CompBio HW3

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1 Problem 1

1.1 a

For multivariate linear regression model $y = x^T\beta + \epsilon,\ \ \epsilon \sim N(0, \sigma^2)$, prove MLE and LSE is equivalent. **LSE** from $y = x^T\beta + \epsilon,\ \ \epsilon\ N(0, \sigma^2)$ and $\hat{y} = x^T\beta$

MSE is equal to:

$$\sum_i (y_i - \hat{y}_i)^2 = \sum_i (y_i - x_i^T\beta)^2 = (Y - X^T\beta)^T(Y - X^T\beta) \tag{1}$$

with $Y = (y_1, y_2, ..., y_n)^T, X = (x_1, x_2, ..., x_n)^T$

according to LSE formulation:

$$\min_\beta (Y - X^T\beta)^T(Y - X^T\beta) \tag{2}$$

derivative of β is 0,

$$\begin{aligned} \frac{\partial(Y - X^T\beta)^T(Y - X^T\beta)}{\partial\beta} &= -2X(Y - X^T\beta) \\ &= -2XY + 2XX^T\beta = 0 \end{aligned} \tag{3}$$

we get

$$\hat{\beta} = (XX^T)^{-1}XY \tag{4}$$

MLE

because $\epsilon \sim N(0, \sigma^2)$

The likelihood function is:

$$\begin{aligned} L(\beta) &= \prod_i P(y_i|x_i, \beta) \\ &= C_1 e^{C_2 \sum_i (y_i - x_i^T\beta)^2} \\ &= C_1 e^{-C_2 (Y - X^T\beta)^T(Y - X^T\beta)} \end{aligned} \tag{5}$$

with $C_1 > 0, C_2 > 0$ being constants.

LL is equal to:

$$\ln L(\beta) = -C(Y - X^T\beta)^T(Y - X^T\beta) \tag{6}$$

with $C > 0$ being a constant

gives the optimization formulation of the problem:

$$\max_\beta -C(Y - X^T\beta)^T(Y - X^T\beta) \tag{7}$$

which is exactly the same as LSE formulation.

1.2 b

In univariate regression model, for one dimension x_j in x, we have $y_j = x_j \beta_j + \epsilon$。

Solve this problem using LSE:

$$\min_{\beta_j} \sum_i (y_{ij} - x_{ij}\beta_j)^2 \tag{8}$$

we get

$$\beta_j^* = \frac{\sum_i x_{ij} y_{ij}}{\sum_i x_{ij}^2} \tag{9}$$

We compare this with the results we got above: $\hat{\beta} = (XX^T)^{-1}XY$.

For the coefficient in multivariate regression $\hat{\beta}$, one dimension $\hat{\beta}_j$ is related to all dimension of data, but for univariate regression, each dimension β_j^* is only related to that corresponding dimension of data.