not spam

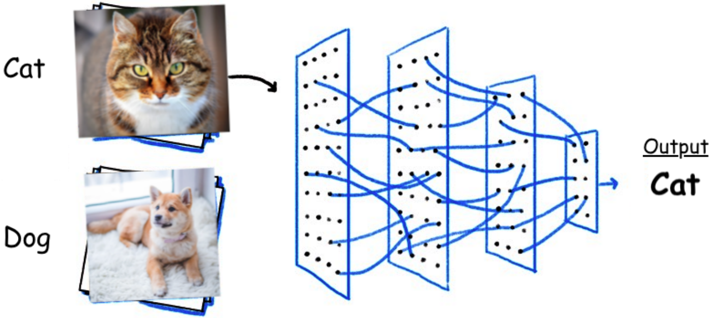


- Handwritten digit recognition: 10 classes - 0, 1, 2, . . ., 9



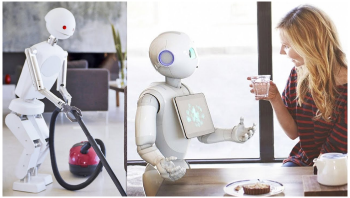
Classification Examples

- Images classification: 3 classes - cat, dog, none of them



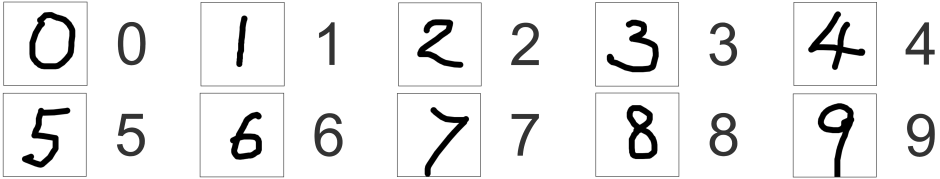
Why Image Classification?

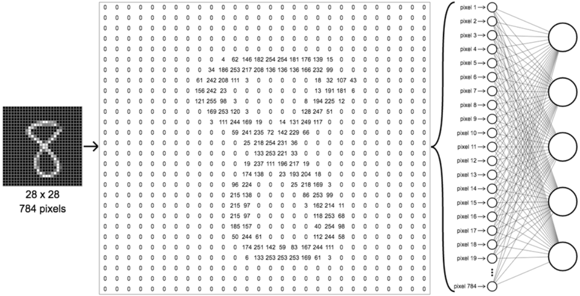
- Image recognition and classification is a subdomain of computer vision. It is an algorithm that looks at an image and assigns it a label from a collection of predefined labels or categories (e.g., a dog, a cat).
- Image classification and recognition are vital components in robotics, such as autonomous vehicles or domestic robots.
- It is also important in security systems such as face recognition, image search engines such as Google or Bing image search, and medical imaging such as cancer detection.



Handwritten Digits Recognition using MLP

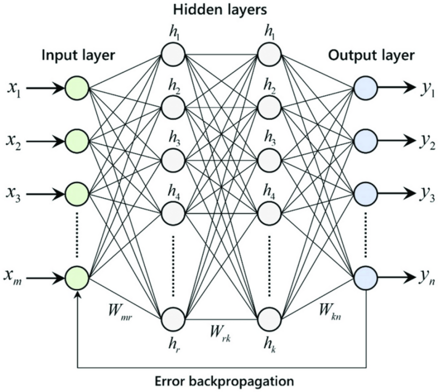
We will build an Artificial Neural Network to recognize/classify handwritten digits.



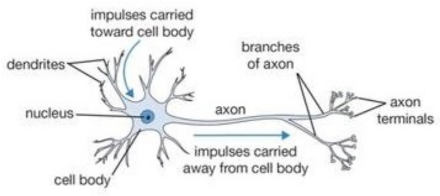


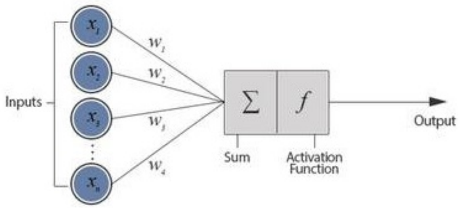
Artificial Neural Network

- Artificial neural networks (ANN) are one of the most powerful artificial intelligence and machine learning algorithms.
- An ANN is considered a universal function approximator that transforms inputs into outputs.
- As the name suggests, it draws inspiration from neurons in our brain and the way they are connected.



Biological Neuron vs Artifical Neuron





Biological Neuron	Artificial Neuron
Dendrites	Inputs
Cell Nucleus (Computation unit)	Node
Axon	Output
Synapse	Weight

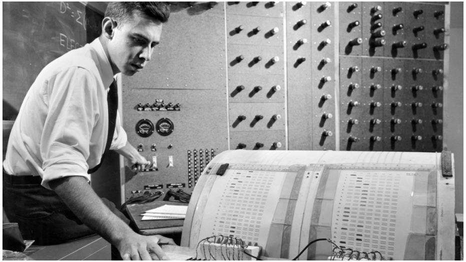
- Node = a linear function & an activation function

Perceptron

What is a Perceptron?

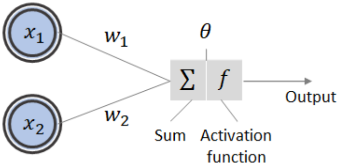
- A perceptron is a simple biological neuron model in an artificial neural network.
- It performs certain calculations to detect input data capabilities.
- It is also the name of an early algorithm for supervised learning of binary classifiers (i.e., only two classes).

- Frank Rosenblatt invented perceptron in 1957.



Frank Rosenblatt works on the "Perceptron" - what he described as the first machine "capable of having an original idea".

An Example of Perceptron

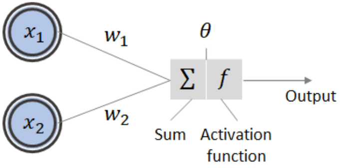


$$output = f(w_1 \times x_1 + w_2 \times x_2 + \theta)$$

Suppose $x_1 = 3,\ x_2 = 5,\ w_1 = 0.2,\ w_2 = 0.3,\ \theta = 0.3,\ f(x) = \frac{1}{1+e^{-x}}$

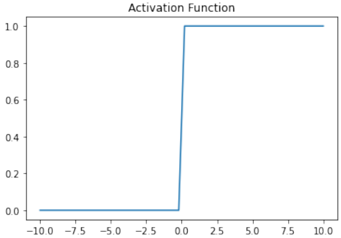
$$\begin{aligned} output &= f(0.2 \times 3 + 0.3 \times 5 + 0.3) \\ &= f(2.4) \\ &= \frac{1}{1 + e^{-2.4}} \\ &= 0.91683 \end{aligned}$$

Perceptron - Logical AND Example



- Suppose we will work on a problem of AND logical operation. The truth table of logical AND is as follows.

x_1	x_2	Output
0	0	0
0	1	0
1	0	0
1	1	1



- Assume the weights and bias are randomly generated, say $w_1 = 0.1$, $w_2 = 0.5$, $\theta = -0.8$. Also, we set learning rate $\eta = 0.2$.
- Activation function is

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 & \text{otherwise} \end{cases}$$

Perceptron Learning Rules

- x_1 and x_2 are inputs
- θ is the bias
- w_1 and w_2 are weights
- O is the computed output
- T is the target

- Δw_1 is the change of w_1
- Δw_2 is the change of w_2
- $\Delta\theta$ is the change of θ
- η is the learning rate

1. If the output is correct (i.e., T is the same as O), the weights w_1 and w_2 are not updated.
2. If the output is incorrect (i.e., T is different to O), the weights w_1 and w_2 are updated according to the following rules such that the output of the perceptron for the new weights is closer to T.

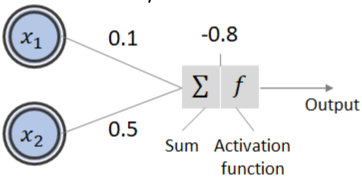
$$\Delta w_i = \eta(T - O)x_i$$
$$\Delta\theta = \eta(T - O)$$
$$w_i = w_i + \Delta w_i$$
$$\theta = \theta + \Delta\theta$$

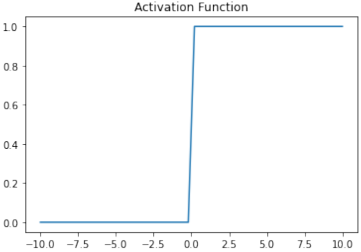
where $i \in \{1, 2\}$.

Initial

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8

$$\eta = 0.2$$

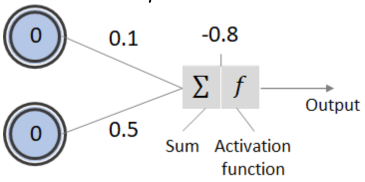


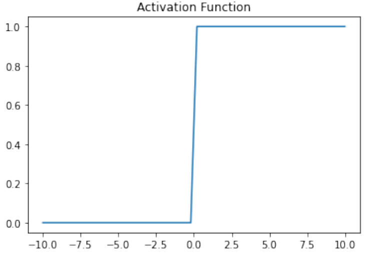


Round 1 - Step 1

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8
0	0	0	0	0	0.1	0	0.5	0	-0.8

$\eta = 0.2$

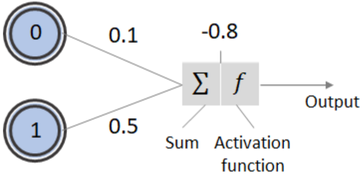


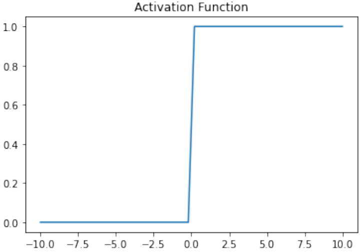


Round 1 - Step 2

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8
0	0	0	0	0	0.1	0	0.5	0	-0.8
0	1	0	0	0	0.1	0	0.5	0	-0.8

$\eta = 0.2$

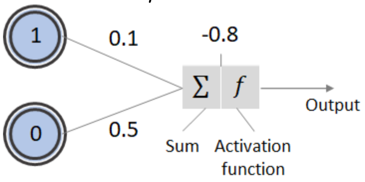


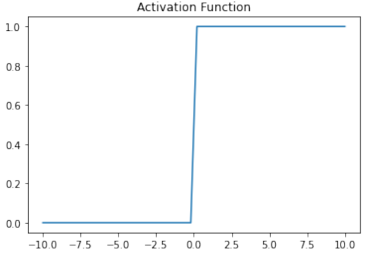


Round 1 - Step 3

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8
0	0	0	0	0	0.1	0	0.5	0	-0.8
0	1	0	0	0	0.1	0	0.5	0	-0.8
1	0	0	0	0	0.1	0	0.5	0	-0.8

$\eta = 0.2$

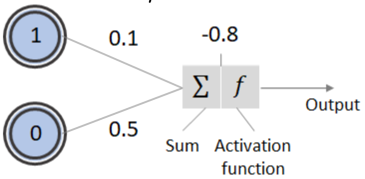


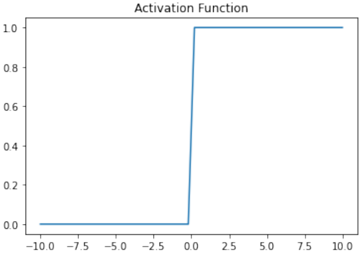


Round 1 - Step 4

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8
0	0	0	0	0	0.1	0	0.5	0	-0.8
0	1	0	0	0	0.1	0	0.5	0	-0.8
1	0	0	0	0	0.1	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.3**	**0.2**	**0.7**	**0.2**	**-0.6**

$$\eta = 0.2$$

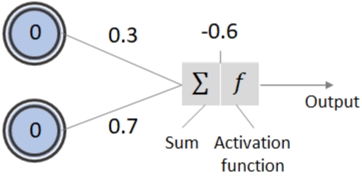


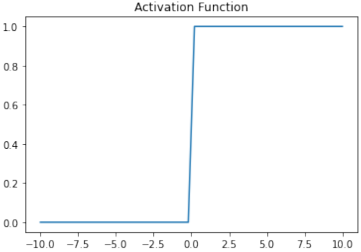


Round 2 - Step 1

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8
0	0	0	0	0	0.1	0	0.5	0	-0.8
0	1	0	0	0	0.1	0	0.5	0	-0.8
1	0	0	0	0	0.1	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.3**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.3	0	0.7	0	-0.6

$$\eta = 0.2$$

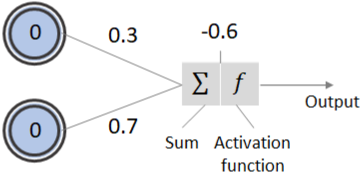


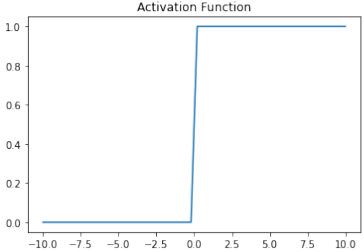


Round 2 - Step 2

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8
0	0	0	0	0	0.1	0	0.5	0	-0.8
0	1	0	0	0	0.1	0	0.5	0	-0.8
1	0	0	0	0	0.1	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.3**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.3	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.3**	**-0.2**	**0.5**	**-0.2**	**-0.8**

$\eta = 0.2$

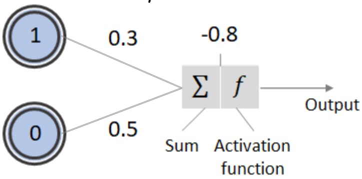


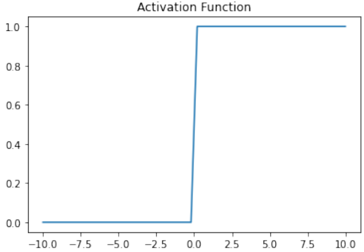


Round 2 - Step 3

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8
0	0	0	0	0	0.1	0	0.5	0	-0.8
0	1	0	0	0	0.1	0	0.5	0	-0.8
1	0	0	0	0	0.1	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.3**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.3	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.3**	**-0.2**	**0.5**	**-0.2**	**-0.8**
1	0	0	0	0	0.3	0	0.5	0	-0.8

$\eta = 0.2$

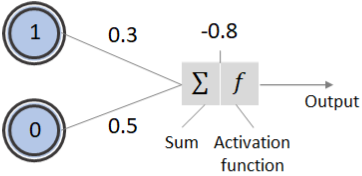


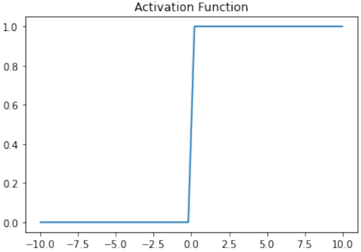


Round 2 - Step 4

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8
0	0	0	0	0	0.1	0	0.5	0	-0.8
0	1	0	0	0	0.1	0	0.5	0	-0.8
1	0	0	0	0	0.1	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.3**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.3	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.3**	**-0.2**	**0.5**	**-0.2**	**-0.8**
1	0	0	0	0	0.3	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.5**	**0.2**	**0.7**	**0.2**	**-0.6**

$\eta = 0.2$

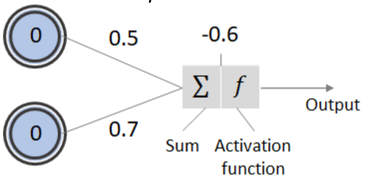


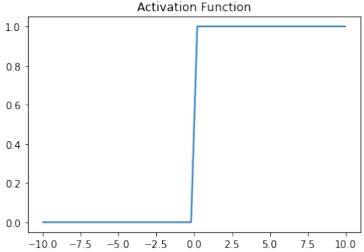


Round 3 - Step 1

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8
0	0	0	0	0	0.1	0	0.5	0	-0.8
0	1	0	0	0	0.1	0	0.5	0	-0.8
1	0	0	0	0	0.1	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.3**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.3	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.3**	**-0.2**	**0.5**	**-0.2**	**-0.8**
1	0	0	0	0	0.3	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.5**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.5	0	0.7	0	-0.6

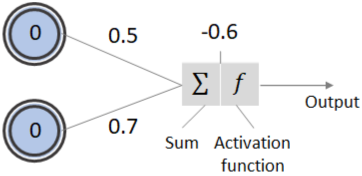
$\eta = 0.2$

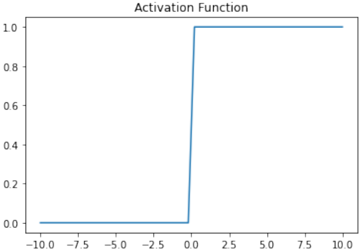




Round 3 - Step 2

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8
0	0	0	0	0	0.1	0	0.5	0	-0.8
0	1	0	0	0	0.1	0	0.5	0	-0.8
1	0	0	0	0	0.1	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.3**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.3	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.3**	**-0.2**	**0.5**	**-0.2**	**-0.8**
1	0	0	0	0	0.3	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.5**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.5	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.5**	**-0.2**	**0.5**	**-0.2**	**-0.8**

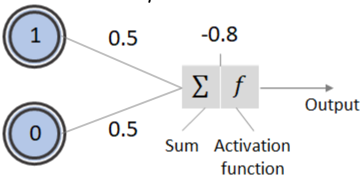


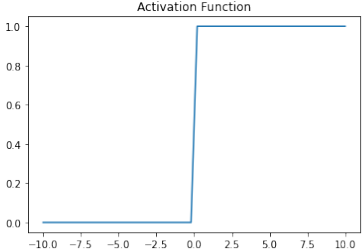


Round 3 - Step 3

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8
0	0	0	0	0	0.1	0	0.5	0	-0.8
0	1	0	0	0	0.1	0	0.5	0	-0.8
1	0	0	0	0	0.1	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.3**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.3	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.3**	**-0.2**	**0.5**	**-0.2**	**-0.8**
1	0	0	0	0	0.3	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.5**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.5	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.5**	**-0.2**	**0.5**	**-0.2**	**-0.8**
1	0	0	0	0	0.5	0	0.5	0	-0.8

$\eta = 0.2$

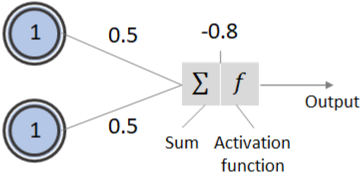


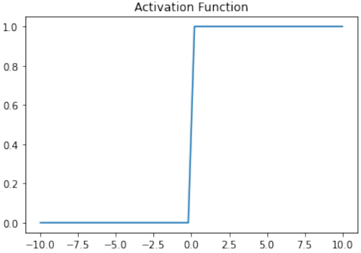


Round 3 - Step 4

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8
0	0	0	0	0	0.1	0	0.5	0	-0.8
0	1	0	0	0	0.1	0	0.5	0	-0.8
1	0	0	0	0	0.1	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.3**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.3	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.3**	**-0.2**	**0.5**	**-0.2**	**-0.8**
1	0	0	0	0	0.3	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.5**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.5	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.5**	**-0.2**	**0.5**	**-0.2**	**-0.8**
1	0	0	0	0	0.5	0	0.5	0	-0.8
1	1	1	1	0	0.5	0	0.5	0	-0.8

$\eta = 0.2$

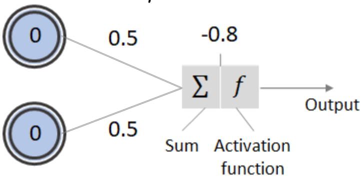


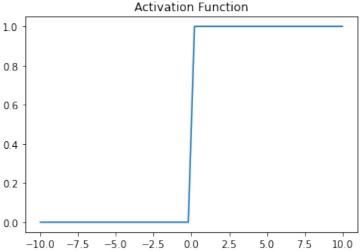


Round 4 - Step 1

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8
0	0	0	0	0	0.1	0	0.5	0	-0.8
0	1	0	0	0	0.1	0	0.5	0	-0.8
1	0	0	0	0	0.1	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.3**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.3	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.3**	**-0.2**	**0.5**	**-0.2**	**-0.8**
1	0	0	0	0	0.3	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.5**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.5	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.5**	**-0.2**	**0.5**	**-0.2**	**-0.8**
1	0	0	0	0	0.5	0	0.5	0	-0.8
1	1	1	1	0	0.5	0	0.5	0	-0.8
0	0	0	0	0	0.5	0	0.5	0	-0.8

$\eta = 0.2$

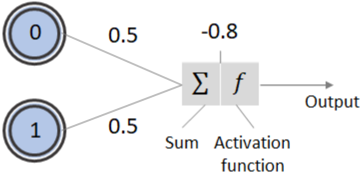


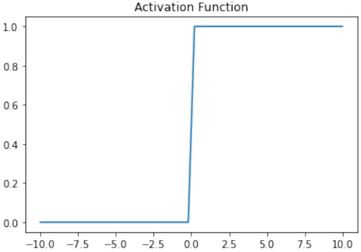


Round 4 - Step 2

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8
0	0	0	0	0	0.1	0	0.5	0	-0.8
0	1	0	0	0	0.1	0	0.5	0	-0.8
1	0	0	0	0	0.1	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.3**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.3	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.3**	**-0.2**	**0.5**	**-0.2**	**-0.8**
1	0	0	0	0	0.3	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.5**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.5	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.5**	**-0.2**	**0.5**	**-0.2**	**-0.8**
1	0	0	0	0	0.5	0	0.5	0	-0.8
1	1	1	1	0	0.5	0	0.5	0	-0.8
0	0	0	0	0	0.5	0	0.5	0	-0.8
0	1	0	0	0	0.5	0	0.5	0	-0.8

$\eta = 0.2$

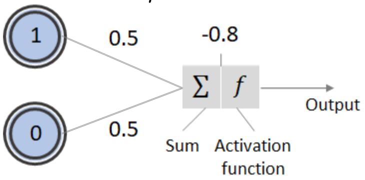


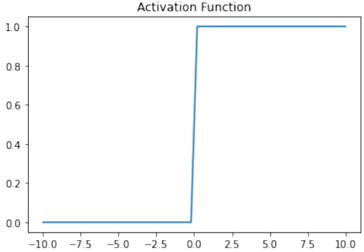


Round 4 - Step 3

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8
0	0	0	0	0	0.1	0	0.5	0	-0.8
0	1	0	0	0	0.1	0	0.5	0	-0.8
1	0	0	0	0	0.1	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.3**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.3	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.3**	**-0.2**	**0.5**	**-0.2**	**-0.8**
1	0	0	0	0	0.3	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.5**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.5	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.5**	**-0.2**	**0.5**	**-0.2**	**-0.8**
1	0	0	0	0	0.5	0	0.5	0	-0.8
1	1	1	1	0	0.5	0	0.5	0	-0.8
0	0	0	0	0	0.5	0	0.5	0	-0.8
0	1	0	0	0	0.5	0	0.5	0	-0.8
1	0	0	0	0	0.5	0	0.5	0	-0.8

$\eta = 0.2$

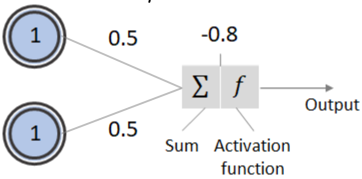


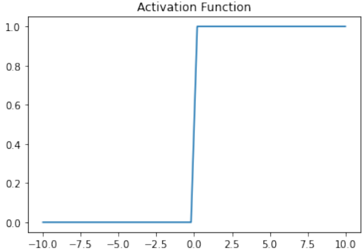


Round 4 - Step 4

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8
0	0	0	0	0	0.1	0	0.5	0	-0.8
0	1	0	0	0	0.1	0	0.5	0	-0.8
1	0	0	0	0	0.1	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.3**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.3	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.3**	**-0.2**	**0.5**	**-0.2**	**-0.8**
1	0	0	0	0	0.3	0	0.5	0	-0.8
1	**1**	**1**	**0**	**0.2**	**0.5**	**0.2**	**0.7**	**0.2**	**-0.6**
0	0	0	0	0	0.5	0	0.7	0	-0.6
0	**1**	**0**	**1**	**0**	**0.5**	**-0.2**	**0.5**	**-0.2**	**-0.8**
1	0	0	0	0	0.5	0	0.5	0	-0.8
1	1	1	1	0	0.5	0	0.5	0	-0.8
0	0	0	0	0	0.5	0	0.5	0	-0.8
0	1	0	0	0	0.5	0	0.5	0	-0.8
1	0	0	0	0	0.5	0	0.5	0	-0.8
1	1	1	1	0	0.5	0	0.5	0	-0.8

$\eta = 0.2$





Perceptron Implementation from Scratch I

```python
import math # Import math module

class Perceptron:
   def __init__(self):
       """ Perceptron initialization """
       self.w = [0.1,0.5]        # Weights
       self.theta = -0.8         # Bias
       self.learningRate = 0.2   # Eta

   def response(self,x):
       """ Perceptron output """
       # Calculate weighted sum
       y = x[0] * self.w[0] + x[1] * self.w[1] + self.theta
       # If weighted sum > 0, return 1. Otherwise return 0
       if y > 0:
          return 1
       else:
          return 0
```

Perceptron Implementation from Scratch II

```
def updateWeights(self,x,iterError):
    """ Weights update """
    # wi = wi + eta * (T-O) * xi
    self.w[0] += self.learningRate * iterError * x[0]
    self.w[1] += self.learningRate * iterError * x[1]

def updateBias(self,iterError):
    """ Bias update """
    # theta = theta + eta * (T-O)
    self.theta += self.learningRate * iterError

def train(self,data):
    """ Training """
    learned = True  # Should perform training
    round = 0       # Initialize round to 0
```

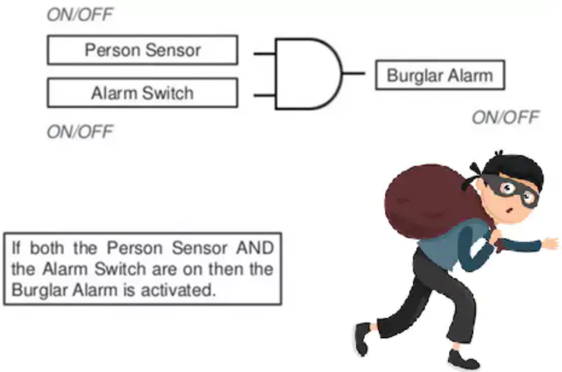
Perceptron Implementation from Scratch III

```python
while learned:                                # While learned is true
    totalError = 0.0                          # Initialize totalError to 0
    for x in data:                            # For each data sample
        r = self.response(x)                  # Calculate perceptron output of x
        if x[2] != r:                         # If the output is different to target
           roundError = x[2] - r              # Error = target - perceptron output
           self.updateWeights(x,roundError)   # Update weights
           self.updateBias(roundError)        # Update bias
           totalError += abs(roundError)      # Update total error
    round += 1

    if math.isclose(totalError, 0) or round >= 100:           # Stopping condition
       print("Total number of rounds (epochs): ", round)      # Print total num of rounds
       print("Final weights: ", self.w)                       # Print final weights
       print("Final bias: ", self.theta)                      # Print final bias
       learned = False                                        # Stop learning
```

Perceptron Implementation from Scratch IV

```
""" Main function """
perceptron = Perceptron()                            # Create Perceptron object
trainset = [[0,0,0], [0,1,0], [1,0,0], [1,1,1]] # Define training set
perceptron.train(trainset)                           # Perform training
```



Perceptron Implementation using Scikit-Learn

```
import numpy as np  # Import NumPy
from sklearn.linear_model import Perceptron # Import Perceptron class from Scikit-Learn

inputs = np.array([[0,0], [0,1], [1,0], [1,1]])  # Inputs
outputs = np.array([0, 0, 0, 1])                 # Expected outputs

# Create and fit a perceptron model
# Set learning rate (eta0)
model = Perceptron(eta0=0.2)
model.fit(inputs, outputs)

# Use the trained model to predict the outputs
predicted_outputs = model.predict([[0,0], [1,0], [1,1], [0,1]])
print(predicted_outputs) # Print the predicated outputs

print(model.coef_)       # Print the final weights
print(model.intercept_)  # Print the bias
```

Stopping Rules

- Use maximum training time
  The training may go a bit beyond the specified time limit in order to complete the current cycle.
- The maximum number of training cycles allowed
  If the maximum number of cycles is exceeded, then training stops.
- Use minimum accuracy
  Training will continue until the specified accuracy is attained.

Terminologies - Learning and Epoch

- Learning is the process of updating weights in the perceptron.
  - We set weights w_1 to 0.1, w_2 to 0.5 initially, but it causes some errors. Then, we update the weight values to 0.5 and predict all instances correctly.
  - The whole process takes 4 rounds/epoches.
- Epoch refers to one cycle through the full training dataset.

Observation

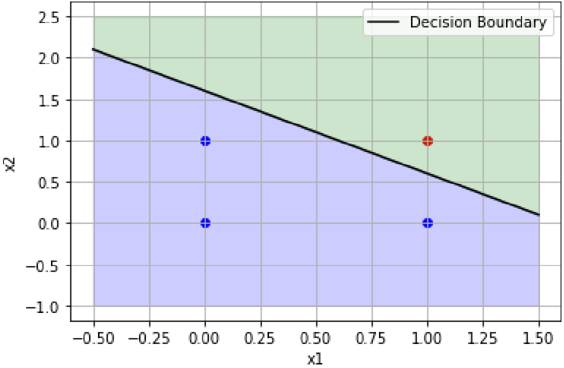
According to the results of the perceptron learning procedure, $w_1 = w_2 = 0.5$. The relationship between the inputs, i.e., x_1, x_2, and the output y is

$$y = \begin{cases} 0 & \text{if } 0.5x_1 + 0.5x_2 - 0.8 \leq 0 \\ 1 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 0 & \text{if } x_1 + x_2 \leq 1.6 \\ 1 & \text{otherwise} \end{cases}$$

Decision Boundary

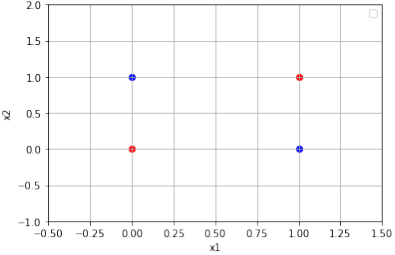
$$y = \begin{cases} 0 & \text{if } x_1 + x_2 \leq 1.6 \\ 1 & \text{otherwise} \end{cases}$$



Problem

- Question: Can we apply the same perceptron learning procedure for the XOR gate, which has the truth table on the right? If so, show all the steps. If not, explain why.
- Answer: No, a perceptron cannot implement XOR. The reason is that the labels in XOR are not linearly separable, i.e. we cannot draw a straight line to separate the points (0,0),(1,1) from the points (0,1),(1,0).

x_1	x_2	Output
0	0	0
0	1	1
1	0	1
1	1	0



To solve the problem, we need a multi-layer perceptron.

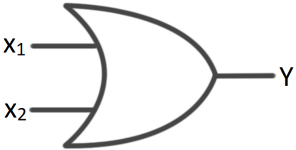
Practice Problem

- Please apply the perceptron learning procedure for the OR gate. The truth table of logical OR is as follows.

x_1	x_2	Output
0	0	0
0	1	1
1	0	1
1	1	1

- Assume the weights and bias are randomly generated, say $w_1 = 0.1$, $w_2 = 0.5$, $\theta = -0.8$. Also, we set learning rate $\eta = 0.2$.
- Activation function is

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 & \text{otherwise} \end{cases}$$



Initial, Round 1, Round 2, & Round 3

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	0.1	-	0.5	-	-0.8
0	0	0	0	0	0.1	0	0.5	0	-0.8
0	1	1	0	0	0.1	0.2	0.7	0.2	-0.6
1	0	1	0	0.2	0.3	0	0.7	0.2	-0.4
1	1	1	1	0	0.3	0	0.7	0	-0.4
0	0	0	0	0	0.3	0	0.7	0	-0.4
0	1	1	1	0	0.3	0	0.7	0	-0.4
1	0	1	0	0.2	0.5	0	0.7	0.2	-0.2
1	1	1	1	0	0.5	0	0.7	0	-0.2
0	0	0	0	0	0.5	0	0.7	0	-0.2
0	1	1	1	0	0.5	0	0.7	0	-0.2
1	0	1	1	0	0.5	0	0.7	0	-0.2
1	1	1	1	0	0.5	0	0.7	0	-0.2

The training converges in 3 epochs if the initial weights are $w_1 = 0.1$, $w_2 = 0.5$, initial bias is $\theta = -0.8$ and learning rate is $\eta = 0.2$. The final weights and bias are: $w_1 = 0.5$, $w_2 = 0.7$, $\theta = -0.2$.

That's all!

Any questions?